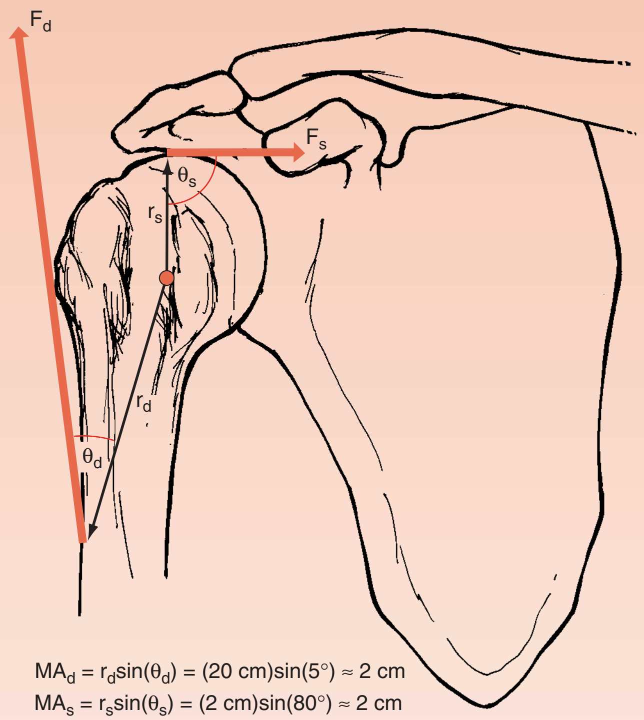
PART

I

Biomechanical Principles



$MA_d = r_d \sin(\theta_d) = (20\ \text{cm})\sin(5°) \approx 2\ \text{cm}$

$MA_s = r_s \sin(\theta_s) = (2\ \text{cm})\sin(80°) \approx 2\ \text{cm}$

PART

I

This part introduces the reader to the basic principles used throughout this book to understand the structure and function of the musculoskeletal system. Biomechanics is the study of biological systems by the application of the laws of physics. The purposes of this part are to review the principles and tools of mechanical analysis and to describe the mechanical behavior of the tissues and structural units that compose the musculoskeletal system. The specific aims of this part are to

- Review the principles that form the foundation of biomechanical analysis of rigid bodies
- Review the mathematical approaches used to perform biomechanical analysis of rigid bodies
- Examine the concepts used to evaluate the material properties of deformable bodies
- Describe the material properties of the primary biological tissues constituting the musculoskeletal system: bone, muscle, cartilage, and dense connective tissue
- Review the components and behavior of joint complexes

By having an understanding of the principles of analysis in biomechanics and the biomechanical properties of the primary tissues of the musculoskeletal system, the reader will be prepared to apply these principles to each region of the body to understand the mechanics of normal movement at each region and to appreciate the effects of impairments on the pathomechanics of movement.

CHAPTER

1

Introduction to Biomechanical Analysis

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Although the human body is an incredibly complex biological system composed of trillions of cells, it is subject to the same fundamental laws of mechanics that govern simple metal or plastic structures. The study of the response of biological systems to mechanical forces is referred to as biomechanics. Although it wasn't recognized as a formal discipline until the 20th century, biomechanics has been studied by the likes of Leonardo da Vinci, Galileo Galilei, and Aristotle. The application of biomechanics to the musculoskeletal system has led to a better understanding of both joint function and dysfunction, resulting in design improvements in devices such as joint arthroplasty systems and orthotic devices. Additionally, basic musculoskeletal biomechanics concepts are important for clinicians such as orthopaedic surgeons and physical and occupational therapists.

Biomechanics is often referred to as the link between structure and function. While a therapist typically evaluates a patient from a kinesiologic perspective, it is often not practical or necessary to perform a complete biomechanical analysis. However, a comprehensive

knowledge of both biomechanics and anatomy is needed to understand how the musculoskeletal system functions. Biomechanics can also be useful in a critical evaluation of current or newly proposed patient evaluations and treatments. Finally, a fundamental understanding of biomechanics is necessary to understand some of the terminology associated with kinesiology (e.g., torque, moment, moment arms).

The purposes of this chapter are to

- **Review some of the basic mathematical principles used in biomechanics**
- **Describe forces and moments**
- **Discuss principles of static analysis**
- **Present the basic concepts in kinematics and kinetics**

The analysis is restricted to the study of rigid bodies. Deformable bodies are discussed in Chapters 2–6. The material in this chapter is an important reference for the force analysis chapters throughout the text.

MATHEMATICAL OVERVIEW

This section is intended as a review of some of the basic mathematical concepts used in biomechanics. Although it can be skipped if the reader is familiar with this material, it would be helpful to at least review this section.

Units of Measurement

The importance of including units with measurements cannot be emphasized enough. Measurements must be accompanied by a unit for them to have any physical meaning. Sometimes, there are situations when certain units are assumed. If a clinician asks for a patient's height and the reply is "5-6," it can reasonably be assumed that the patient is 5 feet, 6 inches tall. However, that interpretation would be inaccurate if the patient was in Europe, where the metric system is used. There are also situations where the lack of a unit makes a number completely useless. If a patient was told to perform a series of exercises for two, the patient would have no idea if that meant two days, weeks, months, or even years.

The units used in biomechanics can be divided into two categories. First, there are the four **fundamental units** of length, mass, time, and temperature, which are defined on the basis of universally accepted standards. Every other unit is considered a **derived unit** and can be defined in terms of these fundamental units. For example, velocity is equal to length divided by time and force is equal to mass multiplied by length divided by time squared. A list of the units needed for biomechanics is found in Table 1.1.

Trigonometry

Since angles are so important in the analysis of the musculoskeletal system, trigonometry is a very useful biomechanics tool. The accepted unit for measuring angles in the clinic is

TABLE 1.1 Units Used in Biomechanics

Quantity	Metric	British	Conversion
Length	meter (m)	foot (ft)	1 ft = 0.3048 m
Mass	kilogram (kg)	slug	1 slug = 14.59 kg
Time	second (s)	second (s)	1 s = 1 s
Temperature	Celsius (°C)	Fahrenheit (°F)	°F = (9/5) × °C + 32°
Force	newton (N = kg × m/s^2)	pound (lb = slug × ft/s^2)	1 lb = 4.448 N
Pressure	pascal (Pa = N/m^2)	pounds per square inch (psi = lb/in^2)	1 psi = 6895 Pa
Energy	joule (J = N × m)	foot pounds (ft-lb)	1 ft-lb = 1.356 J
Power	watt (W = J/s)	horsepower (hp)	1 hp = 7457 W

the degree. There are 360° in a circle. If only a portion of a circle is considered, then the angle formed is some fraction of 360°. For example, a quarter of a circle subtends an angle of 90°. Although in general, the unit degree is adopted for this text, angles also can be described in terms of radians. Since there are 2π radians in a circle, there are 57.3° per radian. When using a calculator, it is important to determine if it is set to use degrees or radians. Additionally, some computer programs, such as Microsoft Excel, use radians to perform trigonometric calculations.

Trigonometric functions are very useful in biomechanics for resolving forces into their components by relating angles to distances in a right triangle (a triangle containing a 90° angle). The most basic of these relationships (**sine, cosine,** and **tangent**) are illustrated in Figure 1.1*A*. A simple mnemonic to help remember these equations is **sohcahtoa—s**ine is the **o**pposite side divided by the **h**ypotenuse, **c**osine is the **a**djacent side divided by the **h**ypotenuse, and **t**angent is the **o**pposite side divided by the **a**djacent side. Although most calculators can be used to evaluate these functions, some important values worth remembering are

$$\sin(0°) = 0,\ \sin(90°) = 1 \qquad \text{(Equation 2.1)}$$

$$\cos(0°) = 1,\ \cos(90°) = 0 \qquad \text{(Equation 2.2)}$$

$$\tan(45°) = 1 \qquad \text{(Equation 2.3)}$$

Additionally, the Pythagorean theorem states that for a right triangle, the sum of the squares of the sides forming the right angle equals the square of the hypotenuse (Fig. 1.1*A*). Although less commonly used, there are also equations that relate angles and side lengths for triangles that do not contain a right angle (Fig. 1.1*B*).

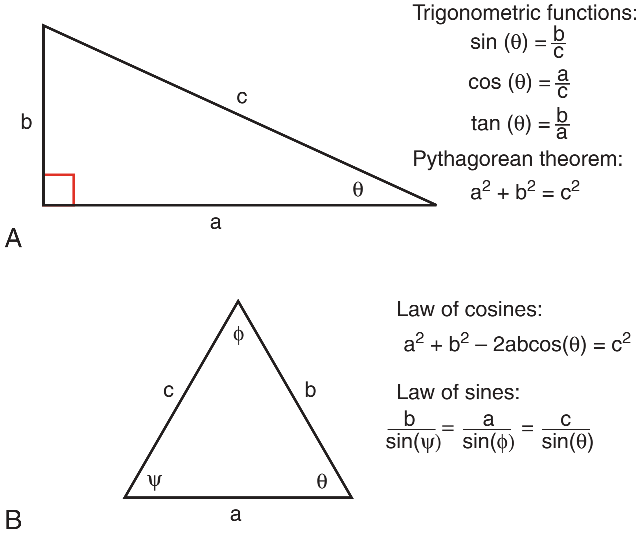


Figure 1.1: Basic trigonometric relationships. These are some of the basic trigonometric relationships that are useful for biomechanics. **A.** A right triangle. **B.** A general triangle.

Vector Analysis

Biomechanical parameters can be represented as either **scalar** or **vector** quantities. A scalar is simply represented by its magnitude. Mass, time, and length are examples of scalar quantities. A vector is generally described as having both **magnitude** and **orientation.** Additionally, a complete description of a vector also includes its **direction** (or **sense**) and **point of application.** Forces and moments are examples of vector quantities. Consider the situation of a 160-lb man sitting in a chair for 10 seconds. The force that his weight is exerting on the chair is represented by a vector with magnitude (160 lb), orientation (vertical), direction (downward), and point of application (the chair seat). However, the time spent in the chair is a scalar quantity and can be represented by its magnitude (10 seconds).

To avoid confusion, throughout this text, bolded notation is used to distinguish vectors **(A)** from scalars (B). Alternative notations for vectors found in the literature (and in classrooms, where it is difficult to bold letters) include putting a line under the letter ($\underline{A}$), a line over the letter ($\overline{A}$), or an arrow over the letter ($\vec{A}$). The **magnitude** of a given vector **(A)** is represented by the same letter, but not bolded (A).

By far, the most common use of vectors in biomechanics is to represent forces, such as muscle and joint reaction and resistance forces. These vectors can be represented graphically with the use of a line with an arrow at one end (Fig. 1.2*A*). The length of the line represents its magnitude,

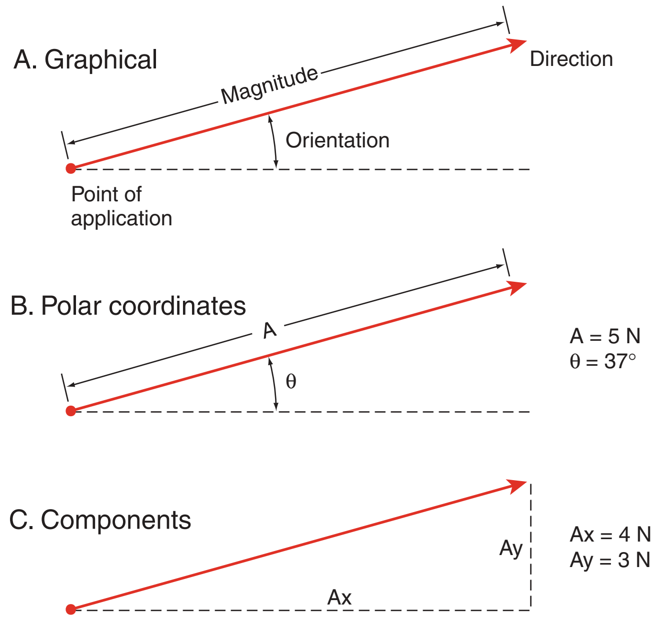


Figure 1.2: Vectors. **A.** In general, a vector has a magnitude, orientation, point of application, and direction. Sometimes the point of application is not specifically indicated in the figure. **B.** A polar coordinate representation. **C.** A component representation.

the angular position of the line represents its orientation, the location of the arrowhead represents its direction, and the location of the line in space represents its point of application. Alternatively, this same vector can be represented mathematically with the use of either **polar coordinates** or **component resolution.** Polar coordinates represent the magnitude and orientation of the vector directly. In polar coordinates, the same vector would be 5 N at 37° from horizontal (Fig. 1.2*B*). With components, the vector is resolved into its relative contributions from both axes. In this example, vector **A** is resolved into its components: $A_X = 4$ N and $A_Y = 3$ N (Fig. 1.2*C*). It is often useful to break down vectors into components that are aligned with anatomical directions. For instance, the *x* and *y* axes may correspond to superior and anterior directions, respectively. Although graphical representations of vectors are useful for visualization purposes, analytical representations are more convenient when adding and multiplying vectors.

Note that the directional information (up and to the right) of the vector is also embedded in this information. A vector with the same magnitude and orientation as the vector represented in Figure 1.2*C*, but with the opposite direction (down and to the left) is represented by $A_X = -4$ N and $A_Y = -3$ N, or 5 N at 217°. The description of the point-of-application information is discussed later in this chapter.

VECTOR ADDITION

When studying musculoskeletal biomechanics, it is common to have more than one force to consider. Therefore, it is important to understand how to work with more than one vector. When adding or subtracting two vectors, there are some important properties to consider. Vector addition is commutative:

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A} \qquad \text{(Equation 2.4)}$$

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B}) \qquad \text{(Equation 2.5)}$$

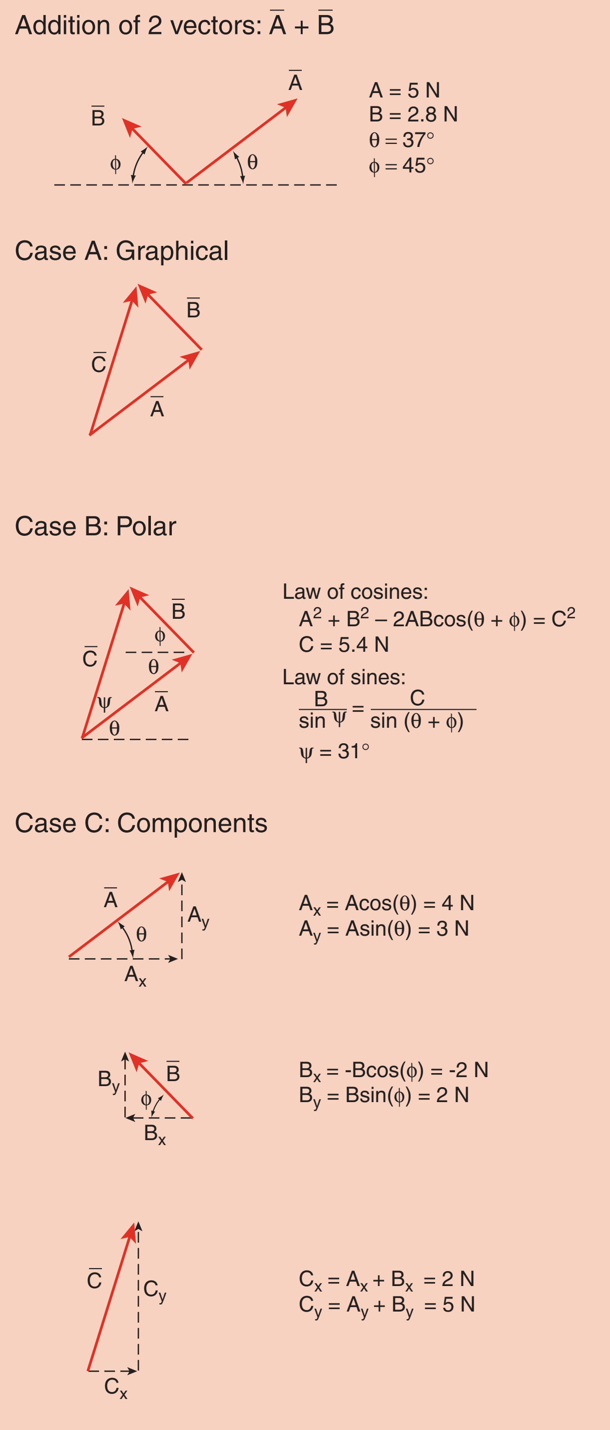
Vector addition is associative:

$$\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C} \qquad \text{(Equation 2.6)}$$

Unlike scalars, which can just be added together, both the magnitude and orientation of a vector must be taken into account. The detailed procedure for adding two vectors **(A + B = C)** is shown in Box 1.1 for the graphical, polar coordinate, and component representation of vectors. The graphical representation uses the "tip to tail" method. The first step is to draw the first vector, **A.** Then the second vector, **B,** is drawn so that its tail sits on the tip of the first vector. The vector representing the sum of these two vectors **(C)** is obtained by connecting the tail of vector **A** and the tip of vector **B.** Since vector addition is commutative, the same solution would have been obtained if vector **B** were the first vector. When using polar coordinates, the vectors are drawn as in the graphical method, and then the law of cosines is used to determine the magnitude of **C** and the law of sines is used to determine the direction of **C** (see Fig 1.1 for definitions of these laws).

EXAMINING THE FORCES BOX 1.1

ADDITION OF TWO VECTORS



For the component resolution method, each vector is broken down into its respective *x* and *y* components. The components represent the magnitude of the vector in that direction. The x and y components are summed:

$$C_X = A_X + B_X \qquad \text{(Equation 2.7)}$$

$$C_Y = A_Y + B_Y \qquad \text{(Equation 2.8)}$$

The vector **C** can either be left in terms of its components, C_X and C_Y, or be converted into a magnitude, C, using the Pythagorean theorem, and orientation, θ, using trigonometry. This method is the most efficient of the three presented and is used throughout the text.

VECTOR MULTIPLICATION

Multiplication of a vector by a scalar is relatively straightforward. Essentially, each component of the vector is individually multiplied by the scalar, resulting in another vector. For example, if the vector in Figure 1.2 is multiplied by 5, the result is $A_X = 5 \times 4\ \text{N} = 20\ \text{N}$ and $A_Y = 5 \times 3\ \text{N} = 15\ \text{N}$. Another form of vector multiplication is the **cross product,** in which two vectors are multiplied together, resulting in another vector (**C = A × B**). The orientation of **C** is such that it is mutually perpendicular to **A** and **B.** The magnitude of **C** is calculated as $C = A \times B \times \sin(\theta)$, where θ represents the angle between A and B, and × denotes scalar multiplication. These relationships are illustrated in Figure 1.3. The cross product is used for calculating joint torques below in this chapter.

Coordinate Systems

A three-dimensional analysis is necessary for a complete representation of human motion. Such analyses require a coordinate system, which is typically composed of anatomically aligned axes: medial/lateral (ML), anterior/posterior (AP), and superior/inferior (SI). It is often convenient to consider only a two-dimensional, or planar, analysis, in which only two of the three axes are considered. In the human body, there are three perpendicular anatomical planes, which are referred to as the **cardinal planes.** The **sagittal plane** is formed by the SI and AP axes, the **frontal (or coronal) plane** is formed by the SI and ML axes, and the **transverse plane** is formed by the AP and ML axes (Fig. 1.4).

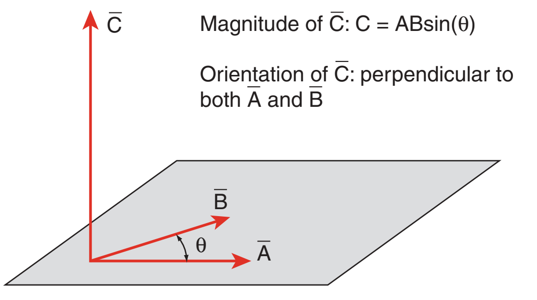


Figure 1.3: Vector cross product. **C** is shown as the cross product of **A** and **B.** Note that **A** and **B** could be any two vectors in the indicated plane and **C** would still have the same orientation.

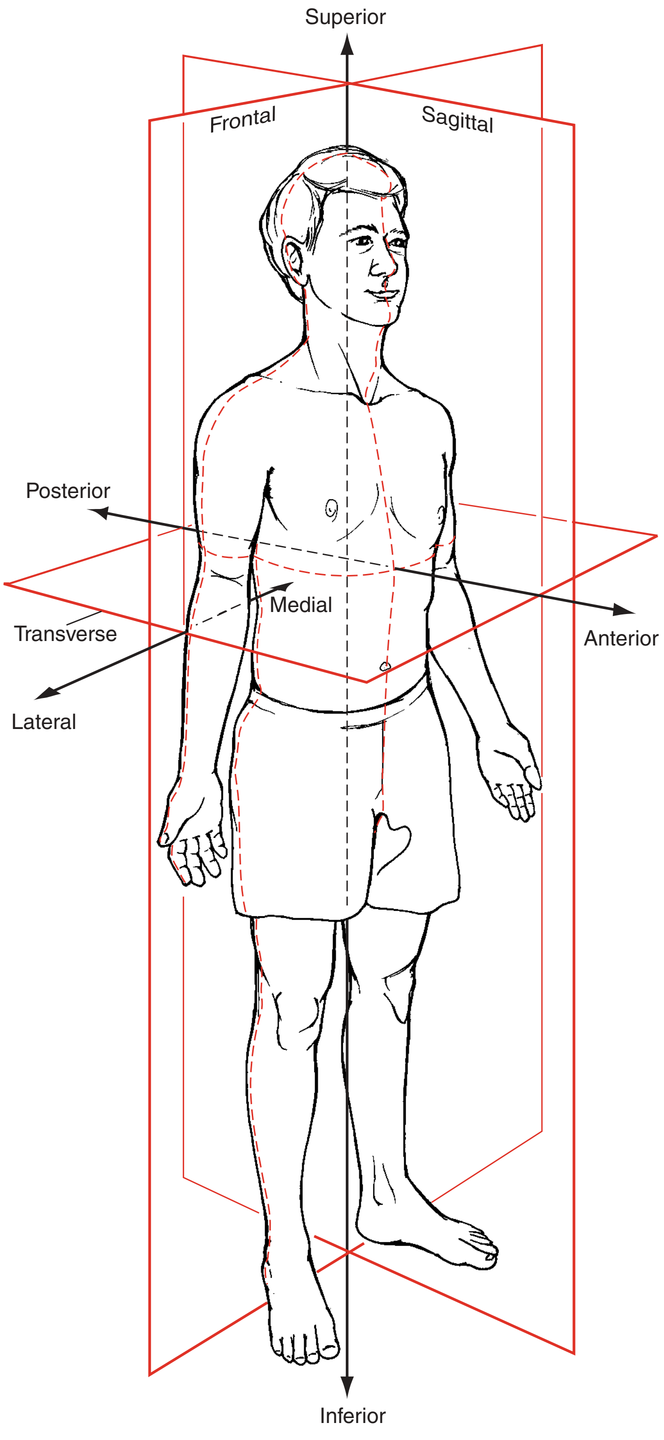


Figure 1.4: Cardinal planes. The cardinal planes, sagittal, frontal, and transverse, are useful reference frames in a three-dimensional representation of the body. In two-dimensional analyses, the sagittal plane is the common reference frame.

The motion of any bone can be referenced with respect to either a **local** or **global** coordinate system. For example, the motion of the tibia can be described by how it moves with respect to the femur (local coordinate system) or how it moves with respect to the room (global coordinate system). Local coordinate systems are useful for understanding joint function and assessing range of motion, while global coordinate systems are useful when functional activities are considered.

Most of this text focuses on two-dimensional analyses, for several reasons. First, it is difficult to display three-dimensional information on the two-dimensional pages of a book. Additionally, the mathematical analysis for a three-dimensional problem is very complex. Perhaps the most important reason is that the fundamental biomechanical principles in a two-dimensional analysis are the same as those in a three-dimensional analysis. It is therefore possible to use a simplified two-dimensional representation of a three-dimensional problem to help explain a concept with minimal mathematical complexity (or at least less complexity).

FORCES AND MOMENTS

The musculoskeletal system is responsible for generating forces that move the human body in space as well as prevent unwanted motion. Understanding the mechanics and pathomechanics of human motion requires an ability to study the forces and moments applied to, and generated by, the body or a particular body segment.

Forces

The reader may have a conceptual idea about what a force is but find it difficult to come up with a formal definition. For the purposes of this text, a **force** is defined as a "push or pull" that results from physical contact between two objects. The only exception to this rule that is considered in this text is the force due to gravity, in which there is no direct physical contact between two objects. Some of the more common force generators with respect to the musculoskeletal system include muscles/tendons, ligaments, friction, ground reaction, and weight.

A distinction must be made between the **mass** and the **weight** of a body. The mass of an object is defined as the amount of matter composing that object. The weight of an object is the force acting on that object due to gravity and is the product of its mass and the acceleration due to gravity ($g = 9.8\ m/s^2$). So while an object's mass is the same on Earth as it is on the moon, its weight on the moon is less, since the acceleration due to gravity is lower on the moon. This distinction is important in biomechanics, not to help plan a trip to the moon, but for ensuring that a unit of mass is not treated as a unit of force.

As mentioned previously, force is a vector quantity with magnitude, orientation, direction, and a point of application. Figure 1.5 depicts several forces acting on the leg in the frontal plane during stance. The forces from the abductor and adductor muscles act through their tendinous insertions,

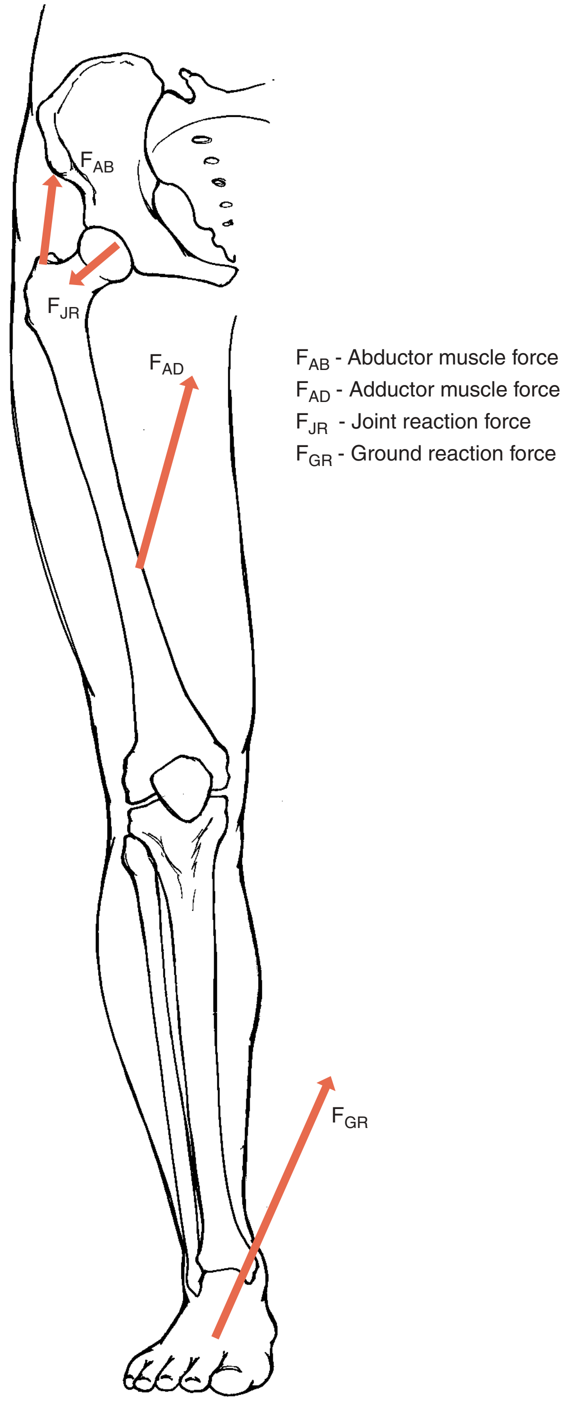


Figure 1.5: Vectors in anatomy. Example of how vectors can be combined with anatomical detail to represent the action of forces. Some of the forces acting on the leg are shown here.

while the hip joint reaction force acts through its respective joint center of rotation. In general, the point of application of a force (e.g., tendon insertion) is located with respect to a fixed point on a body, usually the joint center of rotation. This information is used to calculate the **moment** due to that force.

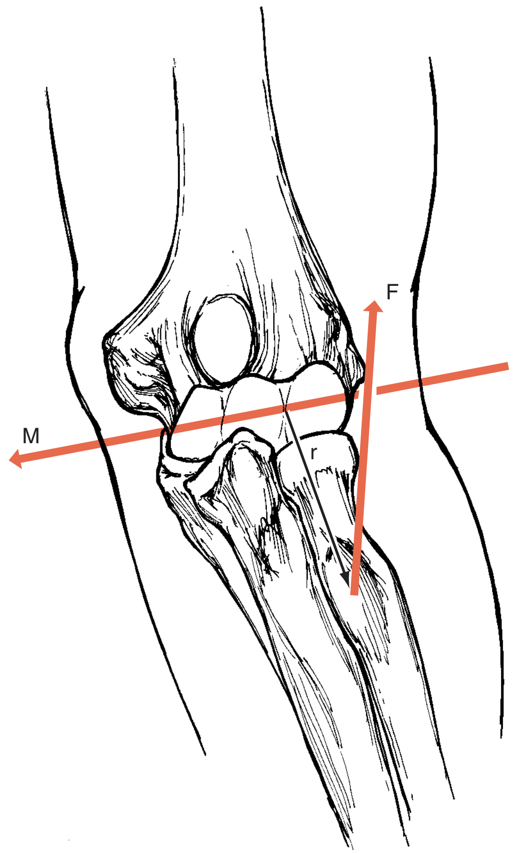


Figure 1.6: Three-dimensional moment analysis. The moment acting on the elbow from the force of the biceps is shown as a vector aligned with the axis of rotation. *F,* force vector; *r,* distance from force vector to joint COR; *M,* moment vector.

Moments

In kinesiology, a moment **(M)** is typically caused by a force **(F)** acting at a distance **(r)** from the center of rotation of a segment. A moment tends to cause a rotation and is defined by the cross product function: $\mathbf{M = r \times F}$. Therefore, a moment is represented by a vector that passes through the point of interest (e.g., the center of rotation) and is perpendicular to both the force and distance vectors (Fig. 1.6). For a two-dimensional analysis, both the force and distance vectors are in the plane of the paper, so the moment vector is always directed perpendicular to the page, with a line of action through the point of interest. Since it has only this one orientation and line of action, a moment is often treated as a scalar quantity in a two-dimensional analysis, with only magnitude and direction. **Torque** is another term that is synonymous with a scalar moment. From the definition of a cross product, the magnitude of a moment (or torque) is calculated as $M = r \times F \times \sin(\theta)$. Its direction is referred to as the direction in which it would tend to cause an object to rotate (Fig. 1.7*A*).

Although there are several different distances that can be used to connect a vector and a point, **the same moment is calculated no matter which distance is selected** (Fig. 1.7*B*).

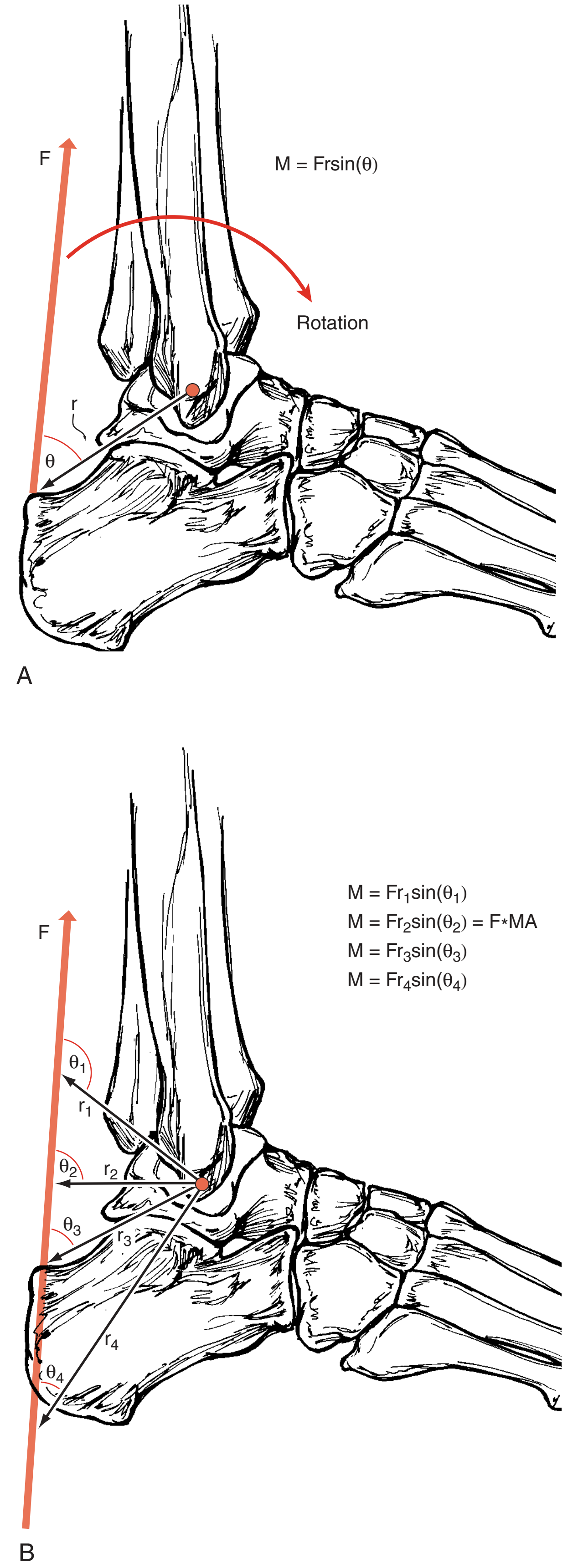


Figure 1.7: *Continued*

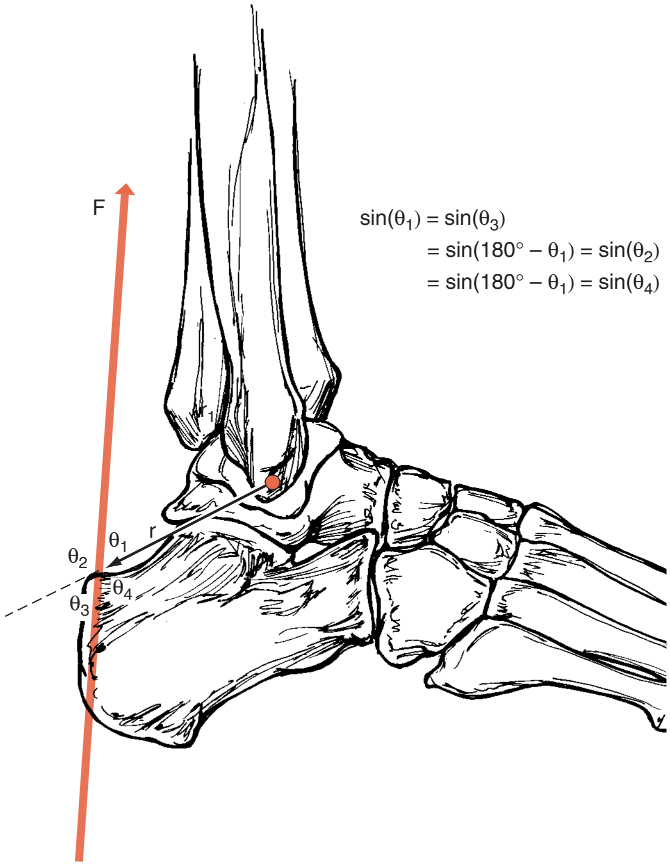


Figure 1.7: Two-dimensional moment analysis. **A.** Plantar flexion moment created by force at the Achilles tendon. **B.** Note that no matter which distance vector is chosen, the value for the moment is the same. **C.** Also, no matter which angle is chosen, the value for the sine of the angle is the same, so the moment is the same.

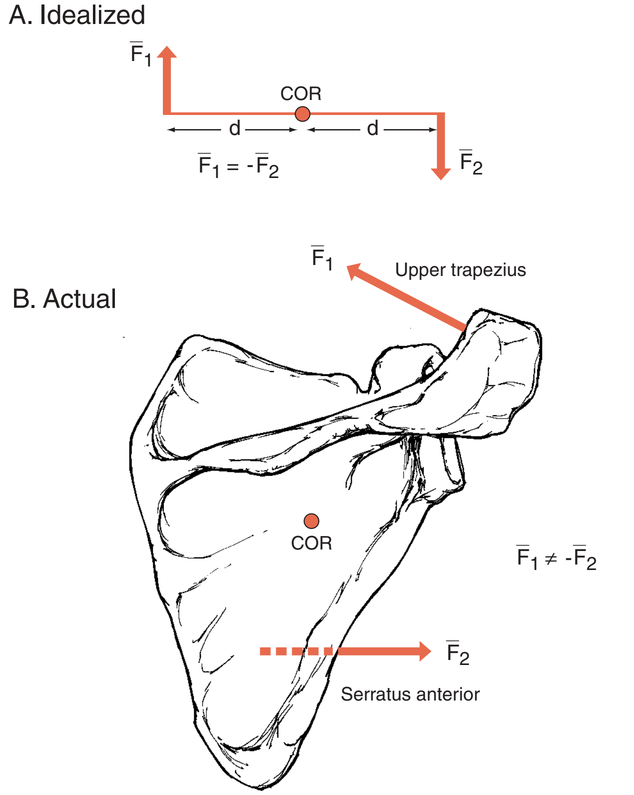


Figure 1.8: Force couples. Distinction between an idealized force couple **(A)** and a more realistic one **(B).** Even though the scapular example given is not a true force couple, it is typically referred to as one. *COR,* center of rotation.

The distance that is perpendicular to the force vector is referred to as the **moment arm** (MA) of that force (r_2 in Fig. 1.7*B*). Since the sine of 90° is equal to 1, the use of a moment arm simplifies the calculation of moment to $M = MA \times F$. The moment arm can also be calculated from any distance as $MA = r \times \sin(\theta)$. Additionally, although there are four separate angles between the force and distance vectors, all four angles result in the same moment calculation (Fig. 1.7*C*).

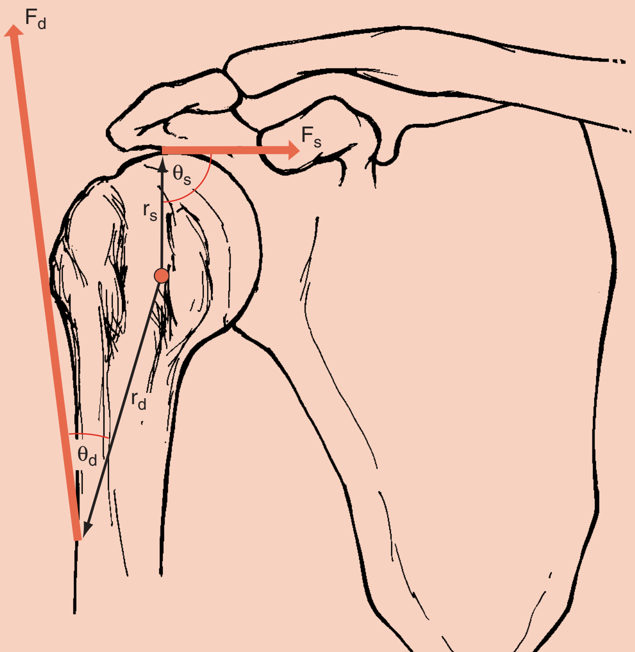
The examples in Figures 1.6 and 1.7 have both force and moment components. However, consider the situation in Figure 1.8*A*. Although the two applied forces create a moment, they have the same magnitude and orientation but opposite directions. Therefore, their vector sum is zero. This is an example of a **force couple.** A pure force couple results in rotational motion only, since there are no unbalanced forces. In the musculoskeletal system, all of these conditions are seldom met, so pure force couples are rare. In general, muscles are responsible for producing both forces and moments, thus resulting in both translational and rotational motion. However, there are examples in the human body in which two or more muscles work in concert to produce a moment, such as the upper trapezius and serratus anterior (Fig. 1.8*B*). Although these muscles do not have identical magnitudes or orientations, this situation is frequently referred to as a force couple.

Muscle Forces

As mentioned previously, there are three important parameters to consider with respect to the force of a muscle: orientation, magnitude, and point of application. With some care, it is possible to measure orientation and line of action from cadavers or imaging techniques such as magnetic resonance imaging (MRI) and computed tomography (CT) [1,3]. This information is helpful in determining the function and efficiency of a muscle in producing a moment. As an example, two muscles that span the glenohumeral joint, the supraspinatus and middle deltoid, are shown in Box 1.2. From the information provided for muscle orientation and point of application in this position, the moment arm of the deltoid is approximately equal to that of the supraspinatus, even though the deltoid insertion on the humerus is much farther away from the center of rotation than the supraspinatus insertion.

EXAMINING THE FORCES BOX 1.2

MOMENT ARMS OF THE DELTOID (MA_d) AND THE SUPRASPINATUS (MA_s)



$MA_d = r_d \sin(\theta_d) = (20\ \text{cm})\sin(5°) \approx 2\ \text{cm}$

$MA_s = r_s \sin(\theta_s) = (2\ \text{cm})\sin(80°) \approx 2\ \text{cm}$

CLINICAL RELEVANCE: MUSCLE FORCES

In addition to generating moments that are responsible for angular motion (rotation), muscles also produce forces that can cause linear motion (translation). This force can be either a stabilizing or a destabilizing force. For example, since the supraspinatus orientation shown in Box 1.2 is primarily directed medially, it tends to pull the humeral head into the glenoid fossa. This compressive force helps stabilize the glenohumeral joint. However, since the deltoid orientation is directed superiorly, it tends to produce a destabilizing force that may result in superior translation of the humeral head.

These analyses are useful, since they can be performed even if the magnitude of a muscle's force is unknown. However, to understand a muscle's function completely, its force magnitude must be known. Although forces can be measured with invasive force transducers [12], instrumented arthroplasty systems [6], or simulations in cadaver models [9], there are currently no noninvasive experimental methods that can be used to measure the in vivo force of intact muscles. Consequently, basic concepts borrowed from freshman physics can be used to predict muscle forces. Although they often involve many simplifying assumptions, such methods can be very useful in understanding joint mechanics, and they are presented in the next section.

STATICS

Statics is the study of the forces acting on a body at rest or moving with a constant velocity. Although the human body is almost always accelerating, a static analysis offers a simple method of addressing musculoskeletal problems. This analysis may either solve the problem or provide a basis for a more sophisticated dynamic analysis.

Newton's Laws

Since the musculoskeletal system is simply a series of objects in contact with each other, some of the basic physics principles developed by Sir Isaac Newton (1642–1727) are useful. Newton's laws are as follows:

First law: An object remains at rest (or continues moving at a constant velocity) unless acted upon by an unbalanced external force.

Second law: If there is an unbalanced force acting on a object, it produces an acceleration in the direction of the force, directly proportional to the force ($f = ma$).

Third law: For every action (force) there is a reaction (opposing force) of equal magnitude but in the opposite direction.

From the first law, it is clear that if a body is at rest, there can be no unbalanced external forces acting on it. In this situation, termed **static equilibrium,** all of the external forces acting on a body must add (in a vector sense) to zero. An extension of this law to objects larger than a particle is that the sum of the external moments acting on that body must also be equal to zero for the body to be at rest. Therefore, for a three-dimensional analysis, there are a total of six equations that must be satisfied for static equilibrium:

$$\sum F_X = 0 \quad \sum F_Y = 0 \quad \sum F_Z = 0$$
$$\sum M_X = 0 \quad \sum M_Y = 0 \quad \sum M_Z = 0 \qquad \text{(Equation 2.9)}$$

For a two-dimensional analysis, there are only two in-plane force components and one perpendicular moment (torque) component:

$$\sum F_X = 0 \quad \sum F_Y = 0 \quad \sum M_Z = 0 \qquad \text{(Equation 2.10)}$$

Under many conditions, it is reasonable to assume that all body parts are in a state of static equilibrium and these three equations can be used to calculate some of the forces acting on the musculoskeletal system. When a body is not in static equilibrium, Newton's second law states that any unbalanced forces and moments are proportional to the acceleration of the body. That situation is considered later in this chapter.

Solving Problems

A general approach used to solve for forces during static equilibrium is as follows:

Step 1 Isolate the body of interest.
Step 2 Sketch this body and all external forces (referred to as a **free body diagram**).
Step 3 Sum the forces and moments equal to zero.
Step 4 Solve for the unknown forces.

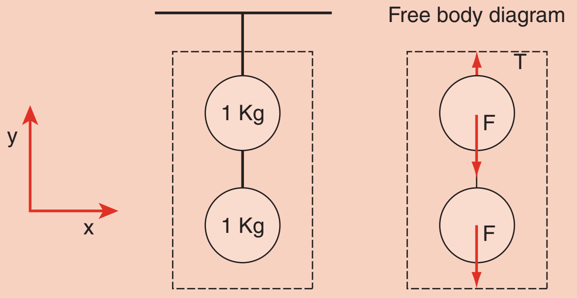
As a simple example, consider the two 1-kg balls hanging from strings shown in Box 1.3. What is the force acting on the top string? Although this is a very simple problem that can be solved by inspection, a formal analysis is presented. Step 1 is to sketch the entire system and then place a dotted box around the body of interest. Consider a box that encompasses both balls and part of the string above the top one, as shown in Box 1.3.

Proceeding to step 2, a free body diagram is sketched. As indicated by Newton's first law, only external forces are considered for these analyses. For this example, everything inside the dotted box is considered part of the body of interest. External forces are caused by the contact of two objects, one inside the box and one outside the box. In this example, there are three external forces: tension in the top string and the weight of each of the balls.

Why is the tension on the top string considered an external force, but not the force on the bottom string? The reason is that the tension on the top string is an **external force** (part of the string is in the box and part is outside the box), and the force on the bottom string is an internal force (the entire string is located inside the box). This is a very important distinction because it allows for isolation of the forces on specific muscles or joints in the musculoskeletal system.

EXAMINING THE FORCES BOX 1.3

A FREE BODY DIAGRAM



$\Sigma F_y = 0$
$T - F - F = 0$
$T = 2F = 2\ (10\ N)$
$T = 20\ N$

$F = mg = (1\ kg)(9.8\ \frac{m}{s^2})$
$= 10\ N$

Why is the weight of each ball considered an external force? Although gravity is not caused by contact between two objects, it is caused by the interaction of two objects and is treated in the same manner as a contact force. One of the objects is inside the box (the ball) and the other is outside the box (the Earth). In general, as long as an object is located within the box, the force of gravity acting on it should be considered an external force.

Why is the weight of the string not considered an external force? To find an exact answer to the problem, it should be considered. However, since its weight is far less than that of the balls, it is considered negligible. In biomechanical analyses, assumptions are often made to ignore certain forces, such as the weight of someone's watch during lifting.

Once all the forces are in place, step 3 is to sum all the forces and moments equal to zero. There are no forces in the x direction, and since all of the forces pass through the same point, there are no moments to consider. That leaves only one equation: sum of the forces in the y direction equal to zero. The fourth and final step is to solve for the unknown force. The mass of the balls is converted to force by multiplying by the acceleration of gravity. The complete analysis is shown in Box 1.3.

Simple Musculoskeletal Problems

Although most problems can be addressed with the above approach, there are special situations in which a problem is simplified. These may be useful both for solving problems analytically and for quick assessment of clinical problems from a biomechanical perspective.

LINEAR FORCES

The simplest type of system, linear forces, consists of forces with the same orientation and line of action. The only things that can be varied are the force magnitudes and directions. An example is provided in Box 1.3. Notice that the only equation needed is summing the forces along the y axis equal to zero. When dealing with linear forces, it is best to align either the x or y axis with the orientation of the forces.

PARALLEL FORCES

A slightly more complicated system is one in which all the forces have the same orientation but not the same line of action. In other words, the force vectors all run parallel to each other. In this situation, there are still only forces along one axis, but there are moments to consider as well.

Levers

A **lever** is an example of a parallel force system that is very common in the musculoskeletal system. Although not all levers contain parallel forces, that specific case is focused on here. A basic understanding of this concept allows for a rudimentary analysis of a biomechanical problem with very little mathematics.

A lever consists of a rigid body with two externally applied forces and a point of rotation. In general, for the musculoskeletal joint, one of the forces is produced by a muscle, one force is provided by contact with the environment (or by gravity) and the point of rotation is the center of rotation of the joint. The two forces can either be on the same side or different sides of the **center of rotation (COR).**

If the forces are on different sides of the COR, the system is considered a **first class lever.** If the forces are on the same side of the COR and the external force is closer to the COR than the muscle force, it is a **second class lever.** If the forces are on the same side of the COR and the muscle force is closer to the COR than the external force, it is a **third class lever.** There are several cases of first class levers; however, most joints in the human body behave as third class levers. Second class levers are almost never observed within the body. Examples of all three levers are given in Figure 1.9.

If moments are summed about the COR for any lever, the resistive force is equal to the muscle force times the ratio of the muscle and resistive moment arms:

$$F_R = F_M \times (MA_M/MA_R) \quad \text{(Equation 2.11)}$$

The ratio of the muscle and resistive moment arms (MA_M/MA_R) is referred to as the **mechanical advantage** of the lever. Based on this equation and the definition of levers, the mechanical advantage is greater than one for a second class lever, less than one for a third class lever, and either for a first class lever. A consequence of this is that since most joints behave as third class levers, muscle forces must always be greater than the force of the resistive load they are opposing. Although this may appear to represent an inefficient design, muscles sacrifice their mechanical advantage to produce large motions and high-velocity motions. This equation is also valid in cases where the two forces are not

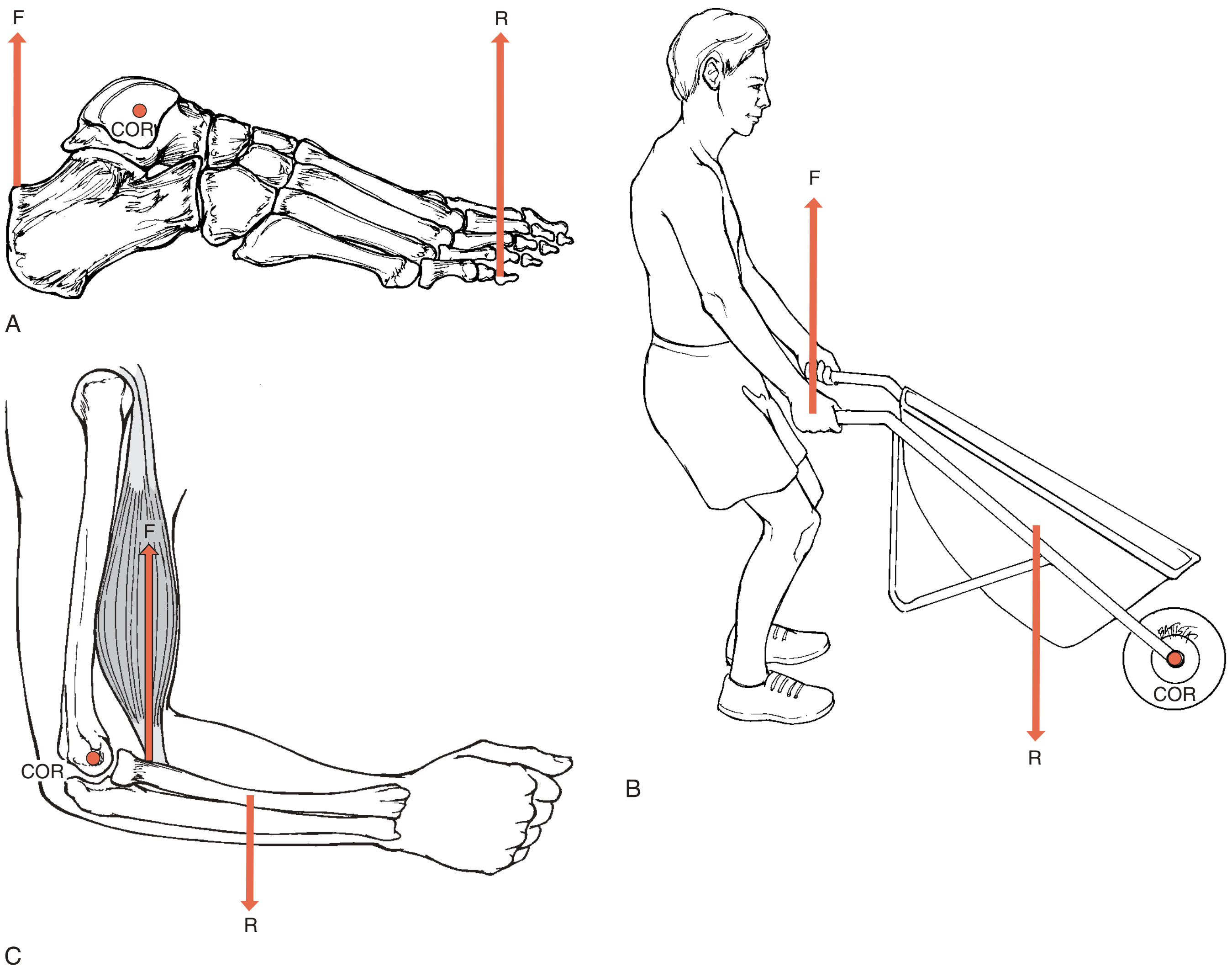


Figure 1.9: Classification of lever systems. Examples of the three different classes of levers, where F is the exerted force, R is the reaction force, and COR is the center of rotation. Most musculoskeletal joints behave as third class levers. **A.** First class lever. **B.** Second class lever. **C.** Third class lever.

parallel, as long as their moment arms are known. The effects of a muscle's moment arm on joint motion is discussed in Chapter 4.

Center of Gravity and Stability

Another example of a parallel force system is the use of the **center of gravity** to determine stability. The center of gravity of an object is the point at which all of the weight of that body can be thought to be concentrated, and it depends on a body's shape and mass distribution. The center of gravity of the human body in the anatomical position is approximately at the level of the second sacral vertebra [8]. This location changes as the shape of the body is altered. When a person bends forward, his or her center of gravity shifts anteriorly and inferiorly. The location of the center of gravity is also affected by body mass distribution changes. For example, if a person were to develop more leg muscle mass, the center of mass would shift inferiorly.

The location of a person's center of gravity is important in athletics and other fast motions because it simplifies the use of Newton's second law. More important from a clinical point of view is the effect of the center of gravity on stability. For motions in which the acceleration is negligible, it can be shown with Newton's first law that the center of gravity must be contained within a person's base of support to maintain stability.

Consider the situation of a person concerned about falling forward. Assume for the moment that there is a ground reaction force at his toes and heel. When he is standing upright, his center of gravity is posterior to his toes, so there is a counterclockwise moment at his toes (Fig. 1.10*A*). This is a stable position, since the moment can be balanced by the ground reaction force at his heel. If he bends forward at his hips to touch the ground and leans too far forward, his center of gravity moves anterior to his toes and the weight of his upper body produces a clockwise moment at his toes (Fig. 1.10*B*). Since there is no further anterior support, this moment is unbalanced and the man will fall forward. However, if in addition to hip flexion he plantarflexes at his ankles while keeping his knee straight, he is in a stable position with his center of gravity posterior to his toes (Fig. 1.10*C*).

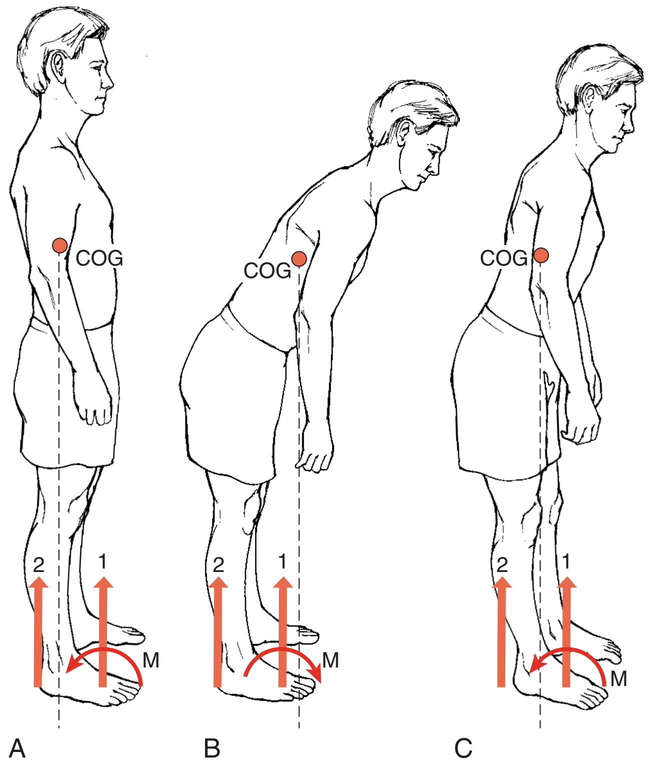


Figure 1.10: Center of gravity. For the man in the figure to maintain his balance, his center of gravity must be maintained within his base of support. This is not a problem in normal standing **(A).** When he bends over at the waist, however, his center of gravity may shift anterior to the base of support, creating an unstable situation **(B).** The man needs to plantarflex at the ankles to maintain his balance **(C).**

Advanced Musculoskeletal Problems

One of the most common uses of static equilibrium applied to the musculoskeletal system is to solve for unknown muscle forces. This is a very useful tool because as mentioned above, there are currently no noninvasive experimental methods that can be used to measure in vivo muscle forces. There are typically 3 types of forces to consider in a musculoskeletal problem: *(a)* the joint reaction force between the two articular surfaces, *(b)* muscle forces and *(c)* forces due to the body's interaction with the outside world. So how many unknown parameters are associated with these forces? To answer this, the location of all of the forces with their points of application must be identified. For the joint reaction force nothing else is known, so there are two unknown parameters: magnitude and orientation. The orientation of a muscle force can be measured, so there is one unknown parameter, magnitude. Finally, any force interaction with the outside world can theoretically be measured, possibly with a handheld dynamometer or by knowing the weight of the segment, so there are no unknown parameters (Table 1.2) [5,8].

Consequently, there are two unknown parameters for the joint reaction force and one unknown parameter for each muscle. However, there are only three equations available from a two-dimensional analysis of Newton's first law. Therefore, if there is more than one muscle force to consider, there are more unknown parameters than available equations. This situation is referred to as **statically indeterminate,** and there are an infinite number of possible solutions. To avoid this problem, only one muscle force can be considered. Although this is an oversimplification of most musculoskeletal situations, solutions based on a single muscle can provide a general perspective of the requirements of a task. Options for solving the statically indeterminate problem are briefly discussed later.

TABLE 1.2 Body Segment Parameters Derived from Dempster [4]

	Percentage of Total Body Weight (%)	Location of the Center of Mass (% of limb segment length from proximal end)	Moment of Inertia about the Center of Mass (kg × m^2)
Head and neck	7.9	43.3	0.029
Trunk	48.6	n.a.	n.a.
Upper extremity	4.9	51.2[a]	0.335
Arm	2.7	43.6	0.040
Forearm and hand	2.2	67.7	0.058
Forearm	1.6	43.0	0.018
Hand	0.6	50.6[b]	0.002
Lower extremity	15.7	43.4[c]	1.785
Thigh	9.6	43.3	0.298
Leg and foot	5.9	43.3	0.339
Leg	4.5	43.4	0.143
Foot	1.4	43.8[d]	0.007

[a] Measured from axis of shoulder to ulnar styloid process.
[b] Measured to PIP joint of long finger.
[c] Measured to medial malleolus.
[d] Measured from heel.

FORCE ANALYSIS WITH A SINGLE MUSCLE

There are additional assumptions that are typically made to solve for a single muscle force:

- Two-dimensional analysis
- No deformation of any tissues
- No friction in the system
- The single muscle force that has been selected can be concentrated in a single line of action
- No acceleration

The glenohumeral joint shown in Box 1.2 is used as an example to help demonstrate the general strategy for approaching these problems. Since only one muscle force can be considered, the supraspinatus is chosen for analysis. The same general approach introduced earlier in this chapter for addressing a system in static equilibrium is used.

Step one is to isolate the body of interest, which for this problem is the humerus. In step two, a free body diagram is drawn, with all of the external forces clearly labeled: the weight of the arm (F_G), the supraspinatus force (F_S), and the glenohumeral joint reaction force (F_J) in Box 1.4. Note that external objects like the scapula are often included in the free body diagram to make the diagram complete. However, the scapula is external to the analysis and is only included for convenience. It is important to keep track of which objects are internal and which ones are external to the isolated body.

The next step is to sum the forces and moments to zero to solve for the unknown values. Since the joint reaction force acts through the COR, a good strategy is to start by summing the moments to zero at that point. This effectively eliminates

EXAMINING THE FORCES BOX 1.4

STATIC EQUILIBRIUM EQUATIONS CONSIDERING ONLY THE SUPRASPINATUS

$\Sigma M = 0$ (at COR)

$(F_S)(R_S)\sin(90°) - (F_G)(R_G)\sin(30°) = 0$

$$F_S = \frac{(28N)(29\text{ cm})\sin 30°}{2\text{ cm}} = 203\text{ N}$$

$\Sigma F_X = 0$

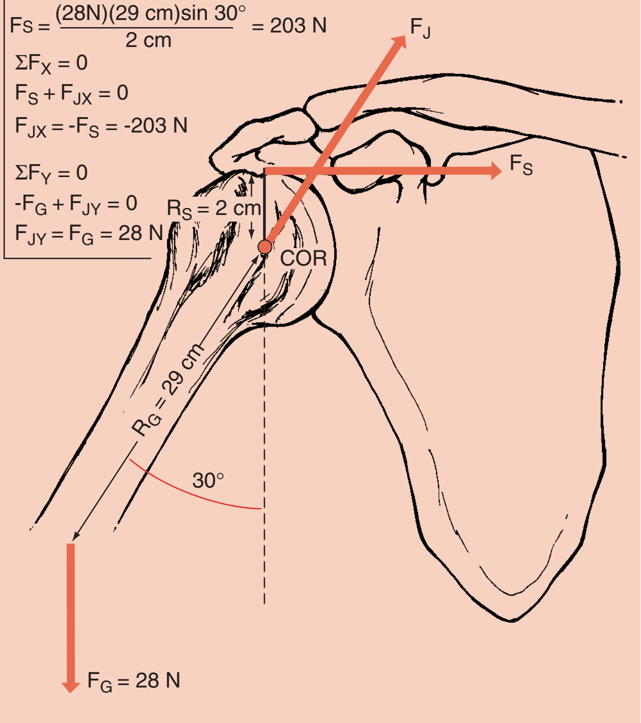
$F_S + F_{JX} = 0$

$F_{JX} = -F_S = -203$ N

$\Sigma F_Y = 0$

$-F_G + F_{JY} = 0$

$F_{JY} = F_G = 28$ N

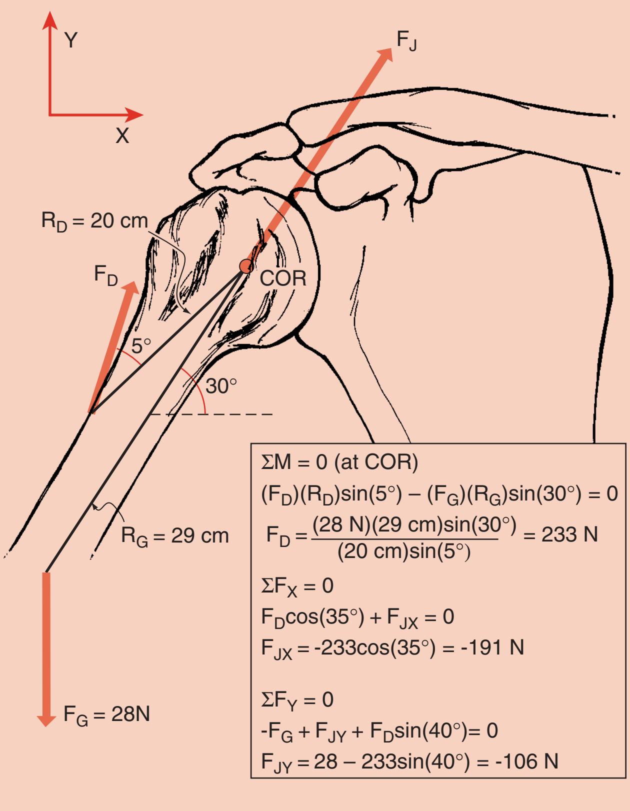


the joint reaction force from this equation because its moment arm is equal to zero. The forces along the *x* and *y* axes are summed to zero to find those components of the joint reaction force. The fourth and final step is to solve for the unknown parameters in these three equations. The details of these calculations are given in Box 1.4. In this example, the magnitude of the supraspinatus force is 203 N, and the joint reaction force is 203 N lateral and 28 N superior. Those components represent the force of the scapula acting on the humerus. Newton's third law can then be used to find the force of the humerus acting on the scapula: 203 N medial and 28 N inferior.

Note that the muscle force is much larger than the weight of the arm. This is expected, considering the small moment arm of the muscle compared with the moment arm of the force due to gravity. While this puts muscles at a mechanical disadvantage for force production, it enables them to amplify their motion. For example, a 1-cm contraction of the supraspinatus results in a 20-cm motion at the hand. This is discussed in more detail in Chapter 4.

EXAMINING THE FORCES BOX 1.5

STATIC EQUILIBRIUM EQUATIONS CONSIDERING ONLY THE DELTOID MUSCLE



$\Sigma M = 0$ (at COR)

$(F_D)(R_D)\sin(5°) - (F_G)(R_G)\sin(30°) = 0$

$$F_D = \frac{(28\text{ N})(29\text{ cm})\sin(30°)}{(20\text{ cm})\sin(5°)} = 233\text{ N}$$

$\Sigma F_X = 0$

$F_D\cos(35°) + F_{JX} = 0$

$F_{JX} = -233\cos(35°) = -191\text{ N}$

$\Sigma F_Y = 0$

$-F_G + F_{JY} + F_D\sin(40°) = 0$

$F_{JY} = 28 - 233\sin(40°) = -106\text{ N}$

The problem can be solved again by considering the middle deltoid instead of the supraspinatus. For those conditions, Box 1.5 shows that the deltoid muscle force is 233 N and the force of the humerus acting on the scapula is 191 N medial and 106 N superior. Notice that although the force required of each muscle is similar (supraspinatus, 203 N vs. deltoid, 230 N), the deltoid generates a much higher superior force and the supraspinatus generates a much higher medial force.

CLINICAL RELEVANCE: SUPRASPINATUS AND DELTOID MUSCLE FORCES

A clinical application of these results is that under normal conditions the supraspinatus serves to maintain joint stability with its medially directed force. However, if its integrity is compromised, as occurs with rotator cuff disease, and the deltoid plays a larger role, then there is a lower medial stabilizing force and a higher superior force that may cause impingement of the rotator cuff in the subacromial region.

The analysis presented above serves as a model for analyzing muscle and joint reaction forces in subsequent chapters. Although some aspects of the problem will clearly vary from joint to joint, the basic underlying method is the same.

FORCE ANALYSIS WITH MULTIPLE MUSCLES

Although most problems addressed in this text focus on solving for muscle forces when only one muscle is taken into consideration, it would be advantageous to solve problems in which there is more than one muscle active. However such systems are statically indeterminate. Additional information is needed regarding the relative contribution of each muscle to develop an appropriate solution.

One method for analyzing indeterminate systems is the **optimization method.** Since an indeterminate system allows an infinite number of solutions, the optimization approach helps select the "best" solution. An optimization model minimizes some cost function to produce a single solution. This function may be the total force in all of the muscles or possibly the total stress (force/area) in all of the muscles. While it might make sense that the central nervous system attempts to minimize the work it has to do to perform a function, competing demands of a joint must also be met. For example, in the glenohumeral example above, it might be most efficient from a force production standpoint to assume that the deltoid acts alone. However, from a stability standpoint, the contribution of the rotator cuff is essential.

Another method for analyzing indeterminate systems is the **reductionist model** in which a set of rules is applied for the relative distribution of muscle forces based on electromyographic (EMG) signals. One approach involves developing these rules on the basis of the investigator's subjective knowledge of EMG activity, anatomy, and physiological constraints

[4]. Another approach is to have subjects perform isometric contractions at different force levels while measuring EMG signals and to develop an empirical relationship between EMG and force level [2,7]. Perhaps the most common approach is based on the assumption that muscle force is proportional to its cross-sectional area and EMG level. This method has been attempted for many joints, such as the shoulder [10], knee and hip. One of the key assumptions in all these approaches is that there is a known relationship between EMG levels and force production.

KINEMATICS

Until now, the focus has been on studying the static forces acting on the musculoskeletal system. The next section deals with **kinematics,** which is defined as the study of motion without regard to the forces that cause that motion. As with the static force analysis, this section is restricted to two-dimensional, or planar, motion.

Rotational and Translational Motion

Pure linear, or **translatory, motion** of an entire object occurs when all points on that object move the same distance (Fig. 1.11*A*). However, with the possible exception of passive manipulation of joints, pure translatory motion does not often occur at musculoskeletal articulations. Instead, **rotational motion** is more common, in which there is one point on a bone that remains stationary (the COR), and all other points trace arcs of a circle around this point (Fig. 1.11*B*). For three-dimensional motion, the **COR** would be replaced by an **axis of rotation,** and there could also be translation along this axis.

Consider the general motion of a bone moving from an initial to a final position. The rotational component of this motion can be measured by tracking the change in orientation of a line on the bone. Although there are an infinite number of lines to choose from, it turns out that no matter which line is selected, the amount of rotation is always the same. Similarly, the translational component of this motion can be measured by tracking the change in position of a point on the bone. In this case, however, the amount of translatory motion is *not* the same for all points. In fact, the displacement of a point increases linearly as its distance from the COR increases (Fig. 1.11*B*). Therefore, from a practical standpoint, if there is any rotation of a bone, a description of joint translation or displacement must refer to a specific point on the bone.

Consider the superior/inferior translation motion of the humerus in Figure 1.12*A*, which is rotated 90°. Point 1 represents the geometric center of the humeral head and does

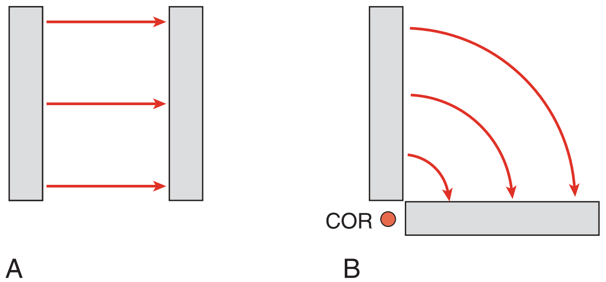


Figure 1.11: Translations and rotations. In biomechanics, motion is typically described in terms of translations and rotations. **A.** In translatory motion, all points on the object move the same distance. **B.** In rotational motion, all points on the object revolve around the center of rotation *(COR),* which is fixed in space.

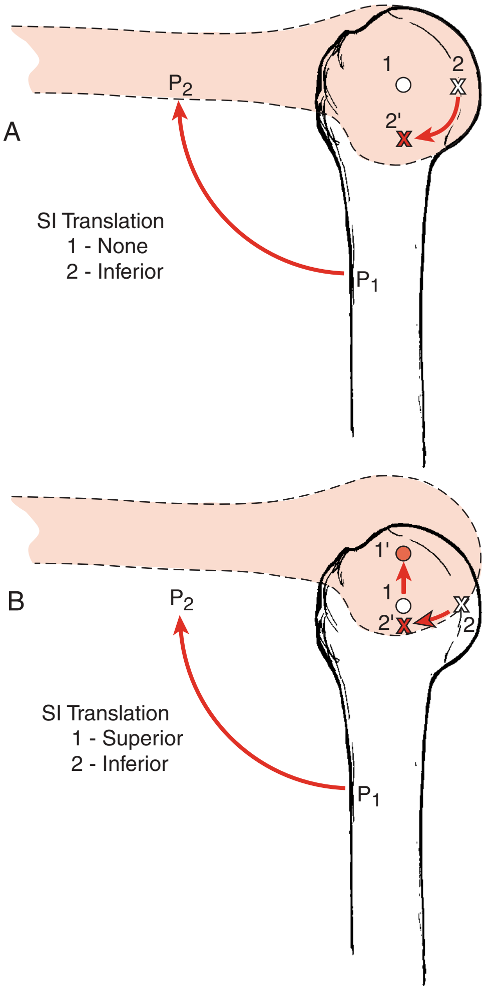


Figure 1.12: Translations and rotations within a joint. For both of these examples, it is fairly straightforward to describe the rotational motion of the humerus—it rotates 90°. However, translational motion is more complicated, and it is important to refer to a specific point. Consider the superior/inferior (SI) translation of two points: point 1 is located at the center of the humeral head and point 2 sits closer to the articular surface. **A.** The center of the rotation of the motion is at point 1, so there is no translation at point 1, but point 2 moves inferiorly. **B.** Point 1 moves superiorly, and point 2 moves inferiorly.

not translate from position 1 to 2. However, point 2 on the articular surface of the humeral head translates inferiorly. The motion in Figure 1.12*B* is similar, except now point 1 translates superiorly, while point 2 still translates inferiorly. This example demonstrates how important the point of reference is when describing joint translations.

Displacement, Velocity, and Acceleration

Both linear and angular displacements are measures of distance. **Position** is defined as the location of a point or object in space. **Displacement** is defined as the distance traveled between two locations. For example, consider the knee joint during gait. If its angular position is 10° of flexion at heel strike and 70° of flexion at toe off, the angular displacement from heel strike to toe off is 60° of flexion.

Change in linear and angular position (displacement) over time is defined as linear and angular **velocity,** respectively. Finding the instantaneous velocity at any given point in time, requires the use of calculus. *Instantaneous velocity* is defined as the differential of position with respect to time. Average velocity may be calculated by simply considering two separate locations of an object and taking the change its position and dividing by the change in time (Table 1.3). As the time interval becomes smaller and approaches zero, the average velocity approaches the instantaneous velocity.

Similarly, changes in linear and angular velocity over time are defined as linear and angular **acceleration.** *Instantaneous acceleration* is defined as the differential of velocity with respect to time. Average acceleration may be calculated by simply considering two separate locations of an object and taking the change in its velocity and dividing by the change in time (Table 1.3). An example of the effect of constant acceleration on velocity and position is shown in Figure 1.13.

KINETICS

Until now, forces and motion have been discussed as separate topics. **Kinetics** is the study of motion under the action of forces. This is a very complex topic that is only introduced here to give the reader some working definitions. The only chapter in this text that deals with these terms in any detail is Chapter 48 on gait analysis.

Inertial Forces

Kinematics and kinetics are bound by Newton's second law, which states that the external force (f) on an object is proportional to the product of that object's mass (m) and linear acceleration (a):

$$f = ma \qquad \text{(Equation 2.12)}$$

For conditions of static equilibrium, there are no external forces because there is no acceleration, and the sum of the external forces can be set equal to zero. However, when an object is accelerating, the so-called **inertial forces** (due to acceleration) must be considered, and the sum of the forces is no longer equal to zero.

Consider a simple example of a linear force system in which someone is trying to pick up a 20-kg box. If this is performed very slowly so that the acceleration is negligible, static equilibrium conditions can be applied (sum of forces equal zero), and the force required is 200 N (Box 1.6). However, if this same box is lifted with an acceleration of 5 m/s^2, then the sum of the forces is *not* equal to zero, and the force required is 300 N (Box 1.6).

There is an analogous relationship for rotational motion, in which the external moment (M) on an object is proportional to that object's **moment of inertia** (I) and angular acceleration (α):

$$M = I\alpha \qquad \text{(Equation 2.13)}$$

Just as mass is a measure of a resistance to linear acceleration, moment of inertia is a measure of resistance to angular acceleration (Table 1.2). It is affected both by the total mass and the distance that mass is from the COR, r, as follows:

$$I = mr^2 \qquad \text{(Equation 2.14)}$$

So the farther the mass of an object is from the COR, the larger its moment of inertia. For example, for a given rotational moment, a figure skater can reduce her moment of inertia by tucking her arms into her body, where they are closer to the COR for that motion. This serves to increase her angular acceleration.

Work, Energy, and Power

Another combination of kinematics and kinetics comes in the form of **work,** which is defined as the force required to move an object a certain distance (work = force × distance).

TABLE 1.3 Kinematic Relationships

	Position	Velocity		Acceleration	
		Instantaneous	Average	Instantaneous	Average
Linear	P	$v = \frac{dP}{dt}$	$v = \frac{P_2 - P_1}{t_2 - t_1}$	$a = \frac{dv}{dt}$	$a = \frac{v_2 - v_1}{t_2 - t_1}$
Angular	θ	$\omega = \frac{d\theta}{dt}$	$\omega = \frac{\theta_2 - \theta_1}{t_2 - t_1}$	$\alpha = \frac{d\omega}{dt}$	$\alpha = \frac{\omega_2 - \omega_1}{t_2 - t_1}$

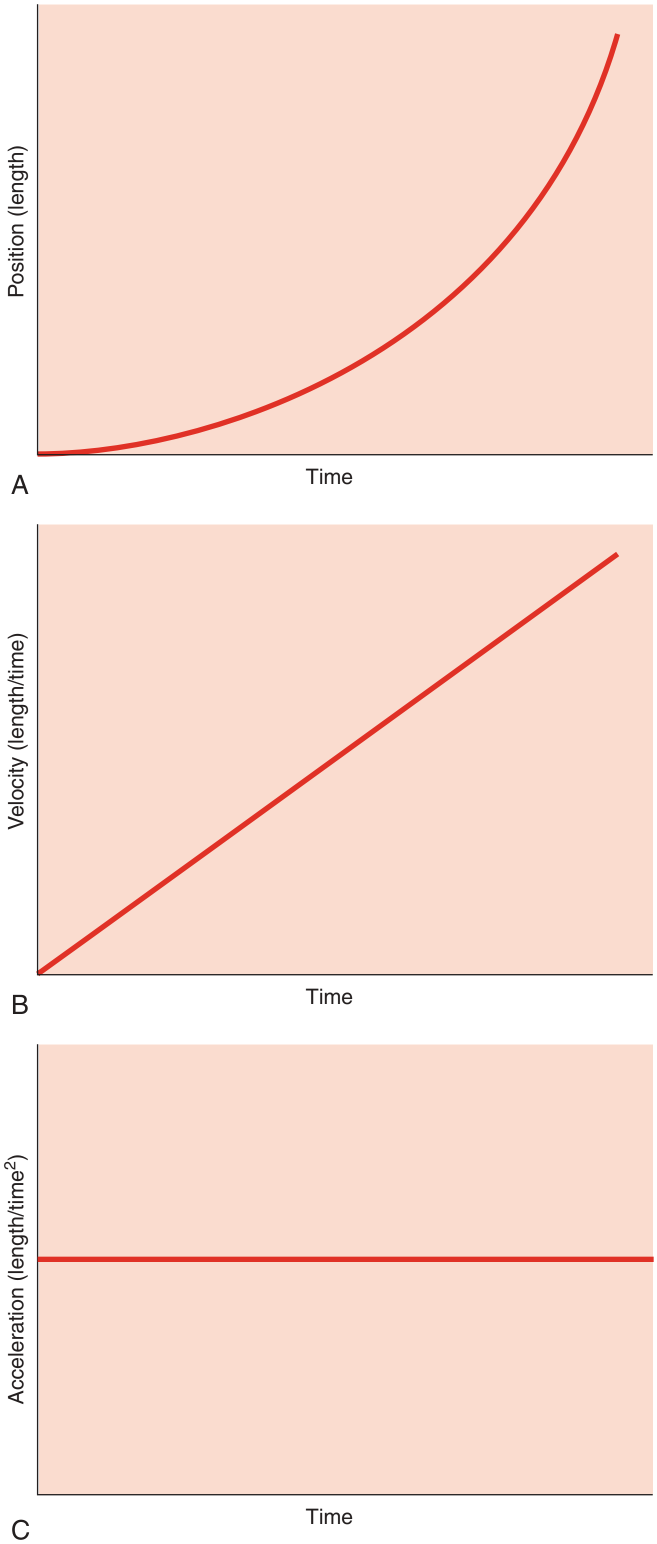
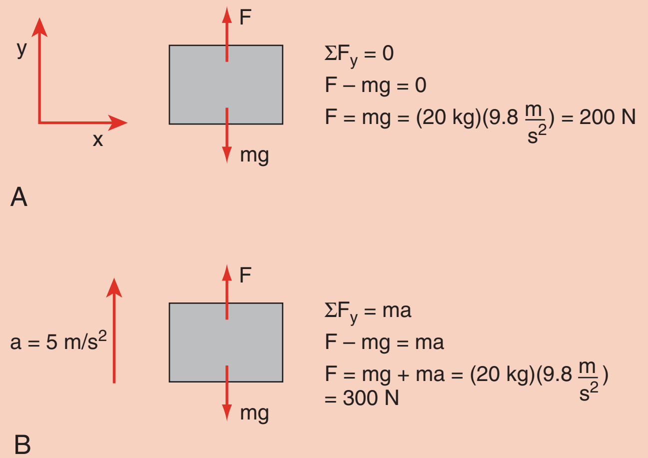


Figure 1.13: Acceleration, velocity, and displacement. Schematic representation of the motion of an object traveling at constant acceleration. The velocity increases linearly with time, while the position increases nonlinearly.

EXAMINING THE FORCES BOX 1.6

STATIC AND DYNAMIC EQUILIBRIUM



The standard unit of work in the metric system is a joule (J; newton × meter). For example, if the 20-kg box in Box 1.6 is lifted 1 m under static equilibrium conditions, the work done is equal to 200 joules (200 N × 1 m). By analogy with the analysis in Box 1.6, under dynamic conditions, the work done is equal to 300 J (300 N × 1 m).

Power is defined as the rate that work is being done (power = work/time). The standard unit of power is a watt (W; watt = newton × meter/second). Continuing with the above example, if the box were lifted over a period of 2 seconds, the average power would be 100 W under static conditions and 150 W under dynamic conditions. In practical terms, the static lift is generating the same amount of power needed to light a 100 W light bulb for 2 seconds.

The ***energy*** of a system refers to its capacity to perform work. Energy has the same unit as work (J) and can be divided into potential and kinetic energy. While **potential energy** refers to stored energy, **kinetic energy** is the energy of motion.

Friction

Frictional forces can prevent the motion of an object when it is at rest and resist the movement of an object when it is in motion. This discussion focuses specifically on Coulomb friction, or friction between two dry surfaces [11]. Consider a box with a weight (W) resting on the ground (Fig. 1.14). If a force (F) applied along the *x* axis is equal to the frictional force (F_f), the box is in static equilibrium. However, if the applied force is greater than the frictional force, the box accelerates to the right because of an unbalanced external force.

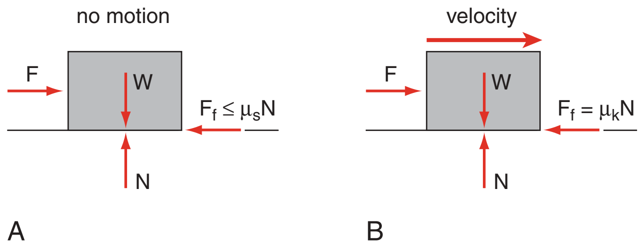


Figure 1.14: Friction. **A.** Under static conditions (no motion), the magnitude of the frictional force *(F_f)* exerted on the box is the same as the applied force *(F)* and cannot be larger than the coefficient of static friction *(μ_s)* multiplied by the normal force *(N)*. If the applied force exceeds the maximum static frictional force, the box will move and shift to dynamic conditions. **B.** Under dynamic conditions, the friction force is equal to the coefficient of dynamic friction (μ_k) multiplied by the normal force.

The frictional force matches the applied force until it reaches a critical value, $F = \mu_s N$, where N is the reaction force of the floor pushing up on the box and μ_s is the coefficient of *static* friction. In this example, N is equal to the magnitude of the force due to the weight of the box. Once this critical value is reached, there is still a frictional force, but it is now defined by: $F = \mu_k N$, where μ_k is the coefficient of *dynamic* friction.

The values for the coefficient of friction depend on several parameters, such as the composition and roughness of the two surfaces in contact. In general, the dynamic coefficient of friction is lower than the static coefficient of friction. As a consequence, it would take less force the keep the box in Figure 1.14 moving than it would take to start it moving.

SUMMARY

This chapter starts with a review of some important mathematical principles associated with kinesiology and proceeds to cover statics, kinematics, and kinetics from a biomechanics perspective. This information is used is used throughout the text for analysis of such activities as lifting, crutch use, and single-limb stance. The reader may find it useful to refer to this chapter when these problems are addressed.

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