# Econ 230A: Public Economics <br> Lecture: Structure of Income Taxation ${ }^{1}$ 

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## Outline

- Optimal Income Taxation
(1) Income Taxation in the US
(2) Simple Model with No Incentive Issues
(3) Mirrlees Model
(9) Intensive vs. Extensive Margins


## Emphasis in our lecture

- Gain an understanding of what affects the optimal income tax structure; how much redistribution to have through the tax code
- Role played by:
(1) responsiveness of labor supply to taxes
(2) distributional objectives
(3) underlying distribution of income
- Also focus on importance of intensive vs extensive margins of labor supply


## 1. Income Taxation in practice

- In the US and Europe, a large share of tax revenue is raised through the income tax
- For example in the U.S., $51.4 \%$ from individual income tax \& 35.9\% from payroll tax
- Labor supply responses to taxation are of fundamental importance for efficiency and equity considerations
- Important developments in 20th century income taxation:
- Most countries have progressive tax systems
- Most countries have reduced the MTR at high incomes (thereby reducing progressivity)
- Traditionally redistribution at the bottom of the income distribution took place through transfer programs. Now, many countries have expanded income tax based redistribution at the bottom (EITC in the US)


## Income Taxation in the US

- Several features of the U.S. system differ from Europe
- Income taxes are levied at the federal and state level (and in a few cases also cities tax).
- Income taxes in the U.S. are applied to FAMILIES not INDIVIDUALS. Based on combined income (if married) and own income (if single).
- Marginal tax rate structure varies by type of household: singles, married, head of household


## - Distribution of tax revenues, US by level of government (Gruber)

U.S. Tax Revenue by Type of Tax (2005, \% of total tax revenue)


State and local governments


Total government


## 1. Income Taxation in the US

- Tax rates change frequently over time.
- Income tax in the US started in 1913 with low rates from $1 \%$ to $7 \%$
- Rates increased a lot in the interwar period (top marginal rates went as high as $80 \%$ in some periods) but before 1942, income tax was paid only by high incomes (large exemption). Less than $10 \%$ of households paid the income tax.
- After 1943, exemptions lowered, big source of revenue for federal govt. Top rates extremely high (around 90\%).
- Top rates have been decreasing in steps to around $30 \%$ after TRA 86.
- Increased to 39.6\% in 1993.
- Reduced to $35 \%$ in 2001-2003
- MTR for married couples

Marginal tax rate if married, filing jointly


## - Top MTR



- Top MTR and top bracket threshold (real)

US Top Marginal Tax Rate and Top Bracket Threshold


## 1. Income Taxation in practice

- US income taxes assessed on the household basis
- Several European countries assess income taxes on individual basis (interesting equity and effic issues)
- Income tax computation: start with Gross Income
- Adjusted Gross Income = Gross Income - some allowable deductions
- taxable income $=$ agi -exemptions - deduction
- Tax schedule applied to taxable income .
- Itemized deductions: about $12 \%$ of AGI lost through itemized deductions: mortgage interest paid, charitable giving, state and local income taxes paid, medical expenses (above $7.5 \%$ of income).
- These are called tax expenditures (and they are huge).
- Differences btw US and Europe in use of VAT (Gruber)
- TAble 18-2



## Modeling: Linear vs Nonlinear Income Taxes

- Canonical linear income tax: $T a x=T+\tau w L$
- tax parameters include $T$ and $\tau$ (marginal tax rate).
- progressive income tax if $T<0$. Negative income tax (illustrate)
- Still limited redistribution because ATR apprach MTR
- Canonical nonlinear income tax: $T(w L)$
- Can allow for varying MTR


## Margins of Labor Supply used in Empirical Literature

- Extensive Margin [work or not]: captures movement in and out of labor market; can also capture retirement
- Intensive Margin [hours of work]: captures variation in work conditional on being in labor market
- Taxable Income: as alternative for hours, used to capture multiple margins of behavior (for analyzing higher income behavior)


## Intensive vs. Extensive Margins

- Trends in LFP rates show big changes for women in recent decades (extensive margin response) [Hoynes 2009]

Figure 4.5 Annual Employment Rates for Women Aged 19-44, by Marital Status and Presence of Children


## Intensive vs. Extensive Margins

- Yet for the same group there is little change in hours worked (intensive margin response) [Eissa and Hoynes 2009]


Figure 3.8
Average Annual Hours Worked for Working Single Women by Presence of Children, 1984-2003.

## Modeling: Linear vs Nonlinear Income Taxes

- Saez (QJE) classificiation of NIT vs EITC for allowing for redistribution at the bottom end of the distribution

b. Earned Income Tax Credit (EITC)


Figure I

## Definitions used in labor taxation/transfers

- Average tax rate vs marginal tax rate vs participation tax rate.
- Let $T(z)$ denore tax liability as function of earnings $z$
(1) Transfer benefit (at zero earning), lumpsum grant: $-T(0)$
(2) Marginal tax rate $T^{\prime}(z)$ : individual keeps $1-T^{\prime}(z)$ for an additional dollar of earnings $->$ relevant for intensive margin of labor supply. [ $w\left(1-T^{\prime}(z)\right)$ is net-of-tax wage]
(3) Average tax rate: Total taxes paid over earnings $T(z) / z$
(9) Participation tax rate $\tau_{p}=[T(z)-T(0)] / z$ : Individual keeps fraction $1-\tau_{p}$ when moving from 0 to $z$ earnings:
$z-T(z)=-T(0)+z-[T(z)-T(0)]=-T(0)+z \cdot\left(1-\tau_{p}\right)$
Relevant for extensive margin labor supply responses.
(5) Break even earnings point $z^{*}$ : point at which tax/transfer function is zero $T\left(z^{*}\right)=0$.


## Statement of optimal tax problem

- As with commodity taxation, we aim to maximize social welfare while taking account that agents will be maximizing (2-part problem)
- Seek to balance equity and efficiency concerns
- [equity] governments value redistribution (from rich to poor)
- [efficiency] redistribution is costly in terms of efficiency loss due to disincentives of taxes (taking from rich) and transfers (giving to poor)
- Hard to make explicit conclusions because it depends on how much you care about these two parts.
- How we will proceed:
- [intensive margin] Illustration: what if we ignore incentive effects
- [intensive margin] Show full problem (Mirrlees)
- [intensive margin] Develop results for top MTR (Saez RESTUD, Diamond and Saez)
- [ext/int margins] Develop results for bottom of the distribution (Saez QJE, Diamond and Saez)


## 2. Simple Model with No Incentive Issues

- Start with simple model in the classic utilitarian tradition with no efficiency issues:
- Variables: $c$ is after tax income, earnings is $z$
- Utility is strictly increasing and concave
- In this simplified model, work is exogenous: earnings $z$ is fixed [ignore efficiency!]
- Therefore utility can be just a function of $c: u(c)$
- Everyone works
- $T$ is the tax function so $c=z-T(z)$.


## 2. Simple Model with No Incentive Issues

- Suppose there are a continuum of agents, with $h(z)$ capturing the distibution of earnings (throughout this literature skills, which here is $z$ and later is $w$, are innate)
- Suppose we have a utilitarian SWF with social welfare being a simple sum of individual utlities
- Assume identical utility functions $u^{h}=u$ for all households $h$
- Therefore the optimal tax problem is to choose $T$ to maximize:

$$
\begin{aligned}
& \int_{0}^{\infty} u(z-T(z)) h(z) d z \\
& \text { st } \int T(z) h(z) d z \geq E, \text { multiplier } \lambda
\end{aligned}
$$

## 2. Simple Model with No Incentive Issues

- $\operatorname{FOC} u^{\prime}(z-T(z))=\lambda$
- So $z-T(z)=$ constant for all $z$.
- So an increase in $z$ is offset perfectly by an increase in $T$
- Conclusion: $100 \%$ marginal tax rate \& perfect equalization of after-tax income!
- Utilitarianism with decreasing marginal utility leads to egalitarianism.
- Obvious missing part?
- redistribution would affect incentives to work and thus the assumption that $z$ is exogenous is unrealistic.


## 3. Mirrlees Model: Setup

- First mathematically rigorous treatment of optimal income tax problem with incentive effects is Mirrlees REStud 1971.
- Central paper in tax policy issues.
- Model setup
- All individuals have the same utility $u(c, I)$ with $c$ after tax income and I labor supply. (So puts aside possibility of differences in preferences.)
- People differ according to their skills or wage rates $w$ :
- Exogenous distribution (and density) of wage rates: $F(w)$ and $f(w)$ (sum to one).
- Individual with skill w supplying / earns wl.
- Everyone works [intensive margin only; no extensive margin]
- Tax is based on earnings, so after-tax income $c=w l-T(w /)$.


## 3. Mirrlees Model: Setup

- Individual chooses $I$ so as to maximize $u(w l-T(w l), I)$
- FOC $w\left(1-T^{\prime}(w /)\right) u_{c}+u_{l}=0$. (e.g., $M R S=$ after tax wage)
- Government social welfare function: $G($.$) , increasing and concave.$
- G increasing: okay
- G concave means that social welfare of increasing utility is decreasing in u (marg increase for rich<marg increase for poor) -> taste for redistribution
- [G(u) $=u$ corresponds to utilitarian case.]


## 3. Mirrlees Model: Setup

- Government program: maximize social welfare subject to budget constraint and individual optimization:

$$
\begin{aligned}
W & \left.=\int G(u(c, l))\right) f(w) d w \\
\text { st } & : \int T(w l) f(w) d w \geq E(\text { multiplier } p) \\
: & w\left(1-T^{\prime}(w l)\right) u_{c}+u_{l}=0 .
\end{aligned}
$$

## 3. Empirical relevance of Mirrlees Model Results

- Mirrlees solves for optimal tax formulas but general formulas very complex
- Hard to interpret and understand what are the economic factors leading to high or low tax rates.
- Not much can be said in general about how rates should vary by income level.
- Unsatisfying. Given this, I like Diamond and Saez's requirement that the following must hold to translate the theory to practice:
(1) Result should be based on empirically relevant and first order object
(2) Result should be robust to changes in modeling assumptions
(3) Result needs to be practical: implementable and socially acceptable


## 3. Diamond and Saez on optimal tax

- Until late 1990s Mirrlees results had little impact on tax policy. Little connection between theoretical optimal tax work and large empirical literature on elasticities.
- Diamond AER 1998 (and later Saez REStud 2001) have re-posed Mirrlees' problem using elasticities. This is really useful because it allows a more direct link between empirical literature (estimating elasticities) and feedback to optimal tax.
- Diamond and Saez argue that the following two features are robust implications of the optimal tax problem:
(1) Top MTR on high earners should not be zero (prior view was that it should be zero at the top ; Seade (JPubE 1977) and Sadka.(REStud 1976))
(2) MTR at the bottom should be negative (subsize low earners) and phase out at high MTR (Saez QJE) [need to introduce extensive margin first]


## 3. Re-posing Mirrlees Model using elasticities (using Saez notation)

- Basic elasticity concepts in the static labor supply model
- max $u(c, z)$ st $c=z(1-\tau)+R$.
- $c$ is consumption ( + in utility function), and $z$ is earnings (- in utility function because earning income requires effort)
- analysis of earnings $z$ is equivalent to labor supply I if wages are fixed $(z=w l)$
- $R$ nonlabor income and $\tau$ marginal tax rate.
- Marshallian labor supply $z=z(1-\tau, R)$
- Hicksian labor supply: $z^{c}(1-\tau, u)$ notation)
- Uncompensated elasticity: $\zeta^{u}=\frac{(1-\tau)}{z} \frac{\partial z}{\partial(1-\tau)}$.
- Compensated elasticity: same using $z^{c}$
- Income effects $\eta=(1-\tau) \frac{\partial z}{\partial R} .<0$
- Slutsky equation: $\zeta^{c}=\zeta^{u}-\eta \geq 0$
- [empirical labor supply literature seeks to estimate these parameters]


## 3. Mirrlees Model: Optimal High Income Tax Rate (Saez

## REStud 2001, Diamond \& Saez)

- So, in the face of these unsatisfying results from Mirrlees, what can we conclude? Saez's paper makes headway on this.
- Return to optimal non-linear income tax rate using elasticities.
- Start with simpler problem: What is the optimal top bracket tax rate?
- Now $35 \%$. Was $39.6 \%$ during Clinton administration. Is $35 \%$ too high or too low?
- Perturbation method: Consider then the effects of a small increase $d \tau$ of the tax rate $\tau$ on social welfare.
- [positive] mechanical effect on revenue: holding constant individual behavior, an increase in the tax leads to an increase in revenue (which can be used for general spending or transfers to poor)
- [negative] behavioral effect on revenue: reduction in labor supply leads to lower revenue
- [negative] utility effect: loss in welfare to high income guys (but this may be small in social welfare terms)
- At optimum, they should sum to 0 .


## 3. Mirrlees Model: Optimal High Income Tax Rate (Saez REStud 2001)

- Model setup
- All individuals have the same utility $u(c, z)$ with $c$ after tax income and $z$ earnings
- People differ according to their skills (or wages, or earnings) $z$.
- Exogenous distribution (CDF, density) of earnings: $H(z)$ and $h(z)$.
- After-tax income $c=z-T(z)$.
- tax rate is constant above a given high income level $\bar{z}$.
- assume there are no income effects: each individual $h$ has a supply function $z^{h}(1-\tau)$.
- maintain intensive margin effect only; eveyone works


## 3. Mirrlees Model: Optimal High Income Tax Rate (Saez REStud 2001)

- Mechanical Effect: a taxpayer with income $z$ (above $\bar{z}$ ) has to pay $(z-\bar{z}) d \tau$ additional taxes. Therefore, summing over the population above $\bar{z}$ and denoting the mean of incomes above $\bar{z}$ by $z_{m}$ and normalizing the total population above $\bar{z}$ to one, the total mechanical effect $M$ is equal to,

$$
M=\left[z_{m}-\bar{z}\right] d \tau
$$

## 3. Mirrlees Model: Optimal High Income Tax Rate (Saez REStud 2001)

- Behavioral Response: No income effects so taxation affects earnings only through the tax rate (or substitution) effect. Using the elasticity $\zeta$, the response to $d \tau$ of a taxpayer earning $z$ (above $\bar{z}$ ) is equal to .

$$
d z=-\frac{\partial z}{\partial(1-\tau)} d \tau=-\zeta z \frac{d \tau}{(1-\tau)}
$$

- This reduction in income $d z$ implies a reduction in tax receipts equal to $\tau d z$.
- The total reduction in tax receipts due to the behavioral response is the sum of the terms $\tau d z$ over all individuals earning more than $z$, which can be written as,

$$
B=-\bar{\zeta} z_{m} \frac{\tau d \tau}{1-\tau}
$$

- $\bar{\zeta}$ is the weighted average of the elasticity in the top bracket weighted by income.


## 3. Mirrlees Model: Optimal High Income Tax Rate (Saez REStud 2001)

- Adding equations $M$ and $B$, the overall effect of the tax reform on government's revenue is obtained,

$$
M+B=\left[\frac{z_{m}}{\bar{z}}-1-\frac{\tau}{1-\tau} \bar{\zeta} \frac{z_{m}}{\bar{z}}\right] \bar{z} d \tau
$$

- The tax reform raises revenue if and only if the expression in square brackets is positive.
- To obtain the optimal tax rate, must equalize the revenue effect obtained from $M+B$ to the welfare effect due to the small tax reform.


## 3. Mirrlees Model: Optimal High Income Tax Rate (Saez REStud 2001)

- To obtain the welfare effect, suppose $\bar{g}$ is the social marginal value of $\$ 1$ in consumption for top bracket taxpayers (relative to government revenue).
- $\bar{g}$ is defined such that the government is indifferent between $\bar{g}$ more dollars of public funds and one more dollar consumed by the taxpayers with income above $\bar{z}$.
- The smaller $\bar{g}$, the less the government values marginal consumption of high incomes.
- Thus $\bar{g}$ is a parameter reflecting the redistributive goals of the government.


## 3. Mirrlees Model: Optimal High Income Tax Rate (Saez REStud 2001)

- So if we take the individual's maximum utility $u\left((1-\tau) z^{*}+R, z^{*}\right)$ the effect of a small change in tax rates $d \tau$ gives (and using the envelope theorem)

$$
d u=u_{c}(-z d \tau+d R)=-u_{c}(z-\bar{z}) d \tau
$$

- Therefore, welfare effects are due uniquely to the mechanical increase in tax liability ( $M=\left[z_{m}-\bar{z}\right] d \tau$ )
- Each additional dollar raised by the government reduces social welfare of people in the top bracket by $\bar{g}$ (the given parameter). Thus the total welfare loss due to the tax reform is equal to $\bar{g} M$.
- Consequently, the government sets the rate $\tau$ such that, $M+B-\bar{g} M=0$.
- Thus the optimal rate is such that,

$$
\frac{\tau_{T O P}^{*}}{1-\tau_{T O P}^{*}}=\frac{(1-\bar{g})\left(z_{m} / \bar{z}-1\right)}{\bar{\zeta} z_{m} / \bar{z}}
$$

## 3. Mirrlees Model: Optimal High Income Tax Rate (Saez REStud 2001)

$$
\frac{\tau_{T O P}^{*}}{1-\tau_{T O P}^{*}}=\frac{(1-\bar{g})\left(\frac{z_{m}}{z}-1\right)}{\bar{\zeta}_{\frac{z_{m}}{z}}^{z}}
$$

- The equation tells us that the optimal tax rate on high income is:
- [redistributive tastes] decreasing in social weight $\bar{g}$ put on high income taxpayers
- [efficiency] decreasing in the elasticity of labor supply among high income earners
- [thickness of tail, potential for revenue] increasing in $\frac{z_{m}}{\bar{z}}\left[\frac{z_{m}}{\bar{z}}\right.$ tells you about the thickness of the tail, and tends to one as you move up the income distribution]


## 3. Mirrlees Model: Optimal High Income Tax Rate (Saez REStud 2001)

- Practical relevance of this result: Saez empirically examines the ratio $\frac{z_{m}}{\bar{z}}$ using wage income reported on tax return data for years 1992 and 1993.
- From $\$ 150,000$ to close to the very top, the ratio $\frac{z_{m}}{\bar{z}}$ is roughly constant around $2->$ therefore the ratio does not approach 1 and the empirical relevance for the top MTR being zero is weak.




## 3. Mirrlees Model: Optimal High Income Tax Rate (Saez REStud 2001)

- Turns out that distributions with constant ratio $\frac{z_{m}}{\bar{z}}$ are well approximated by Pareto distributions
- $\operatorname{Prob}($ Income $>z)=C / z^{a}$ for some constant $C$, parameter $a>1$
- So for us, $\frac{z_{m}}{z}=\frac{a}{a-1}$. The higher $a$ (the lower $\frac{z_{m}}{z}$ ) the thinner is the tail of the income distribution.
- Diamond and Saez report that in the top bracket of $>\$ 400,000$, mean income is $\$ 1.2$ million, so $\frac{z_{m}}{\bar{z}}=3$ or $a=1.5$
- With this, we can restate the optimal tax rate as

$$
\bar{\tau}=\frac{1-\bar{g}}{1-\bar{g}+a \bar{\zeta}}
$$

- As before, we see that the optimal top rate is decreasing in the elasticity and social weight on high income guys, and increasing in thickness of the top tail distirbution.


## 3. Diamond and Saez JEP - calculation of optimal top MTR

- Extreme case $\bar{g}=0$ : Government does not value the marginal consumption of the high incomes and sets the top rate so as to extract as much tax revenue as possible from the high income (soak the rich).
- Implies $\bar{\tau}=\frac{1}{1+a \bar{\zeta}}$
- Plugging in $\bar{g}=0, a=1.5$, and $\bar{\zeta}=0.25$, you get the optimal top MTR $=73$ percent (compared to 42.5 now, including payroll tax and S\&L income tax)
- Current rate can be generated by either $\bar{g}=.72$ or $\bar{\zeta}=0.9$


## 3. Mirrlees Model: General Optimal Non-linear Income Tax (Saez REStud 2001)

- Generalizing to optimal nonlinear income taxes
- He models the non-linear income tax as two parts:
(1) lump-sum amount that is given to everybody and is equal to $-T(0)$.
(2) pattern of marginal tax rates $T^{\prime}(z)$ that describe how the lump-sum amount is taxed away and then how tax liability increases with income.
- FOC for optimal tax rates at income level $z$ :

$$
\frac{T^{\prime}(z)}{1-T^{\prime}(z)}=\frac{1}{\zeta}\left(\frac{1-H(z)}{z h(z)}\right)[1-G(z)]
$$

- $H(z)$ the CDF of income (density $h(z)$ ).
- $g(z)$ denotes the social marginal value of consumption for taxpayers with income $z$ (in terms of public funds). [gov is indifferent btw giving $1 / g\left(z_{1}\right) \$$ to taxpayer with income $z_{1}$ or $1 / g\left(z_{2}\right) \$$ to taxpayer with income $z_{2}$ ]
- $G(z)$ the average social marginal value of consumption for taxpayers with income above $z$.


## 3. Mirrlees Model: General Optimal Non-linear Income Tax (Saez REStud 2001)

- Three elements determine the optimal marginal tax rate $T^{\prime}(z)$ :
(1) Elasticity effects
(2) Shape of the income distribution
(3) Social marginal weights.
- Difference between this and the top rate analysis, change in MTR not just those facing this MTR but all those with incomes above that point.


## 3. Mirrlees Model: General Optimal Non-linear Income Tax (Saez REStud 2001)

- Elasticity effect:
- $T^{\prime}(z)$ is decreasing with the average elasticity $\zeta$ at income level $z$
- Pattern of optimal marginal tax rates depends on the pattern of elasticities by income level. This pattern is not well known empirically.
- Shape of the Income Distribution:
- the behavioral distortion $z$ depends on the density of folks here, and their income, $(z h(z))$
- gain in tax receipts is proportional to the number of people above $z$ $(1-H(z))$. (because the rate applies up the income dist)
- Apply high MTR where density of taxpayers is low relative to number with higher income
- Social Marginal Weights:
- If govt has redistributive tastes then $G(z)$ is decreasing in income and $1-G(z)$ is increasing in income.
- Therefore with tastes for redistribution ( $\mathrm{G}>0$ ) optimal taxes are progressive.


## 3. Mirrlees Model: General Optimal Non-linear Income Tax

 (Saez REStud 2001)- General resulting shape of optimal MTR is sort of $U$ shaped, with higher rates at the bottom
- This is combined with a benefit floor (lump sum if no work). Intuition: SWF likes targeting low income so have the high MTR on earnings to phaseout the grant. This causes an efficiency loss, but this is small because earnings are small. It is key that all labor supply is intensive margin, no movements in and out



## Intensive vs. Extensive Margins

- What type of transfer program is optimal? Or, optimal taxation at the bottom of the income distribution.
- Efficiency costs from labor supply responses to these transfers.
- Two stylized versions of transfer programs: NIT vs EITC (Saez QJE)
- NIT: guaranteed income transfer that is taxed away as earnings increase
- EITC: all earnings below a given threshold are partially matched by the government (negative marginal tax rates at bottom of income distribution); then phased out

a. Negative Income Tax (NIT)

Earnings w
b. Earned Income Tax Credit (EITC)


Earnings w

## Intensive vs. Extensive Margins

- Many countries have NIT-type programs (e.g. welfare); NIT is criticized for bad incentives for work (high MTR) at bottom of income distribution.
- Starting with the US, many countries are adopting EITC type programs
- What type of program is optimal?
- Saez (RESTUD)'s nonlinear tax analysis takes Mirrlees (1971) and concludes MTR shoud be U shaped with income -> NIT type and not EITC type.
- But Mirrlees model assumes that everyone works. Behavioral effects amount to changes in hours worked (intensive margin).
- Strong evidence in the empirical literature that response to tax and transfer programs is also along the extensive margin: participate or not in the labor force.
- Why? Fixed costs (job related expenses, child care), difficult to adjust hours. Empirical Evidence: Eissa TRA86, Meyer \& Rosenbaum QJE 2001, NIT experiments


## Intensive vs. Extensive Margins

- Trends in LFP rates show big changes for women in recent decades (extensive margin response) [Hoynes 2009]

Figure 4.5 Annual Employment Rates for Women Aged 19-44, by Marital Status and Presence of Children


## Intensive vs. Extensive Margins

- Yet for the same group there is little change in hours worked (intensive margin response) [Eissa and Hoynes 2009]


Figure 3.8
Average Annual Hours Worked for Working Single Women by Presence of Children, 1984-2003.

## Intensive vs. Extensive Margins: Saez QJE 2002

- Contribution and context for Saez QJE 2002 paper
- Rethink Mirrlees optimal tax results IF we allow for both intensive and extensive margins
- Turns out that the optimal tax results (for MTR at the bottom) depend critically on whether you allow for an extensive margin response
- Instead they provide support for an EITC like aspect to the taxes


## Intensive vs. Extensive Margins: Saez QJE 2002

- Model of extensive margin response
- Discrete model: $I+1$ possible earnings levels $w_{0}=0<w_{1}<$.. $<w_{l}$
- Each individual has a potential earning level $w_{i}$ and can choose either to work and earn $w_{i}$ or be out of the labor force and earn $w_{0}=0$.
- Government sets tax schedule depending on earnings $T_{i}=T\left(w_{i}\right)$ [ $T$ can be positive or negative, integrated tax and transfer program]
- Consumption in work state $i: c_{i}=w_{i}-T_{i}$
- Individual choice to work depends on difference between $c_{i}=w_{i}-T_{i}$ (work) and $c_{0}=w_{0}-T_{0}$ (nonwork)


## Intensive vs. Extensive Margins: Saez QJE 2002

- In equilibrium, for a given tax schedule, at each skill level, some people choose to work and others don't.
- Resulting earnings outcomes are the share in each outcome $h_{0}, h_{1}, \ldots, h_{1}$ (assume that $\sum h_{i}=1$ ).
- Elasticity $\eta_{i}=\left(c_{i}-c_{0}\right) / h_{i} \cdot \partial h_{i} / \partial\left(c_{i}-c_{0}\right)$.
- Government maximizes social welfare subject to a budget constraint: $\sum h_{i} T_{i} \geq H$
- Social Welfare function is summarized by social marginal welfare weights $g_{i}$ :
- Government is indifferent between $g_{i}$ more dollars of public funds and one more dollar to a worker with wage $w_{i}$
- Redistributive tastes $\Rightarrow g_{i}$ decreasing in $i$
- Possible in US that $g_{0}<g_{1}$ (working poor more "deserving" than nonworking poor)
- No income effects $\Rightarrow$ government indifferent between one more dollar of public funds and one more dollar to all taxpayers $\Rightarrow \sum_{i=0}^{l} h_{i} g_{i}=1$


## Intensive vs. Extensive Margins: Saez QJE 2002

- Consider only extensive margin (choose between work at $w_{i}$ and nonwork $w_{0}$ )
- So the outcomes (e.g. shares at $h_{i}$ ) depend on the difference in after tax income between options $c_{i}-c_{0}$
- Elasticity of participation with respect to income differences $\eta_{i}=\frac{c_{i}-c_{0}}{h_{i}} \frac{\partial h_{i}}{\partial\left(c_{i}-c_{0}\right)}$ (percent change in workers at $i$ who exit the LF when income differential increases by $1 \%$ )
- Optimal tax derivation follows the approach we have seen: pertubation (change $T_{i}$ by $d T_{i}$ ), 3 effects which have to sum to zero at the optimum
(1) Mechanical change in tax revenue: tax increases so revenue increases.
(2) Behavioral Effect of tax revenue: some people are induced to leave the labor market, so revenue falls.
(3) Welfare loss


## Intensive vs. Extensive Margins: Saez QJE 2002

$$
\frac{T_{i}-T_{0}}{c_{i}-c_{0}}=\frac{1}{\eta_{i}}\left(1-g_{i}\right)
$$

- Simple inverse elasticity rule (valid with income effects)
- Implications of redistributive tastes:
- $T_{i}>T_{0}$ for $i$ large (high incomes pay more taxes than non-workers)
- $T_{i}<T_{0}$ for $i$ small (low incomes get a subsidy for working)
- Negative Marginal Tax at the bottom!


## Intensive vs. Extensive Margins: Saez QJE 2002

- Optimal schedule from Saez


Earnings w

- But IF govt only cares about the worst off all SW weights are below 1 but $g_{o}$. In this case $T_{i} \leq T_{0}$ and the negative MTR at the bottom dissappears. Classic welfare program result.


## Intensive vs. Extensive Margins: Saez QJE 2002

- Mixed Model: allow intensive \& extensive margins
- Turns out that optimal taxes (transfers) characterized by budget constraint and

$$
\frac{T_{i}-T_{i-1}}{c_{i}-c_{i-1}}=\frac{1}{\zeta_{i} h_{i}} \sum_{j=i}^{l} h_{j}\left[1-g_{j}-\eta_{j} \frac{T_{j}-T_{0}}{c_{j}-c_{0}}\right]
$$

- $\zeta_{j}$ is the intensive margin elastisticy (as before)
- [This can nest intensive-only case $\eta=0$ or extensive-only case $\zeta=0$ ]


## Intensive vs. Extensive Margins: Saez QJE 2002

- Main conclusions:
- When participation (extensive) elasticity dominates relative to earnings elasticity, optimal schedule favors EITC-type transfer program.
- When earnings (intensive) elasticity dominates relative to participation elasticity, optimal schedule favors NIT-type transfer program.
- Main Intuitions:
- Consider a transfer to low-skilled workers.
- The labor supply response from the participation margin is an increase in labor supply from unemployed/out of the labor force.
- The response from the intensive marginal is a decrease in labor supply from those in (just above) higher income occupations.
- When participation elasticity dominates earnings elasticity, the increase along the extensive margin will dominate the decrease along the intensive margin.

