SATHYABAMA

INSTITUTE OF SCIENCE AND TECHNOLOGY

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SCHOOL OF ELECTRICAL AND ELECTRONICS ENGINEERING

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

CIRCUIT THEORY – SEEA1201

UNIT-I- DC CIRCUITS

UNIT – I

ELECTRICAL AND ELECTRONICS ENGINEERING

Electrical Quantities, Ohm's law, Kirchoff's laws, Resistors in series and parallel combinations, Current and Voltage division rules, Node and Mesh Analysis.

ELECTRICAL QUANTITIES – DEFINITIONS, SYMBOLS AND UNITS

- **Charge:**

A body is said to be charged positively, if it has deficit of electrons. It is said to be charged negatively if it has excess of electrons. The charge is measured in Coulombs and denoted by Q (or) q.

1 Coulomb = Charge on 6.28×10^{18} electrons.

- **Atom:**

To understand the basic concepts of electric current, we should know the Modern Electron Theory. Consider the matter which is in the form of solid, liquid (or) gas. Smallest particle of matter is molecule. Minute Particles are called molecules, which are themselves made up of still minute particles known as Atoms.

Atom: Minute tiny Particles with the central Part Nucleus.

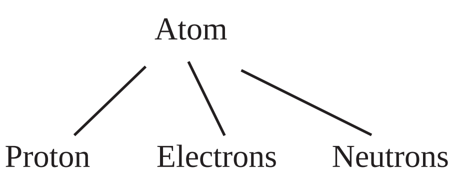


Figure 1.1

These are the types of tiny Particles in an Atom.

Protons: It is charged with positive charge.

Neutron: It is uncharged and hence it is neural.

Electron: It is revolving around nucleus. It is charged with small and constant amount of negative charge.

In an Atom, No of electrons = No of Protons

- **Electric Potential:**

When a body is charged, either electrons are supplied on it (or) removed from it. In both cases the work is done. The ability of the charged body to do work is called electric potential. The charged body has the capacity to do, by moving the other charges by either attraction (or) repulsion.

The greater the capacity of a charged body to do work, the greater is its electric potential. And the work done, to charge a body to 1 Colomb is the measure of electric potential.

$$\text{Electric potential, } V = \frac{\text{Work done}}{\text{Charge}} = \frac{W}{Q}$$

W = Work done per unit charge.
Q = Charge measured in Coulombs.

Unit of electric potential is **Joules / Coulomb (or) Volt**. If W = 1 joule; Q = 1 Coulomb, then V = 1/1 = 1 Volt.

A body is said to have an electric potential of 1 Volt, if one Joule of work is done to charge a body to one Coulomb. Hence greater the Joules / Coulomb on a charged body, greater is electric potential.

- **Potential Difference:**

The difference in the potentials of two charged bodies is called potential difference.

Consider two charged bodies A and B having Potentials of 5 Volts and 3 Volts respectively.

Potential Difference is +2v.
Unit of potential difference is Volts.
Potential difference is sometimes called Voltage.

- **Electric Current:**

Flow of free electrons through a conductor is called electric current. Its unit is Ampere (or) Coulomb / sec.

$$\text{Current, } (I) = \frac{\text{Charge(q)}}{\text{Sec Time(t)}} = \frac{q}{t} \text{ Coulombs /}$$

In differential form, $i = \dfrac{dq}{dt}$ Coulombs / Sec

Consider a conducting material like metal, say Copper. A large number of free electrons are available. They move from one Atom to the other at random, before an electric force is applied. When an electric potential difference is applied across the metallic conductors, free electrons start moving towards the positive terminal of the cell. This continuous flow of electrons forms electric current. According to modern electronic theory, the direction of conventional current is form positive terminal to negative terminal through the external circuit.

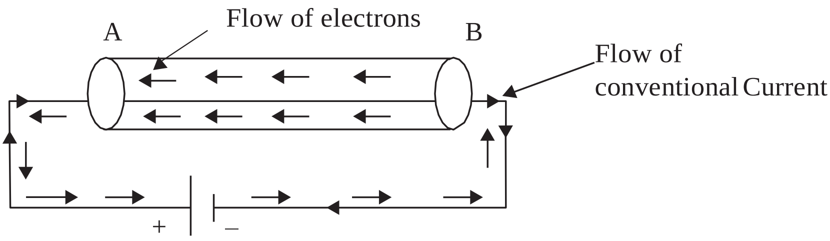


Figure 1.2

Thus, a wire is said to carry a current of 1 Ampere when charge flows through it at the rate of one Coulomb per second.

- **Resistance:**

Consider a conductor which is provided some potential difference. The free electrons start moving in a particular direction. While moving, the free electrons may collide with some Atoms (or) Molecules. They oppose the flow of electrons. Resistance is defined as the property of the substance due to which restricts the flow of electrons through the conductor. Resistance may, also be defined as the physical property of the substance due to which it opposes (or) restricts the flow of electricity (i.e. electrons) through it. Its unit is Ohms.

A wire is said to have a resistance of 1 ohm if a potential difference of 1V across the ends causes current of 1 Amp to flow through it (or) a wire is said to have a resistance of 1 ohm if it releases 1 Joule, when a current of 1A flows through it.

- **Laws of Resistance:**

The electrical resistance (R) of a metallic conductor depends upon the various Factors as given below,

(i) It is directly proportional to length l, ie, $R \alpha l$

(ii) It is inversely proportional to the cross sectional area of the Conductor, ie, $R \alpha \frac{l}{A}$

(iii) It depends upon the nature of the material of the conductor.

(iv) It depends upon the temperature of the conductor.

From the First three points and assuming the temperature to remain constant, we get,

$$R \alpha \frac{l}{A}$$

$$R = \rho \frac{1}{A}$$

ρ (‘Rho’) is a constant of proportionality called **Resistivity** (or) Specific Resistance of the material of the conductor. The value of ρ depend upon the nature of the material of the conductor.

- **Specific Resistance (or) Resistivity:**

Resistance of a wire is given by $R = \rho \frac{1}{A}$

If l = 1metre, A =1m^2 then, R = ρ . The resistance offered by a wire of length 1 metre and across sectional area of Cross-section of 1m^2 is called the Resistivity of the material of the wire.

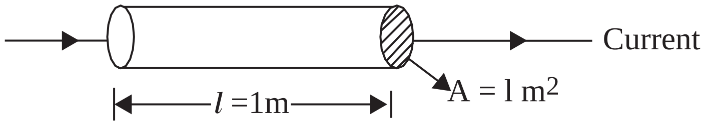


Figure 1.3

If a cube of one meter side is taken instead of wire, ρ is defined as below., Let l = 1 metre, A = 1 m^2, then R = ρ. “Hence, the resistance between the opposite faces of 1 metre cube of the given material is called the resistivity of that material”. The unit of resistivity is ohm-metre

$$\left[\, \rho = \frac{RA}{l} = \frac{\Omega m^2}{m} = \Omega m(\text{ohm-metre}) \,\right]$$

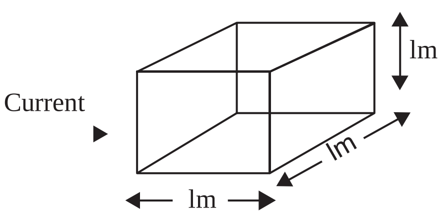


Figure 1.4

- **Conductance (or) Specific Conductance:**

Conductance is the inducement to the flow of current. Hence, Conductance is the reciprocal of resistance. It is denoted by symbol G.

$$G = \frac{1}{R} = \frac{A}{\rho l} = \sigma \frac{A}{l}$$

G is measured in mho Symbol for its unit is (℧)

$$\sigma = \frac{1}{\rho}$$

Here, σ is called the Conductivity (or) Specific Conductance of the material

- **Conductivity (or) Specific Conductance:**

Conductivity is the property (or) nature of the material due to which it allows flow of current through it.

$$G = \sigma \frac{A}{l} \text{ (or) } \sigma = G\frac{l}{A}$$

Substituting the units of various quantities we get

$$\sigma = \frac{\text{mho*m}}{\text{m}^2} = \text{mho/metre}$$

∴ The S.I unit of Conductivity is mho/metre.

- **Electric Power:**

The rate at which the work is done in an electric circuit is called electric power.

$$\text{Electric Power} = \frac{\text{Work done in an electric circuit}}{\text{Time}}$$

When voltage is applied to a circuit, it causes current to flow through it. The work done inmoving the electrons in a unit time is called Electric Power. The unit of Electric Power is Joules/sec (or) Watt. $\left\{P = VI = I^2R = V^2/R\right\}$

- **Electrical Energy:**

The total work done in an electric circuit is called electrical energy.

ie, Electrical Energy = (Electrical Power)*(Time)

$$\text{Electrical Energy} = I^2Rt = \frac{V^2}{R}t$$

Electrical Energy is measured in Kilowatt hour (kwh)

Problem 1.1 The resistance of a conductor 1 mm^2 in cross section and 20 m long is 0.346 Ω. Determine the specific resistance of the conducting material.

Given Data

Area of cross-section A = 1 mm^2
Length, l = 20 m
Resistance, R = 0.346 Ω

Formula used: Specific resistance of the Conducting Material, $R = \frac{\rho l}{A}$

$$\Rightarrow \rho = \frac{RA}{l}$$

Solution: Area of Cross-section, $A = 1mm^2$

$$= 1 * 10^{-6} m^2$$

$$\rho = \frac{1*10^{-6} * 0.346}{20} = 1.738*10^{-8}\ \Omega m$$

Specific Resistance of the conducting Material, $\rho = 1.738*10^{-8}\ \Omega m$.

Problem 1.2 A Coil consists of 2000 turns of copper wire having a cross-sectional area of 1 mm^2. The mean length per turn is 80 cm and resistivity of copper is 0.02 μΩm at normal working temperature. Calculate the resistance of the coil.

Given data:

No of turns = 2000
Length / turn = 80 cm =0.8 m

Resistivity, = 0.02 μΩm = $0.02*10^{-6}$ Ωm = $2*10^{-8}$ Ωm
Cross sectional area of the wire, A= 1mm^2 = $1*10^{-6}m^2$

Solution:
Mean length of the wire, l = 2000*0.8 =1600 m.

We know that, $R = \rho \frac{l}{A}$

Substituting the Values, $R = \frac{2 *10^{-8} *1600}{1*10^{-6}} = 32\Omega$

Resistance of the coil $= 32\Omega$

Problem 1.3 A wire of length 1m has a resistance of 2Ω. What is the resistance of the second wire, whose specific resistance is double that of first, if the length of wire is 3m and the diameter is double that of first?

Given Data:

For the first wire: $l_1 = 1m$, $R_1 =$ 2 Ω, $\rho_1 = \rho$ (say)

$$d_1 = d \ (say)$$

For the Second wire: l_2 = 3m, d_2 = 2d, $\rho_2 = 2\rho$

Solution:

$$R_1 = \rho_1 \frac{l_1}{A_1} = \frac{\rho * 1}{\frac{\pi d^2}{4}} \text{ [Radius of the wire } = \pi r^2 \text{, where } r = \frac{d}{2}\text{]}$$

$$\text{ie, } R_1 = \frac{4\rho}{\pi d^2} = \frac{\rho_1 * 1}{\pi d^2 / 4} \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$R_2 = \rho_2 \frac{l_2}{A_2} = \frac{2\rho * 3}{\frac{\pi (2d)^2}{4}} = \frac{6\rho}{\pi d^2} \qquad (2)$$

Dividing equation (1) by (2),

$$\frac{4\rho}{\pi d^2} * \frac{\pi d^2}{6\rho} \Rightarrow \frac{4}{6} = \frac{R_1}{R_2}$$

$$R_2 = \frac{6R_1}{4} = \frac{6 * 2}{4} = 3\ \Omega$$

$$R_2 = 3\ \Omega$$

The Resistance of the second wire, $R_2 = 3\ \Omega$

Problem 1.4 A Rectangular copper strip is 20 cm long, 0.1 cm wide and 0.4 cm thick. Determine the resistance between (i) opposite ends and (ii) opposite sides. The resistivity of copper is $1.7*10^{-6}$ Ωcm.

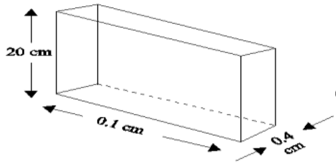


Figure 1.5

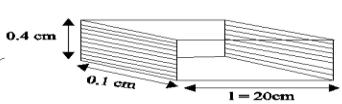


Figure 1.6

(i) Opposite Ends

Wide, w = 0.1cm

Thickness, t = 0.4cm

Length, l = 20cm

(ii) Opposite Sides:

Wide, w = 0.1cm

Thickness, t = 20 cm

Length, l = 0.4 cm

(a) Area = $w*t = 0.1*0.4 = 0.04cm^2$

$$R_1 = \frac{\rho l}{A} = \frac{1.7*10^{-6}*20}{0.04} = 0.85*10^{-3}\,\Omega$$

$R_1 = 0.85\, m\wedge$ [Opposite ends, referring to Figure 1.5]

Area, $A = w*t = 0.1*20 = 20cm^2$

$$R_2 = \frac{1.7*10^{-6}*0.4}{2} = 0.34*10^{-6}\,\Omega$$ [Opposite Sidesi referring to Figure 1.6]

$R_2 = 0.34\mu\,\Omega$

Problem 1.5 A silver wire of length 12m has a resistance of 0.2Ω. Find the specific resistivity of the material. The cross-sectional area of the wire is 0.01cm^2.

$$R = \frac{\rho l}{A} \quad \Rightarrow \quad \text{length, l=12m}$$

Resistance, R= 0.2Ω

A = 0.01cm^2

$$\rho = \frac{RA}{l} = \frac{0.2*0.01*10^{-4}}{12}$$

$$\rho = 1.688*10^{-8}\,\Omega\, m$$

OHM'S LAW AND ITS LIMITATIONS

The relationship between DC potential difference (V) current (I) and Resistance (R) in a DC circuit was first discovered by the scientist George Simon Ohm, is called Ohm's law.

- **Statement:**

The ratio of potential difference between any two points of a conductor to the current following between them is constant, provided the physical condition (eg. Temperature, etc.) do not change.

$$\text{ie, } \frac{V}{I} = \text{Constant}$$

(or)

$$\frac{V}{I} = R$$
$$\Rightarrow V = I * R$$

Where, R is the resistance between the two points of the conductor.

It can also be stated as, provided Resistance is kept constant, current is directly proportional to the potential difference across the ends of the conductor.

$$\text{Power, } P = V * I = I^2 R = \frac{V^2}{R}$$

- **Illustration:**

Let the potential difference between points A and B be V volts and current flowing be I Amp. Then, $\frac{V}{I} = \text{Constant}$,

$$\frac{V}{I} = R \text{ (say)}$$

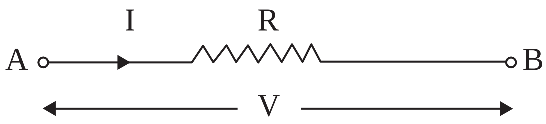


Figure 1.7

We know that, if the voltage is doubled (2V), the current flowing will also be doubled (2I). So, the ratio $\frac{V}{I}$ remains the same (ie, R). Also when voltage is measured in volts, current in ampere, then resistance will be in ohms.

- **Graphical representation of Ohm's law**

[Slope line of the graph represents the resistance]

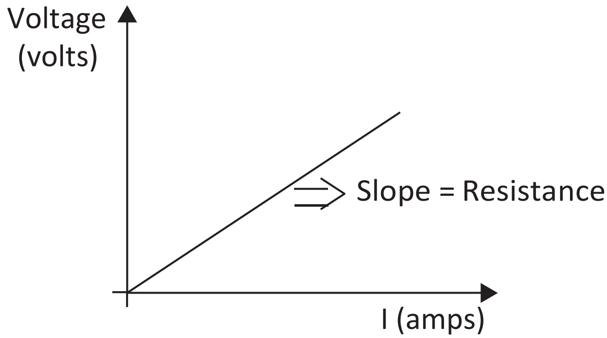


Figure 1.8

- **Limitations in ohm's law**:

(i) Ohm's law does not apply to all non-metallic conductors. For eg. Silico Carbide.
(ii) It also does not apply to non-linear devices such as Zener diode, etc.
(iii) Ohm's law is true for metal conductor at constant temperature. If the temperature changes the law is not applicable.

- **Problems based on ohm's law**:

Problem 1.6. An electric heater draws 8A from 250V supply. What is the power rating? Also find the resistance of the heater element.

Given data:

Current, $I = 8A$
Voltage, $V = 250V$

Solution:

Power rating, $P = VI = 8 * 250 = 2000 Watt$
Resistance (R) = $\frac{V}{I} = \frac{250}{8} = 31.25\ \Omega$

Problem 1.7 What will be the current drawn by a lamp rated at 250V, 40W, connected to a 230 V supply.
Given Data:

Rated Power = 40 W
Rated Voltage = 250 V
Supply Voltage = 230 V

Solution:

Resistance,

$$R = \frac{V^2}{P} = \frac{250^2}{40} = 1562.5\ \Omega$$

$$\text{Current, } I = \frac{V}{P} = \frac{230}{1562.5} = 0.1472\ A$$

Problem 1.8 A Battery has an emf of 12.8 volts and supplies a current of 3.24 A. What is the resistance of the circuit? How many Coulombs leave the battery in 5 minutes?

Solution:

$$\text{Circuit Resistance, R} = \frac{V}{I} = \frac{12.8}{3.24} = 4\ \Omega$$

Charge flowing in 5 minutes = Current × time in seconds

Charge flowing in 5 minutes = 3.24×5×60 = 960 Coulomb

Problem 1.9 If a resistor is to dissipate energy at the rate of 250W, find the resistance for a terminal voltage of 100V.

Given data:

Power = 250W

Voltage = 100V

Solution:

$$\text{Resistance, } R = \frac{V^2}{\rho} = \frac{100^2}{250} = 40\ \Omega$$

$$R = 40\ \Omega\ .$$

Problem 1.10 A voltmeter has a resistance of, 20,200 Ω. When connected in series with an external resistance across a 230 V supply, the instrument reads 160 V. What is the value of external resistance?

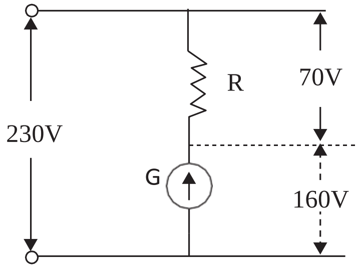


Figure 1.9

The voltage drop across external resistance, R

$$V_R = 230 - 160 = 70V$$

$$\text{Circuit current, } I = \frac{160}{20,000} = \frac{1}{125} A$$

We know that, $V = IR$

$$70 = IR$$

$$70 = \frac{1}{125} \times R$$

$$R = 8750\ \Omega$$

COMBINATION OF RESISTORS

- **Introduction:**

The closed path followed by direct Current (DC) is called a DC Circuit A d.c circuit essentially consist of a source of DC power (eg. Battery, DC generator, etc.) the conductors used to carry current and the load. The load for a DC circuit is usually a resistance. In a DC circuit, loads (i.e, resistances) may be connected in series, parallel, series – parallel. Hence the resistor has to be connected in the desired way for getting the desired resistance.

Resistances in series (or) series combination

The circuit in which resistances are connected end to end so that there is one path for the current flow is called **series circuit**. The voltage source is connected across the free ends. [A and B]

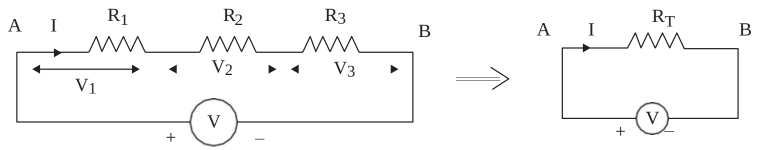


Figure 1.10

In the above circuit, there is only one closed path, so only one current flows through all the elements. In other words, if the Current is same through all the resistors, the combination is called series combination.

- **To find equivalent Resistance**:

Let, V = Applied voltage
I = Source current = Current through each element

V_1, V_2, V_3 are the voltage across R_1, R_2 and R_3 respectively.

By Ohms law, $V_1 = IR_1$

$V_2 = IR_2$ and $V_3 = IR_3$

But $V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3 = I\left(R_1 + R_2 + R_3\right)$

$$V = I\left(R_1 + R_2 + R_3\right)$$

$$V = IR_T$$

$$\frac{V}{I} = R_T$$

The ratio of $\left(V/I\right)$ is the total resistance between points A and B and is called the total (or) equivalent resistance of the three resistances

$$R_T = R_1 + R_2 + R_3$$

Also, $\dfrac{1}{G_T} = \dfrac{1}{G_1} + \dfrac{1}{G_2} + \dfrac{1}{G_3}$ (In terms of conductance)

$\therefore$ Equivalent resistance (R_T) is the sum of all individual resistances.

- **Concepts of series circuit**:

i. The current is same through all elements.
ii. The voltage is distributed. The voltage across the resistor is directly proportional to the current and resistance.
iii. The equivalent resistance (R_T) is greater than the greatest individual resistance of that combination.
iv. Voltage drops are additive.
v. Powers are additive.
vi. The applied voltage is equals to the sum of different voltage drops.

Voltage Division Technique: (or) To find V_1, V_2, V_3 interms of V and R_1, R_2, R_3:

Equivalent Resistance, $R_T = R_1 + R_2 + R_3$

By ohm's low, $I = \dfrac{V}{R_T} = \dfrac{V}{R_1 + R_2 + R_3}$

$$V_1 = IR_1 = \frac{V}{R_T}\ R_1 = \frac{VR_1}{R_1 + R_2 + R_3}$$

$$V_2 = IR_2 = \frac{V}{R_T} R_2 = \frac{VR_2}{R_1 + R_2 + R_3}$$

$$V_3 = IR_3 = \frac{V}{R_T} R_3 = \frac{VR_3}{R_1 + R_2 + R_3}$$

$\therefore$ Voltage across any resistance in the series circuit,

$$\Rightarrow V_x = \frac{R_x}{R_T} V$$

Note: If there are n resistors each value of R ohms in series, then the total Resistance is given by,

$$R_T = n * R$$

- **Applications:**
  - When variable voltage is given to the load, a variable resistance (Rheostat) is connected in series with the load. Example: Fan regulator is connected in series with the fan.
  - The series combination is used where many lamp of low voltages are to be operated on the main supply. Example: Decoration lights.
  - When a load of low voltage is to be operated on a high voltage supply, a fixed value of resistance is connected in series with the load.

- **Disadvantage of Series Circuit:**
  - If a break occurs at any point in the circuit, no current will flow and the entire circuit becomes useless.
  - If 5 numbers of lamps, each rated 230 volts are to be connected in series circuit, then the supply voltage should be 5 x 230 = 1150 volts. But voltage available for lighting circuit in each and every house is only 230 V. Hence, series circuit is not practicable for lighting circuits.
  - Since electrical devices have different current ratings, they cannot be connected in series for efficient operation.

- **Problems based on series combination:**

Problem 1.11 Three resistors 30 Ω, 25 Ω, 45 Ω are connected in series across 200V. Calculate (i) Total resistance (ii) Current (iii) Potential difference across each element.

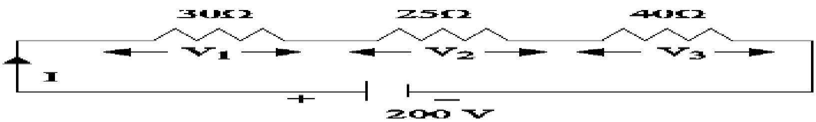


Figure 1.11

(i) Total Resistance (R_T)

$$R_T = R_1 + R_2 + R_3$$
$$R_T = 30 + 25 + 45 = 100\ \Omega$$

(ii) Current, $I = \frac{V}{R_T} = \frac{200}{100} = 2\ A$

(iii) Potential difference across each element,

$$V_1 = IR_1 = 2 * 30 = 60\ V$$
$$V_2 = IR_2 = 2 * 25 = 50\ V$$
$$V_3 = IR_3 = 2 * 45 = 90\ V$$

Problem 1.12 Find the value of 'R' in the circuit diagram, given below.

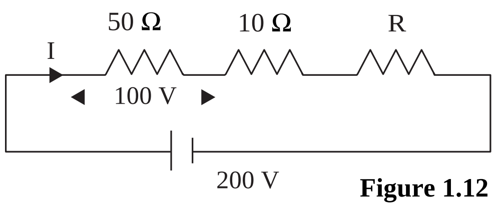


Figure 1.12

We know that, $V_1 = IR_1$

$$I = V_1 / R_1 = 100/50 = 2\ A$$

Similarly, $V_2 = IR_2 = 2 * 10 = 20\ V$

Total voltage drop, $V = V_1 + V_2 + V_3$

$$V_3 = V - (V_1 + V_2) = 200 - (100 + 20)$$
$$V_3 = 80\ V$$
$$V_3 = IR_3,\ R_3 = V_3 / I = 80/2 = 40\ \Omega$$
$$\therefore R_3 = 40\ \Omega$$

Problem 1.13 A 100W, 200V bulb is put in series with a 60W bulb across a supply. What will be the current drawn? What will be the voltage across the 60W bulb? What will be the supply voltage?

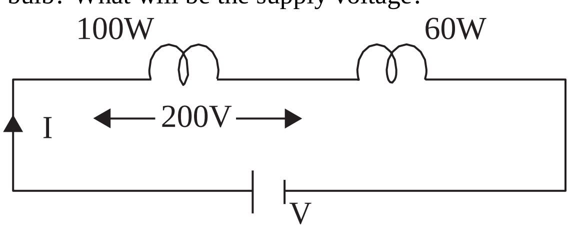


Figure 1.13

Power dissipated in the first bulb, $P_1 = V_1 I$
Current, $I = P_1 / V_1 = 100/200 = 0.5\,A$
Power dissipated in the second bulb, $P_2 = V_2 I$
Voltage across the 60 W bulb,

$$V_2 = \frac{P_2}{I} = \frac{60}{0.5} = 120V$$

The supply voltage, $V = V_1 + V_2 = 200 + 120$

$$V = 320V$$

The supply voltage, V = 320 V.

Problem 1.14 An incandescent lamp is rated for 110V, 100W. Using suitable resistor how can you operate this lamp on 220V mains.

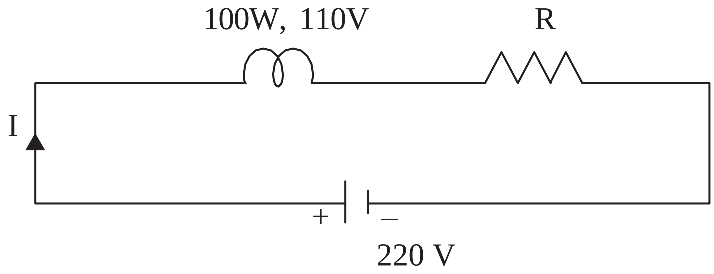


Figure 1.14

Rated current of the lamp, $I = \frac{Power}{Voltage} = \frac{100}{110} = 0.909A$, I = 0.909A

For satisfactory operation of the lamp, Current of 0.909A should flow.
When the voltage across the lamp is 110V, then the remaining voltage must be across R

$$\text{Supply voltage} = V = 220 \text{ Volts}$$

$$\text{Voltage across } R = V - 110 \text{ Volts}$$

$$\text{ie, } V_R = 220 - 110 = 110V$$

$$\text{By ohm's law, } V_R = IR$$

$$110 = 0.909\text{ R}$$

$$R = 121\ \Omega$$

Problem 1.15 The lamps in a set of decoration lights are connected in series. If there are 20 lamps and each lamp has resistance of 25Ω, calculate the total resistance of the set of lamp and hence calculate the current taken from a supply of 230 volts.

Given Data: Supply voltage, $V = 230\ volts$

Resistance of each lamp, $R = 25\ \Omega$

No of lamps in series, $n = 20$

Solution: Total Resistance, $R_T = n * R = 20 * 25$

$$R_T = 500\ \Omega$$

Current from supply. $I = \dfrac{V}{R_T} = \dfrac{230}{500} = 0.46\ A$

Problem 1.16 The field coil of a d.c generator has a resistance of 250Ω and is supplied from a 220 V source. If the current in the field coil is to be limited to 0.44 A. Calculate the resistance to be connected in series with the coil.

Given Data: Source voltage, $V = 220$ volts, $I = 0.44\ A$

Field coil resistance, $R_f = 250\ \Omega$

Solution: Let the resistance in series with R_f be R in Ohms.

Total resistance, $R_T = R_f + R = 250 + R$

Current, $I = 0.44\ A$

By ohm's law, $R_T = \dfrac{V}{I} = \dfrac{220}{0.44} = 500\ \Omega$

$$250 + R = 500\ \Omega$$

$$R = 500 - 250 = 250\ \Omega$$

$$R = 250\ \Omega$$

Resistance in Parallel (or) Parallel Combination

If one end of all the resistors are joined to a common point and the other ends are joined to another common point, the combination is said to be parallel combination. When the voltage source is applied to the common points, the voltage across each resistor will be same. Current in the each resistor is different and is given by ohm's law.

Let R_1, R_2, R_3 be three resistors connected between the two common terminals A and B, as shown in the Figure 1.15(a)..

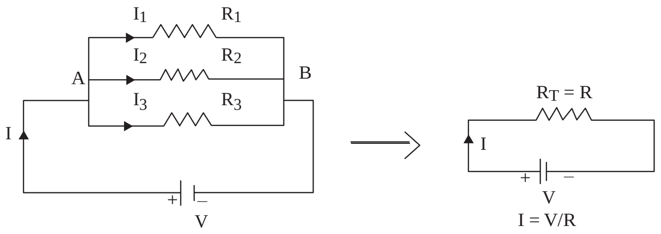


Figure 1.15

$$I = \frac{V}{R} \qquad (1)$$

Let I_1, I_2, I_3 are the currents through R_1, R_2, R_3 respectively. By ohm's law,

$$\left[I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2}, I_3 = \frac{V}{R_3} \right] \qquad (2)$$

Total current is the sum of three individual currents,

$$I_T = I = I_1 + I_2 + I_3 \qquad (3)$$

Substituting the above expression for the current in equation (3),

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Referring to Figure (1.15(b)), $R_T = R$

$$\frac{1}{R} = \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad (4)$$

Hence, in the case of parallel combination the reciprocal of the equivalent resistance is equal to the sum of reciprocals of individual resistances. Multiplying both sides of equation (4) by V^2, we get

$$\frac{V^2}{R} = \frac{V^2}{R_1} + \frac{V^2}{R_2} + \frac{V^3}{R_3}$$

ie, Power dissipated by R = Power dissipated by R_1 + Power dissipated by R_2 + Power dissipated by R_3

We know that reciprocal of Resistance is called as conductance.

Conductance = 1 / Resistance

[G = 1/R]

Equation (4) can be written as,

$$G = G_1 + G_2 + G_3$$

- **Concepts of Parallel Circuit:**
  - Voltage is same across all the elements.
  - All elements will have individual currents, depends upon the resistance of element.
  - The total resistance of a parallel circuit is always lesser than the smallest of the resistance.
  - If n resistance each of R are connected in parallel then,

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots\ldots\ldots n\,terms$$

$$\frac{1}{R_T} = \frac{n}{R}$$

(or)

$$R_T = \frac{R}{n}$$

  - Powers are additive.
  - Conductance are additive.
  - Branch currents are additive.

- **Current Division Technique:**

Case (i) When two resistances are in parallel:

Two resistance R_1 and R_2 ohms are connected in parallel across a battery of V (volts) Current through R_2 is I_2 and through R_2 is I_2 The total current is I.

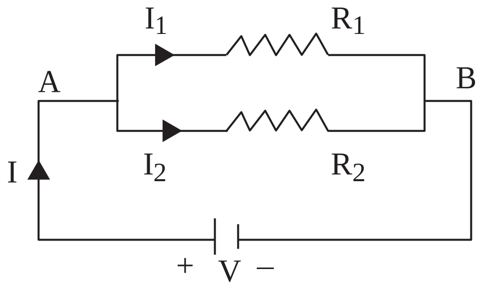


Figure 1.16

To express I_1 and I_2 interms of I, R_1 and R_2 (or) to find branch currents I_1, I_2:

$$I_2 R_2 = I_1 R_1$$

$$I_2 = \frac{I_1 R_1}{R_2} \qquad (1)$$

Also, the total current, $I = I_1 + I_2$ (2)

Substituting (1) in (2), $I_1 + \dfrac{I_1 R_1}{R_2} = I$

$$\frac{I_1 R_2 + I_1 R_1}{R_2} = I$$

$$I_1 (R_1 + R_2) = IR_2$$

$$I_1 = \frac{IR_2}{(R_1 + R_2)}$$

Similarly, $I_2 = \dfrac{IR_1}{(R_1 + R_2)}$

To find the equivalent Resistance, (R_T) :

$$\frac{1}{R} = \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{1}{R_T} = \frac{R_2 + R_1}{R_1 R_2}$$

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

Hence, the total value of two resistances connected parallel is equal to their product divided by their sum i.e.,

$$\text{Equivalent Resistance} = \frac{\text{Product of the two Resistance}}{\text{Sum of the two Resistane}}$$

Case (ii) When three resistances are connected in parallel. Let R_1, R_2 and R_3 be resistors in parallel. Let I be the supply current (or) total current. I_1, I_2, and I_3 are the currents through the resistors R_1, R_2 and R_3.

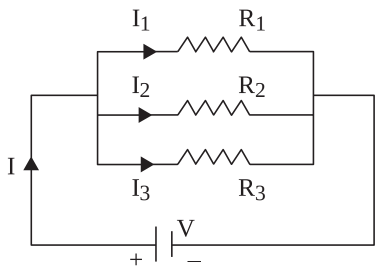


Figure 1.17

To find the equivalent Resistance (R_T):

$$\frac{1}{R} = \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_T} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1R_2R_3}$$

$$R_T = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}$$

To find the branch currents I_1, I_2 and I_3:

We know that, $I_1 + I_2 + I_3 = I$ (1)

Also, $I_3R_3 = I_1R_1 = I_2R_2$

From the above expression, we can get expressions for I_2 and I_3 interms of I_1 and substitute them in the equation (1)

$$I_2 = \frac{I_1R_1}{R_2}; I_3 = \frac{I_1R_1}{R_3}$$

$$I_1 + \frac{I_1R_1}{R_2} + \frac{I_1R_1}{R_3} = I$$

$$I_1\left(1 + \frac{R_1}{R_2} + \frac{R_1}{R_3}\right) = I$$

$$\frac{I_1(R_2R_3 + R_3R_1 + R_1R_2)}{R_2R_3} = I$$

$$I_1 = \frac{I(R_2R_3)}{(R_1R_2 + R_2R_3 + R_3R_1)}$$

Similarly we can express I_2 and I_3 as,

$$I_2 = \frac{I(R_1R_3)}{(R_1R_2 + R_2R_3 + R_3R_1)}$$

$$I_3 = \frac{I(R_1R_3)}{(R_1R_2 + R_2R_3 + R_3R_1)}$$

- **Advantages of parallel circuits:**
  - ∗ The electrical appliances rated for the same voltage but different powers can be connected in parallel without affecting each other's performance.
  - ∗ If a break occurs in any one of the branch circuits, it will have no effect on the other branch circuits.

- **Applications of parallel circuits:**
  - * All electrical appliances are connected in parallel. Each one of them can be controlled individually will the help of separate switches.
  - * Electrical wiring in Cinema Halls, auditoriums, House wiring etc.

Comparison of series and parallel circuits:

Series Circuit	Parallel Circuit
The current is same through all the elements.	The current is divided, inversely proportional to resistance.
The voltage is distributed. It is proportional to resistance.	The voltage is the same across each element in the parallel combination.
The total (or) equivalent resistance is equal to sum of individual resistance, ie. $R_T = R_1 + R_2 + R_3$ Hence, the total resistance is greater than the greatest resistance in the circuit.	Reciprocal of the equivalent resistance is equal to sum of reciprocals of individual resistances, ie, $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ Total resistance is lesser than the smallest resistances in the circuit.
There is only one path for the flow of current.	There are more than one path for the flow of current.

- **Problems based on parallel combinations:**

Problem 1.17 What is the value of the unknown resistor R shown in Figure 1.18. If the voltage drop across the 500Ω resistor is 2.5V. All the resistor are in ohms.

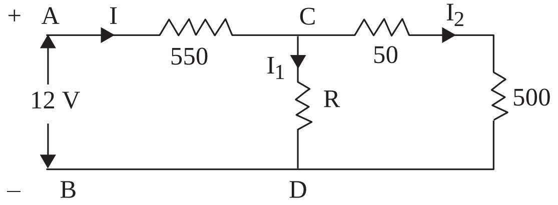


Figure 1.18

Given Data:

$$V_{500} = 2.5V$$

$$I_2 = \frac{V_{500}}{R} = \frac{2.5}{500} = 0.005A$$

V_{50} = Voltage across 50 Ω

$V_{50} = I_2 R = 0.005*50 = 0.25$ V

$V_{CD} = V_{50} + V_{500} = 0.25 + 2.5 = 2.75$ V

V_{550} = Drop across 550Ω = 12 – 2.75 = 9.25 V

$$I = \frac{V_{550}}{R} = \frac{9.25}{550} = 0.0168A$$

$$I = I_1 + I_2 \rightarrow I_1 = I - I_2 = 0.0168 - 0.005$$

$$I_1 = 0.0118A$$

$$R = \frac{V_{CD}}{I_1} = \frac{2.75}{0.0118} = 232.69\ \Omega$$

$$R = 232.69\ \Omega$$

Problem 1.18 Three resistors 2 Ω, 3 Ω and 4 Ω are in parallel. How will be a total current of 8A is divided.

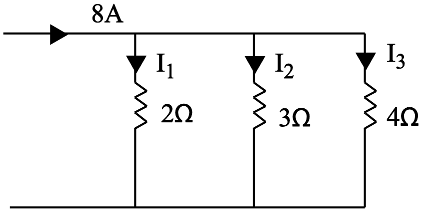


Figure 1.19

This given circuit can be reduced as, 3 Ω and 4 Ω are connected in parallel.

Its equivalent resistances are, $\frac{3*4}{3+4} = \frac{12}{7} = 1.714\Omega$

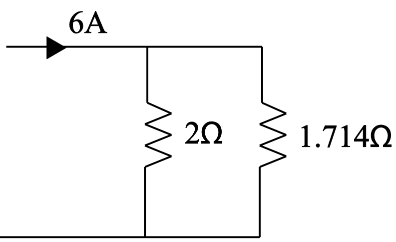


Figure 1.20

1.714 Ω and 2 Ω are connected in parallel, its equivalent resistance is 0.923 Ω

$$\frac{1.714 * 2}{2 + 1.714} = 0.923$$

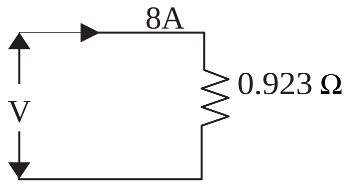


Figure 1.21

$$V = IR = 8 * 0.923$$
$$V = 7.385V$$

Branch currents, $I_1 = \dfrac{V}{R_1} = \dfrac{7.385}{2} = 3.69$ A

$$I_2 = \frac{V}{R_2} = \frac{7.385}{3} = 2.46\,A$$
$$I_3 = \frac{V}{R_3} = \frac{7.385}{4} = 1.84A$$

Problem 1.19 What resistance must be connected in parallel with 10Ω to give an equivalent resistance of 6Ω

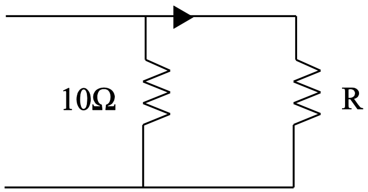


Figure 1.22

R is connected in parallel with 10 Ω Resistor to given an equivalent resistance of 6 Ω.

$$\frac{10 * R}{10 + R} = 6$$
$$10R = (10 + R)6$$
$$10R = 60 + 6R$$
$$10R - 6R = 60$$
$$R = \frac{60}{4} = 15\ \Omega$$
$$R = 15\ \Omega$$

Problem 1.20 Two resistors R_1 and R_2 are connected in Parallel and a Voltage of 200V DC is applied to the terminals. The total current drawn is 20A, R_1=30 Ω. Find R_2 and power dissipated in each resistor, for the figure 1.23.

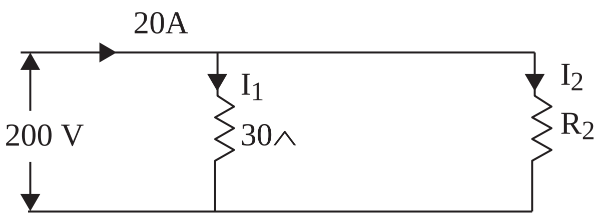


Figure 1.23

Given Data:

V = 200V, I = 20A, R_1 = 30 Ω

Solution: $I_1 = \frac{V}{R_1} = \frac{200}{30} = 6.667\ A$

$$I_1 + I_2 = I$$

$$I_2 = I - I_1$$

$$= 20 - 6.667 = 13.33 \text{ A}$$

$$I_2 = \frac{IR_1}{R_1 + R_2}$$

$$13.33 = \frac{20*30}{30 + R_2}$$

$$(30 + R_2)13.33 = 600$$

$$13.33R_2 = 600 - 400$$

$$13.33R_2 = 200$$

$$R_2 = \frac{200}{13.33} = 15\ \Omega$$

$$R_2 = 15\ \Omega$$

Power dissipated in 30 Ω, $P_1 = VI_1$ = 200*6.667

P_1 = 1333 W

Power dissipated in 15 Ω, $P_2 = VI_2$

P_2 = 200*13.33 = 2667

P_2 = 2667 W

Problem 1.21 Calculate the current supplied by the battery in the given circuit as shown in the figure 1.24.

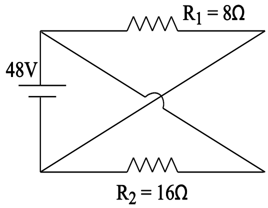


Figure 1.24

Solution: The above given circuit can be redrawn as,

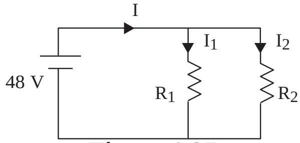


Figure 1.25

R_1 and R_2 are in parallel across the voltage of 48 volts.

$$\text{Equivalent Resistance, } R_T = \frac{R_1 R_2}{R_1 + R_2} = \frac{8*16}{8+16} = \frac{16\ \Omega}{3}$$

$$R_T = 5.33\wedge$$

$$I = \frac{V}{R} = \frac{48}{5.33} = 9\text{A}$$

Problem 1.22 Calculate the total resistance and battery current in the given circuit

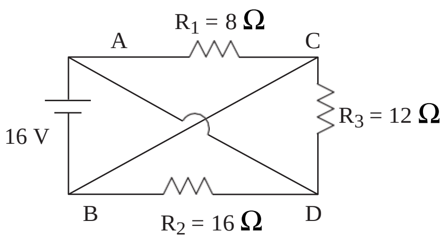


Figure 1.26

The given above circuit can be re-drawn as,

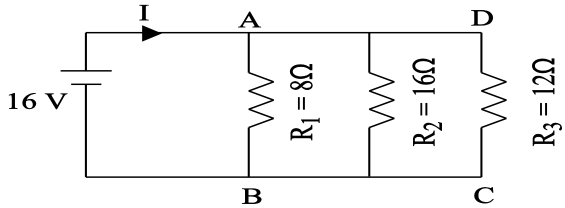


Figure 1.27

8 Ω, 16 Ω, 12 Ω are connected in parallel. Its equivalent resistance,

$$R_T = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}$$

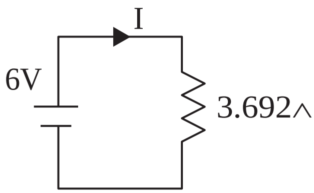


Figure 1.28

$$R_T = \frac{8*6*12}{128+192+96} = 3.692\ \Omega$$

$$R_T = 3.692\ \Omega$$

$$I = \frac{V}{R} = \frac{16}{3.692} = 4.33A$$

Problem 1.23 In the Circuit shown in the figure 1.29, calculate

(i) The current in all resistors.
(ii) The value of unknown resistance ‘x’
(iii) The equivalent resistance between A and B.

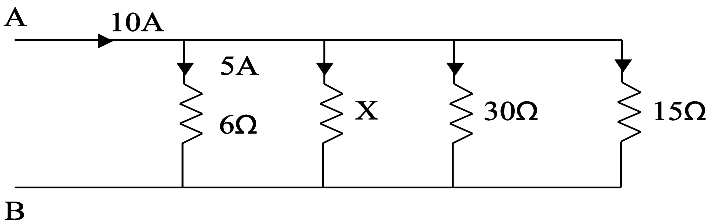


Figure 1.29

Solution: As all the resistors are in parallel, the voltage across each one is same. Give that current through 6 Ω , ie, $I_{6\,\Omega} = 5A$

Voltage across 6 Ω = 5×6 = 30volts.

Hence, current through 30 Ω, $I_{30} = \frac{V_{30}}{R} = \frac{30}{30} = 1A$

Similarly, current through 15 Ω, $I_{15} = \frac{V_{15}}{R} = \frac{30}{15} = 2A$

Total Current, $I = I_6 + I_x + I_{30} + I_{15}$

$$10 = 5 + I_x + 1 + 2$$

$$I_X = 2\,A$$

Hence, the current flowing through the 'X' Resistor is, $I_X = 2\,A$

Value of the Resistor 'X' is given by,

$$X = \frac{30}{I_x} = \frac{30}{2} = 15\ \Omega$$

Let, the equivalent resistance across AB = R_T

$$\frac{1}{R_T} = \frac{1}{6} + \frac{1}{x} + \frac{1}{30} + \frac{1}{15}$$

$$\frac{1}{R_T} = \frac{5 + 2 + 1 + 2}{30} = \frac{1}{3}$$

$$R_T = 3\ \Omega$$

Series — Parallel Combination

As the name suggests, this circuit is a combination of series and parallel circuits. A simple example of such a circuit is illustrated in Figure 1.30. R_3 and R_2 are resistors connected in parallel with each other and both together are connected in series with $R_{1.}$

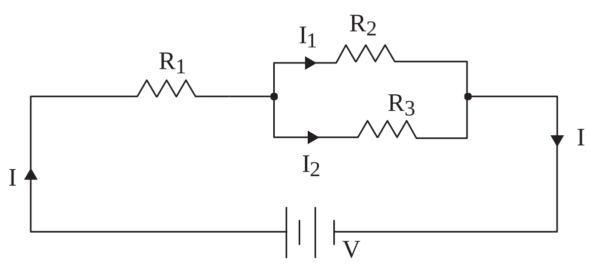


Figure 1.30

Equivalent Resistance: R_T for parallel combination.

$$R_P = \frac{R_2 R_3}{R_2 + R_3}$$

Total equivalent resistance of the circuit is given by,

$$R_T = R_1 + R_P$$

$$R_T = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

Voltage across parallel combination = $I * \frac{R_2 R_3}{R_2 + R_3}$.

- **Problems based on Series – Parallel Combination:**

Problem 1.24 In the circuit, find the current in all the resistors. Also calculate the supply voltage.

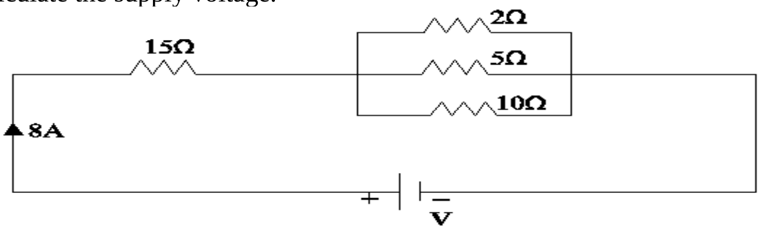


Figure 1.31

Voltage across 15∧, $V_{15} = I_{15} \times R = 8\times15 = 120V$

Resistors 2 Ω, 5 Ω, 10 Ω are connected in parallel, it equivalent resistance is given by,

$$R_P = \frac{2 * 5 * 10}{2\times5 + 5\times10 + 10\times2} = 1.25\ \Omega$$

Voltage across the parallel Combination is given by

$$V_p = V_2 = V_5 = V_{10} = I \times R_P = 8\times1.25 = 10V$$

Total supply Voltage, $V = V_{15}+V_p$

$$V = 120 + 10 = 130\text{V}$$

$$V = 130\text{V}$$

Hence, the Current through the parallel combination of the resistors are given by,

Current through 2 Ω resistor, $I_2 = \frac{V_2}{R} = \frac{10}{2} = 5A$

Current through 5 Ω Resistor, $I_5 = \frac{V_5}{R} = \frac{10}{5} = 2\,A$

Current through 10 Ω Resistor, $I_{10} = \frac{V_{10}}{R} = \frac{10}{10} = 1A$

The Current of 8A across the parallel combination is divided as 5A, 2A, and 1A.

Problem 1.25 Calculate the equivalent resistance offered by the circuit to the voltage source and also find its source current

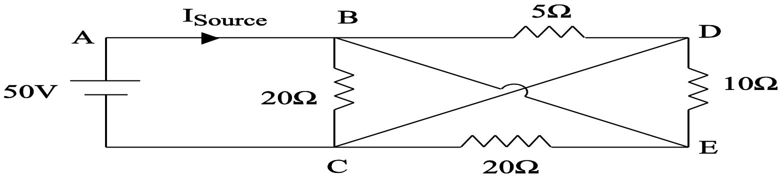


Figure 1.32

Solution: The given above circuit can be re-drawn as

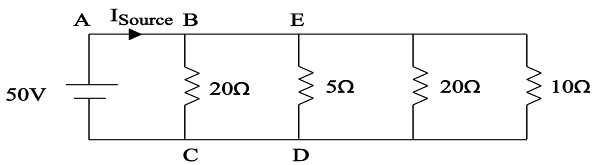


Figure 1.33

20 Ω and 10 Ω resistors are connected in parallel, its equivalent resistance is given by, $\frac{20 * 10}{20 + 10} = 6.667\ \Omega$

The given circuit is reduced as,

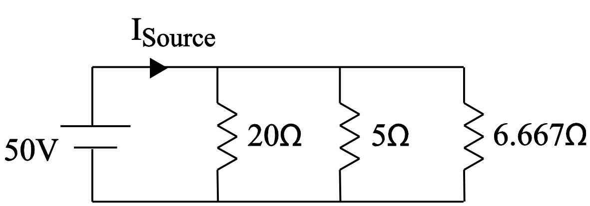


Figure 1.34

6.667 Ω and 5 Ω resistors are connected in parallel, its equivalent resistance is given by, $\frac{6.667 * 5}{6.667 + 5} = 2.857\ \Omega$

The circuit is reduced as,

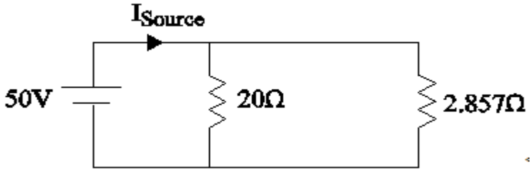


Figure 1.35

20 Ω and 2.857 Ω are connected in parallel. It equivalent resistance is,

$\frac{20 * 2.857}{20 + 2.857} = 2.497\ \Omega$

The Circuit is re-drawn as,

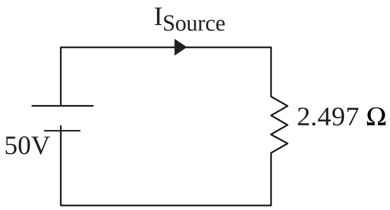


Figure 1.36

Hence the equivalent resistance of the Circuit is R_T= 2.497 Ω = 2.5 Ω

Source Current of the Circuit is given by,

$$I_{source} = \frac{V}{R} = \frac{50}{2.5} = 20A$$

Problem 1.26 Find the equivalent resistance between the terminals A and B.

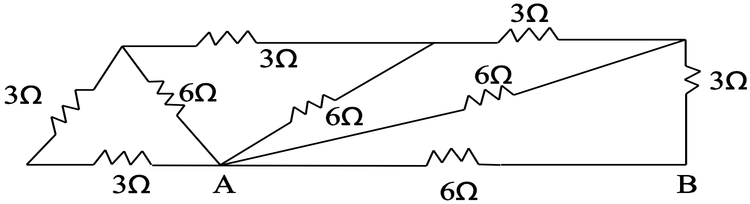


Figure 1.37

Solution:

3 Ω and 3 Ω are connected in Series, it equivalent resistance is, (3 + 3) = 6 Ω. The Circuit gets reduced as

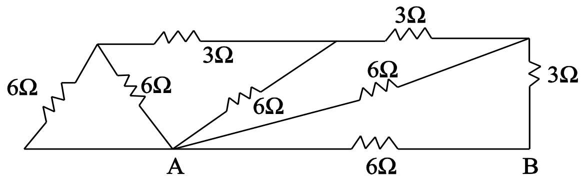


Figure 1.38

6 Ω and 6 Ω are connected in parallel. The circuit gets reduced as,

$$\frac{6*6}{6+6} = 3\text{ohms}.$$

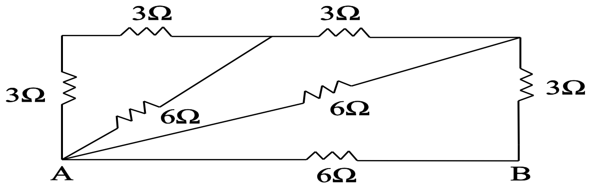


Figure 1.39

3 Ω and 3 Ω are connected in series (3 + 3 = 6 Ω).

The reduced Circuit is,

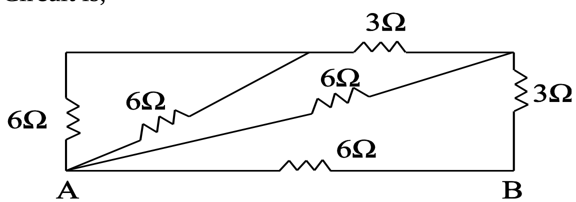


Figure 1.40

6 Ω and 6 Ω are connected in parallel. Its equivalent resistance, $\frac{6*6}{6+6} = 3\ \Omega$

The circuit can be reduced as,

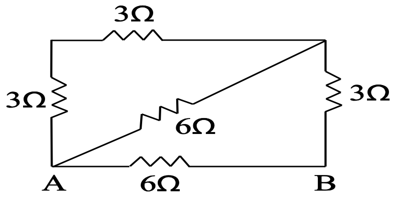


Figure 1.41

3 Ω and 3 Ω are connected in series. (3 + 3 = 6 Ω).

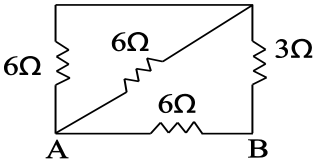


Figure 1.42

6 Ω and 6 Ω are connected in parallel. It equivalent resistance, $\frac{6*6}{6+6} = 3\ \Omega$

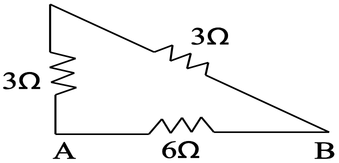


Figure 1.43

3 Ω and 3 Ω are connected in series, the reduced Circuit is 3 + 3 = 6 Ω

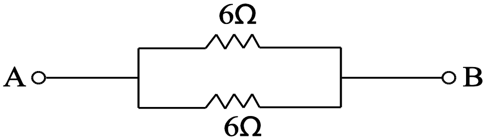


Figure 1.44

6 Ω and 6 Ω are connected in parallel.

$\frac{6*6}{6+6} = 3\ \Omega$. The equivalent resistance between the terminals A and B given by $R_{AB} = 3\ \Omega$.

Figure 1.45

$$\therefore R_{AB} = 3\ \Omega$$

Problem 1.27 Determine the value of R if the power dissipated in 10 Ω resistor is 90 W.

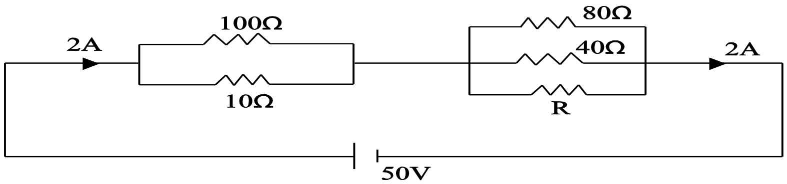


Figure 1.46

Solution:

100 Ω and 10 Ω are connected in parallel.

Its equivalent resistance is, $\frac{100*10}{100+10} = 9.09\ \Omega$

The circuit is reduced as,

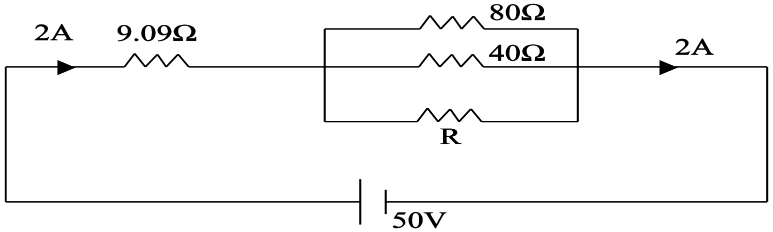


Figure 1.47

Current of 2A flows through the 9.09 Ω resistor. Voltage across 9.09 Ω is given by,

$$V_{9.09} = I_{9.09} \times R$$
$$V_{9.09} = 2 \times 9.09 = 18.18V$$

Similarly voltage across the unknown resistor V_R,

$$V_R = V - V_{9.09} = 50 - 18.18 = 31.818V$$

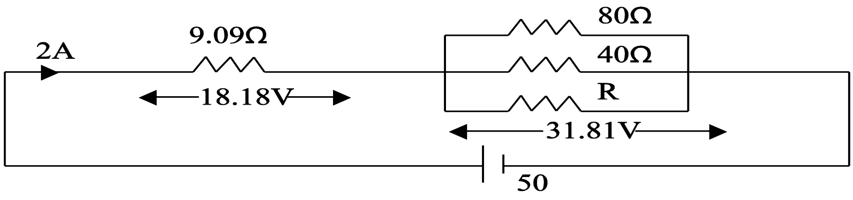


Figure 1.48

Hence the Current through 40 Ω, 80 Ω resistors can be found out with the voltage drop of 31.818V across it.

$$I_{80} = \frac{V_R}{80} = \frac{31.818}{80} = 0.397\ A$$
$$I_{40} = \frac{V_R}{40} = \frac{31.818}{40} = 0.7954\ A$$

Hence current through the unknown resistor R is I_R,

$$I_R = I - [I_{80} + I_{40}]$$
$$I_R = 2 - (0.397 + 0.7954) = 0.8075A$$

Hence, the value of the unknown Resistor R is given by

$$R = \frac{V_R}{I_R} = \frac{31.818}{0.8075} = 39.4\ \Omega$$

The value of the unknown resistor R is given by, R = 39.4 Ω.

Problem 1.28 Calculate the following for the circuits given,

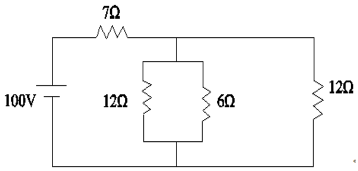


Figure 1.49

(i) Total resistance offered to the Source.
(ii) Total Current from the Source.
(iii) Power Supplied by the Source.

Solution: 12 Ω and 6 Ω are connected in Parallel.

Its equivalent resistance, $\frac{12*6}{12+6} = 4\ \Omega$. The reduced circuit is given as,

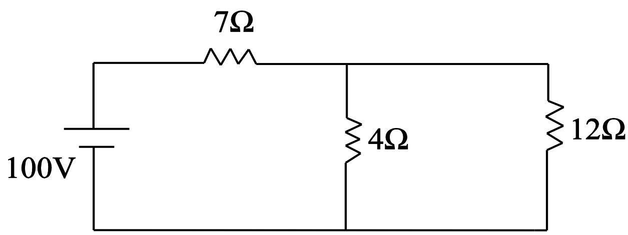


Figure 1.50

4 Ω and 12 Ω are connected in parallel. $\frac{4*12}{12+4} = 3\ \Omega$

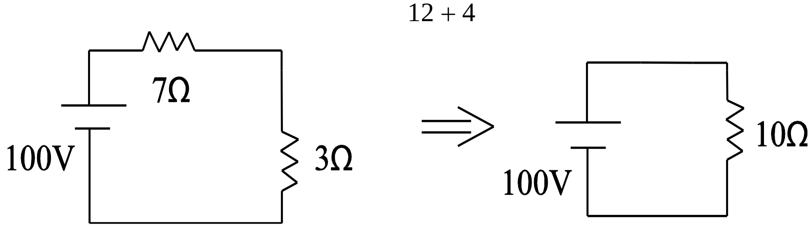


Figure 1.51

7 Ω and 3 Ω are connected in series, $7 + 3 = 10\Omega$

Total resistance offered to the Source, $R = 10\ \Omega$

Total Current from the Source, $I = \frac{100}{10} = 10$ A

$$I = 10A$$

Power supplied by the Source, $P = I^2R = 10^2 \times 10 = 1000W$

P = 1000W.

Problem 1.29 A letter A is Constructed of an uniform wire of 1 Ω resistance per cm. The signs of the letter are 60cm long and the cross piece is 30cm long, Apex angle 60°. Find the resistance of the letter between two ends of the legs.

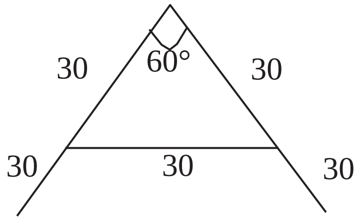


Figure 1.52

Solution:

The given circuit can be redrawn as,

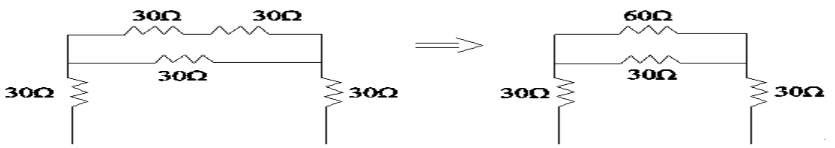


Figure 1.53

60 Ω and 30 Ω are connected in parallel

$$\frac{60 * 30}{60 + 30} = \text{2O ohms}$$

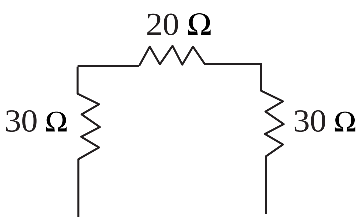


Figure 1.54

Equivalent Resistance = 80 Ω.

Problem 1.30 Find the current supplied by the battery.

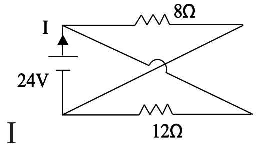


Figure 1.55

Solution:

The given circuit can be re-drawn as,

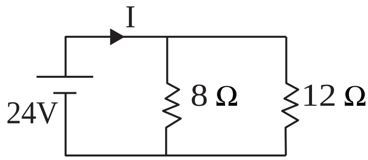


Figure 1.56

8 Ω and 12 Ω connected in parallel.

$$\frac{8*12}{8+12} = 4.8\ \Omega$$

Reduced circuit is,

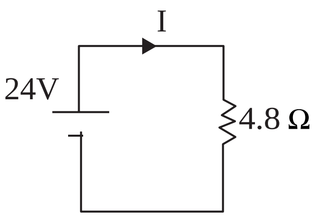


Figure 1.57

Current, $I = \frac{V}{R} = \frac{24}{4.8} = 5A$

$I = 5A$

Problem 1.31 Find the current supplied by the battery for the figure shown below.

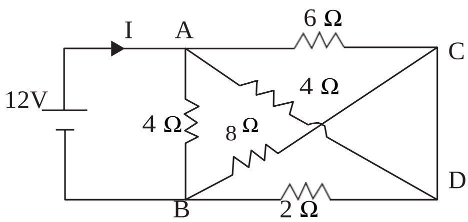


Figure 1.58

Solution:

The given above circuit can be redrawn as,

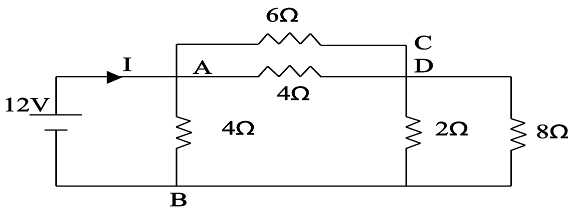


Figure 1.59

4 Ω and 6 Ω are connected in parallel. $\frac{6 * 4}{6 + 4} = 2.4\ \Omega$

Similarly, 2 Ω and 8 Ω are connected in parallel.

$$\frac{2 * 8}{8 + 2} = 1.6\ \Omega$$

The reduced circuit can be re drawn as,

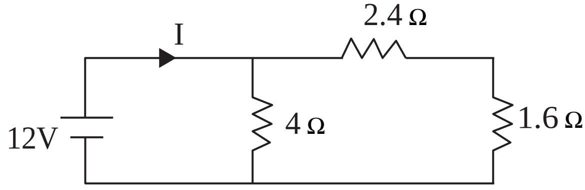


Figure 1.60

2.4 Ω and 1.6 Ω are connected in series. 2.4 +1.6 = 4 Ω

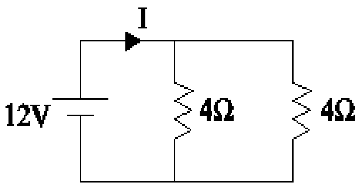


Figure 1.61

4 Ω and 4 Ω are connected in parallel $\frac{4 * 4}{4+4} = 2\ \Omega$

The reduced circuit is,

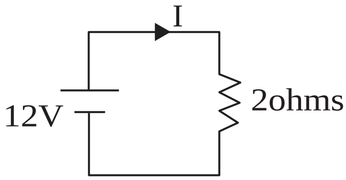


Figure 1.62

$$I = \frac{V}{R} = \frac{12}{2} = 6A$$

Current I, supplied by the battery = 6A.

Problem 1.32 Two Resistors R_1 = 2500 Ω and R_2 = 4000 Ω are joined in series and connected to a 100v supply. The voltage drop across R_1 and R_2 are measured successively by a voltmeter having a resistance of 50,000 Ω. Find the sum of the Reading.

Solution:

Case (i) A voltmeter is connected across 2500 Ω.

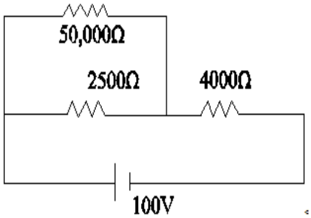


Figure 1.63

2500 Ω and 50,000 Ω are connected is parallel.

$$\frac{2500 * 50000}{2500 + 50000} = 2381\text{ohms}$$

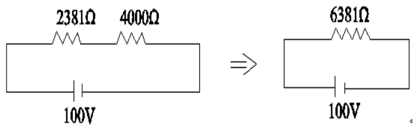


Figure 1.64

2381 Ω and 4000 Ω are connected in series.

$$2381 + 4000 = 6381\ \Omega$$

Current $I = \frac{V}{R} = \frac{100}{6381} = 0.01567\text{A}$

Voltage drop across, the Resister R_1 is measured by connecting a voltmeter having resistance of 50,000 across $R_{1.}$ Hence V_A be voltage drop across R_1

$$V_A = IR = 0.01567 * 2381$$

$$V_A = 37.31\text{V}$$

Case (ii) Voltmeter is connected across 4000 Ω.

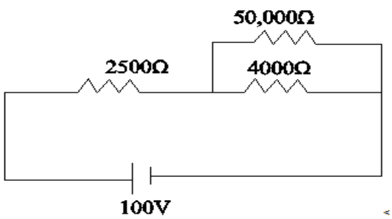


Figure 1.65

4000 Ω and 50,000 Ω are connected in parallel.

$$\frac{4000 * 50000}{4000 + 50000} = 3703.7\text{ohms}$$

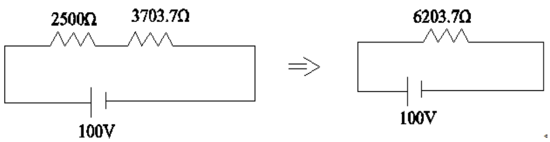


Figure 1.66

Current, $I = \dfrac{V}{R} = \dfrac{100}{6203.7} = 0.0161$ A

Voltage drop across the resistor R_2 is measured by connecting a voltmeter having resistance of 50000 across R_2.Hence, V_B be the voltage drop across R_2.

$$V_B = IR = 0.6161 * 3703.7$$
$$V_B = 59.7\text{V}$$

The total voltage drop = $V_A + V_B$

$$V = 37.31 + 59.7$$
$$V = 97 \text{ V}$$

Problem 1.33 Find the value of 'R' and the total current when the total power dissipated in the network is 16W as shown in the figure.

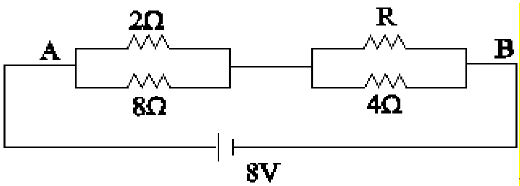


Figure 1.67

Solution:

Total Power (P) = 16w

Total Current, $I = \dfrac{P}{V} = \dfrac{16}{8} = 2\text{A}$

Total Resistance, $(R_{AB}) = \dfrac{P}{I^2} = \dfrac{16}{4} = 4\ \Omega$

Total Resistance between A and B is given by,

$$R_{AB} = \frac{2*8}{2+8} + \frac{4*R}{4+R}$$

$$4 = 1.6 + \frac{4R}{4+R}$$

$$4(4+R) = 1.6(4+R) + 4R \; R = 6\,\Omega.$$

KIRCHOFF'S LAWS

Kirchhoff's current law

The kirchoff's current law states that the algebraic sum of currents in a node is zero.

It can also be stated that "sum of incoming currents is equal to sum of outgoing currents."

Kirchhoff's current law is applied at nodes of the circuit. A node is defined as two or more electrical elements joined together. The electrical elements may be resistors, inductors capacitors, voltage sources, current sources etc.

Consider a electrical network as shown below.

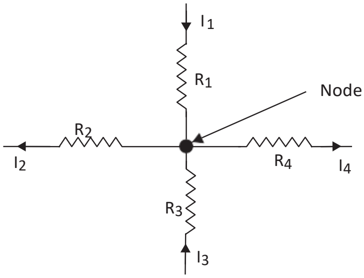


Figure 1.68

Four resistors are joined together to form a node. Each resistor carries different currents and they are indicated in the diagram.

$I_1 \rightarrow$ Flows towards the node and it is considered as positive current. $(+\ I_1)$

$I_2 \rightarrow$ Flows away from the node and it is considered as negative current. $(-\ I_2)$

$I_3 \rightarrow$ Flows towards the node and it is considered as positive current. $(+I_3)$

$I_4 \rightarrow$ Flows away from the node and hence it is considered as negative current $(-I_4)$

Applying KCL at the node, by diffinition-1 algebraic sum of currents in a node is zero.

$$+I_1 - I_2 + I_3 - I_4 = 0 \qquad (1)$$

taking the I_2 & I_4 to other side

$$I_1 + I_3 = I_2 + I_4 \qquad (2)$$

From equation (2) we get the definition – 2. Where I_1 & I_3 are positive currents (Flowing towards the node) I_2 & I_4 are negative currents. (Flowing away from the node).

Kirchoff's voltage Law: (KVL)

Kirchhoff's voltage law states that "sum of the voltages in a closed path (loop) is zero".

In electric circuit there will be closed path called as loops will be present.

The KVL is applied to the closed path only the loop will consists of voltage sources, resistors, inductors etc.

In the loop there will be voltage rise and voltage drop. This voltage rise and voltage drop depends on the direction traced in the loop. So it is important to understand the sign convention and the direction in which KVL is applied (Clock wise Anti clock wise).

- **Sign Conventions**

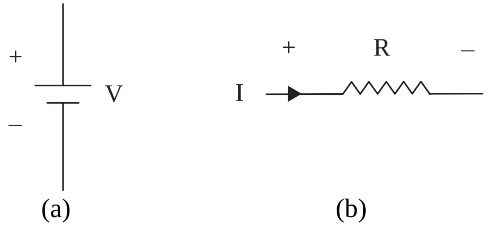
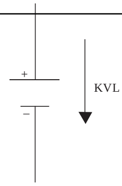


Figure 1.69

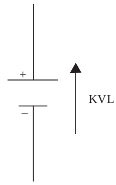
Consider a battery source V as shown in the figure 1.69(a). Here positive of the battery is marked with + sign and negative of the battery is marked with – sign.

When we move from + sign to – sign, it is called voltage drop.

When we move from – sign to + sign, it is called as voltage rise.

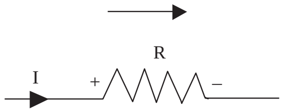


When KVL is applied in Anti clockwise direction as shown above it is called as voltage drop. A voltage drop is indicated in a loop with "—" sign (–V)

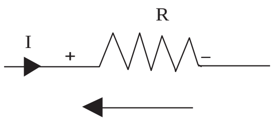


For the same battery source if the KVL is applied in clock wise direction we move from – sign to + sign. Hence it is called as Voltage Rise. A Voltage rise indicated in the loop with + sign. (+V).

Similarly in the resistor the current entry point is marked as positive (+ sign) and current leaving point is marked as negative sign. (– sign).



For the resistor shown in the diagram above, if KVL is applied in clock wise direction then it is called as voltage drop. Voltage drop in KVL equation must be indicated with negative sign (–). $\therefore$ –IR.



For the resistor shown in the diagram above, if KVL is applied in anti clockwise direction then it is called as voltage rise. A voltage rise is indicated in the KVL equation as positive. i.e. + IR.

In short the above explanation is summarized below in a Table.

S.No.	Element	KVL in clockwise	KVL in anticlockwise
1.	+ –	+ – Drop –V	+ – Rise +V
2.	I + R –	I + R – Drop –IR	I + R – Drop +IR

- **Procedure for KVL:**
  - * Identify the loops and Name them.
  - * Mark the branch currents and name them.
  - * Apply the sign convention.
  - * Select a loop & apply KVL either in clockwise or Anticlockwise and frame the equation.
  - * Solve all the equations of the loop.

- **Problems based on Kirchhoff's laws**

Problem 1.34 For the given circuit find the branch currents and voltages by applying KVL.

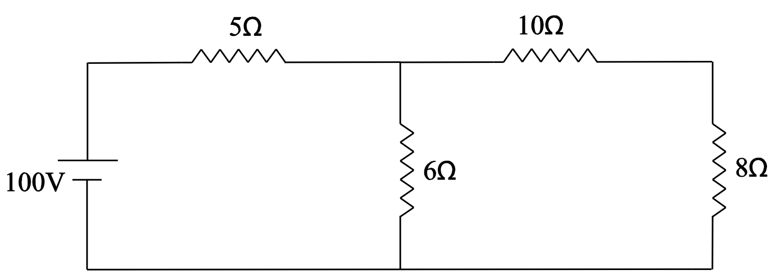


Figure 1.70

Solution:

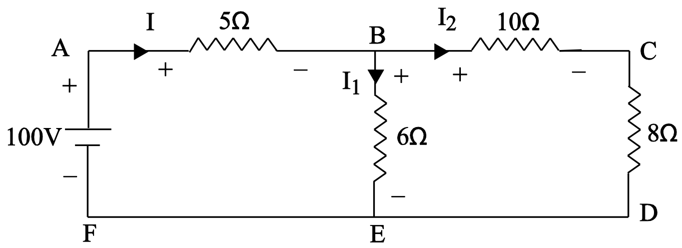


Figure 1.71

Consider loop ABEF & Apply KVL in CLK wise direction

$$100 - 5I - 6I_1 = 0$$

But $I = I_1 + I_2$

$$100 - 5\left(I_1 + I_2\right) - 6I_1 = 0$$
$$100 - 5I_1 - 5I_2 - 6I_1 = 0$$
$$-11I_1 - 5I_2 + 100 = 0$$
$$11I_1 + 5I_2 = 100 \qquad (1)$$

Consider loop BCDEB & Apply KVL in CLK wise direction

$$-10I_2 - 8I_2 + 6I_1 = 0$$
$$-18I_2 + 6I_1 = 0$$
$$6I_1 = 18I_2$$
$$I_1 = 3I_2 \qquad (2)$$

Sub I_1 in equ (1)

$$11\left(3\,I_2\right) + 5\,I_2 = 100$$
$$33\,I_2 + 5\,I_2 = 100$$
$$38\,I_2 = 100$$
$$I_2 = \frac{100}{38} = 2.63\ Amps.$$
$$I_2 = 2.63\ Amps$$

Sub I_2 in equ (2)

$$I_1 = 3(2.63) = 7.89\text{Amps}$$
$$I_1 = 7.89\text{Amps}$$
$$I = I_1 + I_2 = 10.52$$
$$I = 10.52\text{Amps.}$$

Voltage Across 5∧ = $5 \times I = 5 \times 10.52$
$= 52.6\ volts$

Voltage Across 6∧ = $6 \times I_1 = 6 \times 7.89$
$= 47.34\ volts$

Voltage Across 10∧ = $10 \times I_2 = 10 \times 2.63$
$= 26.3\ volts$

Voltage Across 8∧ = $8 \times I_2 = 8 \times 2.63$
$= 21.04\ volts$

(Or)

The above problem can be solved by applying KVL in Anti clock wise directions.

Consider loop ABEF & Apply KVL in anti clock wise direction

$$6I_1 + 5I - 100 = 0$$

But $I = I_1 + I_2$

$$6I_1 + 5(I_1 + I_2) - 100 = 0$$
$$6I_1 + 5I_1 + 5I_2 = 100$$
$$11I_1 + 5I_2 = 100 \qquad (3)$$

Consider loop BCDEB & Apply KVL in anti clockwise direction

$$8I_2 + 10I_2 - 6I_1 = 0$$
$$18I_2 = 6I_1$$
$$I_1 = 3I_2 \qquad (4)$$

equations (3) & (1) are identical

equations (2) & (4) are identical

Hence we get the same answer irrespective of directions of applying KVL.

Problem 1.35 Calculate the branch current in 15 Ω resistor by Applying kirchhoff's law

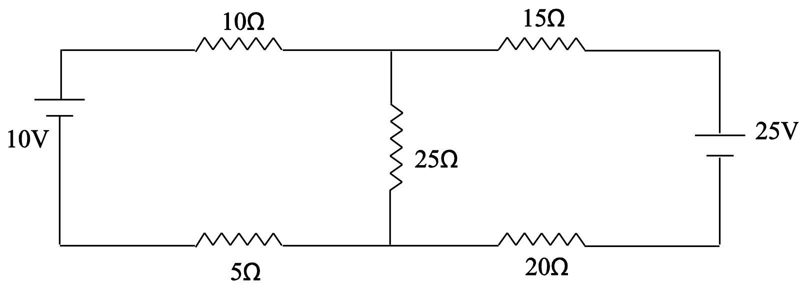


Figure 1.72

Figure 72 battery voltage value 25 volt missing

Solution:

Name the loop and Mark the current directions

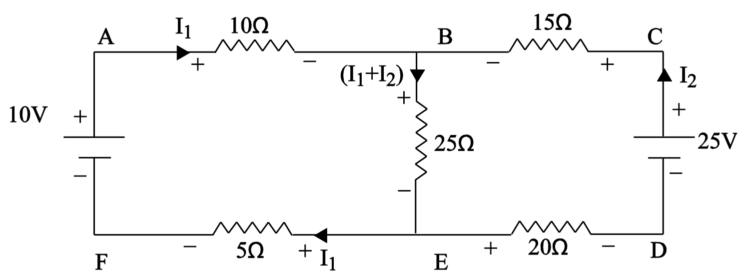


Figure 1.73

Consider the loop ABEFA & apply KVL in CLK wise

$$10 - 10I_1 - 25\left(I_1 + I_2\right) - 5I_1 = 0$$

$$10 - 10I_1 - 25I_1 - 25I_2 - 5I_1 = 0$$

$$-40I_1 - 25I_2 + 10 = 0$$

$$40I_1 + 25I_2 = 10 \qquad (1)$$

Consider the loop BCDEB and Apply KVL in CLK wise direction

Consider the loop BCDEB and Apply KVL in CLK wise direction

$$15I_2 - 25 + 20I_2 + 25\left(I_1 + I_2\right) = 0$$
$$15I_2 - 25 + 20I_2 + 25\left(I_1 + I_2\right) = 0$$
$$15I_2 - 25 + 20I_2 + 25I_1 + 25I_2 = 0$$

$$25I_1 + 60I_2 - 25 = 0$$

$$25I_1 + 60I_2 = 25 \text{……………..(2)}$$

Solve (1) & (2) & find I_2 alone

$(1) \times 25 \Rightarrow 1000\ I_1 + 6\cancel{25I_2 =} 25$

$(2) \times 40 \Rightarrow \cancel{1000}\ I_1 + 2400 I_2$

$(A) - (B) \Rightarrow -1775\ I_2 = -750$

$I_2 = 0.42$ Amps.

Current in 15 Ω resistor is 0.42Amps.

Problem 1.36 For the given network find the branch current in 8 Ω and voltage across the 3 Ω by applying KVL

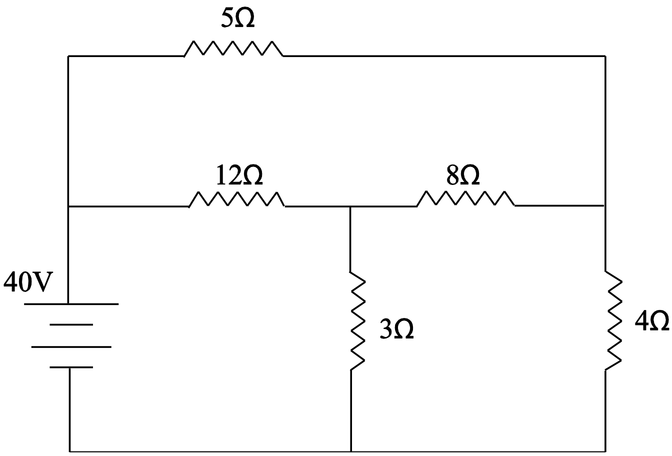


Figure 1.74

Solution:

Name the loop and mark the current directions and apply sign convention.

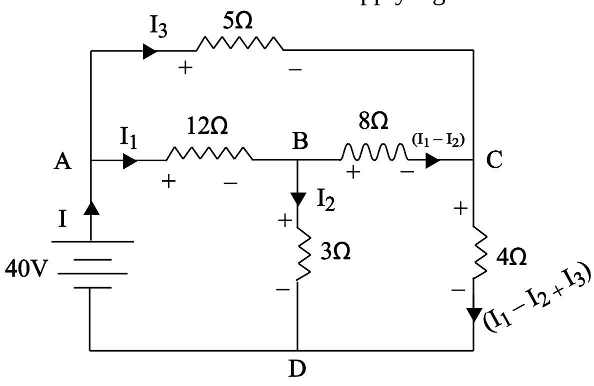


Figure 1.75

Consider loop ABDA and apply KVL $-8I_1 + 8$

$$-12I_1 - 3I_2 + 40 = 0$$

$$12I_1 + 3I_2 = 40$$

Consider loop BCDB and apply KVL

$$-8(I_1 - I_2) - 4(I_1 - I_2 + I_3) + 3I_2 = 0$$

$$I_2 - 4I_1 + 4I_2 - 4I_3 + 3I_2 = 0 \quad (1)$$

$$-12I_1 + 15I_2 - 4I_3 = 0 \quad (2).$$

Consider loop ABCA and apply KVL

$$-12I_1 - 8(I_1 - I_2) + 5I_3 = 0$$
$$-12I_1 - 8I_1 + 8I_2 + 5I_3 = 0$$
$$-20I_1 + 8I_2 + 5I_3 = 0 \quad (3)$$

Solve equ (2) & (3) and cancel out I_3

$$(2) \times 5 \Rightarrow -60I_1 + 75I_2 - 20I_3 = 0$$
$$(3) \times 4 \Rightarrow -80I_1 + 32I_2 + 20I_3 = 0$$

Add the above two equations $\Rightarrow -140I_1 + 107I_2 = 0$ (4)

Solve equ (4) & (1) and find I_1 & I_2

$$12I_1 + 3I_2 = 40 \quad (1)$$
$$-140I_1 + 107I_2 = 0 \quad (4)$$

$$(1) \times 107 \Rightarrow 1284I_1 + 321I_2 = 4280$$
$$(4) \times 3 \Rightarrow -420I_1 + 321I_2 = 0$$

Subtract the above two $1704I_1 = 4280$

$$I_1 = 2.51\ Amps$$

Sub I_1 in (4)

$$-140 \times 2.51 + 107I_2 = 0$$
$$-351.4 + 107I_2 = 0$$
$$107I_2 = 351.4$$
$$I_2 = 3.28\ Amps$$

Current in 8 Ω resistor = $I_1 - I_2$

= 2.51- 3.28

= -0.77 Amps.

Negative sign indicates that current flows in the opposite direction of our assumption.

Voltage in 3 Ω resistor = $3I_2$
$$= 3 \times 3.28 = 9.84 \text{ volts}$$

Note: Since there are 3 loops three unknown currents I_1, I_2 and I_3 should be named in the loop.

Problem 1.37 For the given network shown below find the branch currents by applying KVL and also find the voltage across 5 Ω resistor.

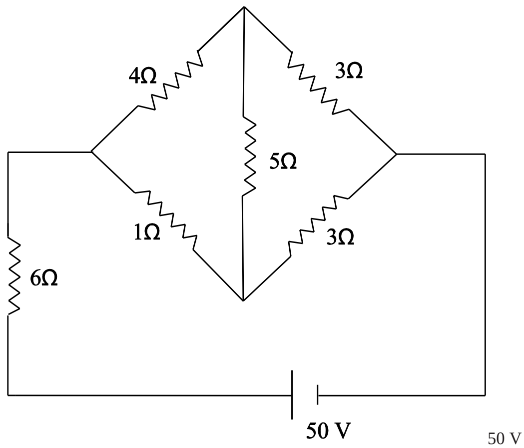


Figure 1.76

Solution:
Name the loop and assume the branch currents.

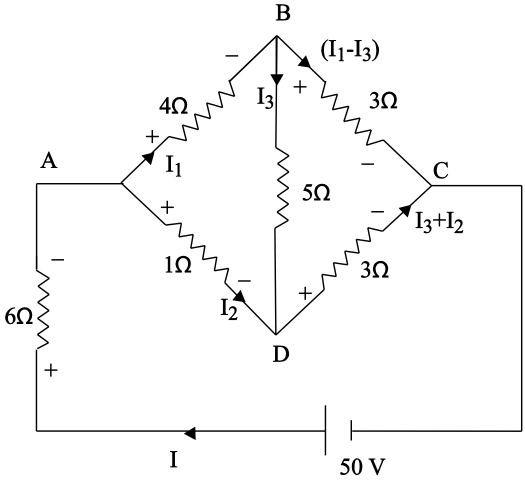


Figure 1.77

Consider the loop ABDA and apply KVL.

$$-4I_1 - 5\,I_3 + I_2 = 0$$

$$-4\,I_1 + I_2 - 5\,I_3 = 0 \qquad (1)$$

Consider the loop BCDB and apply KVL.

$$-3(I_1 - I_3) + 3(I_3 + I_2) + 5I_3 = 0$$

$$-3I_1 + 3I_3 + 3I_3 + 3I_2 + 5I_3 = 0$$

$$-3I_1 + 3I_2 + 11I_3 = 0 \qquad (2)$$

Consider the loop ADCA and apply KVL.

$$-6(I_1 + I_2) - I_2 - 3(I_3 + I_2) + 50 = 0$$

$$-6I_1 - 6I_2 - I_2 - 3I_3 - 3I_2 = -50$$

$$-6I_1 - 10I_2 - 3I_3 = -50$$

$$6I_1 + 10I_2 + 3I_3 = 50 \qquad (3)$$

From eqn is (1) & (2) Cancel I_3

$$-4I_1 + I_2 - 5I_3 = 0 \qquad (4)$$

$$-3I_1 + 3I_2 + 11I_3 = 0 \qquad (5)$$

(4) x 3 $\Rightarrow -12I_1 + 3I_2 - 15I_3 = 0$

(5) x 4 $\Rightarrow -12I_1 + 12I_2 - 44I_3 = 0$

By subtracting the above two equations $-9I_2 - 59I_3 = 0$

$$9I_2 = -59I_3$$

$$I_2 = -6.56 I_3 \qquad (6)$$
$$-3I_1 + 3I_2 + 11I_3 = 0 \qquad (7)$$
$$6I_1 + 10I_2 + 3I_3 = 50 \qquad (8)$$

(7) x 2 $\Rightarrow -6I_1 + 6I_2 + 22I_3 = 0$

(8) $\Rightarrow 6I_1 + 10I_2 + 3I_3 = 50$

By adding the above two equations $16I_2 + 28I_3 = 50$ (9)

Sub eqn (6) in (9)

$$16(-6.56\ I_3) + 25\ I_3 = 50$$
$$-104.96 + 25\ I_3 = 50$$
$$-79.96\ I_3 = 50$$
$$I_3 = -0.625 \text{ Amps} \qquad (10)$$

Sub eqn (10) in (6)

$$I_2 = -6.56 \times (-0.625)$$
$$I_2 = 4.1 \text{ Amps} \qquad (7)$$

Sub (10) & (11) in eqn (8)

$$6I_1 + 10I_2 + 3I_3 = 50$$
$$6I_1 + 10(4.1) + 3(-0.625) = 50$$
$$6I_1 + 41 - 1.875 = 50$$
$$6I_1 = 10.875$$
$$I_1 = 1.81 \text{ Amps}$$

Current in 6 Ω resistor = $(I_1 + I_2) = (1.81 + 4.1) = 5.91$Amps

Current in 4 Ω resistor = $I_1 = 1.81$ Amps

Current in 5 Ω resistor = $I_3 = -0.625$ Amps

Current in 3 Ω resistor = $(I_1 - I_3) = 1.81 + 0.625 = 2.44$ Amps

Current in 3 Ω resistor = $(I_3 + I_2) = 3.475$ Amps

Current in 1 Ω resistor = $I_2 = 4.1$ Amps.

Voltage Across 5 Ω resistor= $5 \times 0.625 = 3.13$ volts.

Problem 1.38 For the Circuit shown below determine voltages (i) V_{df} and (ii)V_{ag}

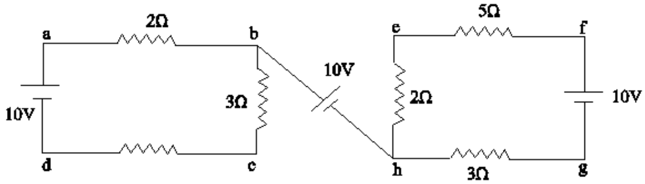


Figure 1.78

Solution:
Mark the current directions and mark the polarity

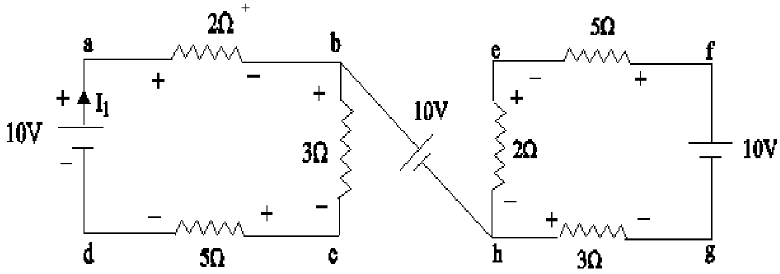


Figure 1.79

Apply KVL to loop abcda
$10 - 2I_1 - 3I_1 - 5I_1 = 0$
$-10I_1 = -10$
$I_1 = 1$ Amps

Apply KVL to loop efghe
$5I_2 - 10 + 3I_2 + 2I_2 = 0$
$10I_2 = 10$
$I_2 = 1$ Amps

To find V_{df}:
Trace the path V_{df}

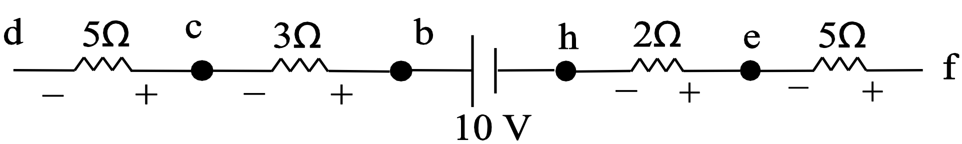


Figure 1.80

$V_{df} = -5(I_1 - 3I_1 + 10 + 2I_2 + 5I_2)$
$V_{df} = -5 - 3 + 10 + 2 + 5$
$V_{df} = 9$ Volts.
$V_{df} = -9$ Volts [$\because$ because - sign on d side + on f side]

To find V_{ag}:

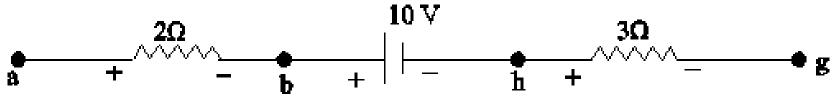


Figure 1.81

Apply KVL to the above Trace
$-2I_1 - 10 - 3I_2 = V_{ag}$
$V_{ag} = -2 - 10 - 3$
$V_{ag} = -15$

$V_{ag} = 15$ Volts. (With a side + w.r.t g)

Problem 1.39 Find the currents through R_2, R_3, R_4, R_5 and R_6 of the network.

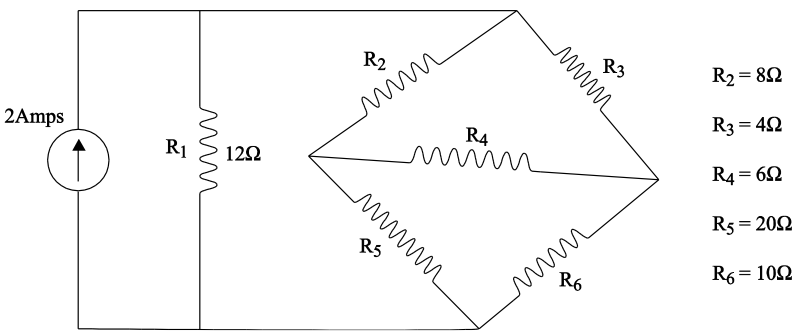


Figure 1.82

$R_2 = 8\,\Omega$
$R_3 = 4\,\Omega$
$R_4 = 6\,\Omega$
$R_5 = 20\,\Omega$
$R_6 = 10\,\Omega$

Solution:
Name the circuit and mark the current directions and polarity as shown below

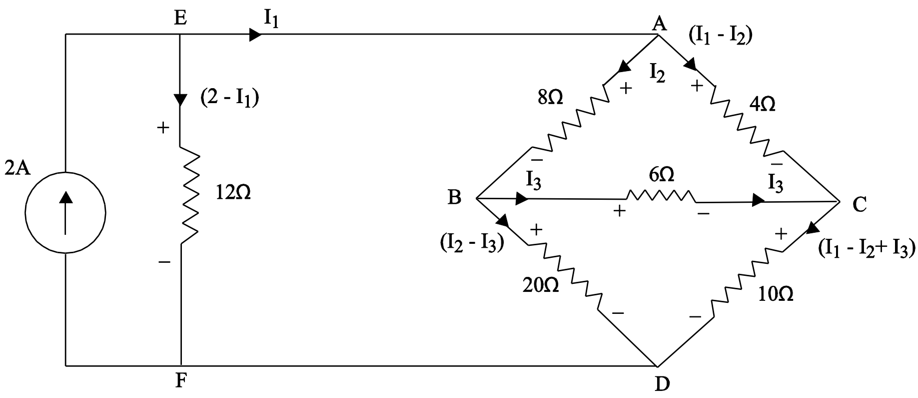


Figure 1.83

Apply KVL to the loop ACBA.

$$-4(I_1 - I_2) + 6I_3 + 8I_2 = 0.$$
$$-4I_1 + 4I_2 + 6I_3 + 8I_2 = 0$$
$$-4I_1 + 12I_2 + 6I_3 = 0 \qquad (1)$$

Apply KVL to the loop BCDB

$$-6I_3 - 10(I_1 - I_2 + I_3) + 20(I_2 - I_3) = 0$$
$$-6I_3 - 10I_1 + 10I_2 - 10I_3 + 20I_2 - 20I_3 = 0$$
$$-10I_1 + 30I_2 - 36I_3 = 0 \qquad (2)$$

Apply KVL to loop EABDFE

$$-8I_2 - 20(I_2 - I_3) + 12(2 - I_1) = 0$$
$$-8I_2 - 20I_2 + 20I_3 + 24 - 12I_1 = 0$$
$$-28I_2 + 20I_3 + 24 - 12I_1 = 0$$
$$-12I_1 - 28I_2 + 20I_3 = -24$$
$$12I_1 + 28I_2 - 20I_3 = 24 \qquad (3)$$

Solving equ. (1) (2) & (3). We get

$I_1 = 1.125$ Amps
$I_2 = 0.375$ Amps
$I_3 = 0$ Amps

$\therefore$ Current in $R_2 = 0.375$ Amps
$R_3 = 0.75$ Amps
$R_4 = 0$ Amps
$R_5 = 0.375$ Amps
$R_6 = 0.75$ Amps

NODAL ANALYSIS

- In nodal analysis, node equations relating node voltages are obtained for a multi node network.
- These node voltages are derived from kirchoff's current law (KCL)

- In this method the number of equations required to be solved is N-1, where N is the number of nodes.
- A node is a junction in a network where three or more branches meet. One of the nodes in a network is regarded as reference (datum) node and the potential of the other nodes are defined with reference to the datum node.

Case I.

Consider figure 1 Let the voltages at nodes a and b be V_a and V_b. Applying Kirchoff's current law (KCL) at node 'a' we get

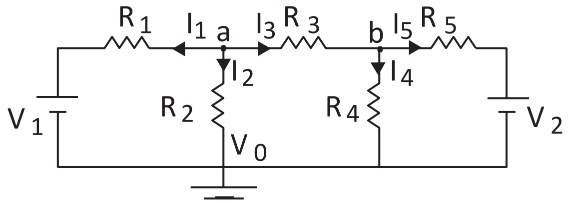


Figure 1.84

$$I_1 + I_2 + I_3 = 0 \qquad (1)$$

Where

$$I_1 = \frac{V_a - V_1}{R_1}; I_2 = \frac{V_a - V_0}{R_2}; I_3 = \frac{V_a - V_b}{R_3};$$

Substituting in equ.(1)

$$\frac{V_a - V_1}{R_1} + \frac{V_a - V_0}{R_2} + \frac{V_a - V_b}{R_3} = 0$$

On simplifying [$V_o = 0$]

$$\frac{V_a}{R_1} - \frac{V_1}{R_1} + \frac{V_a}{R_2} + \frac{V_a}{R_3} - \frac{V_b}{R_3} = 0$$

$$V_a\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] - V_b\left[\frac{1}{R_3}\right] = \frac{V_1}{R} \text{ .... (2)}$$

Similarly for node b we have

$$I_4 + I_5 = I_3 \text{ .... (3)}$$

$$I_4 = \frac{V_b - V_o}{R_4}; I_5 = \frac{V_b - V_2}{R_5}$$

On substituting in equ (3)

$$\frac{V_b - V_o}{R_4} + \frac{V_b - V_2}{R_5} = \frac{V_a + V_b}{R_3}$$

WKT

$V_o = 0$ [reference node]

$$V_b\left[\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right] - V_a\left[\frac{1}{R_3}\right] = \frac{V_2}{R_5} \;\ldots\ldots\ldots\ldots\ldots\ldots\; (4)$$

Solving equations (2) and (4) we get the values as V_a and V_b.

Method for solving V_a and V_b by Cramers rule.

$$\begin{pmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & -\frac{1}{R_3} \\ -\frac{1}{R_3} & \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} \end{pmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} \frac{V_1}{R_1} \\ \frac{V_2}{R_2} \end{bmatrix}$$

$$\Delta = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)\left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right) - \left(\frac{-1}{R_3}\right)\left(\frac{-1}{R_3}\right)$$

To find Δ_1

$$\begin{pmatrix} \frac{V_1}{R_1} & \frac{-1}{R_3} \\ \frac{V_2}{R_5} & \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} \end{pmatrix}$$

$$\Delta_1 = \left(\frac{V_1}{R_1}\right)\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right)-\left(\frac{-1}{R_3}\right)\left(\frac{V_2}{R_5}\right)$$

To find Δ_2:

$$\begin{pmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & \frac{V_1}{R_1} \\ \frac{-1}{R_3} & \frac{V_2}{R_5} \end{pmatrix}$$

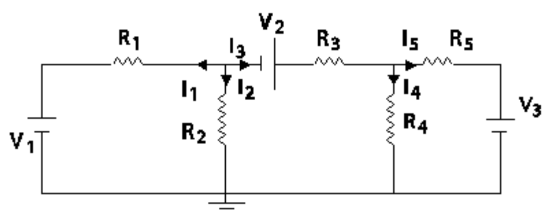
$$\Delta_2 = \left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)\left(\frac{V_2}{R_5}\right)-\left(\frac{-1}{R_3}\right)\left(\frac{V_1}{R_1}\right)$$

To find v_a: To find v_b:

$V_a = \frac{\Delta_1}{\Delta}$; $V_b = \frac{\Delta_2}{\Delta}$

Hence V_a and V_b are found.

CASE II:



Consider fig 2

Let the voltages at nodes a and b be V_a and V_b.

The node equation at node a are

$$I_1 + I_2 + I_3 = 0$$

Where $I_1 = \frac{V_a - V_1}{R_1}$; $I_2 = \frac{V_a}{R_2}$; $I_3 = \frac{V_a + V_2 - V_b}{R_3}$

$$\frac{V_a - V_1}{R_1} + \frac{V_a}{R_2} + \frac{V_a + V_2 - V_b}{R_3} = 0$$

Simplifying

$$\frac{V_a}{R_1} - \frac{V_1}{R_1} + \frac{V_a}{R_2} + \frac{V_a}{R_3} + \frac{V_2}{R_3} - \frac{V_b}{R_3} = 0$$

Combining the common terms.

$$V_a\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]-V_b\left[\frac{1}{R_3}\right]=\frac{V_1}{R_1}-\frac{V_2}{R_3} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The nodal equations at node b are

$$I_3 = I_4 + I_5$$

$$\frac{V_a+V_2-V_b}{R_3}=\frac{V_b}{R_4}+\frac{V_b-V_3}{R_5}$$

On simplifying

$$\frac{V_a}{R_3}+\frac{V_2}{R_3}-\frac{V_b}{R_3}=\frac{V_b}{R_4}+\frac{V_b}{R_5}-\frac{V_3}{R_5}$$

$$\frac{V_a}{R_3}-V_b\left[\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right]=-\frac{V_3}{R_5}-\frac{V_2}{R_3}$$

$$-\frac{V_a}{R_3}+V_b\left[\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right]=\frac{V_3}{R_5}+\frac{V_2}{R_3} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Solving equ (5) and (6) we get V_a and V_b

Method to solve V_a and V_b.

Solve by cramers rule.

$$\begin{pmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & \frac{-1}{R_3} \\ \frac{-1}{R_3} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5} \end{pmatrix}\begin{bmatrix} V_a \\ V_b \end{bmatrix}=\begin{bmatrix} \frac{V_1}{R_1}-\frac{V_2}{R_3} \\ \frac{V_2}{R_3}+\frac{V_3}{R_5} \end{bmatrix}$$

$$\Delta=\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right)-\left(-\frac{1}{R_3}\right)\left(-\frac{1}{R_3}\right)$$

$$\Delta_1=\begin{pmatrix} \frac{V_1}{R_1}-\frac{V_2}{R_3} & -\frac{1}{R_3} \\ \frac{V_2}{R_3}+\frac{V_3}{R_5} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5} \end{pmatrix}$$

$$\left(\frac{V_1}{R_1}-\frac{V_2}{R_3}\right)\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right)-\left(-\frac{1}{R_3}\right)\left(\frac{V_2}{R_3}+\frac{V3}{R_5}\right)$$

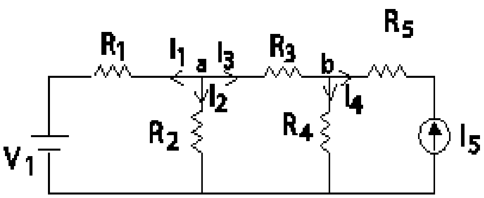
$$\Delta_2 = \begin{pmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & \frac{V_1}{R_1}-\frac{V_2}{R_3} \\ \frac{-1}{R_3} & \frac{V_2}{R_3}+\frac{V_3}{R_5} \end{pmatrix}$$

$$\Delta_2 = \left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)\left(\frac{V_2}{R_3}+\frac{V3}{R_5}\right)-\left(-\frac{1}{R_3}\right)\left(\frac{V_1}{R_1}-\frac{V_2}{R_3}\right)$$

$$\Delta_a = \frac{\Delta_1}{\Delta}; \qquad \Delta_b = \frac{\Delta_2}{\Delta}$$

Hence V_a and V_b are found.

(case iii)



Let the voltages at nodes a and b be V_a and V_b as shown in fig

Node equations at node a are

$$I_1 + I_2 + I_3 = 0$$

$$\frac{V_a - V_1}{R_1}+\frac{V_a}{R_2}+\frac{V_a - V_b}{R_3} = 0$$

$$V_a\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_2}\right]-V_b\left[\frac{1}{R_3}\right]=\frac{V_1}{R_1} \quad \text{..................} (7)$$

Similarly Node equations at node b

$$I_3 + I_5 = I_4$$

$$\frac{V_a - V_b}{R_3} + I_5 = \frac{V_b}{R_4}$$

$$I_5 = V_b\left[\frac{1}{R_3} + \frac{1}{R_4}\right] - V_a\left[\frac{1}{R_3}\right] \quad \text{..................(8)}$$

Solving eqn (7) and (8)

V_a and V_b has been found successfully.

Problems

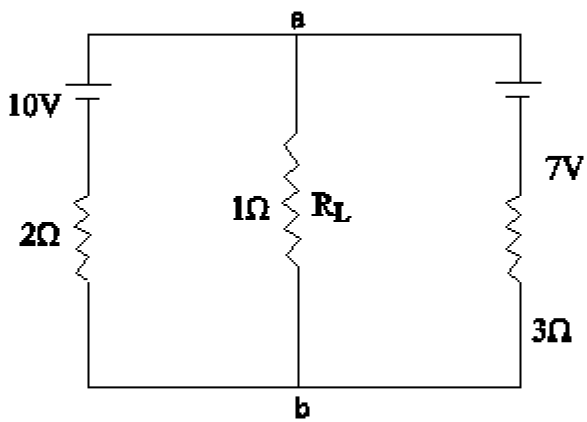
1) Two batteries having emf of 10V and 7V and internal resistances of 2Ω and 3Ω respectively, are connected in parallel across a load of resistance 1Ω . Calculate

(i) The individual battery currents

(ii) The current through the load

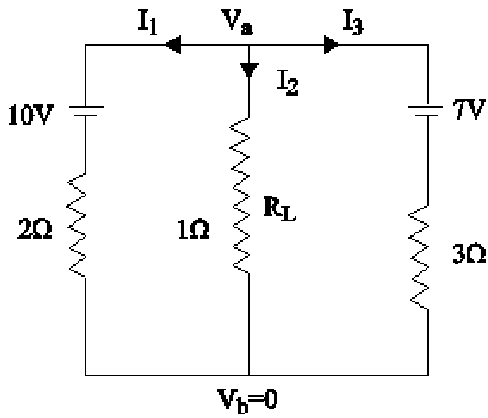
(iii) The Voltage across the load

Solution:



Step 1) Select the nodes and mark the nodes

Step 2) Select the datum or reference node.



<fig 84>

b is the ground node $V_b = 0$

Step 3: Mark the currents I_1, I_2 & I_3

Step 4: Write the node equations for node a and solve for V_a.

$$I_1 + I_2 + I_3 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$I_1 = \frac{V_a - 10}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$I_2 = \frac{V_a}{1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Substituting (2), (3) & (4) in (1)

$$\frac{V_a - 10}{2} + V_a + \frac{V_a - 7}{3} = 0$$

$$V_a\left[\frac{1}{2} + 1 + \frac{1}{3}\right] = \frac{10}{2} + \frac{7}{3}$$

$$1.83\ V_a = 7.33$$

$$V_a = 4\ V$$

(i) Individual battery currents

$$I_1 = \frac{V_a - 10}{2} = \frac{4-10}{2}$$

$$= -3A$$

Ans: I_1=3 A

$$I_3 = \frac{V_a - 7}{3} = \frac{4-7}{3} = -1$$

Ans: I_3 = 1A

(ii) Current through the load

$$I_L = I_{1-2} = \frac{V_a}{1} = 4A$$

(iii)Voltage across the load

$$V_L = V_a - V_b$$
$$= 4 - 0$$
$$V_L = 4\ V$$

2) Write the node voltage equation and calculate the currents in each branch for the network.

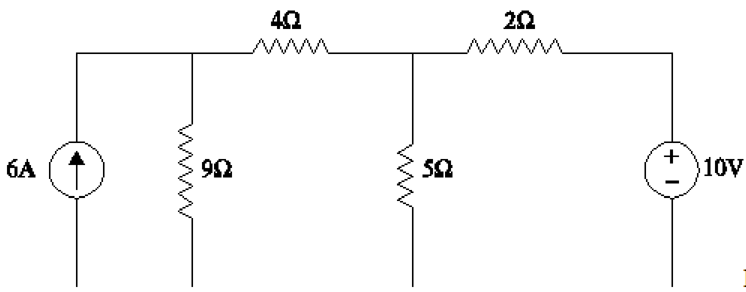


FIG85

Step 1: To assign voltages at each node

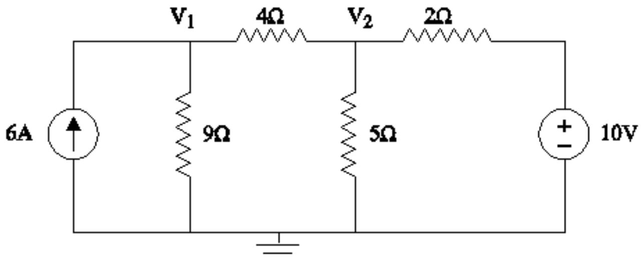


FIG86

V_1 & V_2 are active nodes

V_3 is a reference node on datum node.

Hence $V_3 = 0$.

Step 2: Mark the current directions in all the branches.

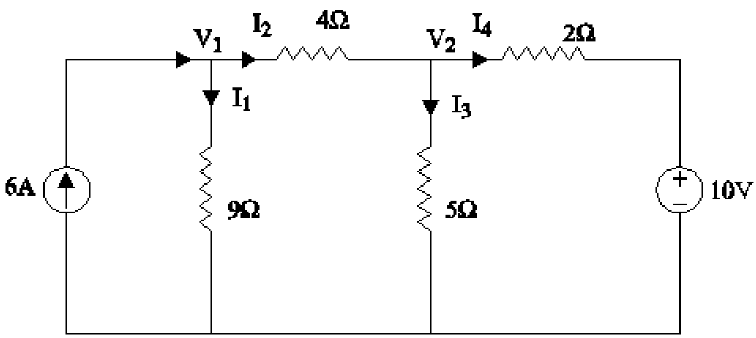


FIG87

Step 3: Write the node equations for node (1) and (2)

Node 1

$I_1 + I_2 = 6$

$$\frac{v_1}{9} + \frac{v_1 - v2}{4} = 6$$

$$V_1\left[\frac{1}{9} + \frac{1}{4}\right] - V_2\left[\frac{1}{4}\right] = 6 \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

Node 2:

$I_2 = I_3 + I_4$

$$\frac{V_1 - V_2}{4} = \frac{V_2}{5} + \frac{V_2 - 10}{2}$$

$$V_1\left[\frac{1}{4}\right] = V_2\left[\frac{1}{4} + \frac{1}{5} + \frac{1}{2}\right] - \frac{10}{2}$$

$$-V_1\left[\frac{1}{4}\right] + V_2\left[\frac{1}{4} + \frac{1}{5} + \frac{1}{2}\right] = \frac{10}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}$$

Step 4: Solving equ (1) and (2) and finding V_1 and V_2 by Cramers rule,

$$\begin{bmatrix} \frac{1}{9}+\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{5}+\frac{1}{4}+\frac{1}{2} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

$$\begin{pmatrix} .36 & -.25 \\ -.25 & .95 \end{pmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

$\Delta = 0.2795$

To find Δ_1

$$\begin{pmatrix} 6 & -.25 \\ 5 & .95 \end{pmatrix} = 6.95$$

$$V_1 = \frac{6.95}{.279} = 24.86\upsilon$$

To find Δ_2

$$\begin{bmatrix} .36 & 6 \\ -.25 & 5 \end{bmatrix} = 3.3$$

$$V_{22}\frac{3.3}{.2795} = 11.8V$$

$$I_{9\Omega} = \frac{V_1}{9} = \frac{24.86}{9} = 2.76A$$

$$I_{4\Omega} = \frac{V_1 - V_2}{4} = \frac{24.86 - 11.8}{4} = 3.26A$$

$$I_{5\Omega} = \frac{V_2}{5} = \frac{11.86}{5} = 2.37A$$

$$I_{2\Omega} = \frac{V_2 - 10}{2} = \frac{11.86 - 10}{2} = 0.93A$$

Hence currents in all the branches are found.

Problem 1.42 Use the Nodal Method to find V_{ba} and current through 30 Ω resistor in the circuit shown

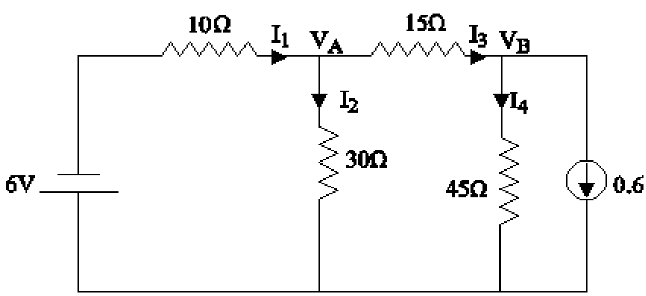


FIG88

At node A

$$\frac{V_A + 6}{10} + \frac{V_A}{30} + \frac{V_A - V_B}{15} = 0$$

$$V_A\left[\frac{1}{10} + \frac{1}{30} + \frac{1}{15}\right] - \frac{V_B}{15} = -0.6$$

At node B

$$\frac{V_B - V_A}{15} + \frac{V_B}{45} \quad +0.6 = 0$$

$$V_B\left[\frac{1}{15} + \frac{1}{45}\right] - \frac{V_A}{15} = -0.6$$

$$\begin{pmatrix} \frac{1}{10} + \frac{1}{30} + \frac{1}{15} & -\frac{1}{15} \\ -\frac{1}{15} & \frac{1}{15} + \frac{1}{45} \end{pmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} -0.6 \\ -0.6 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 0.2 & -0.066 \\ -0.066 & 0.088 \end{bmatrix} = \left[0.0176 - 4.35 \times 10^{-3}\right]$$

$$\Delta = [0.01324]$$
$$\Delta = 0.01324$$

$$\Delta_1 = \begin{vmatrix} -0.6 & -\frac{1}{15} \\ -0.6 & \frac{1}{15} + \frac{1}{45} \end{vmatrix} = -0.093$$

$$\Delta_1 = [-0.053 - 0.04] = \ -0.093$$

$$V_A = \frac{\Delta_1}{\Delta} = -\frac{0.093}{0.01324} = -7.02V$$

$$\Delta_2 = \begin{bmatrix} 0.2 & -0.6 \\ -0.066 & -0.6 \end{bmatrix}$$

$$\Delta_2 = [-0.12 - 0.0396]$$

$$\Delta_2 = -0.1596$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{-0.1596}{0.01324} = -12.05V$$

$$V_{ba} = V_A - V_B = -7 + 12 = 5V$$

$$I_2 = \frac{V_A}{30} = \frac{-7}{30} = -0.233A$$

$$I_2 = -0.233A$$

Maxwell's Mesh method (Loop method).

This method was first proposed by Maxwell simplifies the solution of several networks. In this method, KVL is used. In any network, the number of independent loop equations will be

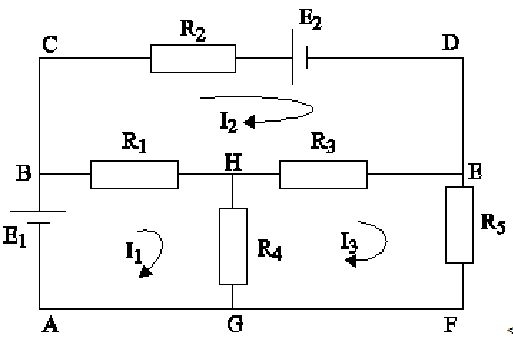
$$m = l - (j - 1)$$

Where l is the number of branches and j is the number of junctions.

Let us consider the circuit shown in fig(). for writing the mesh equations. It has

Number of junctions = 4 (B, H, E, G).

Number of branches = 6 (AB, BC, CD, DE, EF, HG).



<fig89>

In the above figure we shall name the three loop currents I_1, I_2 and I_3. The directions of the loop current are arbitrarily chosen. Note that the actual current flowing through R_4 is $(I_1 - I_3)$ in a downward direction and R_1 is $(I_1 - I_2)$ from left to Right

Apply KVL for the first loop ABHGA,

$$E_1 - R_1 (I_1 - I_2) - R_4 (I_1 - I_3) = 0$$

$$R_1 (I_1 - I_2) + R_4 (I_1 - I_3) = E_1$$

$$\therefore (R_1 + R_4) I_1 - R_1 I_2 - R_4 I_3 = E_1 \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Apply KVL for the loop BEDC,

$$- R_2 I_2 - E_2 - R_3 (I_2 - I_3) - R_1 (I_2 - I_1) = 0$$

$$R_2 I_2 + R_3 (I_2 - I_3) + R_1 (I_2 - I_1) = -E_2$$

$$\therefore - R_1 I_1 + (R_1 + R_2 + R_3) I_2 - R_3 I_3 = -E_2 \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

It can be seen that the diagonal elements of the matrix is the sum of the resistances of the mesh, where as the off diagonal elements are the negative of the sum of the resistance common to the loop.

Thus,

R_{ii} = the sum of the resistances of loop i.

$$R_{ij} = \begin{cases} -\sum(\text{Resistance common to the loop i and loop j,} \\ \quad \text{if } I_i \text{ and } I_j \text{ are in opposite direction in common resistances)} \\ +\sum(\text{Resistance common to the loop i and loop j,} \\ \quad \text{if } I_i \text{ and } I_j \text{ are in same direction in common resistances)} \end{cases}$$

The above equation is only true when all the mesh currents are taken in clockwise direction. The sign of voltage vector is decided by the considered current direction. If the mesh current is entering into the positive terminal of the voltage source, the direction of voltage vector elements will be negative otherwise it will be positive.

Equation (5) can be solved by Cramer's rule as

$$\Delta_1 = \begin{bmatrix} E_1 & R_{12} & R_{13} \\ E_2 & R_{22} & R_{23} \\ E_3 & R_{32} & R_{33} \end{bmatrix}; \quad \Delta_2 = \begin{bmatrix} R_{11} & E_1 & R_{13} \\ R_{21} & E_2 & R_{23} \\ R_{31} & E_3 & R_{33} \end{bmatrix};$$

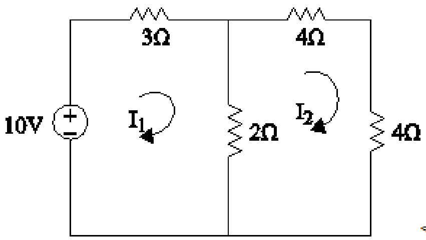
$$\Delta_3 = \begin{bmatrix} R_{11} & R_{12} & E_1 \\ R_{21} & R_{22} & E_2 \\ R_{31} & R_{32} & E_3 \end{bmatrix}; \quad \Delta = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}$$

$$I_1 = \frac{\Delta_1}{\Delta}; \quad I_2 = \frac{\Delta_2}{\Delta}; \quad I_3 = \frac{\Delta_3}{\Delta}$$

Problems:

1) Find the branch currents of fig () using Mesh current method

Solution:



<fig90>

Method 1:

Apply KVL for the first loop,

$$10 - 3I_1 - 2(I_1 - I_2) = 0$$

$$5I_1 - 2I_2 = 10 \text{ .... (1)}$$

Apply KVL for the second loop,

$$- 4I_2 - 4I_2 - 2(I_2 - I_1) = 0$$

$$-2I_1 + 10I_2 = 10 \text{ .... (2)}$$

Solve eqn (1) & (2), we get

$$(1) \text{ X } 5 \Rightarrow 25I_1 - 10I_2 = 50 \text{ ....(3)}$$

$$(2) \quad \Rightarrow -2I_1 + 10I_2 = 0$$

$$(3) + (2) \Rightarrow \quad 23\ I_1 = 50$$

$$I_1 = \frac{50}{23} = 2.174 \text{ A}$$

Sub I_1 in (2)

$$I_2 = \frac{2\times 2.174}{10} = 0.435\text{ A}$$

$$I_3\Omega = 2.174\text{ A}$$

$$I_2\Omega = I_1 - I_2 = 1.739\text{ A}$$

$$I_4\Omega = 0.435\text{ A}$$

Method 2:

R_{11} = Sum of resistances of loop 1 = 3+2 =5Ω

R_{12} = - (common resistance between loop 1 and loop 2) = -2Ω

= R_{21}

R_{22} = Sum of resistance is loop 2 = 4+ 4+ 2 =10

E2 = 0

$$\begin{bmatrix} 5 & -2 \\ -2 & 10 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 5 & -2 \\ -2 & 10 \end{vmatrix} = 50 - 4 = 46$$

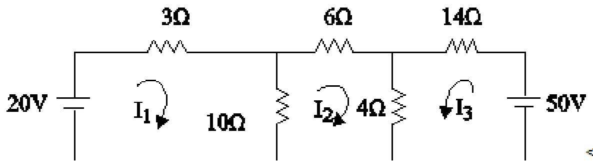
$$\Delta_1 = \begin{vmatrix} 10 & -2 \\ 0 & 10 \end{vmatrix} = 100$$

$$\Delta_2 = \begin{vmatrix} 5 & 10 \\ -2 & 0 \end{vmatrix} = 20$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{100}{46} = 2.174A$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{20}{46} = 0.435A$$

2) Find the loop currents for the network shown in figure below by using Loop Analysis.



<fig91>

Solution

For loop 1,

$$3I_1 + 10(I_1 - I_2) = 20$$

$13I_1 - 10I_2 = 20$ ……………… (1)

For loop 2,

$$10(I_2 - I_1) + 6I_2 + 4(I_2 + I_3) = 0$$

$$10I_2 - 10I_1 + 6I_2 + 4I_2 + 4I_3 = 0$$

$\div 2 \Rightarrow -5I_1 + 10I_2 + 2I_3 = 0$ ……………… (2)

For loop 3,

$$4(I_3 + I_2) + 14I_3 = 50$$

$$4I_2 + 18I_3 = 50$$

$\div 2 \Rightarrow 2I_2 + 9I_3 = 25$ ……………… (3)

$$\therefore \begin{bmatrix} 13 & -10 & 0 \\ -5 & 10 & 2 \\ 0 & 2 & 9 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \\ 25 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 13 & -10 & 0 \\ -5 & 10 & 2 \\ 0 & 2 & 9 \end{vmatrix}$$

$$\Delta = 13(90 - 4) + 10(-45 - 0)$$

$$= 668$$

For loop 1,

$$10I_1 + 5(I_1 + I_2) + 3(I_1 - I_3) = 50$$

$$18I_1 + 5I_2 - 3I_3 = 50 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

For loop 2,

$$2I_2 + 5(I_2 + I_1) + 1(I_2 + I_3) = 10$$

$$5I_1 + 8I_2 + I_3 = 10 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

For loop 3,

$$3(I_3 - I_1) + 1(I_3 + I_2) = -5$$

$$-3I_1 + I_2 + 4I_3 = -5 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$\begin{bmatrix} 18 & 5 & -3 \\ 5 & 8 & 1 \\ -3 & 1 & 4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 50 \\ 10 \\ -5 \end{bmatrix}$$

$$-\ 13\ (250)\ +\ 10\ (-125)\ +\ 20\ (-10)$$

$$\Delta = \begin{bmatrix} 18 & 5 & -3 \\ 5 & 8 & 1 \\ -3 & 1 & 4 \end{bmatrix}$$

$$= 18\ (32-1)\ -5\ (20+3)\ -3(5+24)$$
$$= 356$$

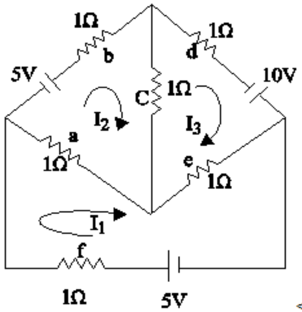
$$\Delta_3 = \begin{vmatrix} 18 & 5 & 50 \\ 5 & 8 & 10 \\ -3 & 1 & -5 \end{vmatrix}$$

$$= 18\ (-40-10)\ -5\ (-25+30)\ +\ 50(5+24)$$
$$= -900 - 25 + 1450$$
$$= 525$$

$$I_3 = \frac{\Delta I_3}{\Delta} = \frac{525}{356} = 1.47A$$

Solution

4) Determine the currents in various elements of the bridge circuit as shown below.



<fig92>

Solution

For loop 1,

$$1I_1 + 1(I_1 - I_2) + 1(I_1 - I_3) = 5$$

$$3I_1 - I_2 - I_3 = 5 \text{ ....} (1)$$

For loop 2,

$$1I_2 + 1(I_2 - I_3) + 1(I_2 - I_1) = 5$$

$$-I_1 + 3I_2 - I_3 = 5 \text{ ....} (2)$$

For loop 3,

$$1I_3 + 1(I_3 - I_1) + 1(I_3 - I_2) = 10$$

$$-I_1 - I_2 + 3I_3 = 10 \text{ ....} (3)$$

$$\rightarrow \begin{bmatrix} -3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 10 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{vmatrix}$$

$= 3(9-1) + 1(-3-1) - 1(1+3)$

$= 16$

$$\Delta_1 = \begin{vmatrix} 5 & -1 & -1 \\ 5 & 3 & -1 \\ 10 & -1 & 3 \end{vmatrix}$$

$= 40 + 25 + 35$

$= 100$

$$\Delta_2 = \begin{vmatrix} 3 & 5 & -1 \\ -1 & 5 & -1 \\ -1 & 10 & 3 \end{vmatrix}$$

$= 3\ (15 + 10)\ -5\ (-3\ -\ 1)\ -1\ \ (-10 + 5)$

$= 100$

$$\Delta_3 = \begin{vmatrix} 3 & -1 & 5 \\ -1 & 3 & 5 \\ -1 & -1 & 10 \end{vmatrix}$$

$= 3\ (30 + 5) + 1\ (-10 + 5) + 5\ (1 + 3)$

$= 120.$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{100}{16} = 6.25A$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{100}{16} = 6.25A$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{120}{16} = 7.5A$$

$I_a = I_1 - I_2 = 6.25 - 6.25 = 0$ A

$I_b = I_2 = 6.25$ A.

$I_c = I_2 - I_3 = 6.25 - 7.5 = -1.25$ A

$I_d = I_3 = 7.5$ A

$I_e = I_1 - I_3 = 6.25 - 7.5 = -1.25$ A.

$I_f = I_1 = 6.25$ A.

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CIRCUIT THEORY – SEEA1201

UNIT-II- AC CIRCUITS

UNIT II

AC CIRCUITS

2.1 INTRODUCTION

We have seen so far about the analysis of DC circuit. A DC quantity is one which has a constant magnitude irrespective of time. But an alternating quantity is one which has a varying magnitude and angle with respect to time. Since it is time varying in nature, at any time it can be represented in three ways 1) By its effective value 2) By its average value and 3) By its peak value.

Some important terms

1. Wave form
   A wave form is a graph in which the instantaneous value of any quantity is plotted against time.

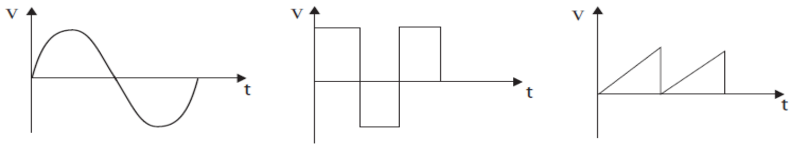

Fig 2.1(a-c)

2. Alternating Waveform
   This is wave which reverses its direction at regularly recurring interval.
3. Cycle

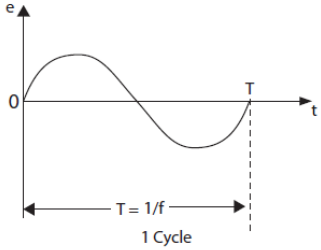

Figure 2.2

It is a set of positive and negative portion of waveforms.

4. Time Period
   The time required for an alternating quantity, to complete one cycle is called the time period and is denoted by T.
5. Frequency
   The number of cycles per second is called frequency and is denoted by f. It is measured in cycles/second (cps) (or) Hertz

$$f = 1/T$$

6. Amplitude
   The maximum value of an alternating quantity in a cycle is called amplitude. It is also known as peak value.
7. R.M.S value [Root Mean Square]
   The steady current when flowing through a given resistor for a given time produces the same amount of heat as produced by an alternating current when flowing through the same resistor for the same time is called R.M.S value of the alternating current.

$$RMS\ Value\ = \sqrt{\frac{Area\ Under\ the\ square\ curve\ for}{one\ complete\ cycle / Period}}$$

8. Average Value of AC
   The average value of an alternating current is defined as the DC current which transfers across any circuit the same change as is transferred by that alternating current during the same time.

   Average Value = Area Under one complete cycle/Period.

9. Form Factor (Kf)
   It is the ratio of RMS value to average value

   Form Factor = RMS value/Average Value

10. Peak Factor (Ka)
    It is the ratio of Peak (or) maximum value to RMS value.

    Peak Factor Ka=Peak Value/RMS value

2.2 Analytical method to obtain the RMS, Average value, Form Factor and Peak factor for sinusoidal current (or) voltage

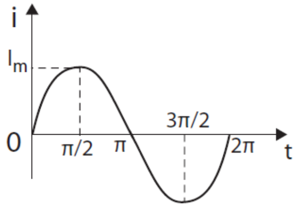

Figure 2.3

$$i = I_m \sin \omega t \ \ ; \omega t=\theta$$

Mean square of AC $I_{RMS}^2 = \frac{1}{2\pi}\int_0^{2\pi} i^2 d\theta$

$$= \frac{1}{\pi}\int_0^{\pi} i^2 d\theta \ \text{[since it is symmetrical]}$$

$$= \frac{I_m^2}{\pi}\int_0^{\pi} \sin^2 d\theta$$

$$= \frac{I_m^2}{\pi}\int_0^{\pi} \frac{1-\cos 2\theta}{2} d\theta$$

$$= \frac{I_m^2}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{I_m^2}{2\pi}\pi$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

Average Value:

$$I_{av} = \int_0^{\pi} \frac{i d\theta}{\pi}$$

$$= \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta \, \mathrm{d}\theta$$

$$= \frac{I_m}{\pi}\int_0^{\pi} \sin\theta \, \mathrm{d}\theta$$

$$= -\frac{\mathrm{I_m}}{\pi}[\cos\theta]_0^{\pi}$$

$$= \frac{I_m}{\pi}\left[\cos\pi - \cos 0\right]$$

$$= \frac{I_m}{\pi}(-1-1)$$

$$= -\frac{2I_m}{\pi}$$

$$Form\ Factor = \frac{RMS}{Avg} = \frac{\mathrm{I_m}/\sqrt{2}}{2\mathrm{I_m}/\pi} = 1.11$$

$$Peak\,Factor = \frac{MAX}{RMS} = \frac{I_m}{RMS} = \frac{I_m}{\frac{I_m}{\sqrt{2}}} = 1.414$$

2.2.1 Expression for RMS, Average, Form Factor, Peak factor for Half wave rectifier

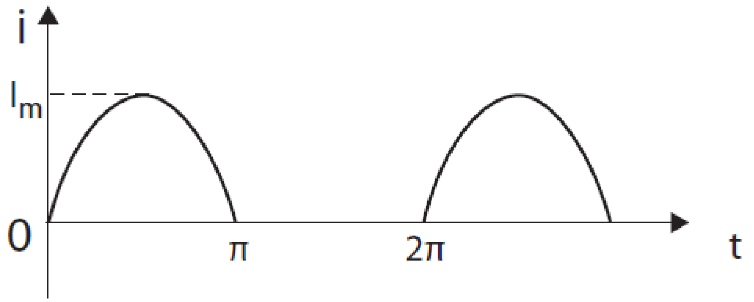

Figure 2.4

1) RMS value

$i = I_m \sin\theta \;;\quad 0 < \theta < \pi$

$i = 0 \;;\quad \pi < \theta \leq 2\pi$

Mean square of AC $I_{RMS}^2 = \frac{1}{2\pi}\int_0^{2\pi} i^2 d\theta$

$$= \frac{1}{2\pi}\int_0^{\pi} i^2 d\theta + \int_{\pi}^{2\pi} i^2 d\theta$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} i^2 d\theta + 0\right]$$

$$= \frac{I_m^2}{2\pi}\int_0^{\pi} \sin^2 d\theta$$

$$= \frac{I_m^2}{2\pi}\int_0^{\pi} \frac{1-\cos 2\theta}{2} d\theta$$

$$= \frac{I_m^2}{4\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{I_m^2}{4\pi}\pi$$

$$I_{RMS} = \frac{I_m}{2}$$

Average Value:

$$I_{av} = \int_0^{\pi} \frac{i d\theta}{2\pi}$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} i d\theta + 0\right]$$

$$= \frac{1}{2\pi}\int_0^{\pi} I_m \sin\theta d\theta$$

$$= \frac{I_m}{2\pi}\int_0^{\pi} I_m \sin\theta d\theta$$

$$= \frac{I_m}{2\pi}\left[\cos\theta\right]_0^{\pi}$$

$$= \frac{I_m}{2\pi}\left[\cos\pi - \cos 0\right]$$

$$= -\frac{I_m}{2\pi}(-1-1)$$

$$= \frac{2I_m}{2\pi} = \frac{I_m}{\pi}$$

$$Form\,Factor = \frac{RMS}{Avg} = \frac{I_m}{2} / \frac{I_m}{\pi} = 1.57$$

$$Peak\ Factor = \frac{MAX}{RMS} = \frac{I_m}{RMS} / \frac{I_m}{\frac{I_m}{2}} = 2$$

Examples:

2.1) The equation of an alternating current is given by

$$i = 40\sin 314\ t$$

Determine

(i) Max value of current
(ii) Average value of current
(iii) RMS value of current
(iv) Frequency and angular frequency
(v) Form Factor
(vi) Peak Factor

Solution:

$$i = 40\sin 314\ t$$

We know that $i = I_m \sin \omega t$

S_o $\quad$ $I_m = 40$

$\omega = 314\ rad/\sec$

(i) Maximum value of current = 40A

(ii) Average value of current

$$I_{Avg} = \frac{2I_m}{\pi} = \frac{2\times 40}{\pi} = 25.464A$$

(iii) RMS value of current

$$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{40}{\sqrt{2}} = 28.28\ Amp$$

(iv) Frequency $f = \frac{\omega}{2\pi} = \frac{314}{2\pi} \approx 50\,Hz$

(*v*) $Form\ Factor \frac{RMS}{Avg} = \frac{28.28}{25.46} = 1.11$

(*vi*) $Peak\ Factor = \frac{\max}{RMS} = \frac{40}{28.28} = 1.414$

2.2) what is the equation of a 50Hz voltage sin wave having an rms value of 50 volt

Solution:

f = 50Hz
V_{rms} = 50V
v = V_m sinωt
ω = 2πf = 2π×50=314 rad/sec
$V_m = V_{rms}\sqrt{2} = 50\times\sqrt{2} = 70.7$ volt
$\therefore v = 70.7\sin 314t$

2.3 PHASOR REPRESENTATION OF SINUSOIDAL VARYING ALTERNATING QUANTITIES

The Phasor representation is more convenient in handling sinusoidal quantities rather than by using equations and waveforms. This vector or Phasor representation of alternating quantity simplifies the complexity of the problems in the AC circuit.

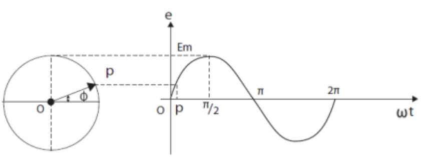

Figure 2.5

$\overline{OP}$ =E_m

E_m – the maximum value of alternating voltage which varies sinusoidally

Any alternating sinusoidal quantity (Voltage or Current) can be represented by a rotating Phasor, if it satisfies the following conditions.

1. The magnitude of rotating phasor should be equal to the maximum value of the quantity.
2. The rotating phasor should start initially at zero and then move in anticlockwise direction. (Positive direction)
3. The speed of the rotating phasor should be in such a way that during its one revolution the alternating quantity completes one cycle.

Phase

The phase is nothing but a fraction of time period that has elapsed from reference or zero position.

In Phase

Two alternating quantities are said to be in phase, if they reach their zero value and maximum value at the same time.

Consider two alternating quantities represented by the equation

$i_1 = Im_1 \sin\theta$
$i_2 = Im_2 \sin\theta$

can be represented graphically as shown in Fig 2.6(a).

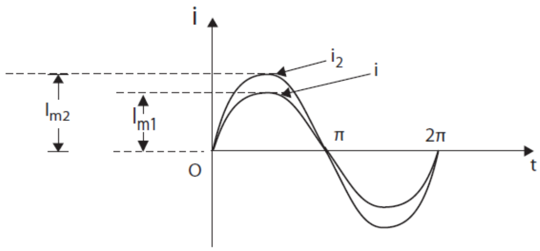

Figure 2.6(a) Graphical representation of sinusoidal current

From Fig 2.6(a), it is clear that both i_1 and i_2 reaches their zero and their maximum value at the same time even though both have different maximum values. It is referred as both currents are in phase meaning that no phase difference is between the two quantities. It can also be represented as vector as shown in Fig 2.6(b).

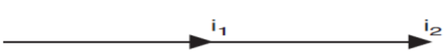

Figure 2.6(b) Vector diagram

Out of Phase

Two alternating quantities are said to be out of phase if they do not reach their zero and maximum value at the same time. The Phase differences between these two quantities are represented in terms of '**lag**' and '**lead**' and it is measured in radians or in electrical degrees.

Lag

Lagging alternating quantity is one which reaches its maximum value and zero value later than that of the other alternating quantity.

Consider two alternating quantities represented by the equation:

$i_1 = Im_1 \sin(\omega t - \Phi)$
$i_2 = Im_2 \sin(\omega t)$

These equations can be represented graphically and in vector form as shown in Fig 2.7(a) and Fig 2.7(b) respectively.

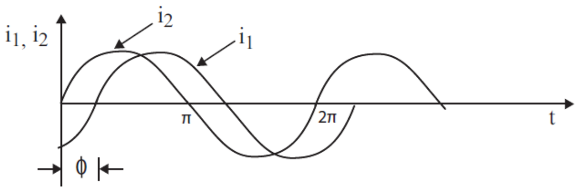

Figure 2.7a

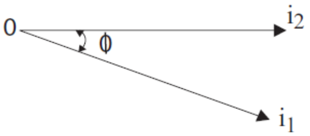

Figure 2.7b

It is clear from the Fig 2.7(a), the current i_1 reaches its maximum value and its zero value with a phase difference of 'Φ' electrical degrees or radians after current i_2. (ie) i_1 lags i_2 and it is represented by a minus sign in the equation.

Lead

Leading alternating quantity is one which reaches its maximum value and zero value earlier than that of the other alternating quantity.

Consider two alternating quantities represented by the equation:
$i_1 = Im_1 \sin(\omega t + \Phi)$
$i_2 = Im_2 \sin(\omega t)$

These equations can be represented graphically and in vector form as shown in Fig 2.8(a) and Fig 2.8(b) respectively.

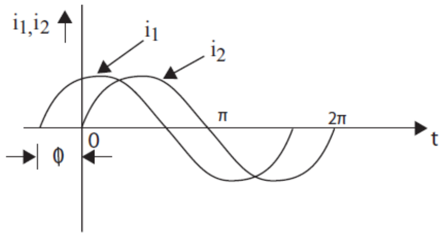

Figure 2.8(a)

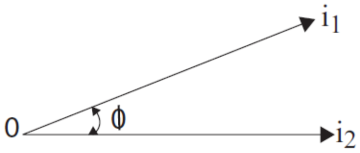

Figure 2.8(b)

The Fig 2.8(a) clearly illustrates that current i_1 has started already and reaches its maximum value before the current i_2. (ie) i_1 leads i_2 and it is represented by a positive sign in the equation.

Note:

1. Two vectors are said to be in quadrature, if the Phase difference between them is 90°.
2. Two vectors are said to be in anti phase, if the phase difference between them is 180°.

2.4 REVIEW OF 'J' OPERATOR

A vector quantity has both magnitude and direction. A vector' A' is represented in two axis plane as shown in Fig 3.10

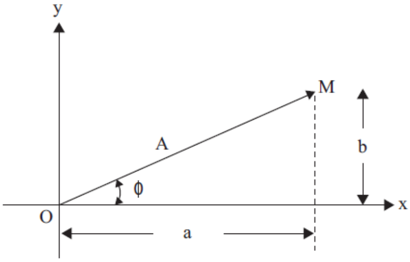

Figure 2.9

In Fig 2.9, OM represents vector A
Φ represents the phase angle of vector A

A = a + jb
a – Horizontal component or active component or in phase component
b – Vertical component or reactive component or quadrature component

The magnitude of vector $'A' = \sqrt{a^2 + b^2}$
Phase angle of Vector ‘A’ = $\alpha = \tan^{-1}$ (b/a)
Features of j – Operator

1. j = $\sqrt{-1}$
   It indicates anticlockwise rotation of Vector through 90°.
2. j^2 = j . j = -1
   It indicates anticlockwise rotation of vector through 180°.
3. j^3 = j . j . j = -j
   It indicates anticlockwise rotation of vector through 270°.
4. j^4 = j . j . j . j = 1
   It indicates anticlockwise rotation of vector through 360°.
5. –j indicates clockwise rotation of vector through 90°.
6. $\frac{1}{j} = \frac{1.j}{j.j} = \frac{j}{j^2} = \frac{j}{-1} = -j$

A vector can be written both in polar form and in rectangular form.
A = 2 + j3

This representation is known as rectangular form.
Magnitude of A = |A| = $\sqrt{2^2 + 3^2}$ =3.606
Phase angle of A = $\alpha = \tan^{-1}$ (3/2) = 56°.31
A=|A| $\angle\alpha°$
A=3.606$\angle$56°.31

This representation is known as polar form.

Note:

1. Addition and Subtraction can be easily done in rectangular form.
2. Multiplication and division can be easily done in polar form.

Examples:

2.3) A= 2 + j3; B = 4 + j5.

Add Vector A and Vector B and determine the magnitude and Phase angle of resultant vector.

Solution:

A + B = 2 + j3 + 4 + j5 = 6 + j8

$\therefore$ Magnitude = | A + B | = $\sqrt{6^2 + 8^2}$ = 10.0

Phase angle = α = $\tan^{-1}$ (B /A) = $\tan^{-1}$ (8/6) = 53°.13

2.4) A= 2 + j5; B = 4 – j2.

Subtract Vector A and Vector B and determine the magnitude and Phase angle of resultant vector.

Solution:

A – B = 2 + j5 – (4 – j2) = 2 + j5 – 4 + j2 = -2 + j7

$\therefore$ Magnitude = | A – B | = $\sqrt{-2^2 + 7^2}$ = 7.280

Phase angle = α = $\tan^{-1}$ (B /A) = $\tan^{-1}$ (7/-2) = -74°.055

2.5) A= 2 + j3; B = 4 – j5.

Perform A x B and determine the magnitude and Phase angle of resultant vector.

Solution:

A= 2 + j3

|A| = $\sqrt{2^2+3^2}$ =3.606

α = $\tan^{-1}$ (3/2) = 56°.310

A= 3.606 $\angle$ 56°.310

B = 4 – j5

|B| = $\sqrt{4^2 + -5^2}$ =6.403

α = $\tan^{-1}$ (-5/4) = -51°.340

B= 6.403$\angle$ -51°.340

A X B = 3.606 $\angle$ 56°.310 X 6.403$\angle$ -51°.340

= 3.606 X 6.403$\angle$ (56°.310 + (-51°.340))

= 23.089 $\angle$4°.970

2.6) A= 4 – j2; B = 2 + j3.

Perform $\frac{A}{B}$ and determine the magnitude and Phase angle of resultant vector.

Solution:

A= 4 – j2

|A| = $\sqrt{4^2 + -2^2}$ = 4.472

α = $\tan^{-1}$ (-2/4) = -26°.565

A= 4.472 $\angle$ -26°.565

B = 2 + j3

$$|B| = \sqrt{2^2 + 3^2} = 3.606$$
$$\alpha = \tan^{-1}(3/2) = 56°.310$$
$$B = 3.606\angle 56°.310$$
$$\frac{A}{B} = \frac{4.472\angle{-26^o}.565}{3.606\angle 56^o.310} = \frac{4.472}{3.606}\angle -26^o.565 - 56^o.310 = 1.240\ \angle{-82.875}$$

2.5 ANALYSIS OF AC CIRCUIT

The response of an electric circuit for a sinusoidal excitation can be studied by passing an alternating current through the basic circuit elements like resistor (R), inductor (L) and capacitor (C).

2.5.1 Pure Resistive Circuit:

In the purely resistive circuit, a resistor (R) is connected across an alternating voltage source as shown in Fig.2.10

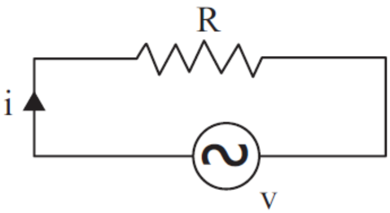

Figure 2.10

Let the instantaneous voltage applied across the resistance (R) be

$$V = V_m \sin\omega t$$

From Ohms law,

$$v = i\,R$$
$$I = \frac{v}{R} = \frac{V_m \sin\omega t}{R}$$
$$\because I_m = \frac{V_m}{R}$$
$$= I_m \sin\omega t$$

where,

$V_m \rightarrow$ Maximum value of voltage (V)
$I_m \rightarrow$ Maximum value of current (A)
$\omega \rightarrow$ Angular frequency (rad/sec)
$t \rightarrow$ Time period (sec)

Phasor Representation:

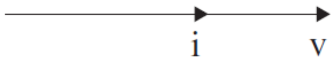

Figure 2.11

Comparing equations, we find that applied voltage and the resulting current are **inphase** with each other. Therefore in a purely resistive circuit there is no phase difference between voltage and current i.e., phase angle is zero (Φ=0).

If voltage is taken as reference, the phasor diagram for purely resistive circuit is shown in Fig.2.11

Waveform Representation:

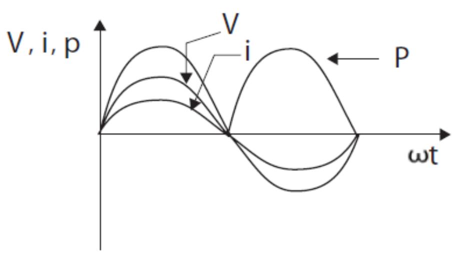

Figure 2.12

The waveform for applied voltage and the resulting current and power were shown in Fig.2.12. Since the current and voltage are inphase the waveforms reach their maximum and minimum values at the same instant.

Impedance:
In an AC circuit, impedance is the ratio of the maximum value of voltage to the maximum value of current.

$$Z = \frac{V_m}{I_m}$$

$$= \frac{V_m}{V_m/R} = R$$

$$\therefore Z = R$$

Power:

(i) Instantaneous power:
It is defined as the product of instantaneous voltage and instantaneous current.

$$p = v\,i$$

$= V_m \sin\omega t \, I_m \sin\omega t = V_m I_m \sin^2\omega t$

$[\because \omega t = \theta]$

$p = V_m I_m \sin^2\theta$

(ii) Average power:

Since the waveform in Fig. is symmetrical, the average power is calculated for one cycle.

$$P = \frac{1}{\pi}\int_0^{\pi} V_m I_m \sin^2\theta d\theta$$

$$= \frac{V_m I_m}{\pi}\int_0^{\pi} \frac{1-\cos 2\theta}{2} d\theta$$

$$\because \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$= \frac{V_m I_m}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{V_m I_m}{2\pi}\left[\pi - \frac{\sin 2\pi}{2} - 0 + \frac{\sin 0}{2}\right]$$

$$= \frac{V_m I_m}{2\pi}[\pi] = \frac{V_m I_m}{2}$$

$$= \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}} = V_{RMS} I_{RMS} = V.I$$

$P = V\,I$

Power Factor:

It is defined as the cosine of the phase angle between voltage and current.

$$\cos\phi = \cos 0 = 1(unity)$$

Problems:

2.7) A voltage of 240 sin 377t is applied to a 6Ω resistor. Find the instantaneous current, phase angle, impedance, instantaneous power, average power and power factor.

Solution:

Given: $v = 240 \sin 377t$

$V_m = 240$ V

$\omega = 377$ rad/sec

$R = 6\Omega$

Instantaneous current:

$$= \frac{V_m \sin\omega t}{R}$$

$$= \frac{240}{6} \sin 377t$$

$$= 40 \sin 377tA$$

I. Phase angle:

$\phi = 0$

II. Impedance:

Z = R = 6Ω

III. Instantaneous power:

IV. $$p = V_m I_m \sin^2 \omega t$$

$$= 240.40.\sin^2 377t$$

$$= 9600 \sin^2 377t$$

V. Average power:

$$P = \frac{V_m I_m}{2} = 4800 \text{watts}$$

VI. Power factor:

cosΦ = cos0 = 1

2.8) A voltage e = 200sinωt when applied to a resistor is found to give a power 100 watts. Find the value of resistance and the equation of current.

Solution:

Given: e = 200sinωt

$V_m = 200$

P = 100w

Average power, P = $\frac{V_m I_m}{2}$

$$100 = \frac{200 I_m}{2}$$

$I_m = 1$ A

Also, $V_m = I_m.R$

R = 200Ω

Instantaneous current, I = I_m sinωt = 1.sinωt A

2.9) A voltage e = 250sinωt when applied to a resistor is found to give a power of 100W. Find the value of R and write the equation for current. State whether the value of R varies when the frequency is changed.

Solution:

Given: e = 250sinωt

$V_m = 250$

P = 100W

I. $P = \frac{V_m I_m}{2}$

$100 = \frac{250 I_m}{2}$

$I_m = 0.8$ A

II. $I_m = \frac{V_m}{R}$

$R = 312.5\Omega$

III. $I = 0.8\sin\omega t$

The resistance is independent of frequency, so the variation of frequency will not affect the resistance of the resistor.

2.5.2 Pure Inductive Circuit:

In this circuit, an alternating voltage is applied across a pure inductor (L) is shown in Fig. 2.13.

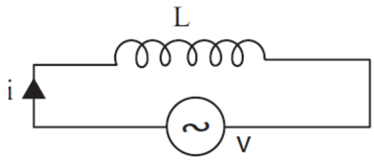

Figure 2.13

Let the instantaneous voltage applied across the inductance (L) be

$$v = V_m \sin\omega t$$

We know that the self induced emf always opposes the applied voltage.

$$V = L\frac{di}{dt}$$

$$i = \frac{1}{L}\int v\,dt = \frac{1}{L}\int V_m \sin\omega t\,dt$$

$$= \frac{V_m}{\omega L}(-\cos\omega t) = \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\left[\because I_m = \frac{V_m}{\omega L}\right]$$

$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

Phasor representation:

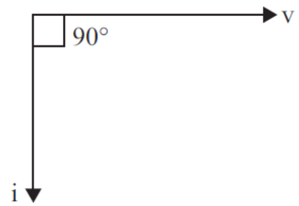

Figure 2.14

Comparing equations, the applied voltage and the resulting current are 90^0 out-of phase. Therefore in a purely inductive circuit there is a phase difference of 90^0 ie., phase angle is 90^0 ($\Phi = 90^0$). Clearly, the current **lags** behind the applied voltage.

Waveform representation:

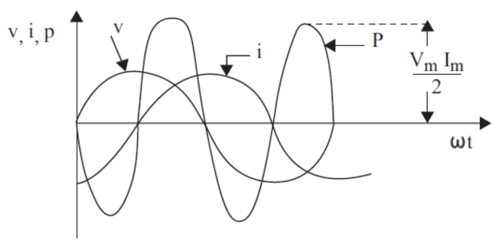

Figure 2.15

The waveform for applied voltage and the resulting current and the power were shown in Fig.2.15. The current waveform is lagging behind the voltage waveform by 90^0.

Impedance (Z):

$$Z = \frac{V_m}{I_m}$$

$$= \frac{V_m}{V_m / \omega L} = \omega L$$

$Z = X_L$ [Impedance is equal to inductive reactance]

Power:

(i)Instantaneous power:

$$p = v\,i$$

$$= V_m \sin\omega t\; I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= V_m I_m \sin\omega t\,(-\cos\omega t)$$

$$= -V_m I_m \sin\omega t \cos\omega t = -V_m I_m \sin\theta \cos\theta$$

(ii) Average power:

Since the waveform in Fig. is symmetrical, the average power is calculated for one cycle.

$$P = -\frac{1}{\pi}\int_0^{\pi} V_m I_m \sin\theta\cos\theta\, d\theta$$

$$= -\frac{V_m I_m}{\pi}\int_0^{\pi}\frac{\sin 2\theta}{2}d\theta$$

$$[\because \sin 2\theta = 2\sin\theta\cos\theta]$$

$$= -\frac{V_m I_m}{2\pi}\left[-\frac{\cos 2\theta}{2}\right]_0^{\pi} = \frac{V_m I_m}{4\pi}[\cos 2\pi - \cos 0]$$

$$= \frac{V_m I_m}{4\pi}[1-1] = 0$$

Thus, a pure inductor does not consume any real power. It is also clear from Fig. that the average demand of power from the supply for a complete cycle is zero. It is seen that power wave is a sine wave of frequency double that of the voltage and current waves. The maximum value of instantaneous power is $\left(\frac{V_m I_m}{2}\right)$.

Power Factor:

In a pure inductor the phase angle between the current and the voltage is 90^0 (lags).

$$\Phi = 90^0;\ \cos\Phi = \cos 90^0 = 0$$

Thus the power factor of a pure inductive circuit is zero lagging.

Problems:

2.10) A coil of wire which may be considered as a pure inductance of 0.225H connected to a 120V, 50Hz source. Calculate (i) Inductive reactance (ii) Current (iii) Maximum power delivered to the inductor (iv) Average power and (v) write the equations of the voltage and current.

Solution:

Given: L = 0.225 H

V_{RMS} = V = 120 V

f = 50Hz

I. Inductive reactance, XL = 2πfL =2π x 50 x 0.225 = 70.68Ω

II. Instantaneous current, i =-I_m cosωt

$\because I_m = \frac{V_m}{\omega L} and V_{RMS} = \frac{V_m}{\sqrt{2}}$, calculate I_m and V_m

$V_m = V_{RMS}\sqrt{2}$ =169.71V

$I_m = \frac{V_m}{\omega L} = \frac{169.71}{70.68} = 2.4A$

Maximum power, $P_m = \frac{V_m I_m}{2}$ = 203.74 W

III. Average power, P=0

IV. Instantaneous voltage, v = Vm sinωt = 169.71 sin 344t volts

Instantaneous current, i = -2.4 cosωt A

2.11) A pure inductance, L = 0.01H takes a current, 10 cos 1500t. Calculate (i) inductive reactance, (ii) the equation of voltage across it and (iii) at what frequency will the inductive reactance be equal to 40Ω.

Solution:

Given: L = 0.01 H

I = 10cos1500t

I_m = 10A

ω = 1500 rad/sec

I. Inductive reactance, X_L = ωL = 1500 x 0.01 = 15Ω

II. The voltage across the inductor, e= $L\frac{di}{dt}$

$= 0.01\frac{d(10\cos 1500t)}{dt}$ = 0.01 x 10[-sin1500t.1500]

= -150 sin 1500t V

III. X_L = 40Ω; 2πfL = 40

f = $\frac{40}{2\pi \times 0.01}$ = 637Hz

2.12) In the circuit, source voltage is v=$200\sin\left(314t+\frac{\pi}{6}\right)$ and the current is i = $20\sin\left(314t-\frac{\pi}{3}\right)$ Find (i) frequency (ii) Maximum values of voltage and current (iii) RMS value of voltage and current (iv) Average values of both (v) Draw the phasor diagram (vi) circuit element and its values

Solution:

Given: $V_m = 200V$
$I_m = 20A$
$\omega = 314$ rad/sec

I. $\omega = 2\pi f$
$f = 50Hz$

II. $V_m = 200V$ and $I_m = 20A$

III. $$V_{RMS} = \frac{V_m}{\sqrt{2}} = 141.42V$$

$$I_{RMS} = \frac{I_m}{\sqrt{2}} = 14.142A$$

IV. For a sinusoidal wave, Average value of current, $I_{av} = \frac{2I_m}{\pi} = 12.732A$

Average value of voltage, $V_{av} = \frac{2V_m}{\pi} = 127.32A$

V. Phasor diagram

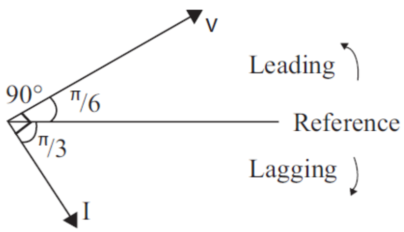

Figure 2.16

VI. From the phasor diagram, it is clear that I lags V by some angle (90^0). So the circuit is purely inductive.

$$I_m = \frac{V_m}{\omega L}$$

$$L = \frac{200}{314 \times 20} = 31.85mH$$

2.5.3 Pure Capacitive Circuit:

In this circuit, an alternating voltage is applied across a pure capacitor(C) is shown in Fig.2.17

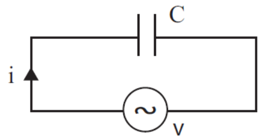

Figure 2.17

Let the instantaneous voltage applied across the inductance (L) be

$$v = V_m \sin\omega t$$

Let at any instant i be the current and Q be the charge on the plates.

So, charge on capacitor, Q = C.v

$$= C.\ V_m \sin\omega t$$

$$\text{Current,i} = \frac{dQ}{dt}$$

$$i = \frac{d}{dt}\left(CV_m \sin\omega t\right) = \omega CV_m \cos\omega t$$

$$= \omega CV_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\left[\because I_m = \omega CV_m\right]$$

$$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

From the above equations , we find that there is a phase difference of 90^0 between the voltage and current in a pure capacitor.

Phasor representation:

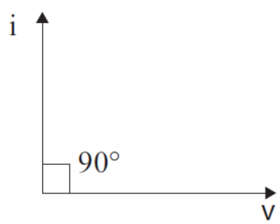

Figure 2.18

In the phasor representation, the current leads the voltage by an angle of 90^0.

Waveform representation:

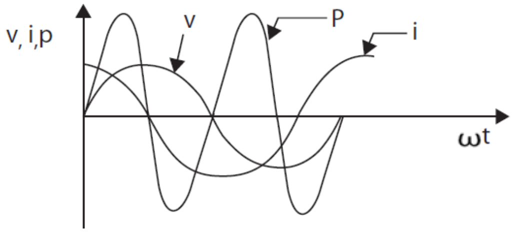

Figure 2.19

The current waveform is ahead of the voltage waveform by an angle of 90^0.

Impedance (Z):

$$Z = \frac{V_m}{I_m}$$

$$= \frac{V_m}{\omega C V_m} = \frac{1}{\omega C}$$

$Z = X_C$ [Impedance is equal to capacitive reactance]

Power:

(i)Instantaneous power:

$$p = v\, i$$

$$= V_m \sin\omega t\ I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= V_m I_m \sin\omega t\ (\cos\omega t)$$

$$= V_m I_m \sin\theta \cos\theta$$

(ii) Average power:

Since the waveform in Fig. is symmetrical, the average power is calculated for one cycle.

$$P = \frac{1}{\pi}\int_0^{\pi} V_m I_m \sin\theta \cos\theta\, d\theta$$

$$= \frac{V_m I_m}{\pi}\int_0^{\pi} \frac{\sin 2\theta}{2} d\theta$$

$$[\because \sin 2\theta = 2\sin\theta\cos\theta]$$

$$= \frac{V_m I_m}{2\pi}\left[-\frac{\cos 2\theta}{2}\right]_0^{\pi} = \frac{V_m I_m}{4\pi}[-\cos 2\pi + \cos 0]$$

$$= \frac{V_m I_m}{4\pi}[-1+1] = 0$$

Thus, a pure capacitor does not consume any real power. It is also clear from Fig. that the average demand of power from the supply for a complete cycle is zero. Again, it is seen that power wave is a sine wave of frequency double that of the voltage and current. The maximum value of instantaneous power is $\left(\frac{V_m I_m}{2}\right)$.

Power Factor:
In a pure capacitor, the phase angle between the current and the voltage is 90^0 (leads).

$$\Phi = 90^0;\ \cos\Phi = \cos 90^0 = 0$$

Thus the power factor of a pure inductive circuit is zero leading.

Problems:

2.13) A 135μF capacitor has a 150V, 50Hz supply. Calculate (i) capacitive reactance (ii) equation of the current (iii) Instantaneous power (iv) Average power (v) RMS current (vi) Maximum power delivered to the capacitor.

Solution:

Given: $V_{RMS} = V = 150V$
$C = 135\mu F$
$f = 50Hz$

I. $X_C = \frac{1}{\omega C} = 23.58\Omega$

II. $i = I_m \sin\left(\omega t + \frac{\pi}{2}\right) \quad \because I_m = \omega C V_m$ and $V_{RMS} = \frac{V_m}{\sqrt{2}}$

$V_m = 150 X \sqrt{2} = 212.13V$

$I_m = 314X135X10^{-6}X212.13 = 8.99A$

$i = 8.99 \sin\left(314t + \frac{\pi}{2}\right) A$

III. $p = V_m I_m \sin\omega t (\cos \omega t) = 212.13X8.99 \sin314t.\cos314t$

$= 66642.6 \sin 314t.\cos314t = 66642.6 \frac{\sin 628t}{2}$

$[\because \sin 2\theta = 2\sin\theta\cos\theta]$

$= 33321.3 \sin628t$ W

IV. Average power, P = 0

V. $I_{RMS} = \frac{I_m}{\sqrt{2}} = 6.36A$

VI. $P_m = \frac{V_m I_m}{2} = 953.52$ W

2.14) A voltage of 100V is applied to a capacitor of 12μF. The current is 0.5 A. What must be the frequency of supply

Solution:

Given: V_{RMS}= V= 100V

C = 12μF

I = 0.5A

I. Find V_m and I_m

$$V_{RMS} = \frac{V_m}{\sqrt{2}}$$

V_m=100X $\sqrt{2}$ =141.42V

$$I_{RMS} = \frac{I_m}{\sqrt{2}}$$

I_m= 0.5X $\sqrt{2}$ = 0.707A

II. $I_m = \omega C V_m = 2\pi f C V_m$

f = 66.3Hz

2.5.4 RL Series Circuit

Let us consider a circuit is which a pure resistance R and a purly inductive coil of inductance L are connected in series as shown in diagram.

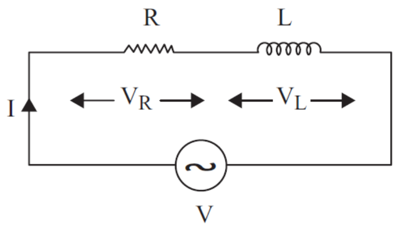

Figure 2.20

Let V = V_m Sin ωt be the applied voltage.

i = Circuit current at any constant.

I = Effective Value of Circuit Current.

V_R= Potential difference across inductor.

V_L= Potential difference across inductor.

F= Frequency of applied voltage.

The same current I flows through R and L hence I is taken as reference vector.

Voltage across resistor V_R= IR in phase with I
Voltage with inductor V_L= IX_L leading I by 90°

The phasor diagram of RL series circuit is shown below.

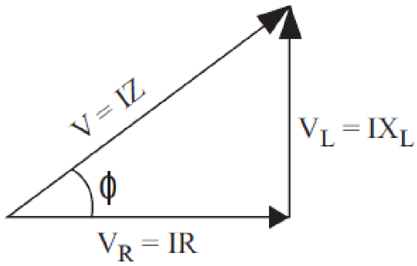

Figure 2.21

At any constant, applied voltage

$$V = V_R + V_L$$
$$V = IR + jIX_L$$
$$V = I\,(R + jx_L)$$
$$\frac{V}{I} = R + jx_L$$
$$= z \text{ impedance of circuit}$$
$$Z = R + j\,x_L$$
$$|z| = \sqrt{R^2 + X_L^2}$$

From phasor disgram,

$$\tan\phi = \frac{x_L}{R}$$
$$\phi = \tan^{-1}\left(\frac{x_L}{R}\right)$$

ϕ is called the phasor angle and it is the angle between V and I, its value lies between 0 to 90^0.

So impedence $Z = R + jX_L$
$$= |Z| < \phi$$

The current and voltage waveform of series RL Circuit is shown below.

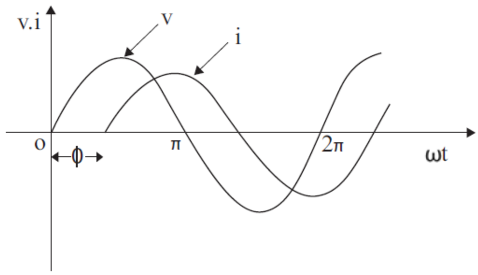

Figure 2.22

$V = V_m \sin\omega t$

$I = I_m \sin(\omega t - \phi)$

The current I lags behind the applied voltage V by an angle ϕ.

From phasor diagram,

Power factor $\cos\ \phi = \frac{R}{Z}$

Actual Power P = VI cos ϕ – Current component is phase with voltage

Reactive or Quadrature Power

Q = VI sinϕ – Current component is quadrature with voltage

Complex or Apparent Power

S = VI – Product of voltage and current

S = P + jQ

Problem

2.15) A series RL Circuit has

$$i(t) = 5\ sin\left(314t + \frac{2\pi}{3}\right) \text{ and } V(t) = 20\ sin\left(314t + \frac{5\pi}{3}\right)$$

Determine (a) the impedence of the circuit

(b) the values of R_1L and power factor

(c) average power of the circuit

Solution:

$$i(t) = 5 \sin(314t + \frac{2\pi}{3})$$

$$V(t) = 20 \sin(314t + \frac{5\pi}{3})$$

Phase angle of current $\theta_i = \frac{2\pi}{3} = \frac{2\times180}{3} = 120°$

Phase angle of voltage $\theta_v = \frac{5\pi}{3} = \frac{5\times180}{3} = 150°$

Phase angle between voltage and current $\theta = \theta_v \sim \theta_i$

$$= 150 - 120$$

$$\theta = 30^0$$

Power factor = cos θ

$$= \cos 30$$

$$= 0.866 \text{ (lagging)}$$

Impedence of the circuit $Z = \dfrac{V_m}{I_m}$

$$= \frac{20}{5}$$

$$Z = 4\Omega$$

(i) But cos $\phi = \dfrac{R}{Z}$

$$0.866 = \frac{R}{4}$$

$$\therefore R = 4 \times 0.866$$

$$R = 3.46\Omega$$

$$|Z| = \sqrt{R^2 + X_L^2}$$

$$X_L = \sqrt{Z^2 + R^2}$$

$$= \sqrt{(4)^2 - (3.46)^2}$$

$$X_L = 2\Omega$$

$$\omega L = 2\Omega$$

$$L = \frac{2}{\omega}$$

$$= \frac{2}{3/4}$$

$$L = 6.37 \times 10^{-3} \text{ H}$$

(ii) Average power = VI cos ϕ

$$= \frac{20}{\sqrt{2}} \cdot \frac{5}{\sqrt{2}} (0.866)$$

$$= 43.3 \text{ watts}$$

2.16) A coil having a resistance of 6Ω and an inductance of 0.03 H is connected across a 100V, 50Hz supply, Calculate.

(i) The current
(ii) The phase angle between the current and the voltage
(iii) Power factor
(iv) Power

Solution:

R = 6Ω
L = 0.03 H
$X_L = 2\pi fL$
$X_L = 2\pi\times50\times0.03$
$X_L = 9.42\Omega$

$$|Z| = \sqrt{(R)^2 + (X_L)^2}$$
$$= \sqrt{(6)^2 + (9.42)^2}$$
$$|Z| = 11.17\Omega$$

(i) $I = \dfrac{V}{Z} = \dfrac{100}{11.17} = 8.95$ amps

(ii) $\phi = tan^{-1}\left(\dfrac{X_L}{R}\right)$

$= tan^{-1}\left(\dfrac{9.42}{6}\right)$

Φ = 57.5 (lagging)

(iii) Power factor = cos ϕ
= cos 57.5
= 0.537 (lagging)

(iv) Power = Average power
= VI cos Φ
= 100 ×8.95×0.537

Power = 480.6 Watts

2.17) A 10Ω resistor and a 20 mH inductor are connected is series across a 250V, 60 Hz supply. Find the impedence of the circuit, Voltage across the resistor, voltage across the inductor, apparent power, active power and reactive power.

Solution:

R = 10Ω
L = 20 mH = 20×10^{-3}H
$X_L = 2\pi fL$
$= 2\pi\times60\times20\times10^{-3}$
$X_L = 7.54\Omega$

(i) $|Z| = \sqrt{R + (X_L)^2} = \sqrt{(10)^2 + (7.54)^2} = 12.5\Omega$

(ii) $I = \dfrac{V}{Z} = \dfrac{250}{12.5} = 20\, amps$

$V_R = IR = 20\times10 = 200$ volts

(iii) $V_L = I\,X_L = 20\times7.54 = 150.8$ volts

(iv) Apparent power S = VI
= 250×20
S= 5000VA

$$\cos\phi = \frac{R}{Z} = \frac{10}{12.5} = 0.8 \text{ (lagging)}$$

Active power = VI cos ϕ

=250×20×0.8

P = 4000 Watts

$$\sin\phi = \sqrt{1-\cos^2\Phi} = \sqrt{1-(0.8)^2} = 0.6$$

Reactive Power Q = VI sin ϕ

= 250×20×0.6

Q= 3000 KVAR

2.18) Two impedances (5+j7)Ω and (10-j7)Ω are connected in series across a 200V supply. Calculate the current, power factor and power.

Solution:

$Z_1 = 5 + j7$
$Z_2 = 10 - j7$
V = 200 volts
$Z_{Total} = Z_1 + Z_2$
$= 5 + j7 + 10 – j7$
$Z_{Total} = 15 < 0^{\cdot}$

$\therefore \phi = 0^{\cdot}$

Taking V as referenve,

V = 200 < 0°· Volts

(i) $I = \frac{V}{Z} = \frac{200\angle 0°}{15\angle 0°} = 13.33\angle 0°\ amps$

(ii) ϕ = 0
PF = cos ϕ = cos 0 = 1

(iii) Power = VI cos ϕ
= 200 ×13.33×1
Power = 2666 watts

2.5.5 RC Series Circuit

Let us consider the circuit shown in diagram in which a pure resistance R and a pure capacitance C are connected in series.

Figure 3.24

Let

$V = V_m \sin\omega t$ be the applied voltage.
I = Circuit current of any instant
I = Effective value of circuit current
V_R = Potential Difference across Resistor
V_c = Potential Difference across Capacitor
f = Frequency of applied voltage
The same Current I flows through R and C
Voltage across R = V_R = IR in phase with I
Voltage across C = V_c = IX_c lagging I by 90^0
Applied voltage V = IR– jIX_c

$$=I\,(R - jx_c)$$

$$\frac{V}{I} = R - jX_c = Z$$

Z – Impedence of circuit

$$|Z| = \sqrt{R^2 + X_c^2}$$

Phasor Diagram of RC series circuit is,

Figure 3.25

From Triangle

$$\tan\phi = \frac{X_c}{R} = \frac{1/\omega c}{R} = \frac{1}{\omega c R}$$

$$\phi = \tan^{-1}\left(\frac{1}{\omega c R}\right)$$

ϕ is called Phase angle and it is angle between V and I. Its value lies between 0 and -90^o.

The current and voltage waveform of series RC Circuit is,

Figure 3.26

$V = V_m \sin \omega t$

$I = I_m \sin (\omega t - \phi)$

The current I leads the applied voltage V by an angle ϕ.

From Phasor Diagram,

Power factor $\cos\phi = \frac{R}{Z}$

Actual or real power $P = VI \cos\phi$

Reactive or Quardrature power $Q = VI \sin\phi$

Complex or Apparent Power $S = P + jQ$

$= VI$

Figure 3.27

PROBLEMS

3.20 A capacitor having a capacitarce of 10 μF is connected in series with a non-inductive resistor of 120Ω across 100V, 50HZ calculate the current, power and the Phase Difference between current and supply voltage.

(Non-inductive Resistor means a Pureresistor)

Solution:

C = 10 μF
R = 120Ω
V = 100V
F = 50Hz

$$X_c = \frac{1}{2\pi fc} = \frac{1}{2\pi\times 50\times 10\times 10^{-6}}$$
$$= 318\,\Omega$$
$$|Z| = \sqrt{R^2 + X_c^{\ 2}}$$
$$= 340\,\Omega$$

(a) $$|I| = \frac{|V|}{|Z|}$$
$$= \frac{100}{340}$$
$$= 0.294\,amps$$

(b) $$Phase Difference\,\phi = \tan^{-1}\left(\frac{X_c}{R}\right)$$
$$= \tan^{-1}\left(\frac{318}{120}\right)$$
$$\phi = 69.3^\circ \quad (Leading)$$
$$\cos\phi = \cos(69.3)^\circ$$
$$= 0.353 \quad (Leading)$$
$$Power = |V||I|\cos\phi$$
$$= 100\times 0.294\times 0.353$$
$$= 10.38\ Watts$$

3.21 The Resistor R in series with capacitance C is connected to a 50HZ, 240V supply. Find the value of C so that R absorbs 300 watts at 100 volts. Find also the maximum charge and the maximum stored energy in capacitance.

Solution:

V = 240 volt
F = 50Hz

Power absorbed by R = 300 watts
Voltage across R = 100 volts

$$|V|^2 = |V_R|^2 + |V_C|^2$$
$$|V_C| = \sqrt{|V|^2 - |V_R|^2}$$
$$= \sqrt{(240)^2 - (100)^2}$$
$$|V_C| = 218.2\, volts$$

For Resistor, Power absorbed = 300 volts

$$|I|^2 R = |V_R||I| = 300$$
$$|I| = \frac{300}{|V_R|} = \frac{300}{100} = 3 amps$$
$$|X_C| = \frac{V_C}{|I|} \quad (Apply\, ohm's\, law\, for\, C)$$
$$= \frac{218.2}{3} = 72.73\Omega$$
$$\frac{1}{2\pi fc} = 72.73$$
$$C = \frac{1}{2\pi \times 50 \times 72.73} = 43.77 \times 10^{-6} F$$
$$C = 43.77 \mu F$$

Maximum charge = Q_m = C × maximum V_c
Maximum stared energy = 1/2 (C × maximum V_c^2)
Maximum $V_c = \sqrt{2}$ × Rms value of V_c
= $\sqrt{2}$ × 218.2 = 308.6 volts

Now

Maximum charge = Q_m = 43.77 x 10^{-6} x 308.6
= 0.0135 Coulomb
Maximum energy stored
= ½ (43.77 x 10^{-6}) $(308.6)^2$
= 2.08 joules.

3.22 A Capacitor and Resistor are connected in series to an A.C. supply of 60 volts, 50HZ. The current is 2A and the power dissipated in the Resistor is 80 watts. Calculate (a) the impedance (b) Resistance (c) capacitance (d) Power factor.

Solution

$$|V| = 60 volts$$

$$F = 50\,Hz$$

$$|I| = 2\,amps$$

Power Dissipated = P = 80 watts

(a) $|Z| = \frac{|V|}{|I|} = \frac{60}{2} = 30\Omega$

(b) $As\, P = I^2 R$

$$R = \frac{P}{I^2} = \frac{80}{4}$$

$$= 20\Omega$$

(c) $Since, |Z|^2 = R^2 + X_c^{\ 2}$

$$X_c = \sqrt{(z)^2 - R^2}$$

$$= \sqrt{30^2 - 20^2} = 22.36\Omega$$

$$\frac{1}{2\pi fc} = 22.36$$

$$c = \frac{1}{2\pi f(22.36)}$$

$$= \frac{1}{2\pi \times 50 \times 22.36}$$

$= 142 \times 10^{-6}$ F

C = 142 µF

(or) Power factor = $\cos\phi = \frac{R}{|Z|}$

$$= = \frac{20}{30}$$

= 0.67(Leading)

It is capacitive circuit.

3.23 A metal filament lamp, Rated at 750 watts, 100V is to be connected in series with a capacitor across a 230V, 60Hz supply. Calculate (i) The capacitance required (ii) The power factor

Solution

Rating of the metal filament W =750watts

V_R = 100 volts

$W = I^2R = V_RI$

$$I = \frac{W}{V_R} = \frac{750}{100} = 7.5\,amps$$

It is like RC Series Circuit

So

$$V^2 = V_R^2 + V_C^2$$
$$V_C = \sqrt{V^2 - V_R^2}$$
$$= \sqrt{(230)^2 - (100)^2}$$
$$= 207\,volts$$

Applying Ohm's Law for C

$$|X_C| = \frac{|V_C|}{|I|} = \frac{207}{7.5}$$
$$= 27.6\Omega$$
$$\frac{1}{2\pi fc} = 27.6$$
$$c = \frac{1}{2\pi \times f \times 27.6} = \frac{1}{2\pi \times 60 \times 27.6}$$
$$= 96.19\,\mu F$$
$$Power\ \ factor = \cos\phi = \frac{R}{|Z|}$$
$$|Z| = \frac{|V|}{|I|} = \frac{230}{7.5} = 30.7\Omega$$
$$R = \frac{W}{I^2} = \frac{750}{(7.5)^2}$$
$$= 13.33\Omega$$
$$Powerfactor = \cos\phi = \frac{R}{Z}$$
$$\cos\phi = \frac{13.33}{30.7}$$
$$= 0.434\text{(Leading)}$$

3.5.6 RLC series circuit

Let v = RMS value of the voltage applied to series combination
I = RMS value of the current flowing
V_R = voltage across R
V_L = voltage across L
V_C = voltage across C

Figure 3.28

A circuit consisting of pure R, pure L and pure C connected in series is known as RLC series circuit.

Phasor diagram

Take I as reference
$V_R = I \times R$
$V_L = I \times X_L$
$V_C = I \times X_C$

Assume $X_L > X_C$

Then $V_L > V_C$

Figure 3.29

The above figure shows the phasor diagram for the combined circuit.
From the voltage triangle

$$|V|^2 = |V_R|^2 + (|V_L| - |V_C|)^2$$
$$= |IR|^2 + (|IX_L| - |IX_C|)^2$$
$$= |I|^2 + [R^2 + (X_L - X_C)^2]$$
$$|V| = |I|\sqrt{R^2 + (X_L - X_C)^2}$$
$$|Z| = \frac{|V|}{|I|}$$
$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{R^2 + X^2} \qquad \because X = (X_L - X_C)$$

Three cases of Z

<u>Case 1</u> If $X_L > X_C$
The circuit behaves like RL circuit. Current lags behind voltage. So power factor is lagging.

<u>Case 2</u> If $X_L < X_C$
The circuit behaves like RC circuit current leads applied voltage power factor is leading.

<u>Case 3</u> When $X_L = X_C$, the circuit behaves like pure resistance. Current is in phase with the applied voltage power factor is unity.
Impedance triangle

Figure 3.30

For $X_L > X_C$ For $X_L > X_C$.

1. If applied voltage
   $V = V_m \sin \omega t$ and ϕ is phase angle then 'i' is given by
   1) $i = I_m \sin(\omega t - \theta)$, for $X_L < X_C$
   2) $i = I_m \sin(\omega t + \theta)$, for $X_L > X_C$
   3) $i = I_m \sin \omega t$ for $X_L = X_C$

2. Impedance for RLC series circuit in complex form (or) rectangular form is given by

$$Z = R + j\,(X_L - X_C)$$

Problems

3.24 In a RLC series circuit, the applied voltage is 5V. Drops across the resistance and inductance are 3V and 1V respectively. Calculate the voltage across the capacitor. Draw the phaser diagram.

$V = 5V \qquad V_R = 3V \qquad V_L = 1V$

$V^2 = V_R^{\,2} + (V_L - V_C)^2$

$(V_L - V_C)^2 = V^2 - V_R^{\,2}$

$= 25 - 9 = 16$

$V_L - V_C = \pm 4$

$V_C = V_L \pm 4 = 1+4$

$V_C = 5V$

3.25 A coil of resistance 10Ω and in inductance of 0.1H is connected in series with a capacitance of 150μF across a 200v, 50HZ supply. Calculate

a) the inductive reactance of the coil.
b) the capacitive reactance
c) the reactance
d) current
e) power factor

$R = 10\Omega$

$L = 0.1\ H$

$C = 150\ \mu F \qquad = 150 \times 10^{-6}\ F$

$V = 200V \qquad f = 50\ Hz$

a) $X_L = 2\pi f L = 2\pi\,(50)\,0.1$

$= 31.4\ \Omega$

b) $$X_C = \frac{1}{2\pi fc} = \frac{1}{2\pi(50)(150\times10^{-6})}$$

$= 21.2\Omega$

c) the reactance $X = X_L - X_C$

$= 31.4 - 21.2$

$= 10.2\ \Omega$ (Inductive)

d) $$|Z| = \sqrt{R^2 + X^2}$$

$$= \sqrt{10^2 + (10.2)^2}$$

$$= 14.28\Omega\,(Inductive)$$

$$I = \frac{|V|}{|Z|} = \frac{200}{14.28} = 14 amps$$

e) $P.F = \cos\phi = \frac{R}{|Z|} = \frac{100}{14.28}$

$= 0.7$ (lagging) (I lags behind V)

3.5.7 Parallel AC circuit

When the impedance and connected in parallel and the combination is excited by AC source it is called parallel AC circuit.

Consider the parallel circuit shown in figure.

$$X_{C1} = \frac{1}{2\pi fc_1} = \frac{1}{\omega c_1}$$

$$X_{C2} = 2\pi fL_2 = \omega L_2$$

Impedance $|Z_1| = \sqrt{R_1^{\,2} + X_{C1}^{\,2}}$

$$\phi_1 = \tan^{-1}\left(\frac{X_{C1}}{R_1}\right)$$

$$|Z_2| = \sqrt{R_2^{\,2} + X^2_{\,L2}}$$

$$\phi_2 = \tan^{-1}\left(\frac{X_{L2}}{R_2}\right)$$

Conductance = g

Susceptance = b

Admittance = y

Branch 1

Conductance $g_1 = \frac{R_1}{|Z_1|^2}$

$$b_1 = \frac{X_{C1}}{|Z_1|^2}\ (positive)$$

$$|Y_1| = \sqrt{g_1^{\,2} + b_1^{\,2}}$$

Branch 2

$$g_2 = \frac{R_2}{|Z_2|^2}$$

$$b_2 = \frac{X_{C2}}{|Z_2|^2}\ \text{(Negative)}$$

$$|Y_2| = \sqrt{g_2^{\,2} + b_2^{\,2}}$$

Total conductance $G = g_1 + g_2$
Total Suceptance $B = b_1 - b_2$

Total admittance $|Y| = \sqrt{G^2 + B^2}$

Branch current $|I_1| = |V||Y_1|$

$|I_2| = |V||Y_2|$

$|I| = |V||Y|$

Phase angle $= tan^{-1}\left(\frac{B}{G}\right)$ lag if B-negative

Power factor $\cos\phi = \frac{|G|}{|Y|}$

Problems:

3.26 Two impedances of parallel circuit can be represented by (20 + j15) and (10 – j60) Ω. If the supply frequency is 50 Hz, find the resistance, inductance or capacitance of each circuit.

$Z_1 = 20 + j15\ \Omega$
$Z_2 = 10 - j60\ \Omega$
$F = 50$ Hz
$Z_1 = R_1 + jX_L$
$Z_2 = R_2 - jX_C$

J term positive for in inductive
J term negative for capacitive.

For circuit 1, $R_1 = 20\Omega$

$X_1 = X_L = 2\pi fL = 2\pi\ (50)\ (L)$

$X_L = 15$

$2\pi\ (50)\ L = 15$

$$L = \frac{15}{2\pi(50)}$$

L = 48 mH

For circuit 2

$Z_2 = 10 - j60$
$R_2 = 10$
$X_2 = X_C = 60\ \Omega$

ie, $\frac{1}{2\pi fC} = 60$

$$C = \frac{1}{2\pi(50)60}$$

C = 53 μF.

2.3.27 Two circuits, the impedances of which are Z_1 = (10 + j15) Ω and Z_2 = (6 – j8) Ω are connected in parallel. If the total current supplied is 15A. What is the power taken by each branch.

$$Z1 = (10 + j15)\Omega = 18.03\angle 56.3$$
$$Z2 = (6 - \mathrm{j}8)\Omega = 10\angle -53.13$$
$$\mathrm{I} = 15\ \mathrm{A}$$
$$I_1 = I\frac{Z_2}{Z_1 + Z_2} \qquad \text{(Current divider rule)}$$
$$= \frac{15\angle 0^0 \times 10\angle -53.13^0}{16 + j7}$$
$$(Z_1 + Z_2 = 10 + j15 + 6 - j8)$$
$$I_1 = \frac{150\angle -53.13^0}{17.46\angle 23.63}$$
$$I_1 = 8.6\angle -76.76\ A$$

By KCL $I_2 = I - I_1$

$$= 15\angle 0 - 8.6\angle -76.76$$
$$= 15 - (1.97 - j8.37)$$
$$= 15.5 - 32.7A$$

Power taken by branch 1

= power dissipated in resistance of branch 1

$$= |I_1|^2 R_1 = (8.6)^2 \times 10$$
$$= 739.6 \text{ watts}$$

Power taken by branch 2

$$= |I_2|^2 R_2$$
$$= (15.5)^2 \times 6$$
$$= 1442 \text{ watts}$$

3.28 A 100Ω resistance and 0.6H inductance are connected in parallel across a 230v 50 Hz supply. Find the line current, impedance, power dissipated and parameter of the equivalent series circuit.

$Z_1 = R = 100\Omega$

$Z_2 = j X_L = j2\pi fL$

$= j\ (2\pi \times 50 \times 0.6)$

$= j\ 188.5\Omega$

$= 188.5 \angle 90$

$$Z_T = Z_1 * Z_2$$

$$= \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{100\angle 0 \times 188.5\angle 90}{100 + j188.5}$$

$$= \frac{18850\angle 90}{213.4\angle 62}$$

$= 88.33 \angle 28$

$= 78 + j41.46 \Rightarrow R + jX_L$

$Total\ impedance\ |Z_T| = 88.33\Omega$

R = 78Ω, X_L = 41.46Ω

X_L = 2πfLeq

41.46 = 2π × 50 × Leq

$$Leq = \frac{41.46}{2\pi \times 50}$$

Leq = 132 mH

= 30 – j40 + 24 +j32

= 54 – j8

= 54.6 $\angle$ – 8.43 A

Comparing ‘V’ and ‘I_T’ current I_T lag voltage ‘V’

∴ ϕ = 8.43° lag

Power factor = cos ϕ= cos 8.43

= 0.99 lag

True Power $= W = |V||I|\cos\phi$

=200 × 54.6 × cos 8.43

= 10802 watts

= 10.802 KW

Apparent Power $= |V|\, I$

= 200× 54.6

= 10920 VA = 10.920 KVA

Reactive Power $= |V|\, I \sin\phi$

= 200 × 54.6 × sin 8.43

= 1601 VAR

= 1.601 KVAR

Let Z_{total} = Total impedance

$$Z_{Total} = \frac{V}{I_{total}} = \frac{200\angle 0^0}{54.6\angle -8.43}$$

= 3.663 $\angle$ 8.43

= 3.623 + j0.54

= R +j X_L

R = 3.623Ω X_L = 0.54 Ω

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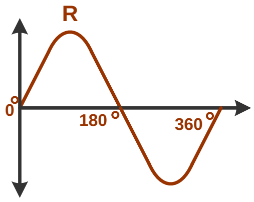
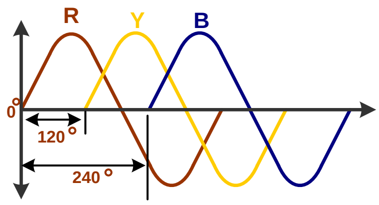
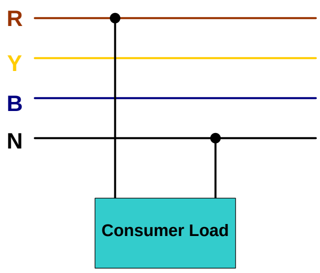
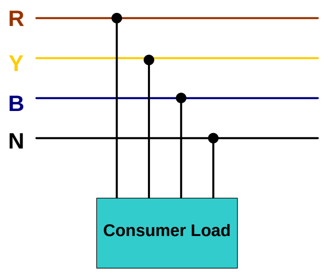
CIRCUIT THEORY – SEEA1201

UNIT-III- 3-PHASE AC CIRCUITS

Comparison between single phase and three phase

Basis for Comparison	Single Phase	Three Phase
Definition	The power supply through one conductor.	The power supply through three conductors.
Wave Shape		
Number of wire	Require two wires for completing the circuit	Requires four wires for completing the circuit
Voltage	Carry 230V	Carry 415V
Phase Name	Split phase	No other name
Network	Simple	Complicated
Loss	Maximum	Minimum
Power Supply Connection		
Efficiency	Less	High
Economical	Less	More
Uses	For home appliances.	In large industries and for running heavy loads.

Generation of three phase EMF

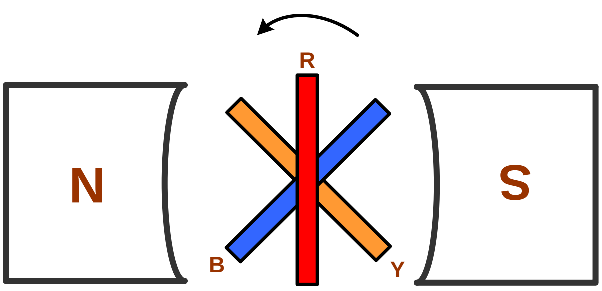


Figure 5.1 Generation of three phase emf

- According to Faraday's law of electromagnetic induction, we know that whenever a coil is rotated in a magnetic field, there is a sinusoidal emf induced in that coil.

- Now, we consider 3 coil C_1(R-phase), C_2(Y-phase) and C_3(B-phase), which are displaced 120^0 from each other on the same axis. This is shown in fig. 5.1.
- The coils are rotating in a uniform magnetic field produced by the N and S pols in the counter clockwise direction with constant angular velocity.
- According to Faraday's law, emf induced in three coils. The emf induced in these three coils will have phase difference of 120^0. i.e. if the induced emf of the coil C_1 has phase of 0^0, then induced emf in the coil C_2 lags that of C_1 by 120^0 and C_3 lags that of C_2 120^0.

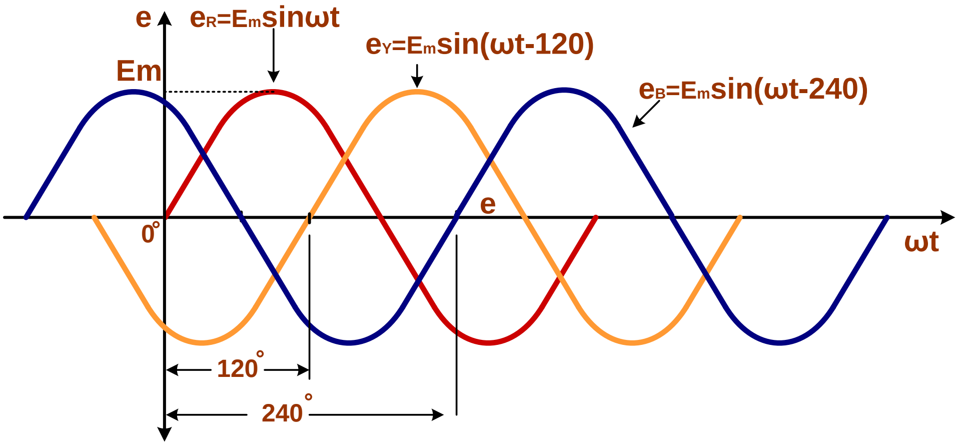


Figure 5.2 Waveform of Three Phase EMF

- Thus, we can write,

$$e_R = E_m \sin \omega t$$

$$e_Y = E_m \sin\left(\omega t - 120^0\right)$$

$$e_B = E_m \sin\left(\omega t - 240^0\right)$$

- The above equation can be represented by their phasor diagram as in the Fig. 5.3.

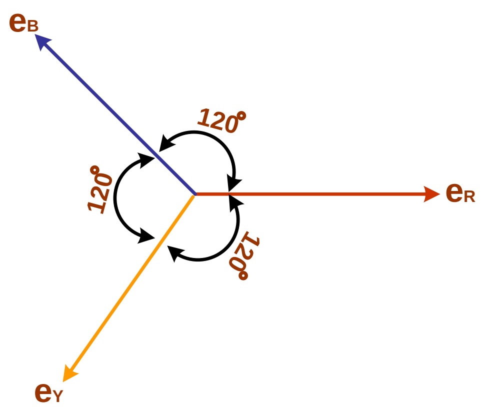


Figure 5.3 Phasor Diagram of Three Phase EMF

Important definitions

➢ **Phase Voltage**

It is defined as the voltage across either phase winding or load terminal. It is denoted by V_{ph}.

Phase voltage V_{RN}, V_{YN} and V_{BN} are measured between R-N, Y-N, B-N for star connection and between R-Y, Y-B, B-R in delta connection.

- **Line voltage**

  It is defined as the voltage across any two-line terminal. It is denoted by V_L.

  Line voltage V_{RY}, V_{YB}, V_{BR} measure between R-Y, Y-B, B-R terminal for star and delta connection both.

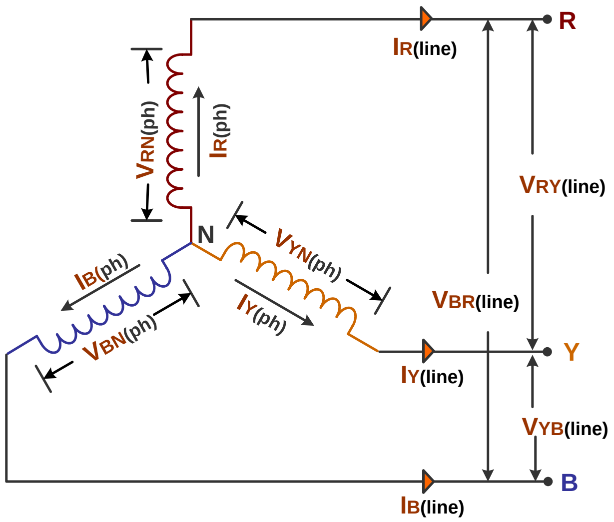


Figure 5.4 Three Phase Star Connection System

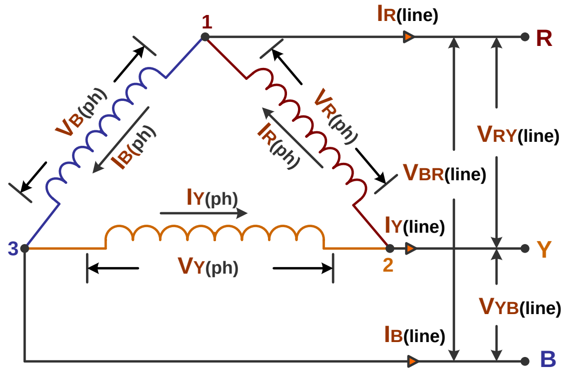


Figure 5.5 Three Phase Delta Connection System

- **Phase current**

  It is defined as the current flowing through each phase winding or load. It is denoted by I_{ph}.

  Phase current $I_{R(ph)}$, $I_{Y(ph)}$ and $I_{B(Ph)}$ measured in each phase of star and delta connection. respectively.

- **Line current**

  It is defined as the current flowing through each line conductor. It denoted by I_L.

  Line current $I_{R(line)}$, $I_{Y(line)}$, and $I_{B((line)}$ are measured in each line of star and delta connection.

- **Phase sequence**

  The order in which three coil emf or currents attain their peak values is called the phase sequence. It is customary to denoted the 3 phases by the three colours. i.e. red (R), yellow (Y), blue (B).

- **Balance System**

  A system is said to be balance if the voltages and currents in all phase are equal in magnitude and displaced from each other by equal angles.

- **Unbalance System**

  A system is said to be unbalance if the voltages and currents in all phase are unequal in magnitude and displaced from each other by unequal angles.

- **Balance load**

  In this type the load in all phase are equal in magnitude. It means that the load will have the same power factor equal currents in them.

- **Unbalance load**

  In this type the load in all phase have unequal power factor and currents.

Relation between line and phase values for voltage and current in case of balanced delta connection.

➢ **Delta (Δ) or Mesh connection**, starting end of one coil is connected to the finishing end of other phase coil and so on which giving a closed circuit.

Circuit Diagram

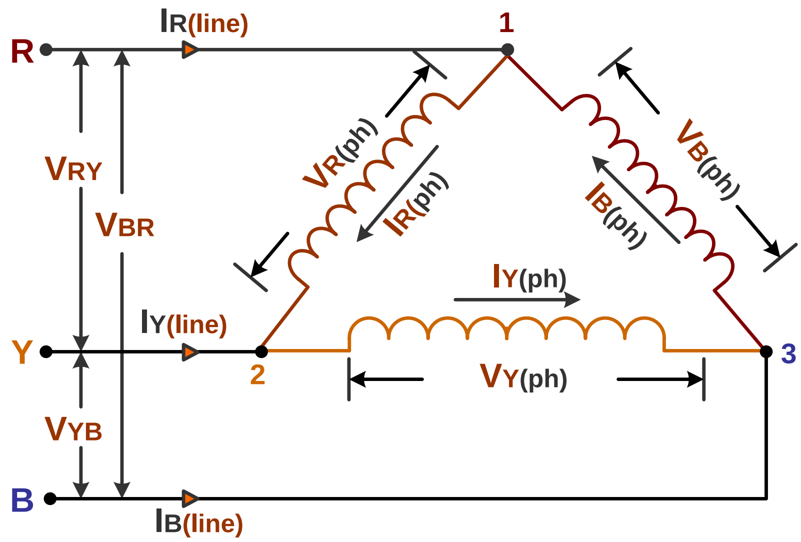


Figure 5.6 Three Phase Delta Connection

- Let,

$$\text{Line voltage, } V_{RY} = V_{YB} = V_{BR} = V_L$$

$$\text{Phase voltage, } V_{R(ph)} = V_{Y(ph)} = V_{B(ph)} = V_{ph}$$

$$\text{Line current, } I_{R(line)} = I_{Y(line)} = I_{B(line)} = I_{line}$$

$$\text{Phase current, } I_{R(ph)} = I_{Y(ph)} = I_{B(ph)} = I_{ph}$$

Relation between line and phase voltage

- For delta connection line voltage V_L and phase voltage V_{ph} both are same.

$$V_{RY} = V_{R(ph)}$$

$$V_{YB} = V_{Y(ph)}$$

$$V_{BR} = V_{B(ph)}$$

$$\therefore V_L = V_{ph}$$

Line voltage = Phase Voltage

Relation between line and phase current

- For delta connection,

$$\mathrm{I}_{R(line)} = \mathrm{I}_{R(ph)} - \mathrm{I}_{B(ph)}$$

$$\mathrm{I}_{Y(line)} = \mathrm{I}_{Y(ph)} - \mathrm{I}_{R(ph)}$$

$$\mathrm{I}_{B(line)} = \mathrm{I}_{B(ph)} - \mathrm{I}_{Y(ph)}$$

- i.e. current in each line is vector difference of two of the phase currents.

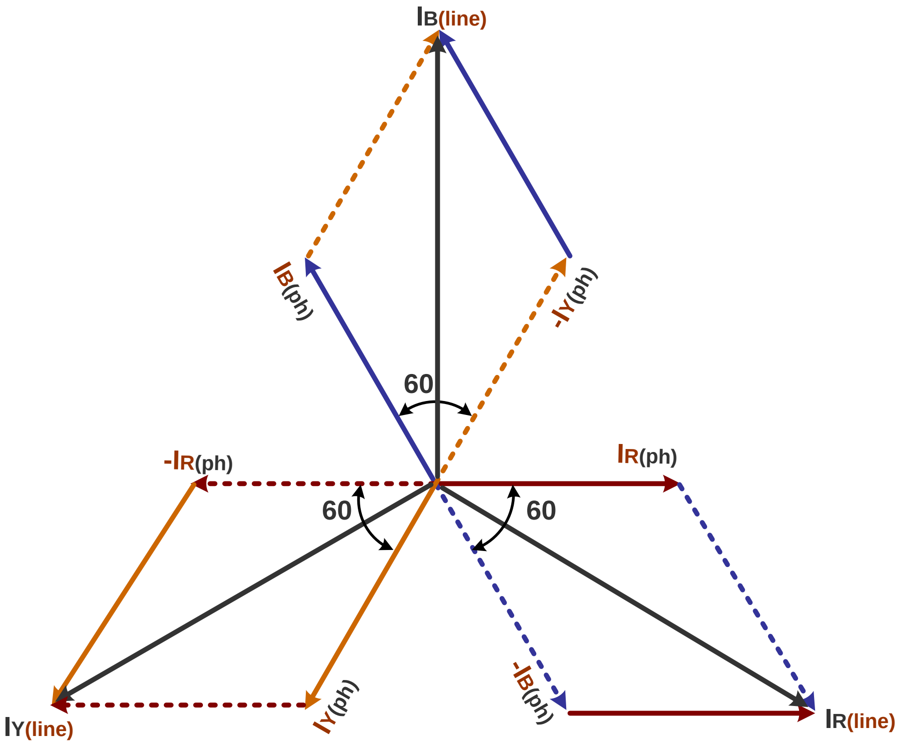


Figure 5.7 Phasor Diagram of Three Phase Delta Connection

- So, considering the parallelogram formed by I_R and I_B.

$$I_{R(line)} = \sqrt{I_{R(ph)}^2 + I_{B(ph)}^2 + 2I_{R(ph)}I_{B(ph)}\cos\theta}$$

$$\therefore I_L = \sqrt{I_{ph}^2 + I_{ph}^2 + 2I_{ph}I_{ph}\cos 60^\circ}$$

$$\therefore I_L = \sqrt{I_{ph}^2 + I_{ph}^2 + 2I_{ph}^2 \times \left(\frac{1}{2}\right)}$$

$$\therefore I_L = \sqrt{3I_{ph}^2}$$

$$\therefore I_L = \sqrt{3}I_{ph}$$

- Similarly, $I_{Y(line)} = I_{B(line)} = \sqrt{3}\ I_{ph}$
- Thus, in delta connection Line current = $\sqrt{3}$ Phase current

Power

$$P = V_{ph}I_{ph}\cos\phi + V_{ph}I_{ph}\cos\phi + V_{ph}I_{ph}\cos\phi$$

$$P = 3V_{ph}I_{ph}\cos\phi$$

$$P = 3V_L\left(\frac{I_L}{\sqrt{3}}\right)\cos\phi$$

$$\therefore P = \sqrt{3}V_L I_L\cos\phi$$

Relation between line and phase values for voltage and current in case of balanced star connection.

➢ In the **Star Connection**, the similar ends (either start or finish) of the three windings are connected to a common point called star or neutral point.

Circuit Diagram

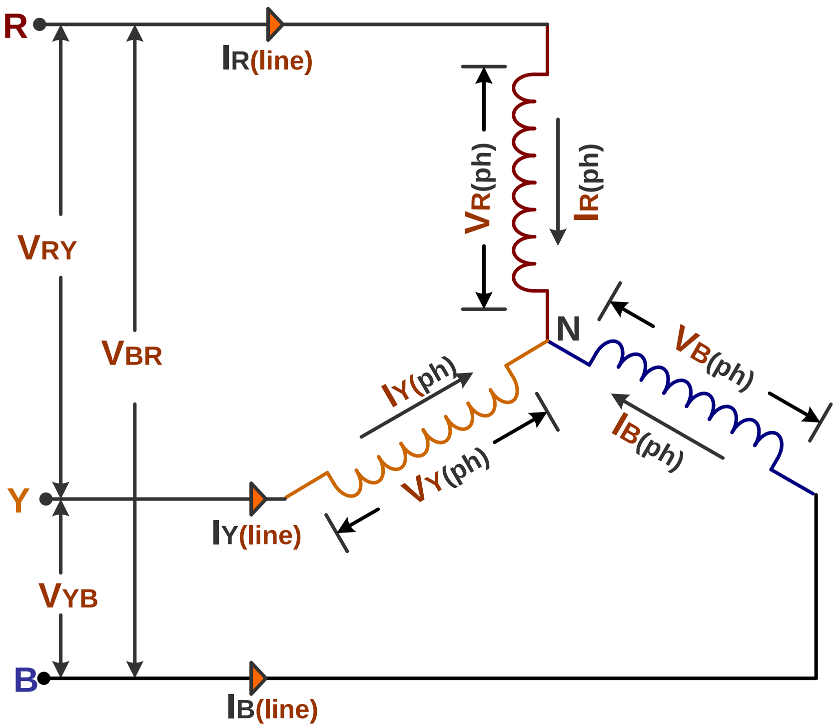


Figure 5.8 Circuit Diagram of Three Phase Star Connection

- Let,

  line voltage, $V_{RY} = V_{BY} = V_{BR} = V_L$

  phase voltage, $V_{R(ph)} = V_{Y(ph)} = V_{B(ph)} = V_{ph}$

  line current, $I_{R(line)} = I_{Y(line)} = I_{B(line)} = I_{line}$

  phase current, $I_{R(ph)} = I_{Y(ph)} = I_{B(ph)} = I_{ph}$

Relation between line and phase voltage

- For star connection, line current I_L and phase current I_{ph} both are same.

  $I_{R(line)} = I_{R(ph)}$

  $I_{Y(line)} = I_{Y(ph)}$

  $I_{B(line)} = I_{B(ph)}$

  $\therefore \quad I_L = I_{ph}$

  Line Current = Phase Current

Relation between line and phase voltage

- For delta connection,

$$V_{RY} = V_{R(ph)} - V_{Y(ph)}$$
$$V_{YB} = V_{Y(ph)} - V_{B(ph)}$$
$$V_{BR} = V_{B(ph)} - V_{R(ph)}$$

- i.e. line voltage is vector difference of two of the phase voltages. Hence,

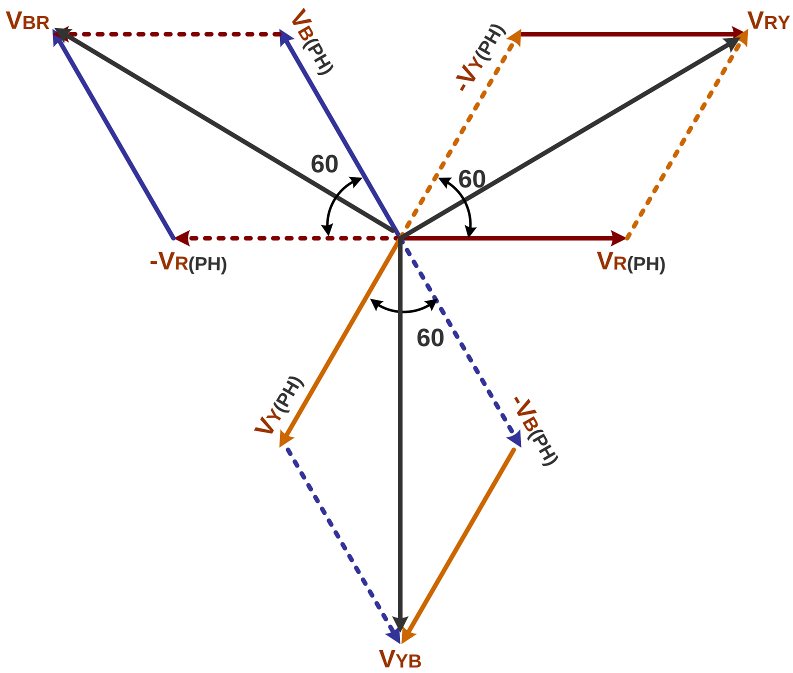


Figure 5.9 Phasor Diagram of Three Phase Star Connection

From parallelogram,

$$V_{RY} = \sqrt{V_{R(ph)}^{2} + V_{Y(ph)}^{2} + 2V_{R(ph)}V_{Y(ph)}\cos\theta}$$
$$\therefore V_{L} = \sqrt{V_{ph}^{2} + V_{ph}^{2} + 2V_{ph}V_{ph}\cos 60^{\circ}}$$
$$\therefore V_{L} = \sqrt{V_{ph}^{2} + V_{ph}^{2} + 2V_{ph}^{2} \times \left(1/2\right)}$$
$$\therefore V_{L} = \sqrt{3V_{ph}^{2}}$$
$$\therefore V_{L} = \sqrt{3}V_{ph}$$

- Similarly, $V_{YB} = V_{BR} = \sqrt{3}\ V_{ph}$
- Thus, in star connection Line voltage = $\sqrt{3}$ Phase voltage

Power

$$P = V_{ph}I_{ph}\cos\phi + V_{ph}I_{ph}\cos\phi + V_{ph}I_{ph}\cos\phi$$
$$P = 3V_{ph}I_{ph}\cos\phi$$
$$P = 3\left(\frac{V_L}{\sqrt{3}}\right)I_L\cos\phi$$
$$\therefore P = \sqrt{3}V_L I_L \cos\phi$$

Measurement of power in balanced 3-phase circuit by two-watt meter method

- This is the method for 3-phase power measurement in which sum of reading of two wattmeter gives total power of system.

Circuit Diagram

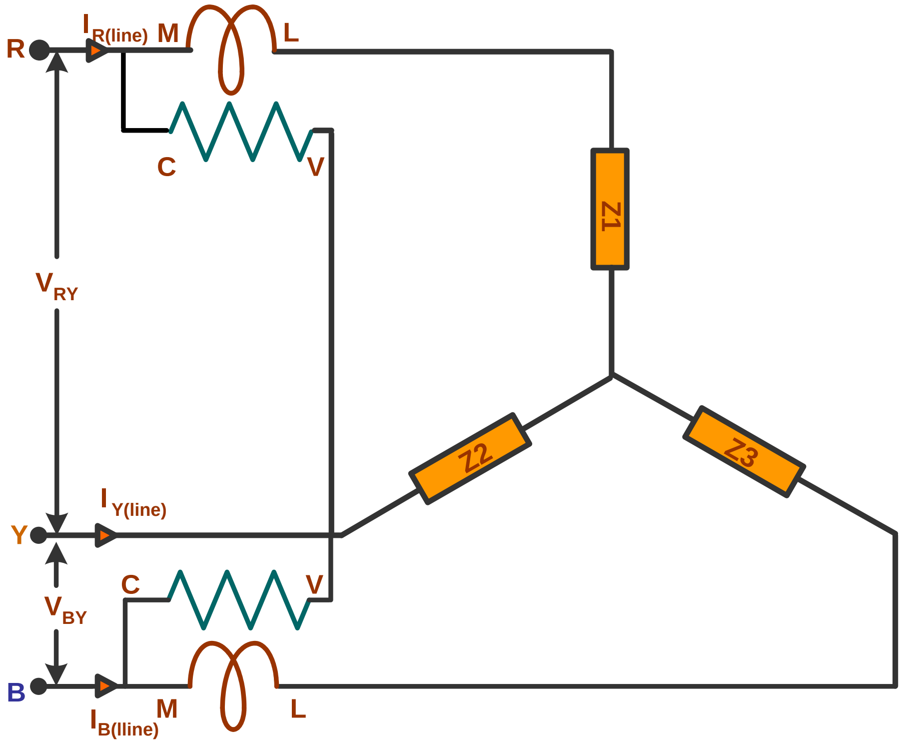


Figure 5.10 Circuit Diagram of Power Measurement by Two-Watt Meter in Three Phase Star Connection

- The load is considered as an inductive load and thus, the phasor diagram of the inductive load is drawn below in Fig. 5.11.

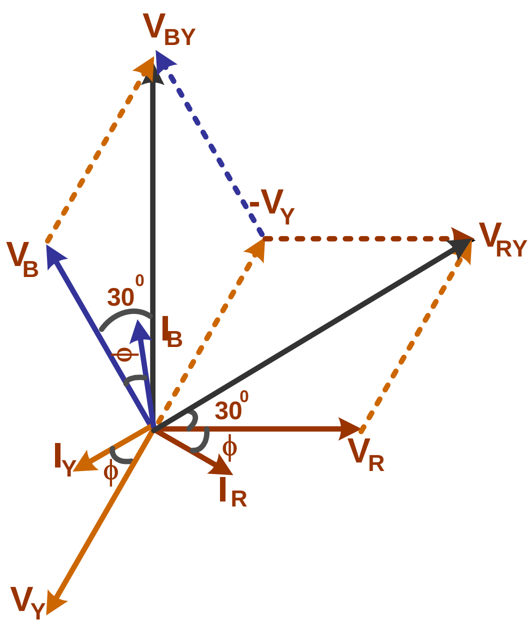


Figure 5.11 Phasor Diagram of Power Measurement by Two-Watt Meter in Three Phase Star Connection

- The three voltages V_{RN}, V_{YN} and V_{BN}, are displaced by an angle of 120^0 degree electrical as shown in the phasor diagram. The phase current lag behind their respective phase voltages by an angle ϕ. The power measured by the Wattmeter, W_1 and W_2.

  Reading of wattmeter, $W_1 = V_{RY} I_R \cos\phi_1 = V_L I_L \cos(30+\phi)$

  Reading of wattmeter, $W_2 = V_{BY} I_B \cos\phi_2 = V_L I_L \cos(30-\phi)$

  Total power, P = W_1+W_2

$$\therefore P = V_L I_L \cos(30+\phi) + V_L I_L \cos(30-\phi)$$
$$= V_L I_L \left[\cos(30+\phi) + \cos(30-\phi)\right]$$
$$= V_L I_L \left[\cos 30 \cos\phi + \sin 30 \sin\phi + \cos 30 \cos\phi - \sin 30 \sin\phi\right]$$
$$= V_L I_L \left[2\cos 30 \cos\phi\right]$$
$$= V_L I_L \left[2\left(\frac{\sqrt{3}}{2}\right)\cos\phi\right]$$
$$= \sqrt{3} V_L I_L \cos\phi$$

- Thus, the sum of the readings of the two wattmeter is equal to the power absorbed in a 3-phase balanced system.

Determination of Power Factor from Wattmeter Readings

- As we know that

$$W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi$$

Now,

$$W_1 - W_2 = V_L I_L \cos(30+\phi) - V_L I_L \cos(30-\phi)$$
$$= V_L I_L \left[\cos 30 \cos\phi + \sin 30 \sin\phi - \cos 30 \cos\phi + \sin 30 \sin\phi\right]$$
$$= V_L I_L \left[2\sin 30 \sin\phi\right]$$
$$= V_L I_L \left[2\left(\frac{1}{2}\right)\sin\phi\right] = V_L I_L \sin\phi$$

$$\therefore \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)} = \frac{\sqrt{3} V_L I_L \sin\phi}{\sqrt{3} V_L I_L \cos\phi} = \tan\phi$$

$$\therefore \tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}$$

- Power factor of load given as,

$$\therefore \cos\phi = \cos\left(\tan^{-1}\frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)}\right)$$

Effect of power factor on wattmeter reading:

- From the Fig. 5.6, it is clear that for lagging power factor $\cos\phi$, the wattmeter readings are

$$W_1 = V_L I_L \cos(30+\phi)$$

$$W_2 = V_L I_L \cos(30-\phi)$$

- Thus, readings W_1 and W_2 will very depending upon the power factor angle ϕ.

p.f	ϕ	$W_1 = V_L I_L \cos(30+\phi)$	$W_2 = V_L I_L \cos(30-\phi)$	Remark
$\cos\phi = 1$	0^0	$\frac{\sqrt{3}}{2} V_L I_L$	$\frac{\sqrt{3}}{2} V_L I_L$	Both equal and +ve
$\cos\phi = 0.5$	60^0	0	$\frac{\sqrt{3}}{2} V_L I_L$	One zero and second total power
$\cos\phi = 0$	90^0	$-\frac{1}{2} V_L I_L$	$\frac{1}{2} V_L I_L$	Both equal but opposite

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CIRCUIT THEORY – SEEA1201

UNIT-IV- TRANSIENT RESPONSE ANALYSIS

Unit IV: TRANSIENT ANALYSIS

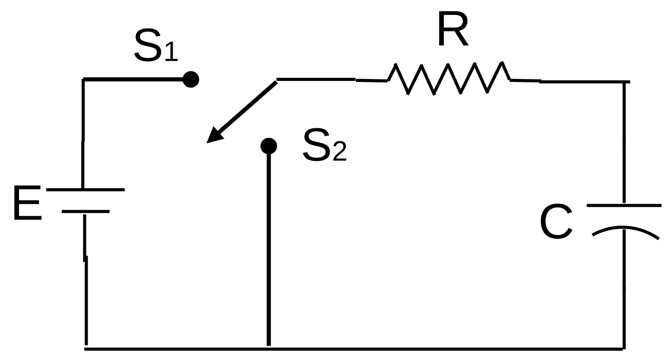
7.1 INTRODUCTION

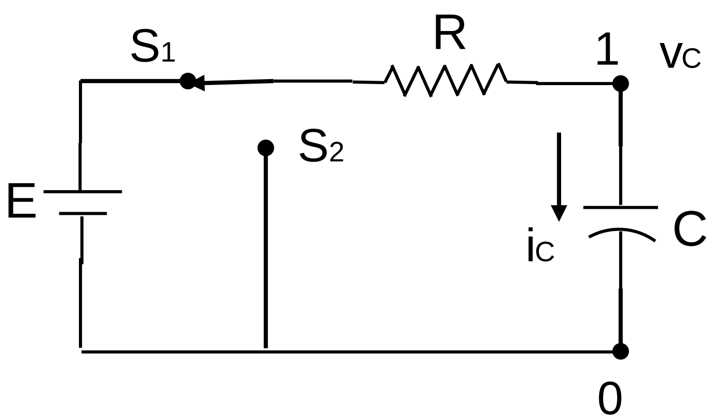
So far steady state analysis of electric circuits was discussed. Electric circuits will be subjected to sudden changes which may be in the form of opening and closing of switches or sudden changes in sources etc. Whenever such a change occurs, the circuit which was in a particular steady state condition will go to another steady state condition. Transient analysis is the analysis of the circuits during the time it changes from one steady state condition to another steady state condition.

Transient analysis will reveal how the currents and voltages are changing during the transient period. To get such time responses, the mathematical models should necessarily be a set of differential equations. Setting up the mathematical models for transient analysis and obtaining the solutions are dealt with in this chapter.

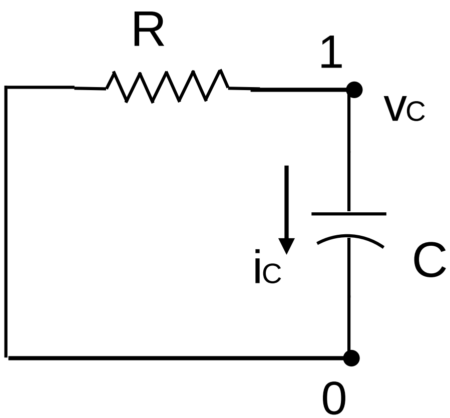
A quick review on various test signals is presented first. Transient response of simple circuits using classical method of solving differential equations is then discussed. Laplace Transform is a very useful tool for solving differential equations. After introducing the Laplace Transform, its application in getting the transient analysis is also discussed.

What is TRANSIENT ANALYSIS?





With steady state condition, at time $t = 0$ switch position is changed from S_1 and S_2



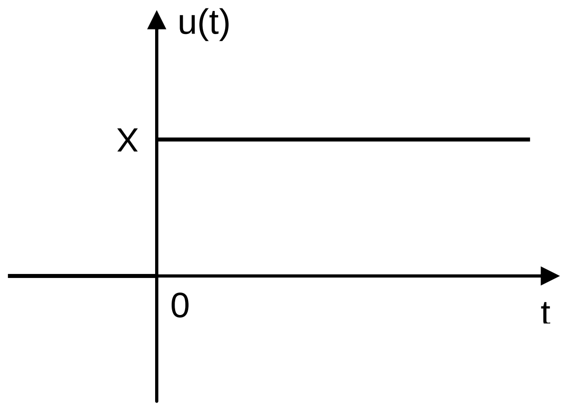
For $t \geq 0$, both v_C and i_C change with respect time.

Step function

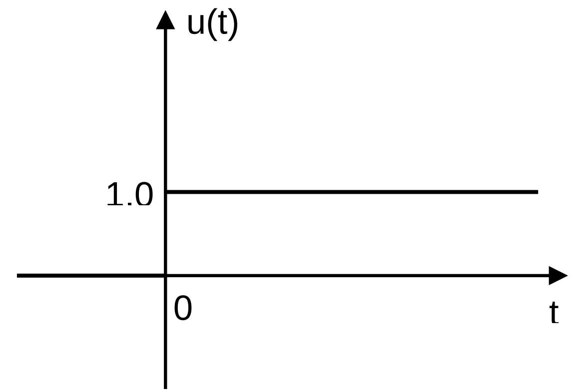
Step function is denoted as u(t) and is described by

$$u(t) = \begin{cases} X & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases} \qquad (7.2)$$

Fig. (a) shows a step function.



(a)



(b)

The step function with X = 1 is called as unit step function. It is described as

$$u(t) = \begin{cases} 1.0 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases} \qquad (7.3)$$

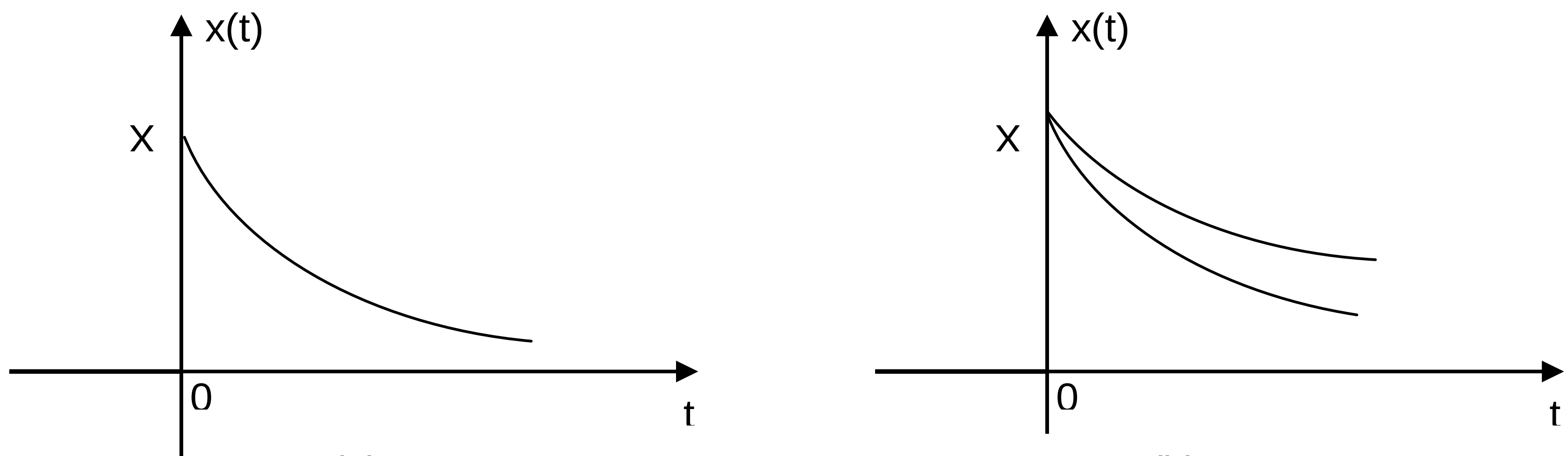
Unit step function is shown in Fig. (b).

Exponentially decaying function

Exponentially decaying function is described by

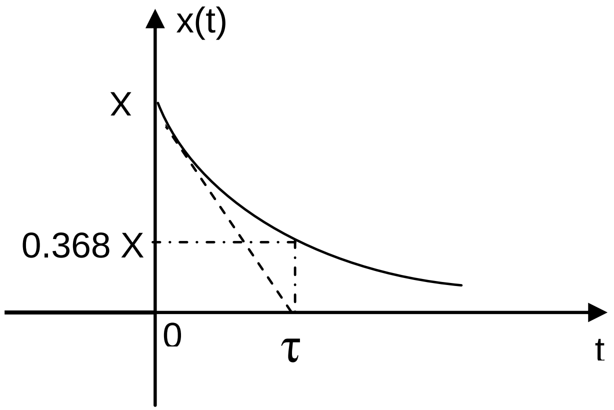
$$\left.\begin{aligned} x(t) &= X\,e^{-\alpha t} \quad \text{for } t \geq 0 \\ &= 0 \quad \text{for } t < 0 \end{aligned}\right\} \qquad (7.4)$$

The value of this function decreases exponentially with time as shown in Fig. below.



(a) (b)

For exponentially decaying function, the time required for the signal to reach zero value, when it is decreased at a constant rate, equal to the rate of decay at time t = 0, is called TIME CONSTANT. Time constant is the measure of rate of decay.



Consider the exponentially decaying signal shown and described by

$$x(t) = X\, e^{-\alpha t} \qquad (7.5)$$

Its slope at time t = 0 is given by

$$\left.\frac{dx}{dt}\right|_{t=0} = -\alpha X e^{-\alpha t}\Big|_{t=0} = -\alpha X \qquad (7.6)$$

Minus sign indicates that the function value is decreasing with increase in time. Then, as stated by the definition, time constant τ is given by $\tau = \dfrac{X}{\alpha X} = \dfrac{1}{\alpha}$ (7.7)

For this exponentially decaying function, knowing $\alpha\tau = 1$, the value of x(t) at time $t = \tau$ is obtained as

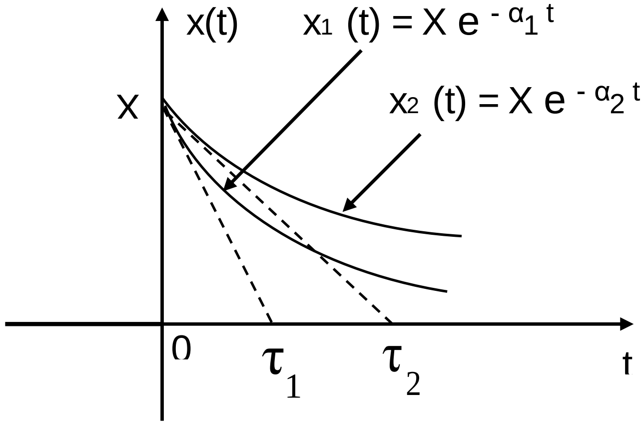
$$x(t)\Big|_{t=\tau} = X e^{-\alpha t}\Big|_{t=\tau} = X e^{-1} = 0.368\,X$$

Therefore, for exponentially decaying function, time constant τ is also defined as the time required for the function to reach 36.8 % of its value at time t = 0. This aspect is shown in previous Fig.

Now consider the two exponentially decaying signals shown. They are described by

$$x_1(t) = X\ e^{-\alpha_1 t}$$

$$x_2(t) = X\ e^{-\alpha_2 t}$$

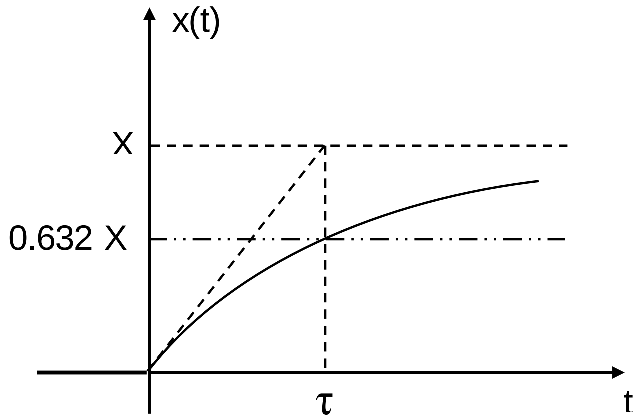


Their time constants are τ_1 and τ_2 respectively. It is seen that $\tau_1 < \tau_2$ and hence $\alpha_1 > \alpha_2$. Further, it can be noted that, smaller the time constant faster is the rate of decay.

Exponentially increasing function

The plot of $x(t) = X\,(1 - e^{-\alpha t})$ (7.36)

is shown in the Fig. It is to be seen that at time t = 0, the function value is zero and the function value tends to X as time t tends to ∞. This is known as exponentially increasing function



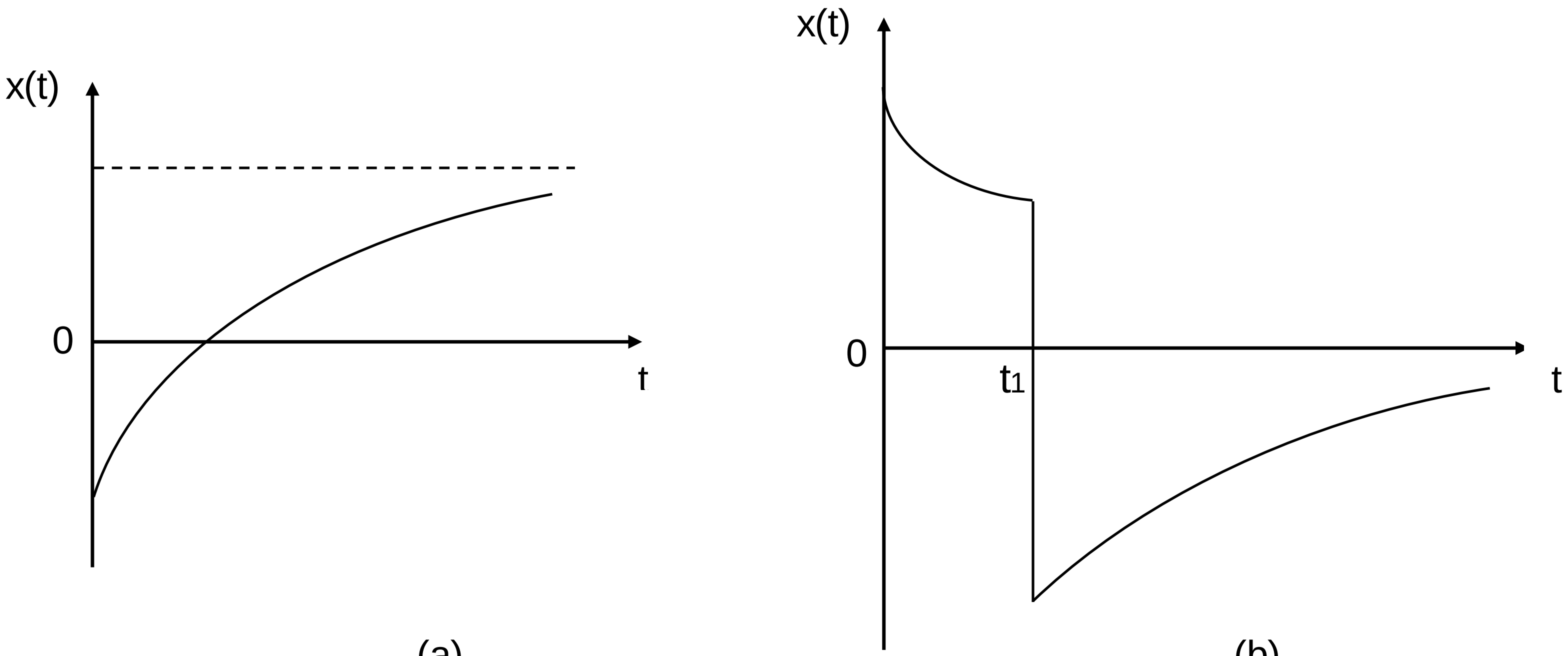
For such exponentially increasing function, time constant, τ is the time required for the function to reach the final value, if the function is increasing at the rate given at time t = 0.

$$\left.\frac{dx}{dt}\right|_{t=0} = 0 + \alpha X e^{-\alpha t}\Big|_{t=0} = \alpha X \qquad \text{Therefore} \quad \tau = \frac{X}{\alpha X} = \frac{1}{\alpha} \qquad (7.37)$$

The value of x(t) at time t = τ is obtained as $x(t) = X\,(1 - e^{-1}) = 0.632\,X$ (7.38)

Thus, for exponentially increasing function, time constant τ is also defined as the time taken for the function to reach 63.2 % of the final value. This is shown in Fig. above.

In the Fig. (a) shown below, x(t) is continuous.



(a) (b)

In Fig. (b) shown, x(t) has discontinuity at time $t = t_1$. The value of $\frac{dx}{dt}$ at time $t = t_1$ tends to infinity.

7.3 CERTAIN COMMON ASPECTS OF RC AND RL CIRCUITS

While doing transient analysis on simple RC and RL circuits, we need to make use of the following two facts.

1. **The voltage across a capacitor as well as the current in an inductor cannot have discontinuity.**

2. **With dc excitation, at steady state, capacitor will act as an open circuit and inductor will act as a short circuit.**

These two aspects can be explained as follows.

The current through a capacitor is given by $i_C = C\ (dv\ /\ dt)$. If the voltage across the capacitor has discontinuity, then at the time when the discontinuity occurs, dv / dt becomes infinity resulting the current i_C to become infinity. However, in physical system, we exclude the possibility of infinite current. Then, we state that in a capacitor, the voltage cannot have discontinuity. Suppose, if the circuit condition is changed at time t = 0, the capacitor voltage must be continuous at time t = 0 and hence $v_C(0^+) = v_C(0^-)$. (7.14)

where time 0^+ refers the time just after t = 0 and time 0^- refers the time just before t = 0.

Similarly the voltage across an inductor is v_L = L (di / dt). If the current through the inductor has discontinuity, then at the time when the discontinuity occurs, di / dt becomes infinity resulting the voltage v_L to become infinity. However, in physical system, we exclude the possibility of infinite voltage. Then, we state that in an inductor, the current cannot have discontinuity. Suppose, if the circuit condition is changed at time t = 0, the inductor current must be continuous at time t = 0 and hence

$$i_L(0^+) = i_L(0^-) \qquad (7.15)$$

With dc excitation, at steady state condition, all the element currents and voltages are of dc in nature. Therefore, both di / dt and dv / dt will be zero. Since i_C = C (dv / dt) and v_L = L (di / dt), with dc excitation, at steady state condition, the current through the capacitor as well as the voltage across the inductor will be zero. <u>In other words, with dc excitation, at steady state condition, the capacitor will act as an open circuit and the inductor will act as a short circuit.</u>

Switching occurs at time $t = 0$

$v_C(0^+) = v_C(0^-)$ $i_L(0^+) = i_L(0^-)$

With DC excitation, at steady state

capacitor acts as OPEN CIRCUIT and

inductor acts as SHORT CIRCUIT

7.4 TRANSIENT IN RC CIRCUIT

While studying the transient analysis of RC and RL circuits, we shall encounter with two types of circuits namely, source free circuit and driven circuit.

Source free circuit

A circuit that does not contain any source is called a source free circuit. Consider the circuit shown in Fig. 7.7 (a). Let us assume that the circuit was in steady state condition with the switch is in position S_1 for a long time. Now, the capacitor is charged to voltage E and will act as open circuit.

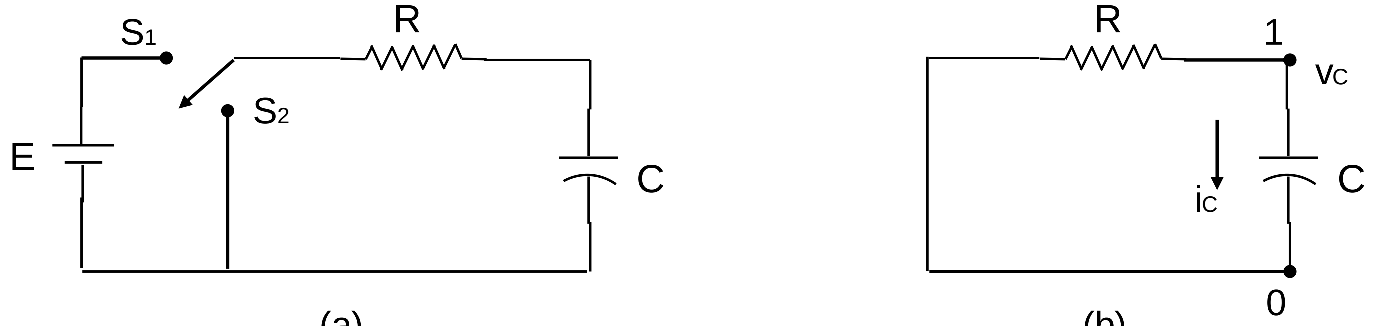


Fig. 7.7 Source free RC circuit.

Suddenly, at time t = 0, the switch is moved to position S_2. The voltage across the capacitor and the current through the capacitor are designated as v_C and i_C respectively. The voltage across the capacitor will be continuous. Hence

$$v_C(0^+) = v_C(0^-) = E \qquad (7.16)$$

The circuit for time t > 0 is shown in Fig. 7.7 (b). We are interested in finding the voltage across the capacitor as a function of time. Later, if required, current through the capacitor can be calculated from $i_C = C\frac{dv}{dt}$. Voltage at node 1 is the capacitor voltage v_C. The node equation for the node 1 is

$$\frac{v_C}{R} + C\frac{dv_C}{dt} = 0 \qquad (7.17)$$

i.e. $$\frac{dv_C}{dt} + \frac{v_C}{RC} = 0 \qquad (7.18)$$

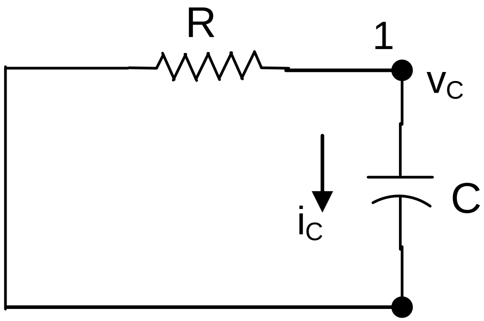


Fig. 7.7 (b)

We have to solve this first order differential equation (DE) with the initial condition

$$v_C(0^+) = E \qquad (7.19)$$

We notice that DE in Eq. (7.18) is a **homogeneous equation** and hence will have **only complementary solution**. Let us try $v_C(t) = K\,e^{st}$ (7.20)

as a possible solution of Eq. (7.18).

$$\frac{dv_C}{dt} + \frac{v_C}{RC} = 0$$ with the initial condition $v_C(0^+) = E$

A possible solution is: $v_C(t) = K e^{st}$

Substituting the solution in the DE. we get

$$s K e^{st} + \frac{1}{RC} K e^{st} = 0 \qquad \text{i.e. } K e^{st} \left(s + \frac{1}{RC} \right) = 0$$

The above equation will be satisfied if

$$K e^{st} = 0 \text{ and or } \left(s + \frac{1}{RC}\right) = 0$$

From Eq. (7.20) it can be seen that $K e^{st} = 0$ will lead to the trivial solution of $v_C(t) = 0$. We are looking for the non-trivial solution of Eq. (7.18). Therefore

$$s + \frac{1}{RC} = 0 \tag{7.21}$$

$$s + \frac{1}{RC} = 0 \qquad (7.21)$$

This is the characteristic equation of the DE given in Eq. (7.18). Its solution $s = -\frac{1}{RC}$ is called the root of the characteristic equation. It is also called as the natural frequency because it characterizes the response of the circuit in the absence of any external source. Thus the solution of the DE (7.18) is obtained by substituting $s = -\frac{1}{RC}$ in the solution $v_C(t) = K\,e^{st}$. Therefore,

$$v_C(t) = K\,e^{-\frac{1}{RC}t} \qquad (7.22)$$

The constant K can be found out by using the initial condition of $v_C(0) = E$ Substituting t = 0 in the above equation, we get

$$v_C(0) = K = E \qquad (7.23)$$

Thus the solution is $$v_C(t) = E\,e^{-\frac{1}{RC}t} \qquad (7.24)$$

Thus the solution is $$v_C(t) = E\, e^{-\frac{1}{RC}t} \tag{7.24}$$

It can be checked that this solution satisfy

$$\frac{dv_C}{dt} + \frac{v_C}{RC} = 0 \text{ with the initial condition } v_C(0^+) = E$$

Obtained solution is sketched in Fig. 7.8. It is an exponentially decaying function.

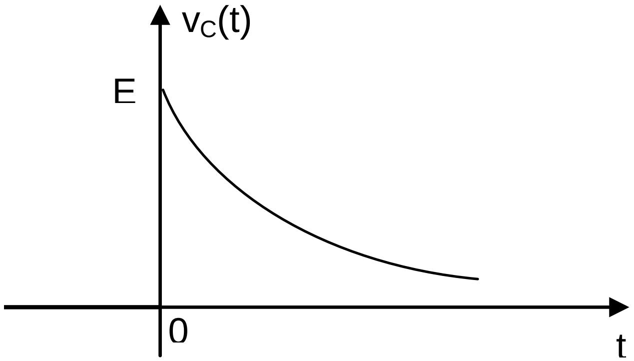


Fig. 7.8 Plot of $v_C(t)$ as given by Equation (7.24).

In this case, the time constant τ = RC. By varying values of R and C, we can get different exponentially decaying function for $v_C(t)$. The dimension of time constant RC can be verified as time as shown below.

$$RC = \frac{\text{volt}}{\text{amp.}}\ \frac{\text{coulomb}}{\text{volt}} = \frac{\text{amp. sec.}}{\text{amp.}} = \text{sec.}$$

$$v_C(t) = E\ e^{-\frac{1}{RC}t} \tag{7.24}$$

The current through the capacitor, in the direction as shown in Fig. 7.7 (b), is given by

$$i_C(t) = C\,\frac{dv_C}{dt} = CE\left(-\frac{1}{RC}\right) e^{-\frac{1}{RC}t}$$

$$= -\frac{E}{R}\, e^{-\frac{1}{RC}t} \tag{7.25}$$

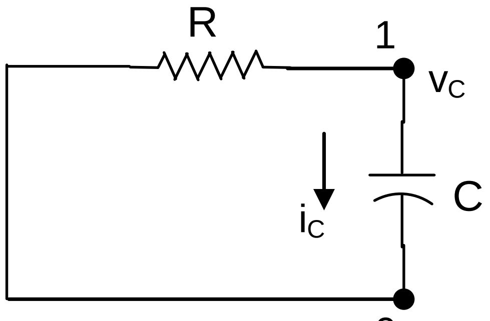


Fig. 7.7 (b)

Since the capacitor is discharging, the current is negative in the direction shown in Fig. 7.7 (b). The plot of capacitor current $i_C(t)$ is shown in Fig. 7.9.

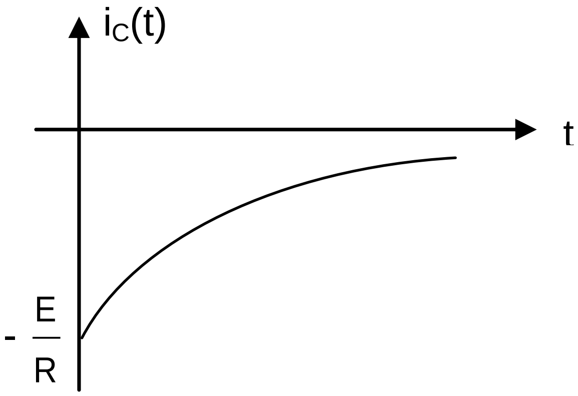


Fig. 7.9 Plot of $i_C(t)$ as given by Equation (7.25).

Driven circuit

Again consider the circuit shown in Fig. 7.7 (a) which is reproduced in Fig. 7.10 (a). Let us say that the switch was in position S_2 long enough so that $v_C(t) = 0$ and $i_C(t) = 0$ i.e. all the energy in the capacitor is dissipated and the circuit is at rest. Now, the switch is moved to position S_1. We shall measure time from this instant. As discussed earlier, since the capacitor voltage cannot have discontinuity,

$$v_C(0^+) = v_C(0^-) = 0 \qquad (7.26)$$

The circuit applicable for time t > 0, is shown in Fig. 7.10 (b).

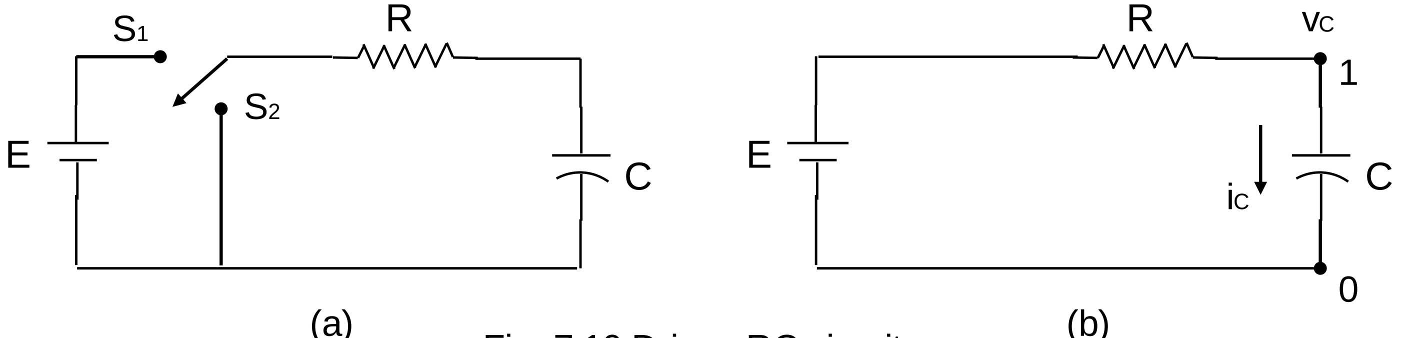


Fig. 7.10 Driven RC circuit.

Node equation for the node 1 gives

$$\frac{v_C - E}{R} + C\frac{dv_C}{dt} = 0 \qquad (7.27)$$

$$\text{i.e.} \frac{dv_C}{dt} + \frac{v_C}{RC} = \frac{E}{RC} \qquad (7.28)$$

$$\frac{dv_C}{dt} + \frac{v_C}{RC} = \frac{E}{RC} \quad (7.28)$$

Unlike in the previous case, now the right hand side is not zero, but contains a term commonly called the forcing function. For this reason, this circuit is classified as driven circuit. The initial condition for the above DE is

$$v_C(0^+) = 0 \quad (7.29)$$

The complete solution is given by

$$v_C(t) = v_{cs}(t) + v_{ps}(t) \quad (7.30)$$

where $v_{cs}(t)$ is the complementary solution and $v_{ps}(t)$ is the particular solution.

The complementary solution $v_{cs}(t)$ is the solution of the homogeneous equation

$$\frac{dv_C}{dt} + \frac{v_C}{RC} = 0 \quad (7.31)$$

Recalling that Eq. (7.22) is the solution of Eq. (7.18), we get

$$v_{cs}(t) = K\, e^{-\frac{1}{RC}t} \quad (7.32)$$

Since the forcing function is a constant, the particular solution can be taken as

$v_{ps}(t) = A$

Since it satisfies the non-homogeneous equation given by Eq. (7.28), $\frac{dv_C}{dt} + \frac{v_C}{RC} = \frac{E}{RC}$

on substitution, we get

$0 + \frac{A}{RC} = \frac{E}{RC}$ i.e. $A = E$.

Thus $\quad v_{ps}(t) = E \qquad (7.33)$

Addition of $v_{cs}(t)$ and $v_{ps}(t)$ yields $\qquad v_C(t) = K\, e^{-\frac{1}{RC}t} + E \qquad (7.34)$

To determine the value of K, apply the initial condition of $v_C(0) = 0$ to the above equation. Thus

$0 = K + E \qquad$ i.e. $K = - E$

Thus, the complete solution is $\quad v_C(t) = - E\, e^{-\frac{1}{RC}t} + E \;=\; E\,(1 - e^{-\frac{1}{RC}t}) \qquad (7.35)$

The plot of capacitor voltage $v_C(t) = E\,(1 - e^{-\frac{1}{RC}t})$ is shown in Fig. 7.11.

For this function, time constant τ is = RC.

The current through the capacitor is calculated as

$$i_C(t) = C\,\frac{dv_C}{dt} = C\,\frac{E}{RC}\,e^{-\frac{1}{RC}t}$$

$$= \frac{E}{R}\,e^{-\frac{1}{RC}t} \qquad (7.39)$$

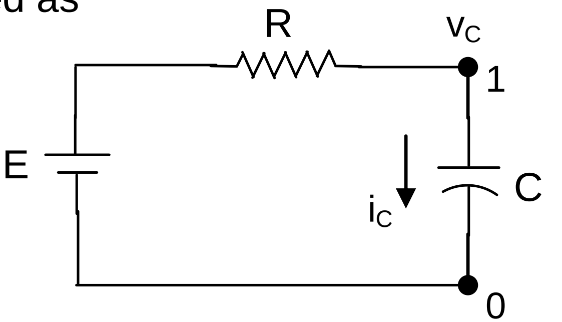


Fig. 7.10 (b)

Now, the capacitor current as marked in Fig. 7.10 (b), is positive and the capacitor gets charged. This capacitor current is plotted as shown in Fig. 7.12.

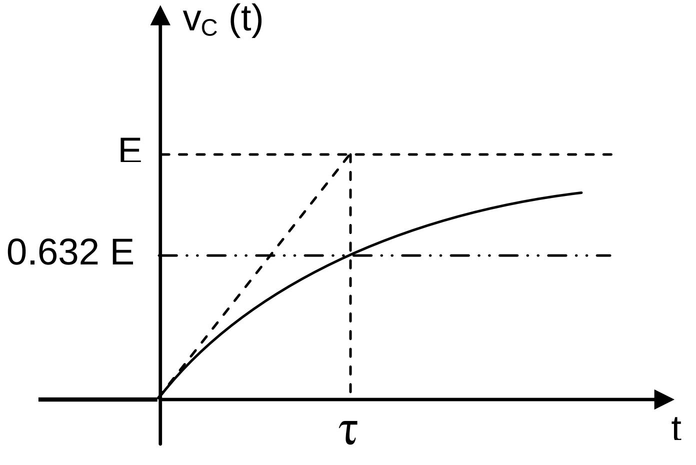


Fig. 7.11 Plot of $v_C(t)$ as given by Eqn. (7.35).

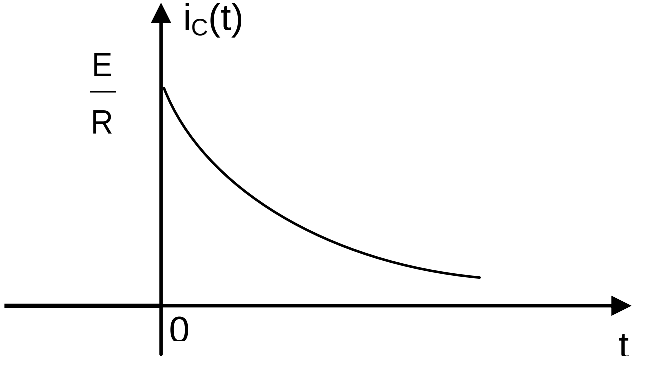


Fig. 7.12 Plot of $i_C(t)$ as given by Eqn. (7.39).

We have solved the circuits shown in Fig. 7.10 (b) and the resulting solutions are shown in Figs. 7.11 and 7.12. They are reproduced in Fig. 7.13.

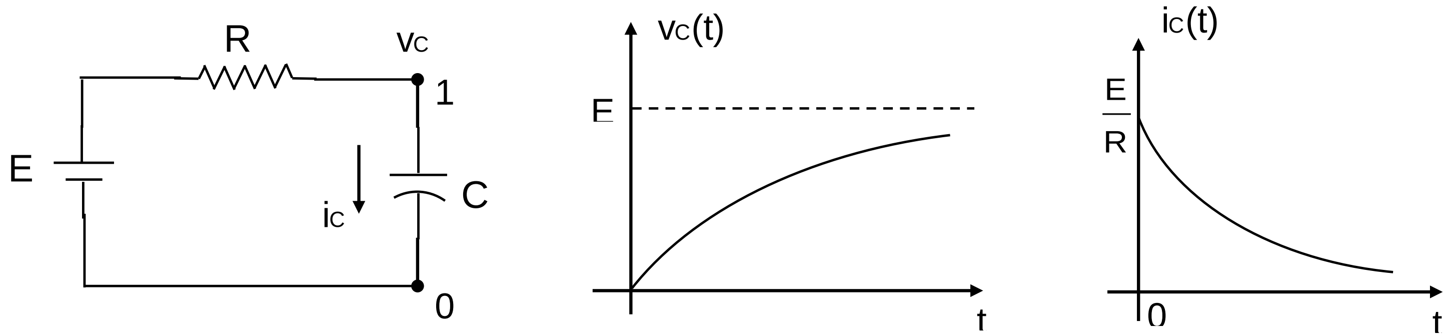


Fig. 7.13 RC driven circuit and voltage and current responses.

These results can be obtained straight away recognizing the following facts.

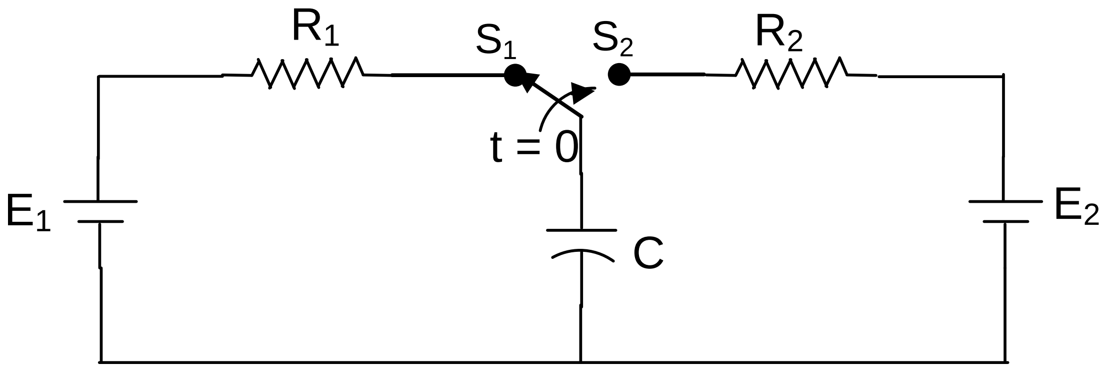
The solution of first order differential equation will be either exponentially decreasing or exponentially increasing. It is known that $v_C(0^+) = 0$. With dc excitation, at steady state, the capacitor will act as open circuit and hence $v_C(\infty) = E$. Thus, the capacitor voltage exponentially increases from 0 to E.

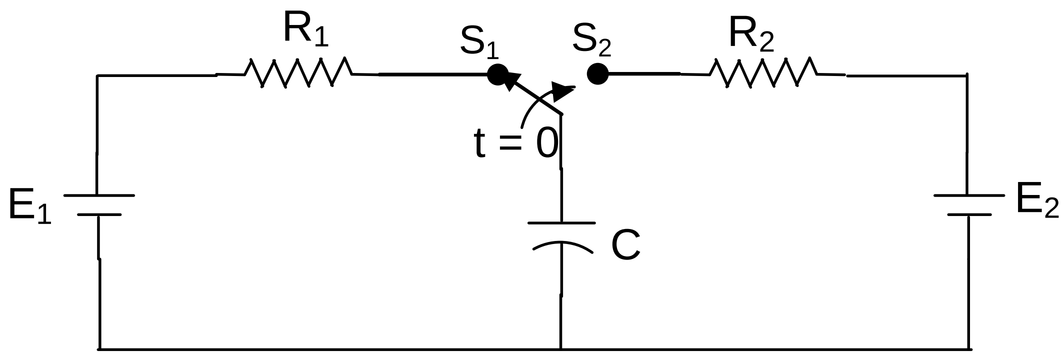
Since $v_C(0^+) = 0$, initially the capacitor is short circuited and hence $i_C(0) = \frac{E}{R}$. With dc excitation, at steady state, the capacitor will act as open circuit and hence $i_C(\infty) = 0$. Thus the capacitor current exponentially decreases from $\frac{E}{R}$ to zero.

Similar reasoning out is possible, in other cases also, to obtain the responses directly.

More general case of finding the capacitor voltage

In the previous discussion, it was assumed that the initial capacitor voltage $v_C(0) = 0$. There may be very many situations wherein initial capacitor voltage is not zero. There may be initial charge in the capacitor resulting non-zero initial capacitor voltage (Example 7.8). Further, the circuit arrangements can also cause non-zero initial capacitor voltage. For this purpose consider the circuit shown below. The switch was in position S_1 for a long time. It is moved from position S_1 to S_2 at time $t = 0$.





We shall assume the following:

1. At time $t = 0^-$ the circuit was at steady state condition with the switch in position S_1

2. After switching to position S_2, the circuit is allowed to reach the steady state condition

Thus, we are interested about the transient analysis for one switching period only.

Initial capacitor voltage $v_C(0)$ is E_1 and the final capacitor voltage $v_C(\infty)$,will be E_2.

The more general expression for the capacitor voltage can be obtained as

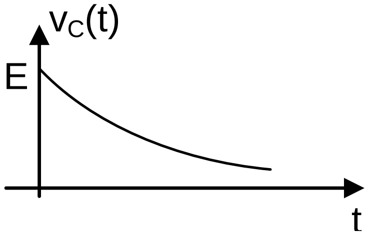
$$v_C(t) = v_C(\infty) + [v_C(0) - v_C(\infty)]\ e^{-\frac{1}{R_2 C}t} \qquad (7.47)$$

Summary of formulae useful for transient analysis on RC circuits

1. Time constant $\tau = RC \qquad \alpha = 1 / RC$

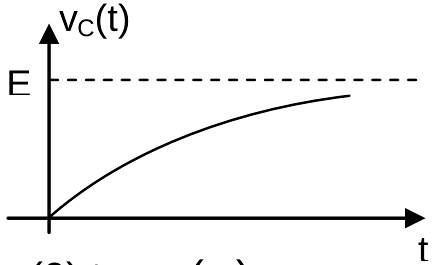
2. When the capacitor is discharging from the initial voltage of E

$$v_C(t) = E\, e^{-\frac{1}{RC}t}$$



3. When the capacitor is charged from zero initial voltage to final voltage of E

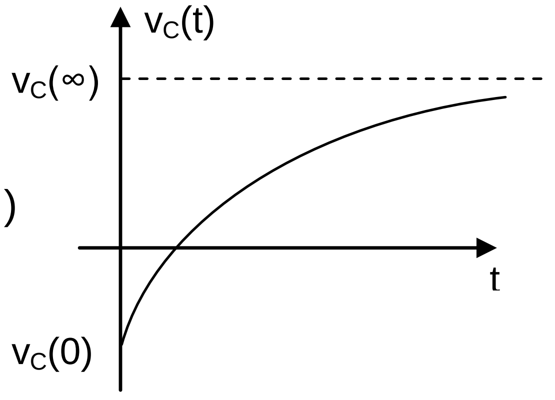
$$v_C(t) = E\left(1 - e^{-\frac{1}{RC}t}\right)$$



4. When the capacitor voltage changes from $v_C(0)$ to $v_C(\infty)$

$$v_C(t) = v_C(\infty) + [v_C(0) - v_C(\infty)]\, e^{-\frac{1}{RC}t}$$

Plot of $v_C(t)$ depends on values of $v_C(0)$ and $v_C(\infty)$



5. Capacitor current $i_C(t) = C\dfrac{dv_C(t)}{dt}$

Example 7.1 An RC circuit has R = 20 Ω and C = 400 μF. What is its time constant?

Solution For RC circuit, time constant τ = RC.

Therefore, $\tau = 20 \times 400 \times 10^{-6}$ s = 8 ms

Example 7.2 A capacitor in an RC circuit with R = 25 Ω and C = 50 μF is being charged with initial zero voltage. What is the time taken for the capacitor voltage to reach 40 % of its steady state value?

Solution With R = 25 Ω and C = 50 μF, $\tau = RC = 1.25 \times 10^{-3}$ s; hence $1/RC = 800\ s^{-1}$.

Taking the capacitor steady state voltage as E, $v_C(t) = E\left(1 - e^{-\frac{1}{RC}t}\right)$

Let t_1 be the time at which the capacitor voltage becomes 0.4 E. Then

$$0.4\,E = E\,(1 - e^{-800\,t_1}) \text{ i.e. } 0.4 = 1 - e^{-800\,t_1}$$

$$e^{-800\,t_1} = 0.6 \text{ i.e. } -800\,t_1 = \ln 0.6 = -0.5108$$

$$\text{Therefore, } t_1 = \frac{0.5108}{800}\,\text{s} = 0.6385 \times 10^{-3}\ \text{s} = 0.6385\ \text{ms}$$

Example 7.3 In an RC circuit, having a time constant of 2.5 ms, the capacitor discharges with initial voltage of 80 V. (a) Find the time at which the capacitor voltage reaches 55 V, 30 V and 10 V (b) Calculate the capacitor voltage at time 1.2 ms, 3 ms and 8 ms.

Solution (a) Time constant RC = 2.5 ms; Thus $\frac{1}{RC} = \frac{1000}{2.5} = 400\ s^{-1}$

During discharge, capacitor voltage is given by $v_C(t) = 80\ e^{-400\,t}$ V

Let t_1, t_2 and t_3 be the time at which capacitor voltage becomes 55 V, 30 V and 10 V.

$$55 = 80\,e^{-400 t_1};\ -400\ t_1 = \ln \frac{55}{80} = -0.3747;\ \text{Thus } t_1 = 0.93765\ \text{ms}$$

$$30 = 80\ e^{-400 t_2};\ -400\ t_2 = \ln \frac{30}{80} = -0.9808;\ \text{Thus } t_2 = 2.452\ \text{ms}$$

$$10 = 80\ e^{-400 t_3};\ -400\ t_3 = \ln \frac{10}{80} = -2.0794;\ \text{Thus } t_3 = 5.1985\ \text{ms}$$

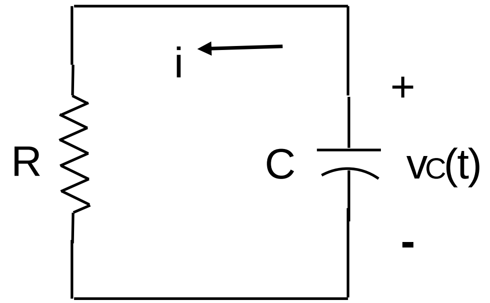
(b)

$$v_C(1.2\text{x } 10^{-3}) = 80\ e^{-400\,t} = 80\ e^{-0.48} = 49.5027\ \text{V}$$

$$v_C(3\text{x } 10^{-3}) = 80\ e^{-400\,t} = 80\ e^{-1.2} = 24.0955\ \text{V}$$

$$v_C\ (8\text{x } 10^{-3}) = 80\ e^{-400\,t} = 80\ e^{-3.2} = 3.261\ \text{V}$$

Example 7.4 Consider the circuit shown below.



Given

$v_C(t) = 56\ e^{-250\,t}$ V for $t > 0$

$i(t) = 7\ e^{-250\,t}$ mA for $t > 0$

(a) Find the values of R and C. (b) Determine the time constant.

(c) At what time the voltage $v_C(t)$ will reach half of its initial value?

Solution (a) Given that $v_C(t) = 56\ e^{-250\,t}$ V. Therefore $\tau = RC = \dfrac{1}{250}$ s

Resistance $R = \dfrac{v_C(t)}{i(t)} = 8000\,\Omega$; Thus capacitance $C = \dfrac{1}{250 \text{ X } 8000}$ F $= 0.5\,\mu\text{F}$

(b) Time constant = RC = 4×10^{-3} s = 4 ms

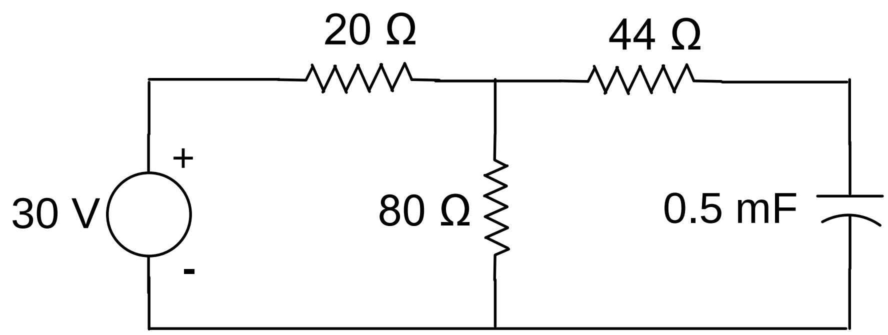
(c) Let t_1 be the time taken for the voltage to reach half of its initial value of 56 V.

Then, $56\,e^{-250\,t_1} = 28$; i.e. $e^{-250\,t_1} = 0.5$ i.e. $-250\ t_1 = \ln 0.5 = -0.6931$;

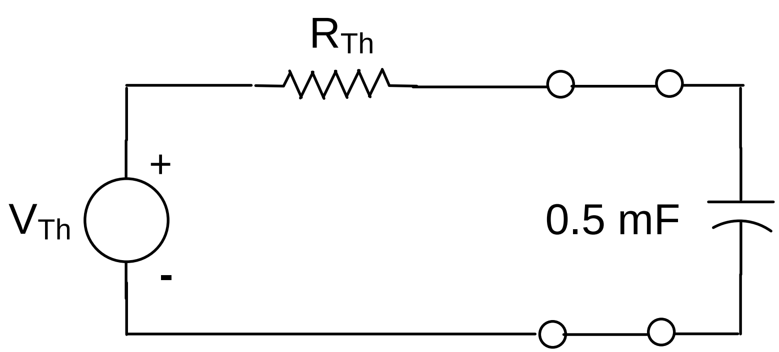
$$\text{Time } t_1 = \frac{0.6931}{250}\,\text{s} = 2.7724 \times 10^{-3}\ \text{s} = 2.7724\ \text{ms}$$

Example 7.5

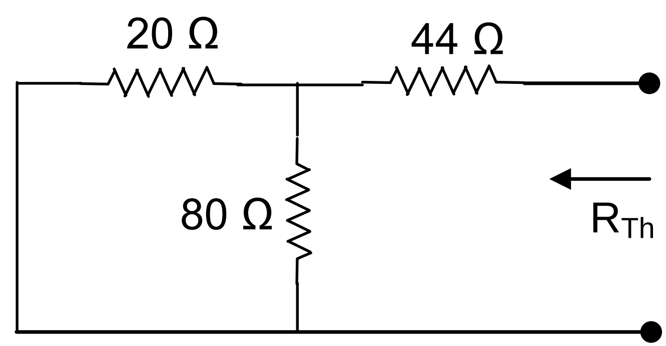
Find the time constant of the RC circuit shown in below.



Solution Thevenin's equivalent across the capacitor, is shown below.



(a)

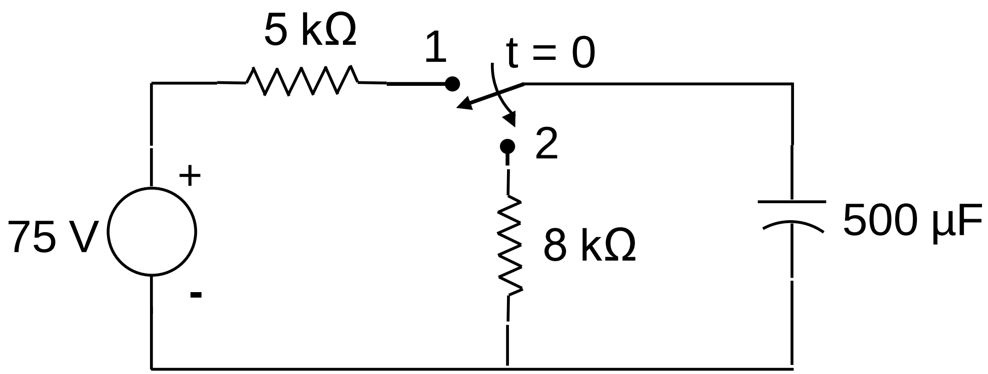


(b)

Referring to Fig. (b) above, $R_{Th} = 44 + (20 || 80) = 60\ \Omega$

Time constant $\tau = RC = 60 \times 0.5 \times 10^{-3}$ s = 30 ms

Example 7.6 The switch in circuit shown was in position1 for a long time. It is moved from position 1 to position 2 at time $t = 0$. Sketch the wave form of $v_C(t)$ for $t > 0$.



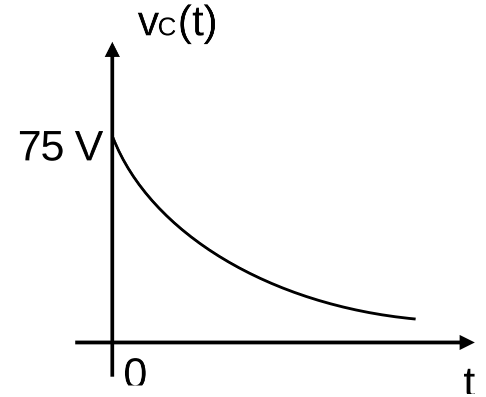
Solution With switch is in position 1, capacitor gets charged to a voltage of 75 V. i.e. $v_C(0^+) = 75$ V. The switch is moved to position 2 at time $t = 0$.

Time constant $RC = 8 \times 10^3 \times 500 \times 10^{-6} = 4$ s

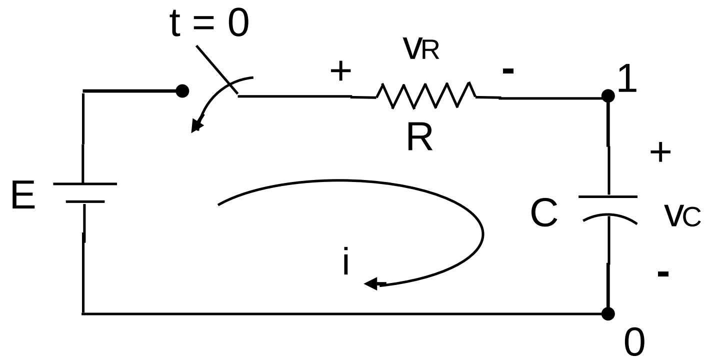
Finally the capacitor voltage decays to zero. Thus,

$$v_C(t) = 75\ e^{-0..25\ t}$$

Wave form of the capacitor voltage is shown.



Example 7.7 A series RC circuit has a constant voltage of E, applied at time t = 0 as shown in Fig. below. The capacitor has no initial charge. Find the equations for i, v_R and v_C. Sketch the wave shapes.



E = 100 V

R = 5000 Ω

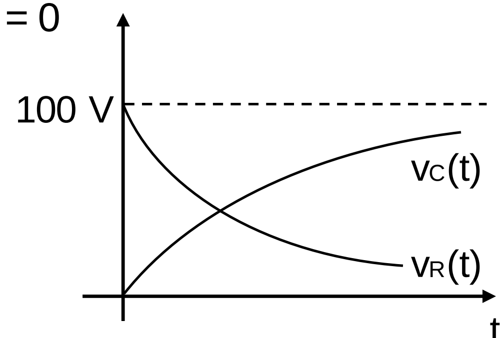
C = 20 µF

Solution Since there is no initial charge, $v_C(0^+) = v_C(0^-) = 0$

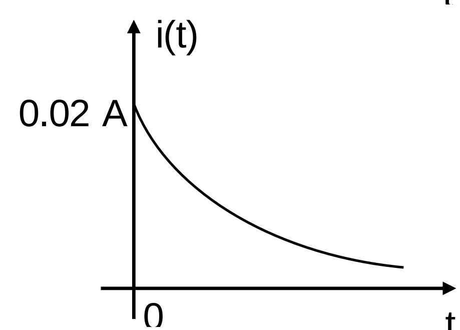
For t > 0, capacitor is charged to final voltage of 100 V.

Time constant $RC = 5000 \times 20 \times 10^{-6} = 0.1$ sec.

$v_C(t) = E\,(1 - e^{-\frac{1}{RC}t})$. Thus, $v_C(t) = 100\,(1 - e^{-10\,t})$ V



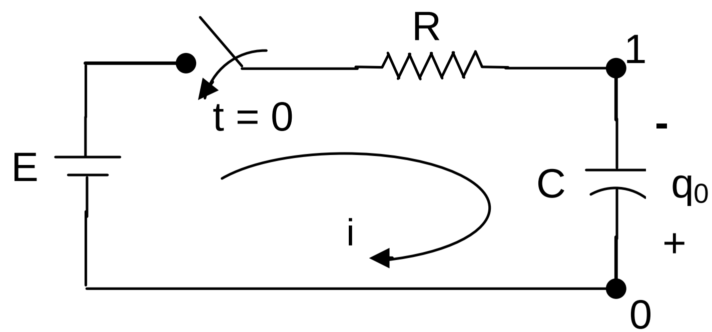
$$i(t) = C\,\frac{dv_C}{dt} = 20 \times 10^{-6} \times 100 \times 10\, e^{-10\,t} = 0.02\, e^{-10\,t} \text{ A}$$



Voltage across the resistor is $v_R(t) = R\, i(t) = 100\, e^{-10\,t}$ V

Wave shapes of i, $v_R(t)$ and $v_C(t)$ are shown.

Example 7.8 A 20 μF capacitor in the RC circuit shown has an initial charge of q_0 = 500 μC with the polarity as shown. The switch is closed at time t = 0. Find the current transient and the voltage across the capacitor. Find the time at which the capacitor voltage is zero. Also sketch their wave shape.



E = 50 V

R = 1000 Ω

C = 20 μF

Solution Initial charge of q_0 in the capacitor is equivalent to initial voltage of

$$v_C(0) = -\frac{q_0}{C} = -\frac{500 \text{ X } 10^{-6}}{20 \text{ X } 10^{-6}} = -25 \text{ V};$$

Further, $v_C(\infty) = E = 50$ V

Time constant RC = 1000 X 20 X 10^{-6} = 20 X 10^{-3} s. Thus 1/RC = 50 s^{-1}

$$v_C(t) = v_C(\infty) + [v_C(0) - v_C(\infty)]\ e^{-\frac{1}{RC}t}$$

$$v_C(t) = 50 + [-25 - 50]\ e^{-50t} = 50 - 75\ e^{-50t}$$

$$\text{Current } i(t) = C\frac{dv_C}{dt} = 20 \text{ X } 10^{-6} \text{ X } 75 \text{ X } 50\ e^{-50t} \text{ A} = 0.075\ e^{-50t} \text{ A}$$

Let t_1 be the time at which the capacitor voltage becomes zero. Then

$50 - 75\, e^{-50\, t_1} = 0$ i.e. $e^{-50\, t_1} = 0.6667$

$-50\, t_1 = -0.4054$ i.e. $t_1 = 8.108 \times 10^{-3}$ s

The capacitor voltage becomes zero at time $t_1 = 8.108$ ms

Wave forms are shown in Fig. 7.24

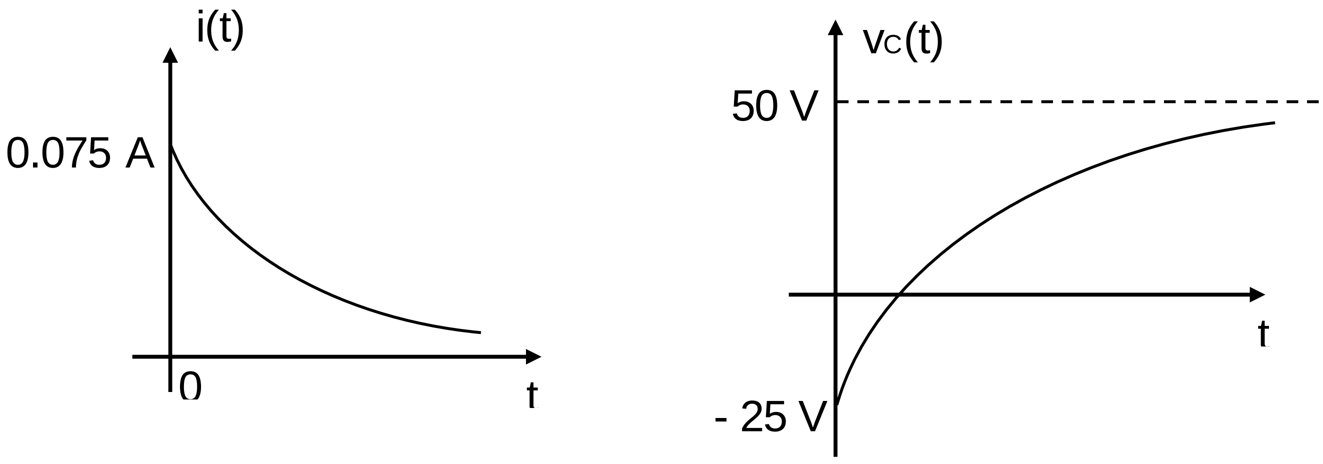
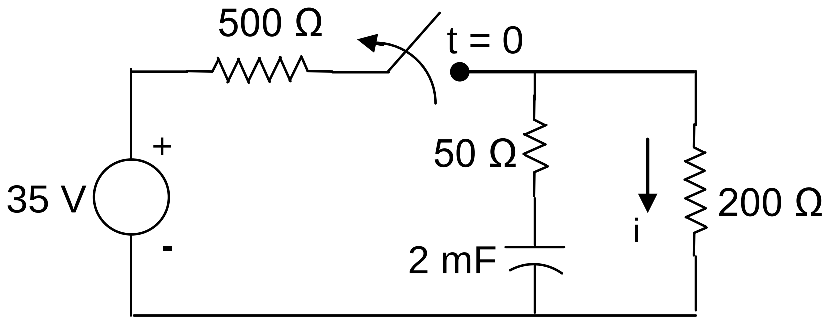


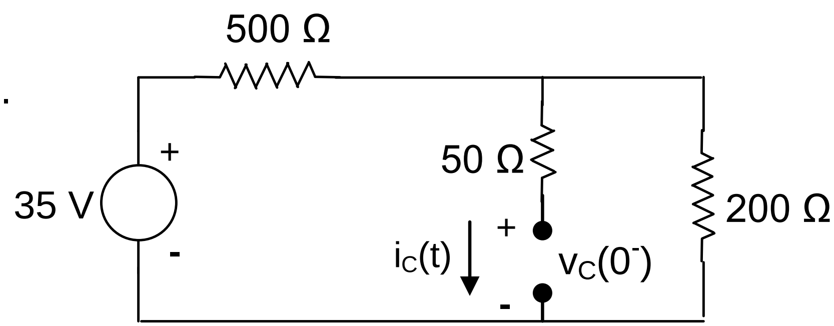
Fig. 7.24 Wave forms - Example 7.8.

Example 7.9

Consider the circuit shown below. The switch was in closed position for a long time. It is opened at time t = 0. Find the current i(t) for t > 0.



Solution Circuit at time $t = 0^-$ is shown.



$$v_C(0^-) = 35 \text{ X } \frac{200}{200 + 500} = 10 \text{ V}$$

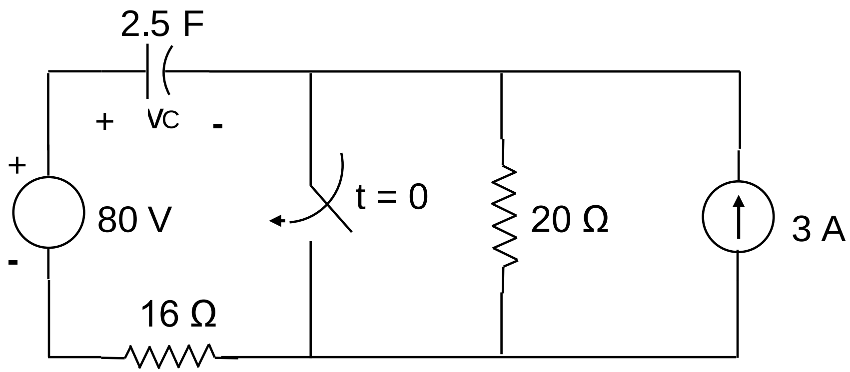
For time t > 0, capacitor voltage of 10 V is discharged through a resistor of 250 Ω.

Time constant $RC = 250 \text{ X } 2 \text{ X } 10^{-3} = 0.5$ s; $v_C(t) = 10\, e^{-2t}$ V

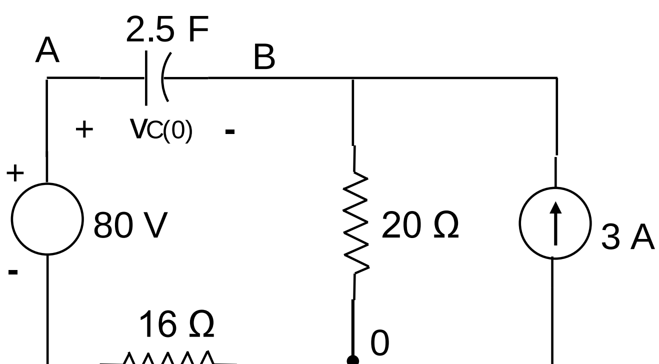
$$i_C(t) = C\,\frac{dv_C}{dt} = 2 \text{ X } 10^{-3} \text{ X } (-20)\, e^{-2t} \text{ A} = -40 \text{ X } 10^{-3}\, e^{-2t} \text{ A} = -0.04\, e^{-2t} \text{ A}$$

Thus $i(t) = -i_C(t) = 0.04\, e^{-2t}$ A

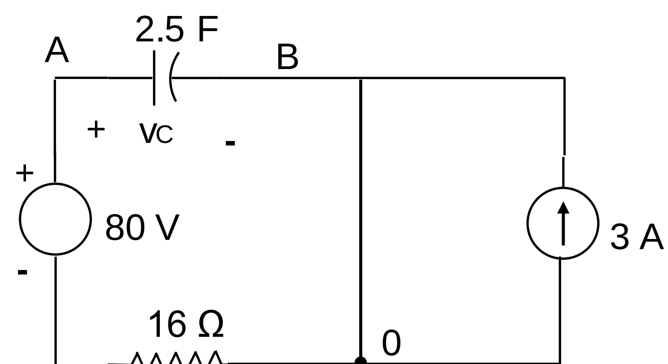
Example 7.10 Consider the circuit shown. The switch was in open position for a long time. It is operated as shown. Compute and plot the capacitor voltage for $t > 0$. Also find the time at which the capacitor voltage is 50 V.



Solution Circuit at time $t = 0^-$ is shown in Fig. (a).



(a)



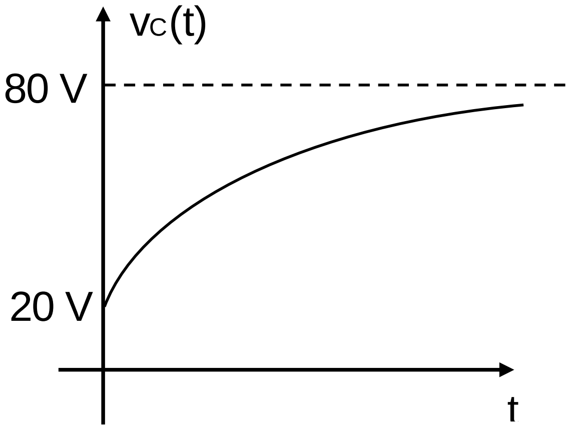
(b)

Capacitor acts as open circuit. $I_{16\,\Omega} = 0$. Voltage $V_A = 80$ V and voltage $V_B = 60$ V

Thus $v_C(0) = 20$ V

With the switch is in closed position, the circuit will be as shown in Fig. (b). With the steady state reached, Capacitor acts as open circuit. $I_{16\,\Omega} = 0$.

Voltage $V_A = 80$ V and voltage $V_B = 0$ V. Thus $v_C(\infty) = 80$ V



RC = 16 X 2.5 = 40 s

Using $v_C(t) = v_C(\infty) + [v_C(0) - v_C(\infty)]\ e^{-\frac{1}{RC}t}$ we get

$v_C(t) = 80 + [20 - 80]\ e^{-0.025\,t} = 80 - 60\ e^{-0.025\,t}$ V

Plot of the capacitor voltage is shown.

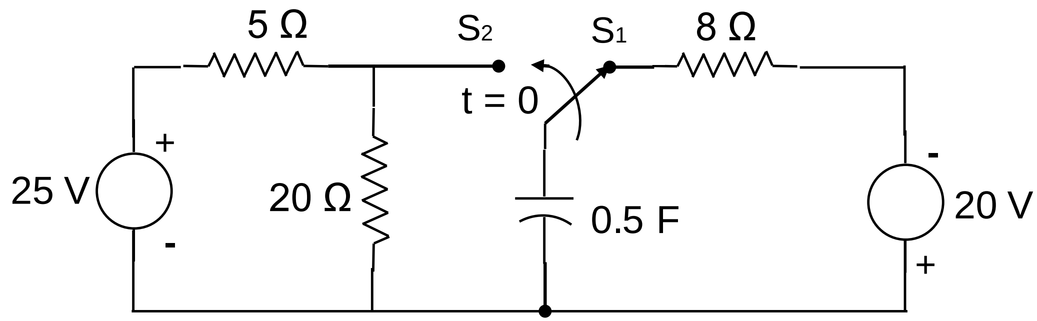
Let t_1 be the time at which the capacitor voltage = 50 V. Then

$80 - 60\ e^{-0.025t_1} = 50$ i.e. $60\ e^{-0.025t_1} = 30$ i.e. $e^{-0.025t_1} = 0.5$ i.e. $-0.025\,t_1 = -0.6932$

Thus $t_1 = 27.728$ s

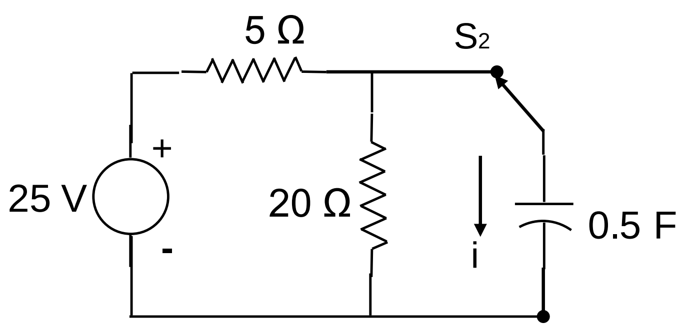
Capacitor voltage becomes 50 V at time $t_1 = 27.728$ s

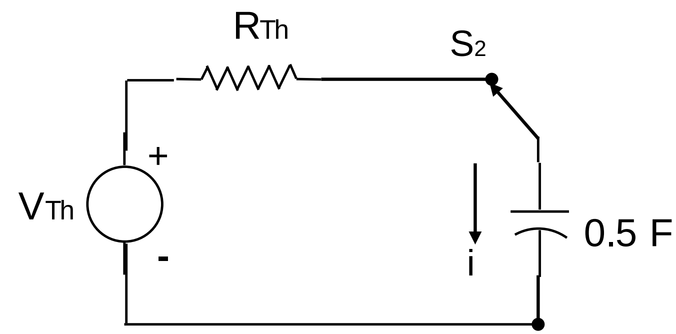
Example 7.11 Consider the circuit shown below. The switch was in position S_1 for a long time. It is operated as shown. Compute and plot the capacitor voltage for $t > 0$. Also find the time at which the capacitor voltage becomes zero.



Solution Voltage $v_C(0) = -20$ V

Circuit for time $t > 0$ and its Thevenin's equivalent are shown below.





$$V_{Th} = \frac{20}{20+5} \times 25 = 20 \text{ V}$$

$R_{Th} = 5 \,||\, 20 = 4\ \Omega$; Thus $RC = 4 \times 0.5 = 2$ s

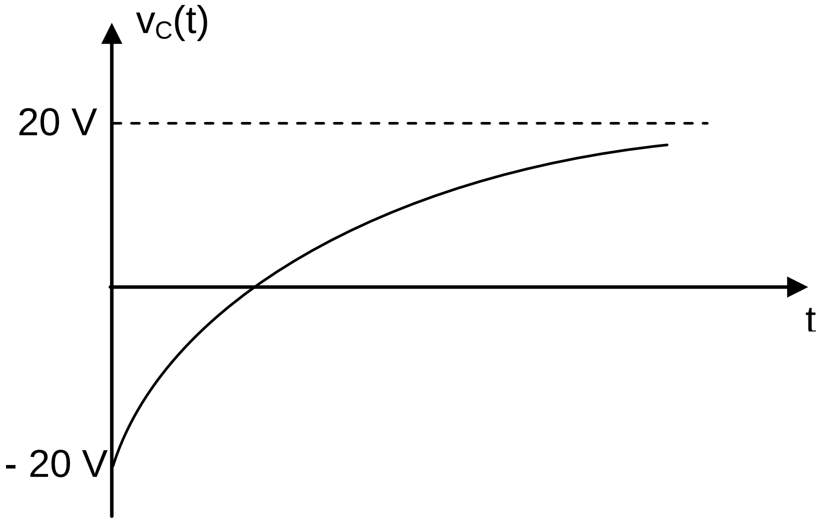
Using $v_C(t) = v_C(\infty) + [v_C(0) - v_C(\infty)]\ e^{-\frac{1}{RC}t}$ we get

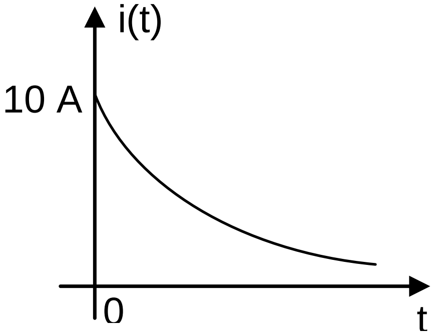
$$v_C(t) = 20 + [-20 - 20]\, e^{-0.5t} = 20 - 40\, e^{-0.5t} \text{ V}$$

$v_C(t) = 20 - 40\, e^{-0.5\,t}$ V

$$i_C(t) = C\,\frac{dv_C}{dt} = 0.5 \times 20\, e^{-0.5\,t} \text{ A} = 10\, e^{-0.5\,t} \text{ A}$$

Wave shapes of $v_C(t)$ and $i_C(t)$ are shown below.





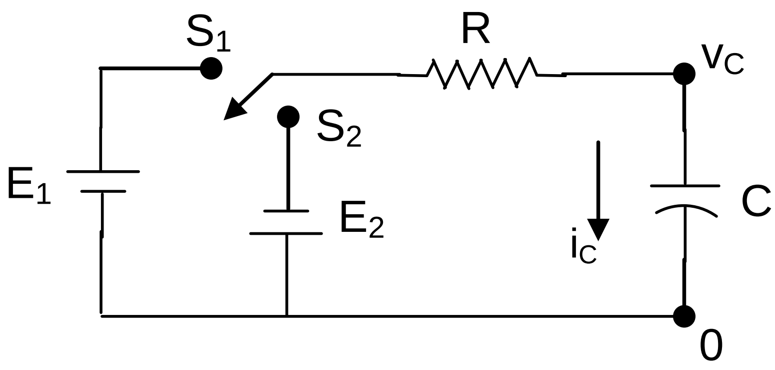
Let t_1 be the time at which the capacitor voltage reaches zero value. Then

$20 - 40\; e^{-0.5t_1} = 0$; i.e. $e^{-0.5t_1} = 0.5$; i.e. $-0.5\, t_1 = -0.6931$; Thus $t_1 = 1.3863$ s

Capacitor voltage reaches zero value at time $t_1 = 1.3863$ s

So far we have done transient analysis for one switching period. Now we shall illustrate how to carry out transient analysis for two switching period through an example.

Example 7.12 In the initially relaxed RC circuit shown the switch is closed on to position S_1 at time t = 0. After one time constant, the switch is moved on to position S_2. Find the complete capacitor voltage and current transients and show their wave forms.



E_1 = 20 V; E_2 = 40 V

R = 500 Ω

C = 0.5 µF

Solution RC = 500 X 0.5 X 10^{-6} s = 0.25 X 10^{-3} s = 0.25 ms 1/RC = 4000 s^{-1}

During the first switching period, capacitor gets charged from zero volt. Its voltage exponentially increases towards 20 V. Thus

$$v_C(t) = 20\,(1 - e^{-4000\,t})\ \text{V}$$

At t = 1 time constant, $v_C = 20\,(1 - e^{-1}) = 12.64$ V

For the second switching operation, there is initial capacitor voltage of 12.64 V.

Let the second switching occurs at time $t' = 0$. Time $t' = 0$ implies time $t = 0.25 \times 10^{-3}$ s i.e. $t' = t - 0.25 \times 10^{-3}$. For $t' > 0$, capacitor voltage changes from its initial value, $v_C(0)$, of 12.64 V to final value, $v_C(\infty)$, of - 40 V. Knowing that

$$v_C(t) = v_C(\infty) + [v_C(0) - v_C(\infty)]\ e^{-\frac{1}{RC}t}$$ we get

$$v_C(t') = -40 + [12.64 + 40]\ e^{-4000\,t'} = 52.64\ e^{-4000\,t'} - 40 \text{ V}$$

Therefore, capacitor voltages for the two switching periods are

$$v_C(t) = 20\,(1 - e^{-4000\,t}) \text{ V} \quad \text{for } t > 0 \text{ and } \leq 0.00025 \text{ s}$$

$$v_C(t) = 52.64\ e^{-4000\,(t - 0.00025)} - 40 \text{ V for } t \geq 0.00025 \text{ s}$$

with $v_C(0.00025^-) = v_C(0.00025^+) = 12.64$ V

(Note that the capacitor voltage shall maintain continuity)

Knowing that

$v_C(t) = 20\,(1 - e^{-4000\,t})$ V for $t > 0$ and ≤ 0.00025 s

$v_C(t) = 52.64\, e^{-4000\,(t - 0.00025)} - 40$ V for $t \geq 0.00025$ s

For the first switching period

Capacitor current $i_C(t) = C\,\dfrac{dv_C}{dt} = 0.5 \text{ X } 10^{-6} \text{ X } 20 \text{ X } 4000\, e^{-4000\,t} = 0.04\, e^{-4000\,t}$ A

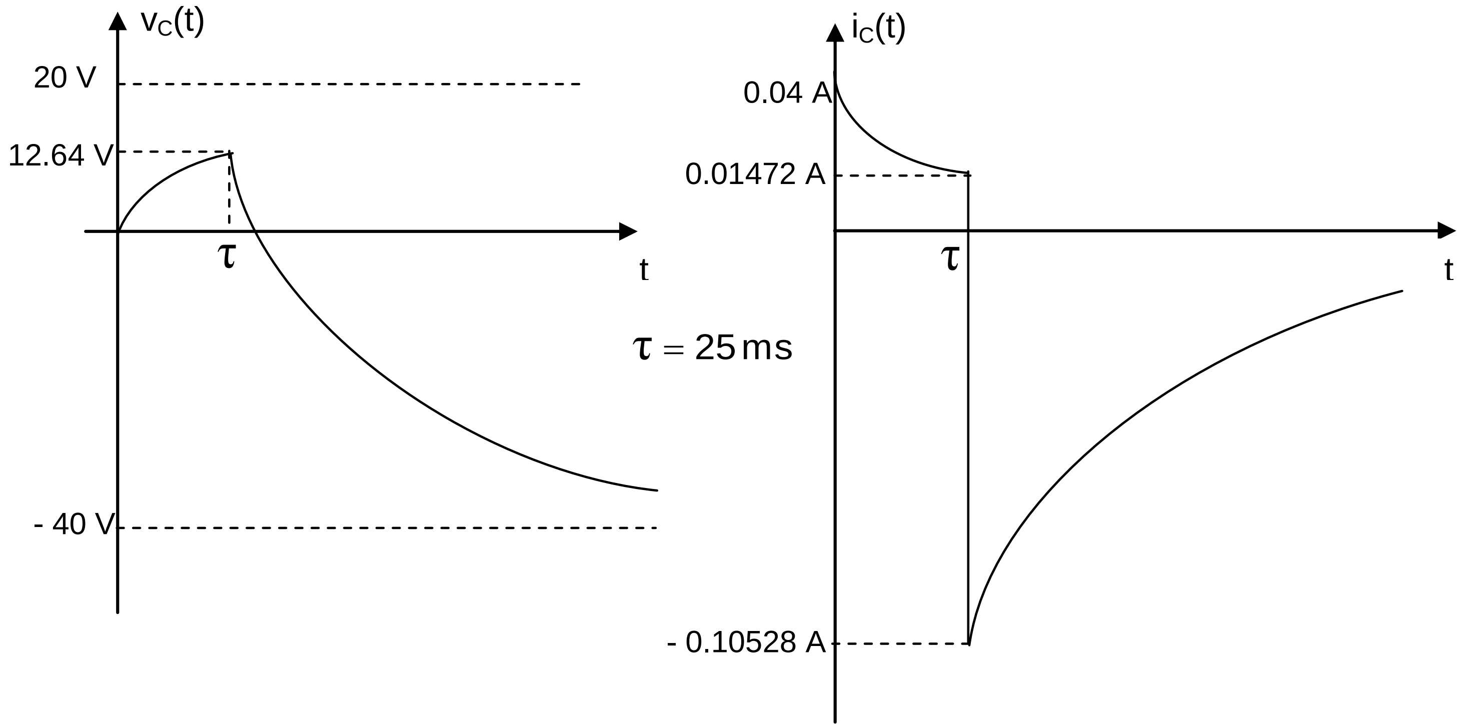
$i_C(0.00025^-) = 0.04\, e^{-1} = 0.01472$ A

For the second switching period,

$v_C(t') = 52.64\, e^{-4000\,t'} - 40$ V

$i_C(t') = 0.5 \text{ X } 10^{-6} \text{ X } (-52.64 \text{ X } 4000\, e^{-4000\,t'}) = -0.10528\, e^{-4000\,t'}$ A

i.e. $i_C(t - 0.00025) = -0.10528\, e^{-4000\,(t - 0.00025)}$ A $\qquad i_C(0.00025^+) = -0.10528$ A



Note: At the switching time, voltage across the capacitor does not have discontinuity i.e. $v_C(0.25 \text{ X } 10^{-3})^- = v_C(0.25 \text{ X } 10^{-3})^+$. On the other hand, the current through the capacitor has discontinuity at the instant of switching. The current just before switching and just after switching can be calculated by considering the circuit conditions at the respective time. At time $t = (0.25 \text{ X } 10^{-3})^-$, current $i = \dfrac{20 - 12.64}{500} = 0.01472$ A

At time $t = (0.25 \text{ X } 10^{-3})^+$, current $i = \dfrac{-40 - 12.64}{500} = -0.10528$ A

RC Circuit	RL Circuit
$\tau = RC \qquad \alpha = 1 / RC$	$\tau = L / R \qquad \alpha = R / L$
Switching at t = 0 $\quad v_C(0^+) = v_C(0^-)$	Switching at t = 0 $\quad i_L(0^+) = i_L(0^-)$
With DC, at SS capacitor acts as open circuit	With DC, at SS inductor acts as short circuit
$v_C(0) \neq 0;\ v_C(\infty) = 0;$ Then $v_C(t) = v_C(0)\ e^{-\frac{1}{RC}t}$	$i_L(0) \neq 0;\ i_L(\infty) = 0;$ Then $i_L(t) = i_L(0)\ e^{-\frac{R}{L}t}$
$v_C(0) = 0;\ v_C(\infty) \neq 0;$ Then $v_C(t) = v_C(\infty)\ (1 - e^{-\frac{1}{RC}t})$	$i_L(0) = 0;\ i_L(\infty) \neq 0;$ Then $i_L(t) = i_L(\infty)\ (1 - e^{-\frac{R}{L}t})$
$v_C(0) \neq 0;\ v_C(\infty) \neq 0;$ Then $v_C(t) = v_C(\infty) + [v_C(0) - v_C(\infty)]\ e^{-\frac{1}{RC}t}$	$i_L(0) \neq 0;\ i_L(\infty) \neq 0;$ Then $i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]\ e^{-\frac{R}{L}t}$
$i_C(t) = C\dfrac{dv_C(t)}{dt}$	$v_L(t) = L\dfrac{di_L(t)}{dt}$

7.5 TRANSIENT IN RL CIRCUIT

Now we shall consider RL circuit for the transient analysis. As stated earlier,

1. The current in an inductor cannot have discontinuity at the time when switching occurs.

2. With dc excitation, at steady state, inductor will act as a short circuit.

Now also we shall end up with first order DE whose solution will be exponential in nature.

<u>Source free circuit</u>

A circuit that does not contain any source is called a source free circuit. Consider the circuit shown in Fig. 7.35 (a). Let us assume that the circuit was in steady state condition with the switch is in position S_1 for a long time. Now the inductor acts as short circuit and it carries a current of $\frac{E}{R}$.

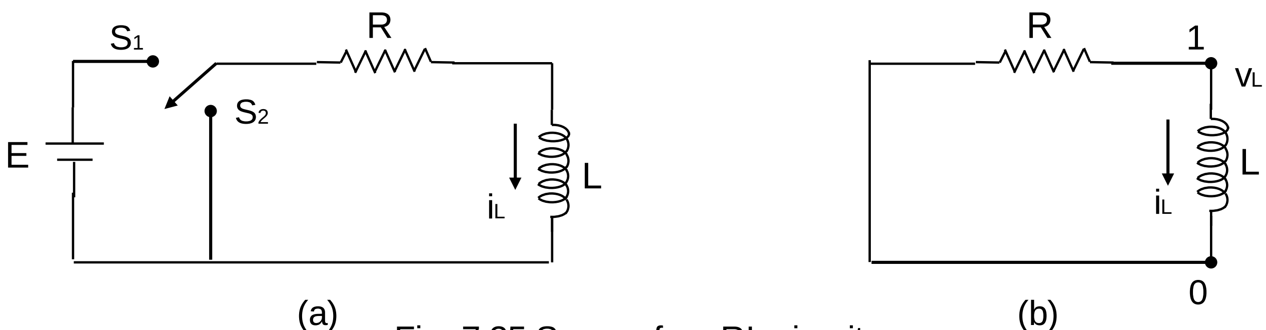
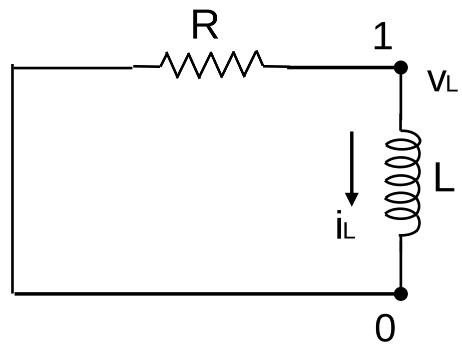


Fig. 7.35 Source free RL circuit.

Suddenly, at time $t = 0$, the switch is moved to position S_2. The current through the inductor and the voltage across the inductor are designated as i_L and v_L respectively. The current through the inductor will be continuous. Hence



$$i_L(0^+) = i_L(0^-) = \frac{E}{R} \tag{7.49}$$

The circuit for time $t > 0$ is shown above. We are interested in finding the current through the inductor as a function of time. Later, if required, voltage across the inductor can be calculated from $v_L = L\frac{di}{dt}$. The mesh equation for the circuit is

$$R\,i_L + L\frac{di_L}{dt} = 0 \tag{7.50}$$

i.e. $$\frac{di_L}{dt} + \frac{R}{L}\,i_L = 0 \tag{7.51}$$

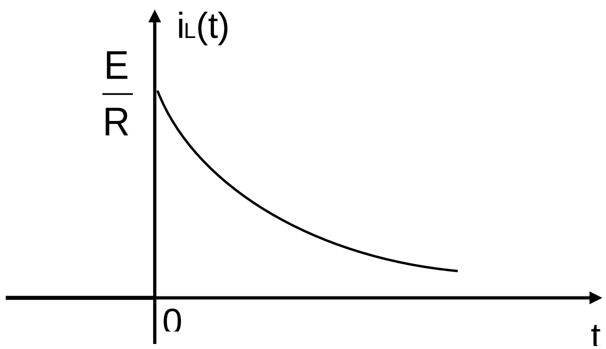
We need to solve the above equation with the initial condition

$$i_L(0^+) = \frac{E}{R} \tag{7.52}$$

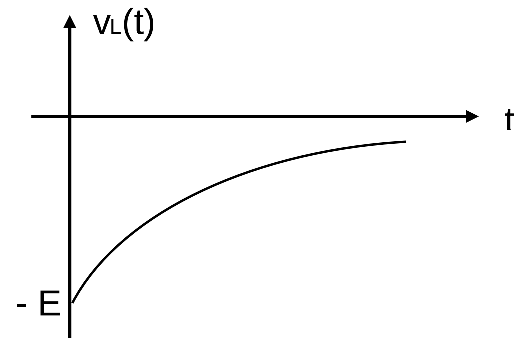
The structure of the equation (7.51) is the same as Eq. (7.18). In this case, the time constant, τ is $\frac{L}{R}$. The inductor current exponentially decays from the initial value of $\frac{E}{R}$ to the final value of zero. Thus the solution of equation 7.51 yields

$$i_L(t) = \frac{E}{R}\, e^{-\frac{R}{L}t} \tag{7.53}$$

The plot of inductor current is shown in Fig. (a).



(a)



(b)

It can be seen that the dimension of L / R is time. Dimensionally

$$\frac{L}{R} = \frac{\text{Flux linkage}}{\text{amp.}}\ \frac{\text{amp.}}{\text{volt}} = \frac{\text{Flux linkage}}{\text{Flux linkage / sec}} = \text{sec.}$$

The voltage across the inductor is: $v_L(t) = L\frac{di}{dt} = L\,\frac{E}{R}\left(-\frac{R}{L}\right) e^{-\frac{R}{L}t} = -E\, e^{-\frac{R}{L}t}$ (7.54)

The plot of the voltage across the inductor is shown in Fig. (b).

Driven circuit

Consider the circuit shown in Fig. 7.37 (a). After the circuit has attained the steady state with the switch in position S_2, the switch is moved to position S_1 at time t = 0. We like to find the inductor current for time t > 0.

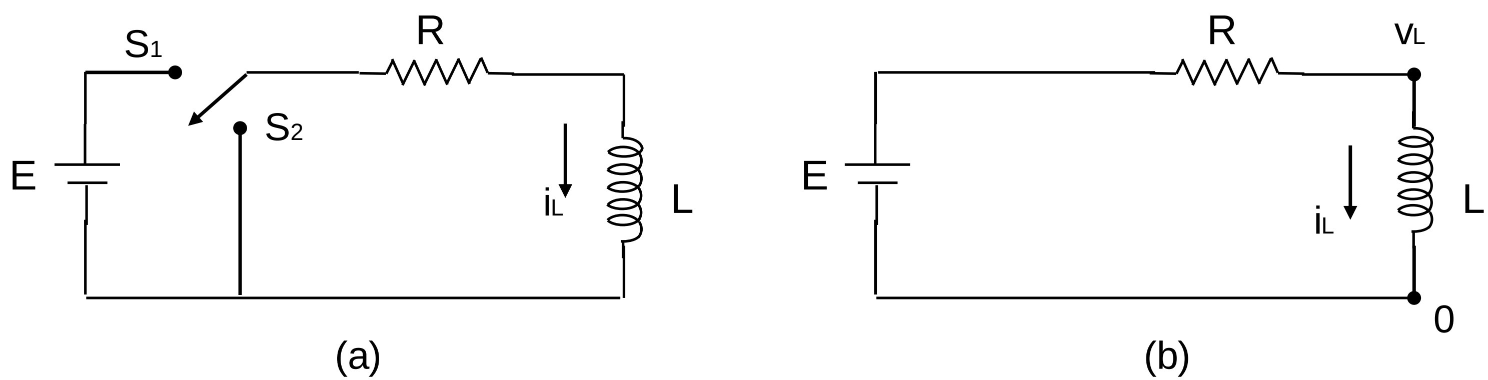


Fig. 7.37 Driven RL circuit.

Since the current through the inductor must be continuous

$$i_L(0^+) = i_L(0^-) = 0 \quad (7.55)$$

The circuit for time t > 0 is shown in Fig. 7.37 (b). The mesh equation is

$$R\, i_L + L\frac{di_L}{dt} = E \quad (7.56) \qquad \text{i.e.} \quad \frac{di_L}{dt} + \frac{R}{L}\, i_L = E \quad (7.57)$$

We need to solve the above DE with the initial condition $i_L(0) = 0$

$$\frac{di_L}{dt} + \frac{R}{L} i_L = E$$

$$i_{cs} = K \; e^{-\frac{R}{L}t} \quad \text{and} \quad i_{ps} = A$$

Substituting i_{ps} in the DE, we get

$$0 = -\frac{R}{L} A = \frac{E}{L} \text{ and hence } A = \frac{E}{R}$$

This gives, $i_{ps} = E / R$

The total solution is $i_L(t) = K \; e^{-\frac{R}{L}t} + \frac{E}{R}$

Using the initial condition in the above, we get

$$0 = K + \frac{E}{R} \text{ i.e. } K = -\frac{E}{R}$$

Therefore the inductor current is

$$i_L(t) = -\frac{E}{R} e^{-\frac{R}{L}t} + \frac{E}{R} \qquad = \frac{E}{R}(1 - e^{-\frac{R}{L}t}) \qquad (7.58)$$

Inductor current $i_L(t)$ exponentially increases from 0 to $\frac{E}{R}$ with time constant, $\tau = \frac{L}{R}$ as shown in Fig. 7.38 (a).

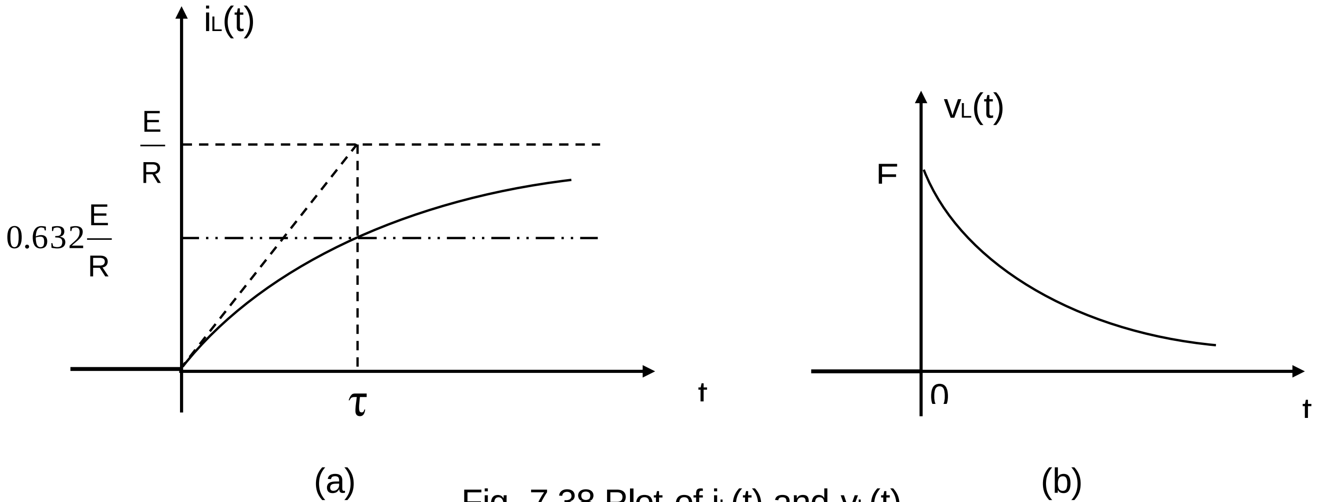


Fig. 7.38 Plot of $i_L(t)$ and $v_L(t)$.

Now, the voltage across the inductor is obtained as

$$v_L(t) = L\,\frac{di}{dt} = L\,\frac{E}{R}\,\frac{R}{L}\,e^{-\frac{R}{L}t} = E\,e^{-\frac{R}{L}t} \tag{7.59}$$

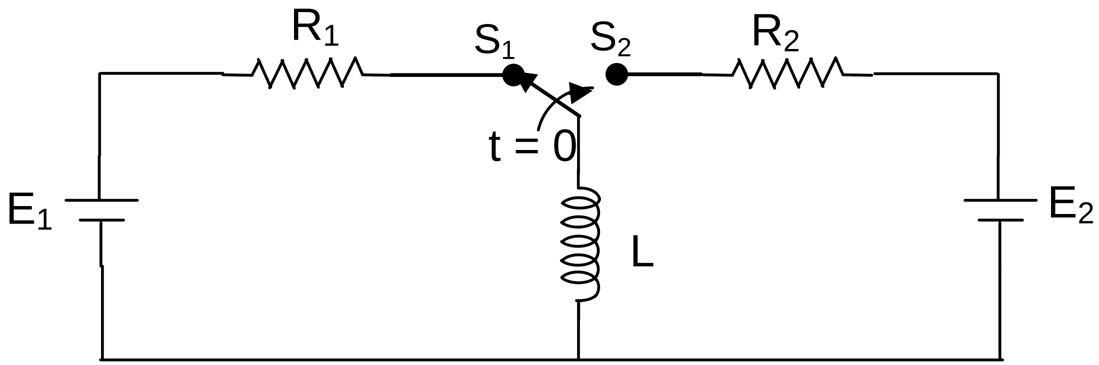
It can be seen that the voltage $v_L(t)$ exponentially decreases from E to zero with the time constant, $\tau = \frac{L}{R}$ as shown in Fig. 7.38 (b).

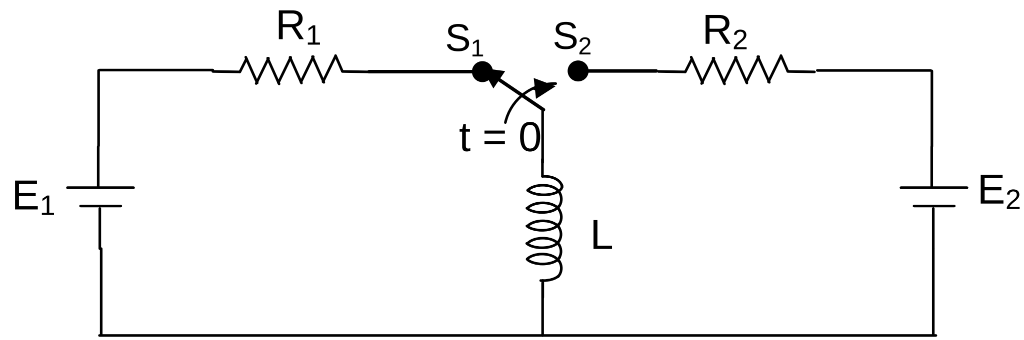
It is to be noted that the initial and the final values of the inductor current and the voltage across it can be readily computed by considering the circuit condition at that time.

More general case of finding the inductor current

In the previous discussion, it was assumed that the initial inductor current $i_L(0) = 0$. There may be very many situations wherein initial inductor current is not zero.

The circuit arrangements can cause non-zero initial inductor current. For this purpose consider the circuit shown below. The switch was in position S_1 for a long time. It is moved from position S_1 to S_2 at time $t = 0$.





We shall assume the following:

1. At time $t = 0^-$ the circuit was at steady state condition with the switch in position S_1

2. After switching to position S_2, the circuit is allowed to reach the steady state condition

Thus, we are interested about the transient analysis for one switching period only.

Initial inductor current $i_L(0)$ is E_1 / R_1 and the final inductor current $i_L(\infty)$, will be E_2 / R_2.

The more general expression for the inductor current can be obtained as

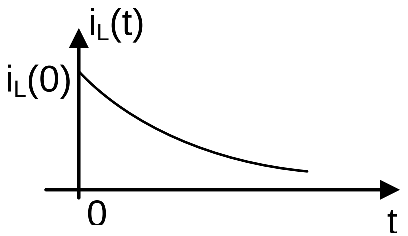
$$i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]\, e^{-\frac{R_2}{L}t} \qquad (7.63)$$

Summary of formulae useful for transient analysis on RL circuits

1. Time constant $\tau = L / R$ Hence $\alpha = R / L$

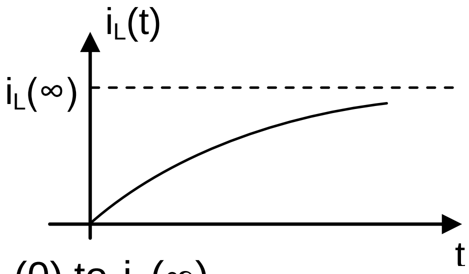
2. When the inductor current is decaying from the initial value of $i_L(0)$ to zero

$$i_L(t) = i_L(0)\ e^{-\frac{R}{L}t}$$



3. When the inductor current is exponentially increasing from zero to $i_L(\infty)$

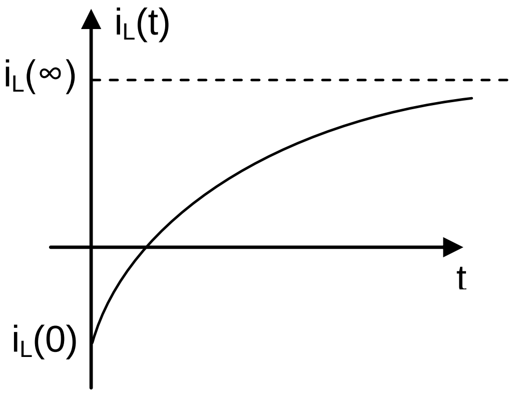
$$i_L(t) = i_L(\infty)\ (1 - e^{-\frac{R}{L}t})$$



4. When the inductor current changes from $i_L(0)$ to $i_L(\infty)$

$$i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]\ e^{-\frac{R}{L}t}$$

Plot of $i_L(t)$ depends on values of $i_L(0)$ and $i_L(\infty)$



5. Inductor voltage $v_L(t) = L \dfrac{di_L(t)}{dt}$

Example 7.13 An RL circuit with R = 12 Ω has time constant of 5 ms. Find the value of the inductance.

Solution R = 12 Ω; Time constant, $L / R = 5 \times 10^{-3}$ s

Inductance $L = 12 \times 5 \times 10^{-3} = 60$ mH

Example 7.14

In an RL circuit having time constant 400 ms the inductor current decays and its value at 500 ms is 0.8 A. Find the equation of $i_L(t)$ for t > 0.

Solution $L / R = 400 \times 10^{-3}$ s; $R / L = 2.5\ s^{-1}$; As $i_L(t)$ decays, $i_L(t) = i_L(0)\, e^{-\frac{R}{L}t}$

When t = 500 ms, $i_L(t) = 0.8$ A. Using this

$$0.8 = i_L(0)\, e^{-2.5 \times 0.5} = i_L(0)\, e^{-1.25} = 0.2865\, i_L(0)$$

Thus $i_L(0) = 0.8 / 0.2865 = 2.7923$ A

Therefore $i_L(t) = 2.7923\, e^{-2.5t}$

Example 7.15 In a RL circuit with time constant of 1.25 s, inductor current increases from the initial value of zero to the final value of 1.2 A.

(a) Calculate the inductor current at time 0.4 s, 0.8 s and 2 s.

(b) Find the time at which the inductor current reaches 0.3 A, 0.6 A and 0.9 A.

Solution $L / R = 1.25$ s $i_L(0) = 0$ $i_L(\infty) = 1.2$ A $\alpha = 1/1.25 = 0.8\ s^{-1}$

(a) $i_L(t) = 1.2\ (1 - e^{-0.8\,t})$ A

When time t = 0.4 s, $i_L = 1.2(1 - e^{-0.32}) = 0.3286$ A

When time t = 0.8 s, $i_L = 1.2(1 - e^{-0.64}) = 0.5672$ A

When time t = 2 s, $i_L = 1.2(1 - e^{-1.6}) = 0.9577$ A

(b) Let t_1, t_2 and t_3 be the time at which current reaches 0.3 A, 0.6 A and 0.9 A.

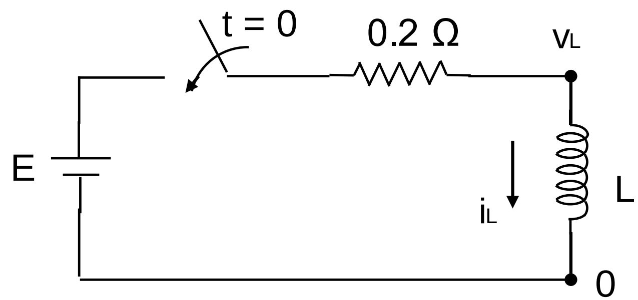
$0.3 = 1.2\ (1 - e^{-0.8\,t_1})$ i.e. $e^{-0.8\,t_1} = 0.75$ i.e. $0.8\ t_1 = 0.2877$ i.e. $t_1 = 0.3596$ s

$0.6 = 1.2\ (1 - e^{-0.8\,t_2})$ i.e. $e^{-0.8\,t_2} = 0.5$ i.e. $0.8\ t_2 = 0.6931$ i.e. $t_2 = 0.8664$ s

$0.9 = 1.2\ (1 - e^{-0.8\,t_3})$ i.e. $e^{-0.8\,t_3} = 0.25$ i.e. $0.8\ t_3 = 1.3863$ i.e. $t_3 = 1.7329$ s

Example 7.16

In the RL circuit shown in Fig. below, the voltage across the inductor for t > 0 is given by $v_L(t) = 0.16\ e^{-200t}\ V$. Determine the value of the inductor L and obtain the equation for current $i_L(t)$. Also compute the value of voltage E.



Solution $v_L(t) = 0.16\ e^{-200t}\ V$; $R = 0.2\ \Omega$ $\alpha = \frac{R}{L} = 200;$ i.e. $L = \frac{0.2}{200} H = 1\,mH$

When the switch is closed inductor current exponentially increases from 0 to $i_L(\infty)$. It is

$i_L(t) = i_L(\infty)\ (1 - e^{-\frac{R}{L}t})$ Also $v_L(t) = L\frac{di_L}{dt} = L\ i_L(\infty)\frac{R}{L}\ e^{-\frac{R}{L}t} = R\ i_L(\infty)\ e^{-\frac{R}{L}t}$

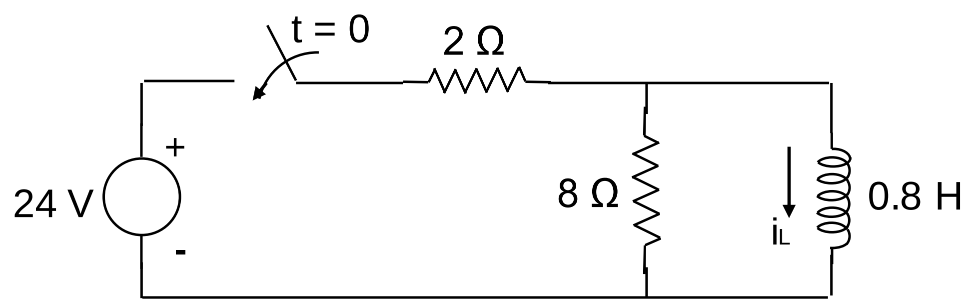
Comparing $v_L(t) = = R\ i_L(\infty)\ e^{-\frac{R}{L}t}$ with $v_L(t) = 0.16\ e^{-200t}\ V$

Therefore, $0.2\ i_L(\infty) = 0.16$ i.e. $i_L(\infty) = 0.16 / 0.2 = 0.8\ A$

Thus, $i_L(t) = 0.8\ (1 - e^{-200t})$

Also $i_L(\infty) = \frac{E}{0.2}$ Therefore, $\frac{E}{0.2} = 0.8;$ Thus $E = 0.16\ V$

Example 7.17 The switch in the circuit shown was in open position for a long time. It is closed at time t = 0. Find $i_L(t)$ for time t > 0.



Solution Current $i_L(0) = 0$

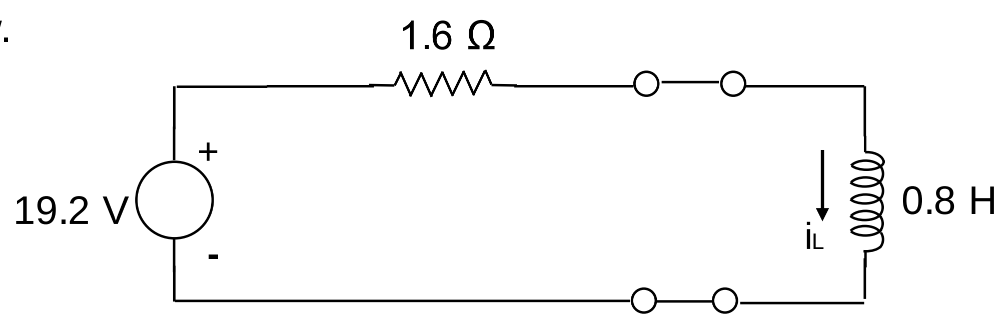
When the switch is closed, Current $i_L(\infty) = 24 / 2 = 12$ A

Thevenin's resistance = $8 || 2 = 1.6\ \Omega$ $\tau = L / R = 0.8 / 1.6 = 0.5$ s ; $\alpha = 2\ s^{-1}$

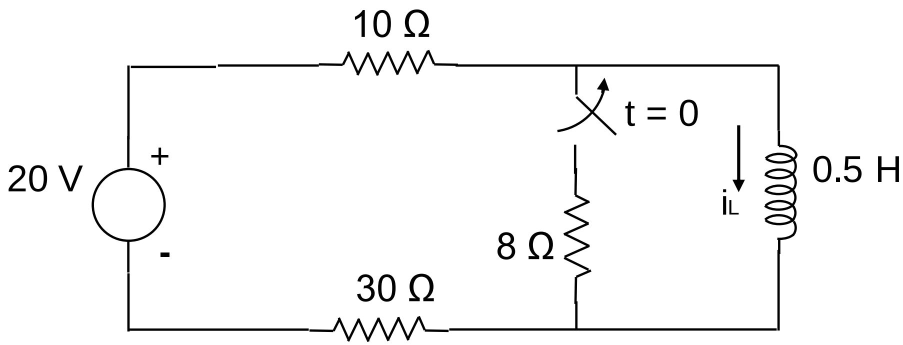
Inductor current exponentially increases from 0 to 12 A.

Current $i_L(t) = 12\,(1 - e^{-2t})$ A

Same result can be obtained by getting the Thevenin's equivalent circuit for time t > 0 as shown in Fig. below.

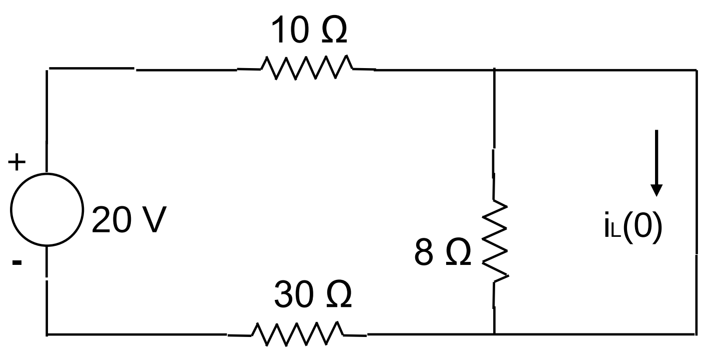


Example 7.18 The switch in the circuit shown was in closed position for a long time. Find current $i_L(t)$ for time $t > 0$.

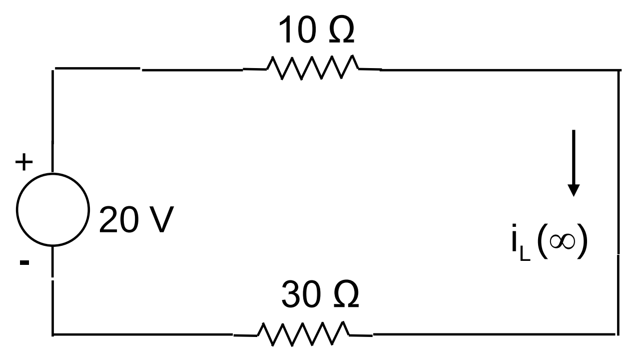


Solution

Circuit for $t = 0^-$ and $t = \infty$ are shown in Fig. (a) and (b) below.



(a)

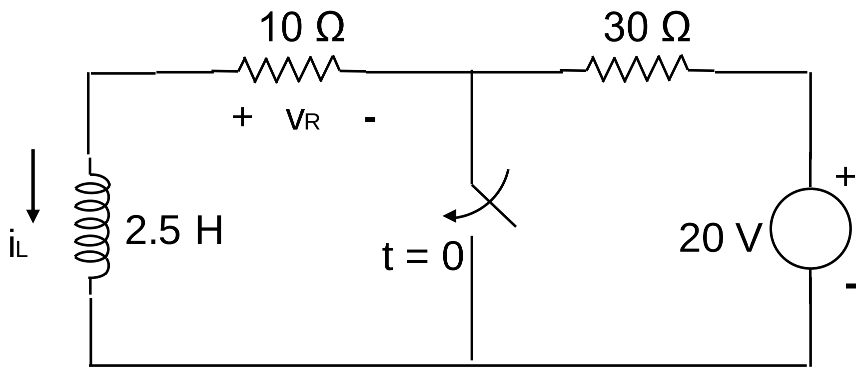


(b)

Current $i_L(0) = 20 / 40 = 0.5$ A Further, current $i_L(\infty) = 20 / 40 = 0.5$ A

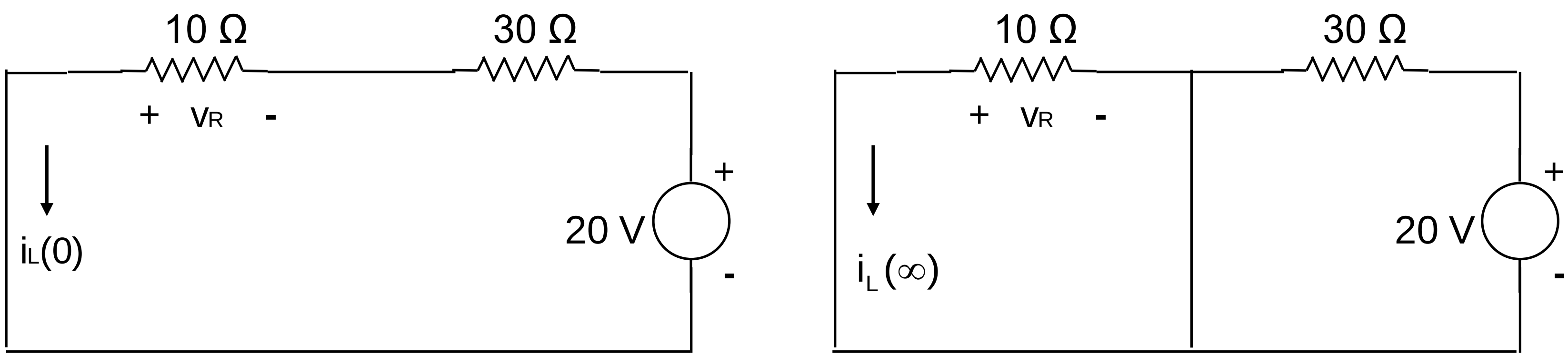
Therefore, current $i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]\, e^{-\frac{R}{L}t} = 0.5$ A

Example 7.19 In the circuit shown the switch was in open position for a long time. Determine the current $i_L(t)$ and the voltage $v_R(t)$ for time $t > 0$.



Solution

Circuit for $t = 0^-$ and $t = \infty$ are shown in Fig. (a) and (b) below.



(a) (b)

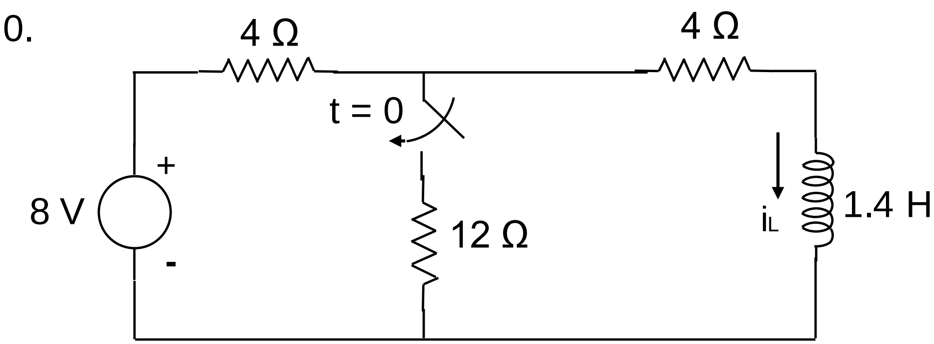
Current $i_L(0) = 20 / (10 + 30) = 0.5$ A; Current $i_L(\infty) = 0$: Thevenin's resistance = 10 Ω

Time constant = $L / R = 2.5 / 10 = 0.25$ s; $\alpha = 4\ s^{-1}$

Thus $i_L(t) = 0.5\ e^{-4t}$ A Voltage $v_R(t) = -10\ i_L(t) = -5\ e^{-4t}$ V

Example 7.20

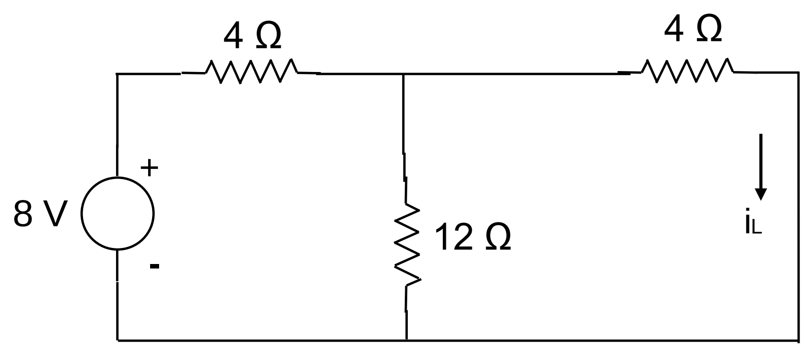
The circuit shown was in steady state condition with the switch open. Find the inductor current for time $t > 0$.



t = 0

Solution

Current $i_L(0) = 8 / (4 + 4) = 1$ A



Circuit for $t = \infty$ is

$i_T = 8 / 7 = 1.1429$ A

$i_L(\infty) = (12/16)\ 1.1429$ A

$= 0.8571$ A

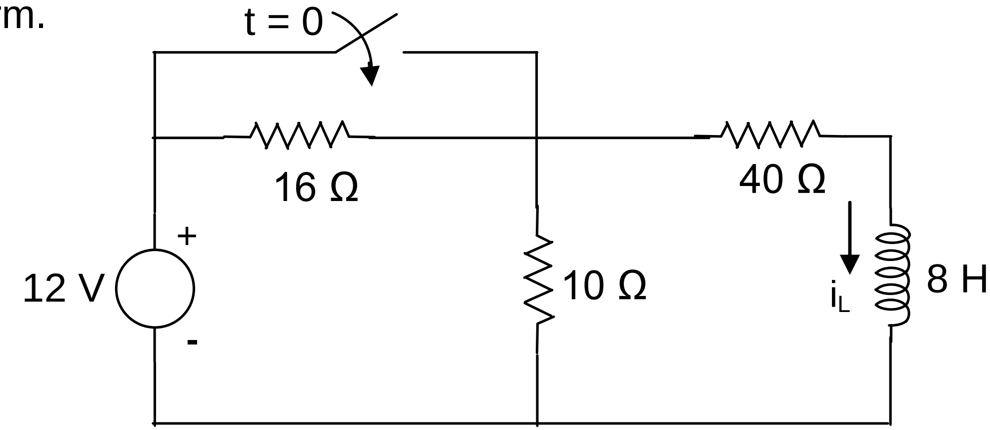
Thevenin's resistance wrt inductor = 4 + 3 = 7 Ω

Time constant $L / R = 1.4 / 7 = 0.2$ s; $\alpha = 5\ s^{-1}$

Current $i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]\ e^{-\frac{R}{L}t} = 0.8571 + [1 - 0.8571]\ e^{-5t}$ A

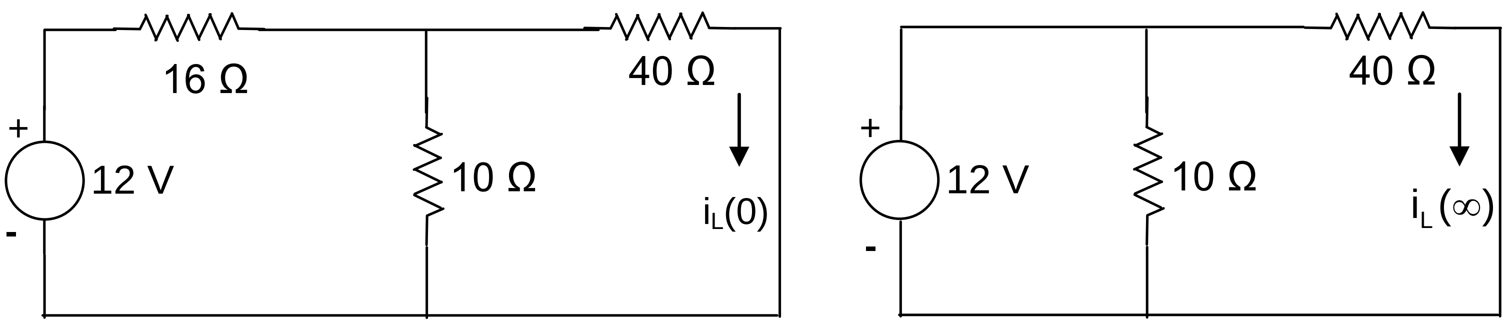
$= 0.8571 + 0.1429\ e^{-5t}$ A

Example 7.21   With the switch open, the circuit shown below was in steady state condition. At time t = 0, the switch is closed. Find the inductor current for time t > 0 and sketch its wave form.



Solution

Circuit for $t = 0^-$ and $t = \infty$ are shown in Fig. (a) and (b).



To find $i_L(0)$: $R_T = 16 + 8 = 24\ \Omega$; $I_T = 12 / 24 = 0.5$ A; $i_L(0) = 0.5 \times \dfrac{10}{50} = 0.1$ A

To find $i_L(\infty)$; $12 / 40 = 0.3$ A; Further, $R_{Th} = 40\ \Omega$

Time constant = L / R_{Th} = 8 / 40 = 0.2 s $\alpha = 5\ s^{-1}$

Current $i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]\ e^{-\frac{R}{L}t} = 0.3 + [0.1 - 0.3]\ e^{-5t}$

$= 0.3 - 0.2\ e^{-5t}$ A

Current wave form is shown in Fig. 7,51.

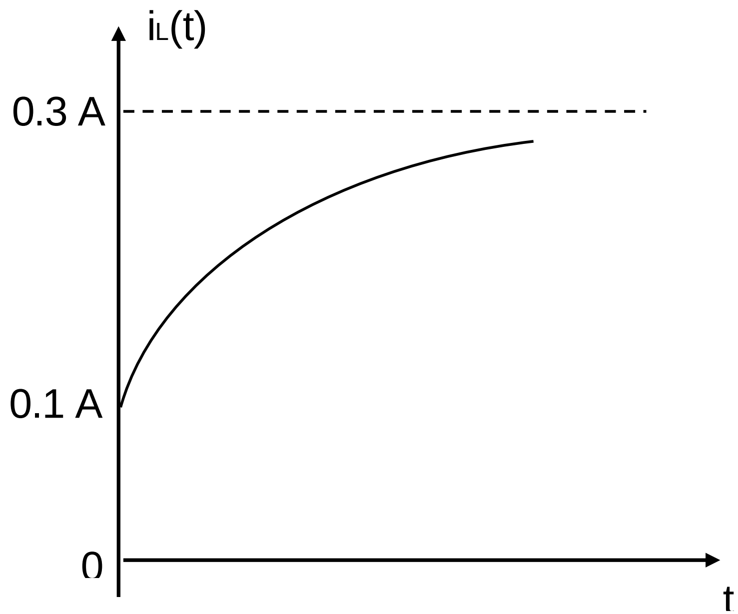
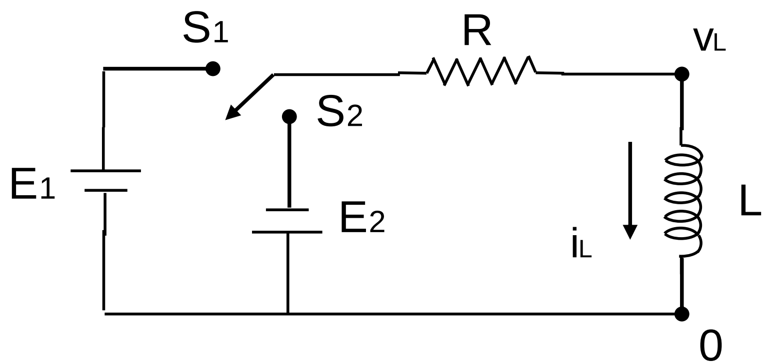


Fig. 7.51 Wave form of $i_L(t)$ - Example 7.21.

Example 7.22

For the initially relaxed circuit shown, the switch is closed on to position S_1 at time t = 0 and changed to position S_2 at time t = 0.5 ms. Obtain the equation for inductor current and voltage across the inductor in both the intervals and sketch the transients.



E_1 = 100 V; E_2 = 50 V

R = 100 Ω

L = 0.2 H

Solution

With the switch is in position S_1, inductor current exponentially increases from zero to the steady state value of 100 / 100 = 1 A. Knowing the time constant as L / R =

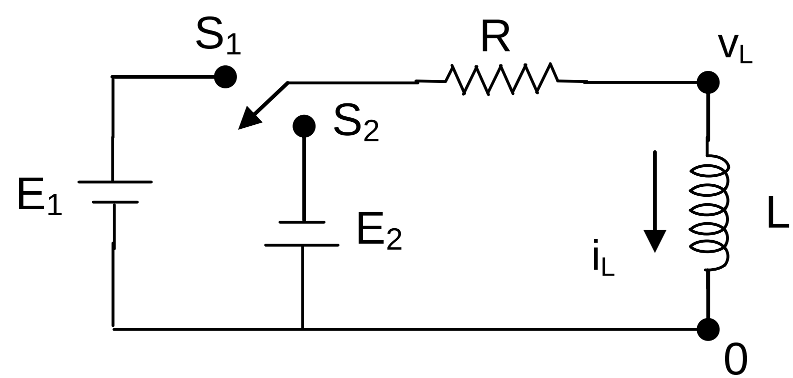
0.2 / 100 = 1 / 500 s, equation of inductor current in the first switching interval is

$i_L(t) = 1 - e^{-500t}$ A Corresponding voltage is

$$v_L(t) = L\frac{di_L}{dt} = 0.2 \times 500\, e^{-500t}\text{ V} = 100\, e^{-500t}\text{ V} \qquad \text{for } 0.5 \times 10^{-3} \geq t > 0$$

Therefore $\quad i_L(0.5 \times 10^{-3}) = 1 - e^{-0.25} = 0.2212$ A

$\quad v_L(0.5 \times 10^{-3}) = 100\, e^{-0.25} = 77.88$ V



Let the second switching occurs at time t' = 0.

Then, $t' = t - 0.5 \times 10^{-3}$

For time t' > 0, the mesh equation is

$$R\, i_L(t') + L\frac{di_L}{dt'} = -E_2 \quad \text{i.e.} \quad \frac{di_L}{dt'} + \frac{R}{L}\, i_L(t') = -\frac{E_2}{L} \quad \text{with } i(0) = 0.2212 \text{ A}$$

$$\frac{di_L}{dt'} + \frac{R}{L}\, i_L(t') = - \frac{E_2}{L} \text{ with } i(0) = 0.2212 \text{ A}$$

$i_{cs} = K\, e^{-\frac{R}{L}t'}$ and $i_{ps} = A$

Substituting the particular solution to the non-homogeneous DE, we get

$$\frac{R}{L} A = - \frac{E_2}{L} \text{ i.e. } A = - \frac{E_2}{R} = - 0.5$$

Complete solution is

$i_L(t') = K\, e^{-500\, t'} - 0.5$

Using the initial condition

K - 0.5 = 0.2212 i.e. K = 0.7212. Thus

$i_L(t') = 0.7212\, e^{-500\, t'} - 0.5$ A

$v_L(t') = 0.2 \times (-\,0.7212 \times 500)\, e^{-500\, t'} = -\,72.12\, e^{-500\, t'}$ V

When t’ = 0, inductor voltage = - 72.12 V

The current and voltage transients are shown in Fig. 7.53.

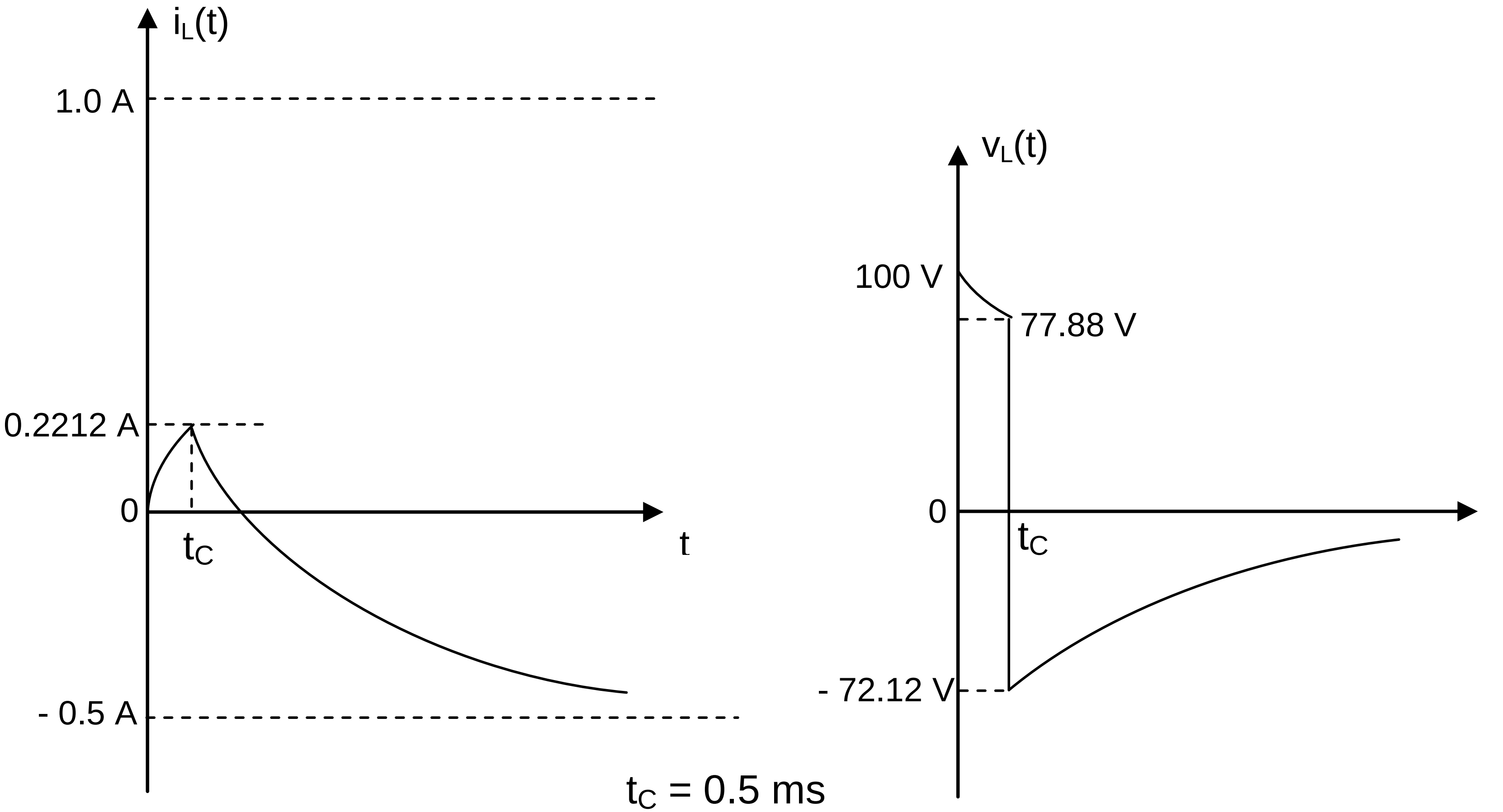


Fig. 7.53 Wave forms - Example 7.22.

7.6 LAPLACE TRANSFORM

In circuits with several capacitances and inductors, we often come across with integro-differential equations. Such equations can be rewritten as higher order DEs. The classical method of solving the DEs is rather involved. Here, the complimentary solution and the particular solution have to be determined and finally the arbitrary constants have to be obtained from the initial conditions. The Laplace Transform (LT) method is much superior to the classical method due to the following reasons.

1. Laplace transformation transforms exponential and trigonometric functions into algebraic functions.

2. Laplace transformation transforms differentiation and integration into multiplication and division respectively.

3. It transforms integro-differential equations into algebraic equations which are much simpler to handle.

4. The arbitrary constants need not be determined separately. Complete solution will be obtained directly.

The LT of f(t) is defined by $$F(s) = \int_0^{\infty} f(t)\, e^{-st}\, dt \qquad (7.65)$$

The following Table 7.1 gives the LT of some important functions used quite often in transient analysis.

Table 7.1 Laplace transform of certain time functions.

Time function f(t)	Laplace transform F(s)	Time function f(t)	Laplace transform F(s)
u(t)	$\frac{1}{s}$	E	$\frac{E}{s}$
e^{-at}	$\frac{1}{s+a}$	e^{at}	$\frac{1}{s-a}$
sin ωt	$\frac{\omega}{s^2+\omega^2}$	sin (ωt + θ)	$\frac{s\sin\theta+\omega\cos\theta}{s^2+\omega^2}$
cos ωt	$\frac{s}{s^2+\omega^2}$	cos (ωt + θ)	$\frac{s\cos\theta-\omega\sin\theta}{s^2+\omega^2}$
$\frac{df}{dt}$	$s\,F(s) - f(0^+)$	$\frac{d^2f}{dt^2}$	$s^2\,F(s) - s\,f(0^+\} - f'(0^+)$
$\int_0^\infty f(t)\,dt$	$\frac{F(s)}{s}$	$e^{-\alpha t}\ f(t)$	$F(s+\alpha)$
$f(t - t_1)$	$e^{-t_1 s}\,F(s)$	t	$\frac{1}{s^2}$

While finding inverse Laplace Transform, in many cases, as a first step, F(s) is to be split into sum of functions in s. This is done using partial fraction method. The results of two cases that are used quite often are furnished below.

1. $$F(s) = \frac{s^2 + ps + q}{(s+a)(s+b)(s+c)} = \frac{K_1}{s+a} + \frac{K_2}{s+b} + \frac{K_3}{s+c} \qquad (7.66)$$

Here $K_1 = (s+a)\,F(s)\Big|_{s=-a}$

$K_2 = (s+b)\,F(s)\Big|_{s=-b}$

$K_3 = (s+c)\,F(s)\Big|_{s=-c}$ (7.67)

2. $$F(s) = \frac{A}{s\,(s+B)} = \frac{k_1}{s} + \frac{k_2}{s+B} = \frac{A}{B}\frac{1}{s} - \frac{A}{B}\frac{1}{s+B} = \frac{A}{B}\left(\frac{1}{s} - \frac{1}{s+B}\right)$$

7.7 TRANSFORM IMPEDANCE AND TRANSFORM CIRCUIT

When LT method is used for transient analysis, **Transform Circuit shall be arrived first**. In the transform circuit, all the currents and voltages are the transformed quantities of the currents and voltages. Further, all the element parameters are replaced by their Transform Impedances. Transform impedances of the individual element shall be arrived at as discussed below.

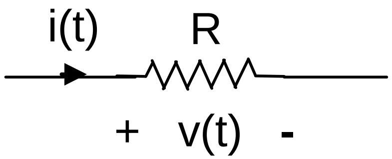
Resistor

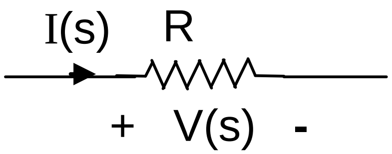
The terminal relationship for the resistor, in time domain is

$$v(t) = R\, i(t) \tag{7.68}$$

Taking LT on both sides, $V(s) = R\, I(s)$ (7.69)

Fig. below shows the terminal relationships of resistor in time and transform domains.





<u>Inductor</u> For an inductor, v-i relationships in time domain are

$$v(t) = L\,\frac{di}{dt} \qquad (7.70)$$

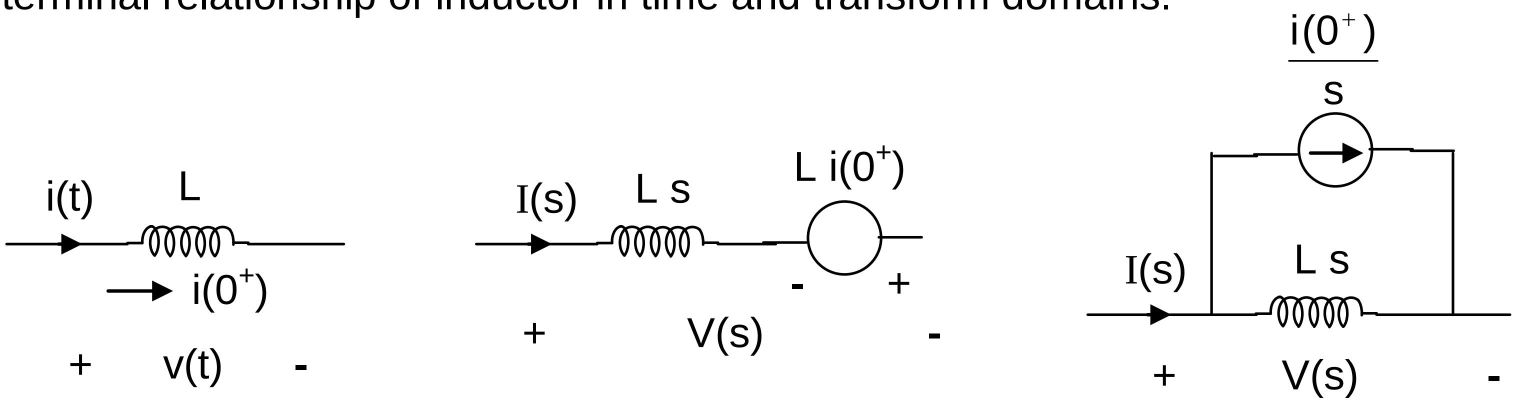
$$i(t) = \frac{1}{L}\int_0^t v\,dt + i\,(0^+) \qquad (7.71)$$

where $i(0^+)$ is the current flowing through the inductor at time $t = 0^+$. On taking LT of these equations, we get

$$V(s) = L\,s\,I(s) - L\,i(0^+) \qquad (7.72)$$

$$I(s) = \frac{V(s)}{L\,s} + \frac{i(0^+)}{s} \qquad (7.73)$$

Note that above two equations are not different. Fig. below shows the representation of the terminal relationship of inductor in time and transform domains.



It is to be noted that both the transform domain circuits shown above are equivalent of each other. One can be obtained from the other using source transformation.

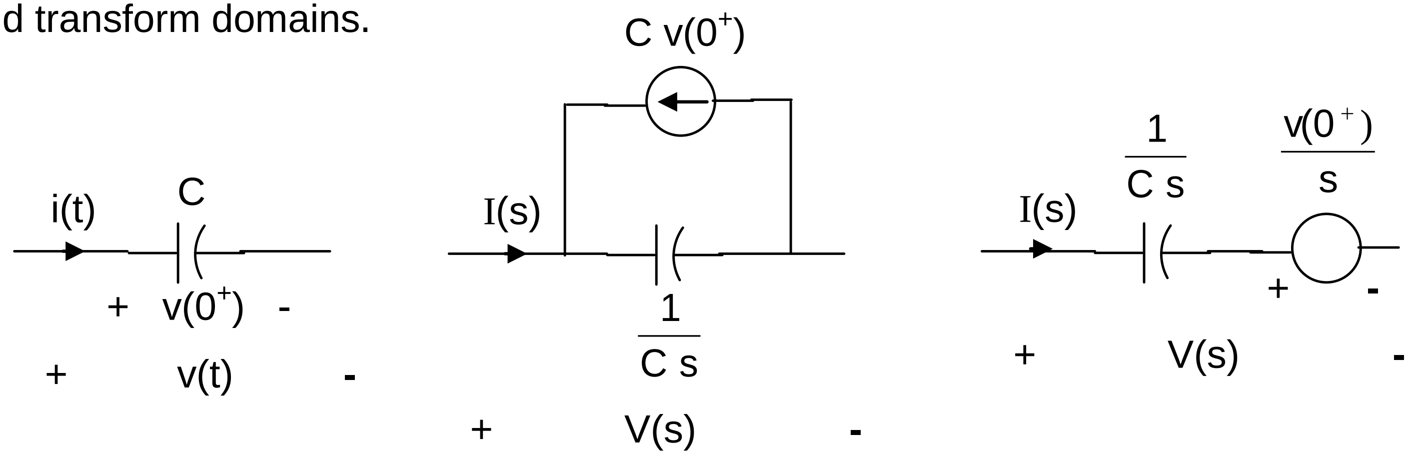
Capacitor   For a capacitor, v-i relationships in time domain are

$$i(t) = C\,\frac{dv}{dt} \qquad (7.74) \qquad\qquad v(t) = \frac{1}{C}\int_0^t i\,dt + v\,(0^+) \qquad (7.75)$$

where $v(0^+)$ is the voltage across the capacitor at time $t = 0^+$. On taking LT of these equations, we get

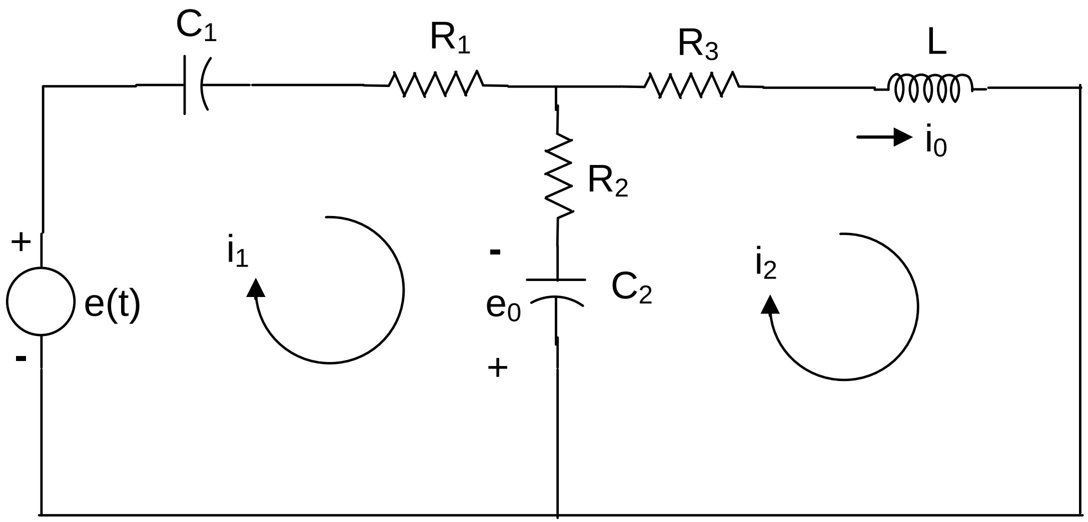
$$I(s) = C\,s\,V(s) - C\,v(0^+) \qquad (7.76) \qquad\qquad V(s) = \frac{I(s)}{C\,s} + \frac{v\,(0^+)}{s} \qquad (7.77)$$

Note that the above two equations are not different. They are written in different form. Fig. below shows the representation of the terminal relationship of capacitor in the time and transform domains.

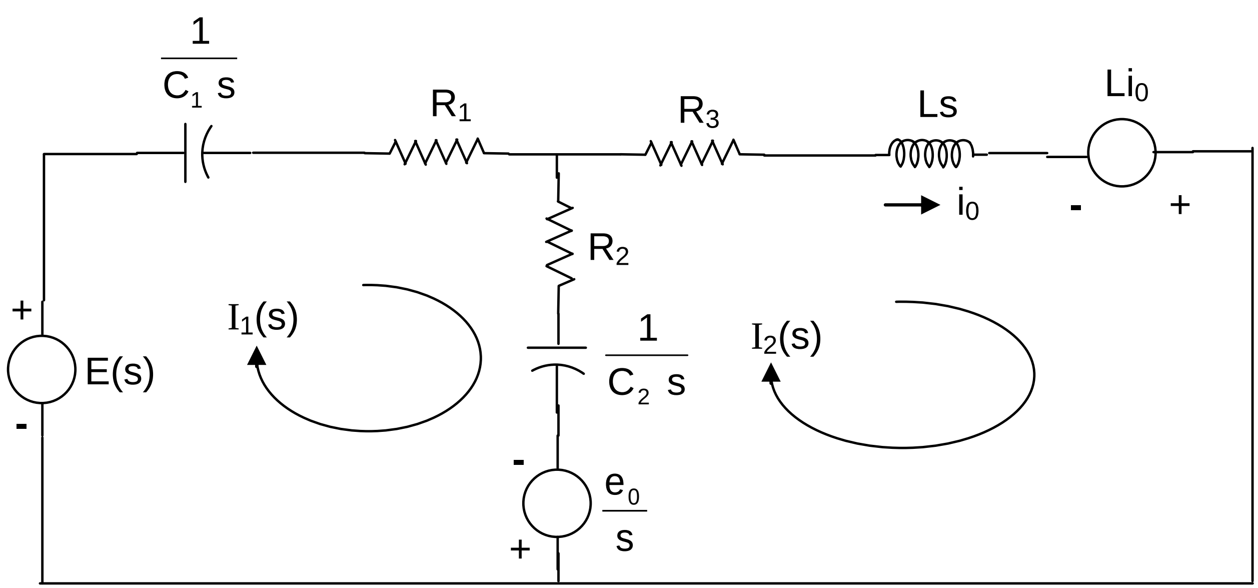


Here again, both the transform domain circuits shown are equivalent of each other. One can be obtained from the other using source transformation.

Example 7.23 For the circuit shown below, obtain the transform circuit.



Solution Fig. below shows the transform circuit.



7.8.1 RL CIRCUIT

Consider the RL circuit shown in Fig. 7.59(a). Assume that the switch is closed at time t = 0 and assume that the current i at the time of switching is zero.

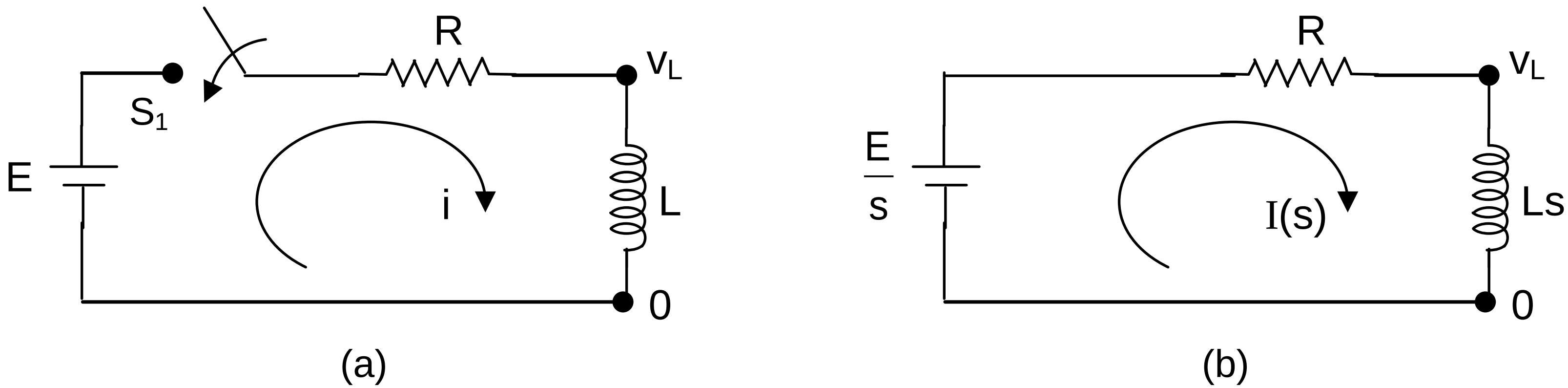


Fig. 7.59 Time domain and s domain - R-L circuit.

The transform circuit in s domain is shown in Fig. 7.59 (b). From this,

$$I(s) = \frac{E/s}{R + Ls} = \frac{E/L}{s\,(s + \frac{R}{L})} = \frac{E/L}{R/L}\left(\frac{1}{s} - \frac{1}{s + \frac{R}{L}}\right) = \frac{E}{R}\left(\frac{1}{s} - \frac{1}{s + \frac{R}{L}}\right) \tag{7.78}$$

Taking inverse LT $\quad i(t) = \frac{E}{R}\,(1 - e^{-\frac{R}{L}t})$ (7.79)

Thus, inductor current rises exponentially with time constant L / R.

Voltage across the inductor is given by

$$V(s) = L\, s\, I(s) = \frac{E}{s + \frac{R}{L}} \tag{7.80}$$

Taking inverse LT $\qquad v_L(t) = E\ e^{-\frac{R}{L}t} \qquad (7.81)$

Inductor voltage increases exponentially with time constant L / R. The current and voltage transients are shown in Fig. 7.60.

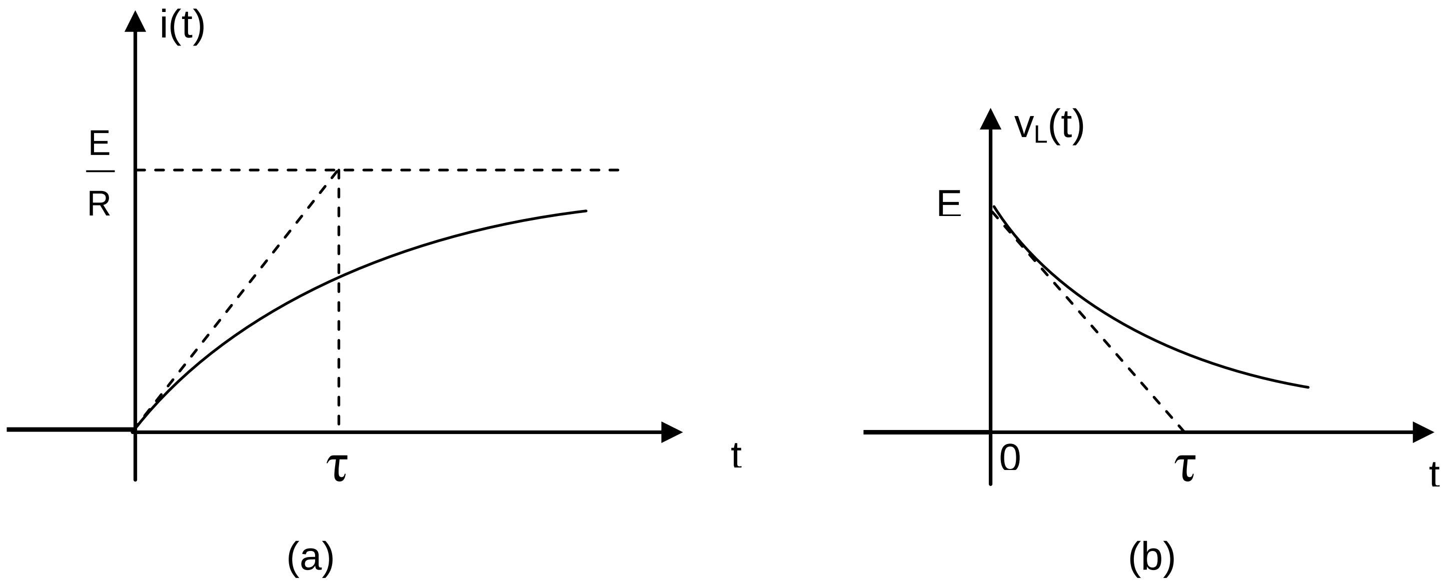
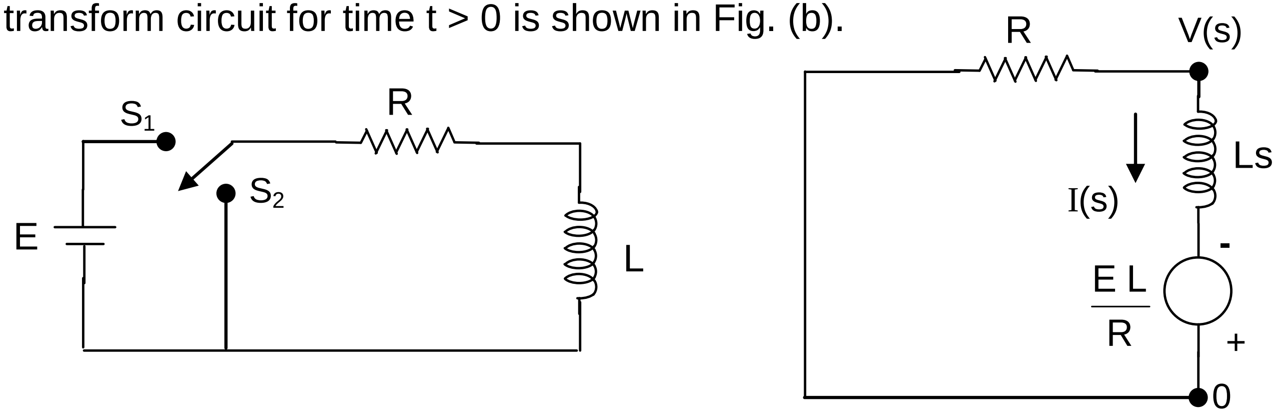


Fig. 7.60 Plot of $i_L(t)$ and $v_L(t)$.

Consider the circuit shown in Fig.(a). Let us say that with the switch in position S_1, steady state condition is reached. The current flowing through the inductor is E / R. At time t = 0, the switch is turned to position S_2. Then

$i(0^+) = i(0^-) = E / R$

The transform circuit for time t > 0 is shown in Fig. (b).



(a) (b)

Considering the transformed circuit $$I(s) = \frac{\frac{EL}{R}}{R + Ls} = \frac{\frac{E}{R}}{s + \frac{R}{L}} \qquad (7.82)$$

Taking inverse LT $$i(t) = \frac{E}{R}\, e^{-\frac{R}{L}t} \qquad (7.83)$$

The current decays exponentially with time constant L / R.

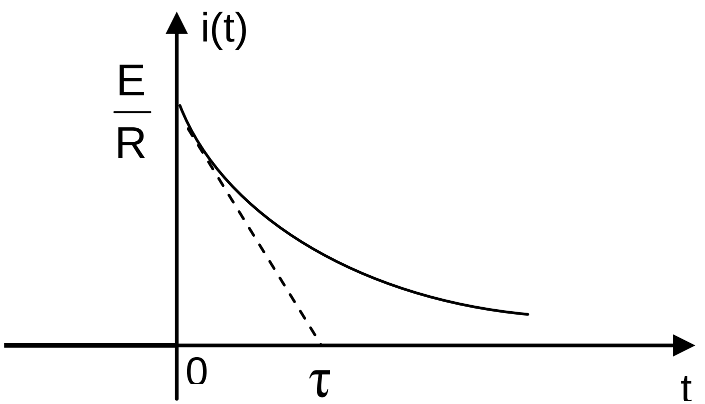
Since R I(s) + V(s) = 0 the voltage across the inductor is

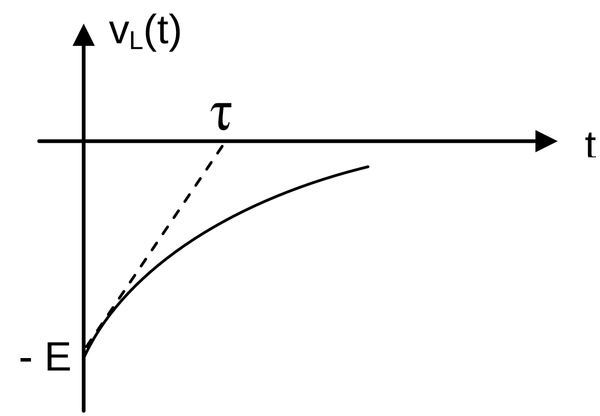
$$V(s) = -R\,I(s) = -\frac{E}{s + \frac{R}{L}} \qquad (7.84)$$

Taking inverse LT $$v_L(t) = -E\ e^{-\frac{R}{L}t} \qquad (7.85)$$

The inductor voltage exponentially changes from - E to zero with time constant L / R.

The current and voltage transients are given by the above two equations are shown.

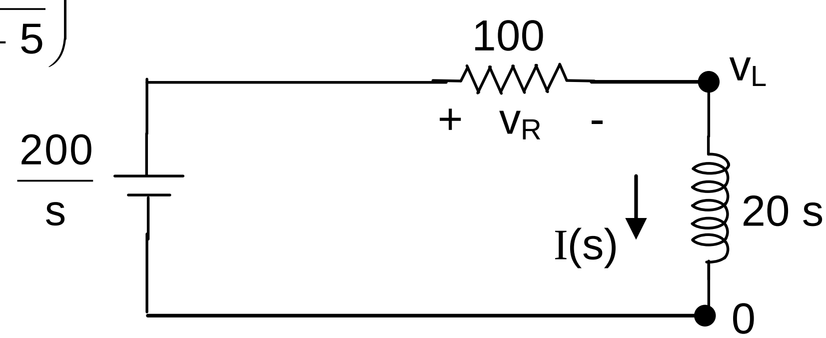




<u>Example 7.24</u> Initially relaxed series RL circuit with R = 100 Ω and L = 20 H has dc voltage of 200 V applied at time t = 0. Find (a) the equation for current and voltages across different elements (b) the current at time t = 0.5 s and 1.0 s (c) the time at which the voltages across the resistor and inductor are equal.

<u>Solution</u> Transform circuit for time t > 0 is shown.

(a) $I(s) = \dfrac{\frac{200}{s}}{100 + 20s} = \dfrac{10}{s(s+5)} = 2\left(\dfrac{1}{s} - \dfrac{1}{s+5}\right)$



Therefore, current $i(t) = 2\,(1 - e^{-5t})$ A

Voltage $v_R(t) = R\,i(t) = 200\,(1 - e^{-5t})$ V

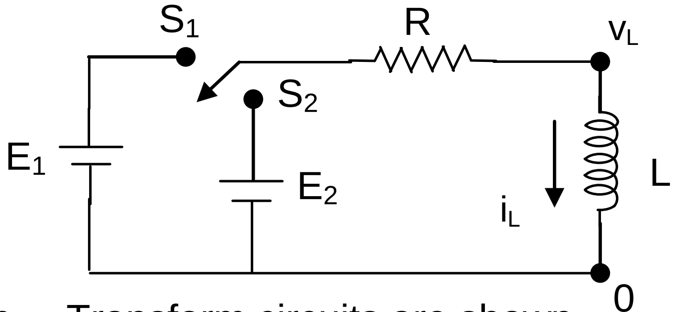
Voltage $v_L(t) = L\dfrac{di}{dt} = 20 \times 2 \times 5\, e^{-5t} = 200\, e^{-5t}$ V

(b) $i(0.5) = 2\,(1 - e^{-2.5}) = 1.8358$ A $\qquad i(1.0) = 2\,(1 - e^{-5}) = 1.9865$ A

(c) Let t_1 be the time at which $v_R(t) = v_L(t)$. Then

$200\,(1 - e^{-5t_1}) = 200\, e^{-5t_1}$ i.e. $e^{-5t_1} = 0.5$ This gives $t_1 = 0.1386$ s

Example 7.25 For the circuit shown, with zero inductor current the switch is closed on to position S_1 at time t = 0. At one mille second it is moved to position S_2 Obtain the equation for the currents in both the intervals.

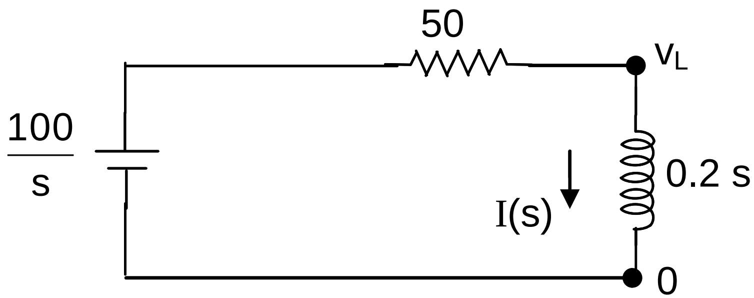


E_1 = 100 V; E_2 = 50 V

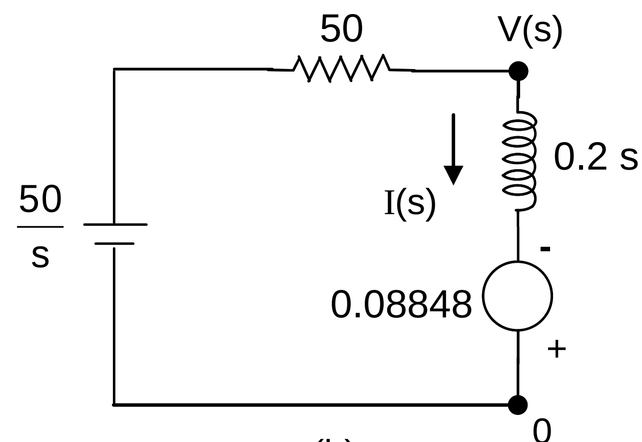
R = 50 Ω

L = 0.2 H

Solution Transform circuits are shown.



(a)



(b)

The transform circuit for the first interval is shown in Fig. 7.65 (a). From this

$$I(s) = \frac{\frac{100}{s}}{50 + 0.2s} = \frac{500}{s(s+250)} = 2\left(\frac{1}{s} - \frac{1}{s+250}\right)$$

Thus, $i(t) = 2\,(1- e^{-250t})$ A $\qquad i(0.001) = 2\,(1- e^{-0.25}) = 0.4424$ A

At time t = 0.001 s, the switch is moved to position S_2. We shall say that this is done at time t' = 0. Thus t' = 0 implies that t = 0 and hence t' = t - 0.001.

The transform circuit for time t' > 0 is shown in Fig. 7.65 (b) in which

$L\, i(0^+) = 0.2 \times 0.4424 = 0.08848$

$$\text{Now, } I(s) = \frac{\frac{50}{s} + 0.08848}{50 + 0.2s} = \frac{50 + 0.08848\,s}{s(50 + 0.2s)} = \frac{250 + 0.4424s}{s\,(s + 250)} = \frac{K_1}{s} + \frac{K_2}{s + 250}$$

$$K_1 = \left.\frac{250 + 0.4424s}{s + 250}\right|_{s = 0} = 1 \qquad K_2 = \left.\frac{250 + 0.4424s}{s}\right|_{s = -250} = -0.5576$$

$$\text{Thus, } I(s) = \frac{1}{s} - \frac{0.5576}{s + 250}$$

Taking inverse LT we get, current $i(t') = 1 - 0.5576\, e^{-250t'}$

Thus for the two intervals currents are given by

$i(t) = 2\,(1 - e^{-250t})$ A $\quad 0.001 \geq t > 0$

$i(t) = 1 - 0.5576\, e^{-250\,(t - 0.001)}$ A $\quad t > 0.001$

Example 7.26 In the previous example, compute the voltage across the inductor in both the intervals and sketch the wave form.

Solution In the first interval, $i(t) = 2\,(1- e^{-250\,t})$ A

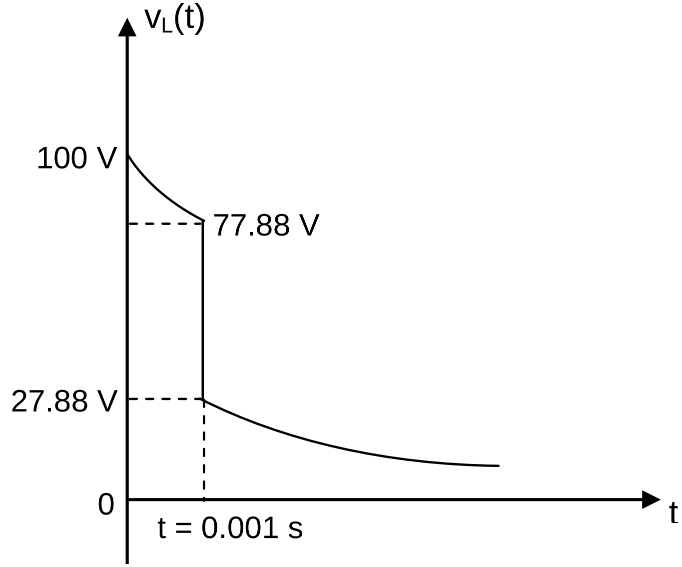
$$v_L(t) = L\,\frac{di}{dt} = 0.2 \text{ X } 2 \text{ X } 250\, e^{-250t} = 100\, e^{-250t} \text{ V} \qquad v_L(0.001) = 100\, e^{-0.25} = 77.88 \text{ V}$$

In the second interval, $i(t') = 1 - 0.5576\, e^{-250t'}$

$$v_L(t') = L\,\frac{di}{dt'} = 0.2 \text{ X } 0.5576 \text{ X } 250\, e^{-250t'} = 27.88\, e^{-250t'} = 27.88\, e^{-250(t\,-\,0.001)} \text{ V}$$

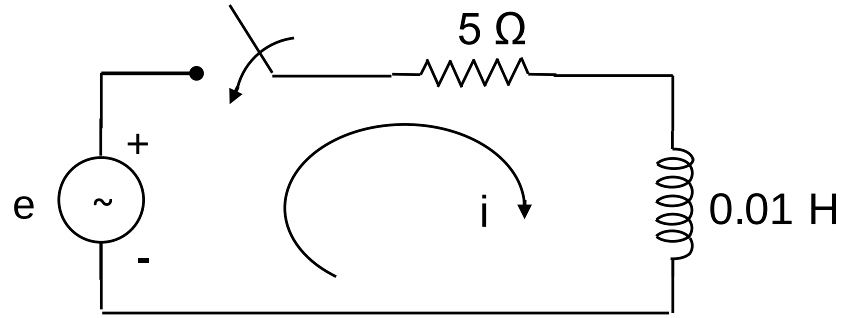
$$v_L(0.001) = v_L(t')\Big|_{t' = 0} = 27.88 \text{ V}$$

The wave form of the voltage across the inductor is shown below.



Example 7.27

In the initially relaxed RL circuit shown, the sinusoidal source of e = 100 sin (500 t) V is applied at time t = 0. Determine the resulting transient current for time t > 0.



Solution

e = 100 sin (500 t) V; Its LT is

$$E(s) = \frac{100 \text{ X } 500}{s^2 + 250000} = \frac{5 \text{ X } 10^4}{s^2 + 25 \text{X} 10^4}$$

Impedance = 5 + j 0.01 s

Current $I(s) = \dfrac{5 \times 10^4}{(s^2 + 25 \times 10^4)(5 + 0.01s)} = \dfrac{5 \times 10^6}{(s^2 + 25 \times 10^4)(s + 500)}$

$$= \frac{K_1 s + K_2}{s^2 + 25 \times 10^4} + \frac{K_3}{s + 500}$$

$$K_3 = \left. \frac{5 \times 10^6}{s^2 + 25 \times 10^4} \right|_{s = -500} = 10$$

Since $\dfrac{5 \times 10^6}{(s^2 + 25 \times 10^4)(s + 500)} = \dfrac{K_1 s + K_2}{s^2 + 25 \times 10^4} + \dfrac{10}{s + 500}$

$5 \times 10^6 = (K_1 s + K_2)(s + 500) + 10\,(s^2 + 25 \times 10^4)$

$= (K_1 + 10)\, s^2 + (500\, K_1 + K_2)\, s + (500\, K_2 + 25 \times 10^5)$

Comparing the coefficients, in LHS and RHS

$K_1 + 10 = 0$ i.e. $K_1 = -10$

$500\, K_1 + K_2 = 0$ i.e. $K_2 = -500\, K_1$. Thus $K_2 = 5000$

Therefore, $I(s) = \left[\dfrac{-10\, s}{s^2 + 25 \times 10^4} + \dfrac{5000}{s^2 + 25 \times 10^4} + \dfrac{10}{s + 500} \right]$

On taking inverse LT, we get $i(t) = 10\,[-\cos 500\, t + \sin 500\, t + e^{-500t}]$ A

$= 14.14 \sin(500\, t - 45^0) + 10\, e^{-500t}$ A

7.8.2 RC CIRCUIT Consider the RC circuit shown in Fig. 7.68 (a). Assume that the switch is closed at time $t = 0$ and assume that the voltage across the capacitor at the time of switching is zero.

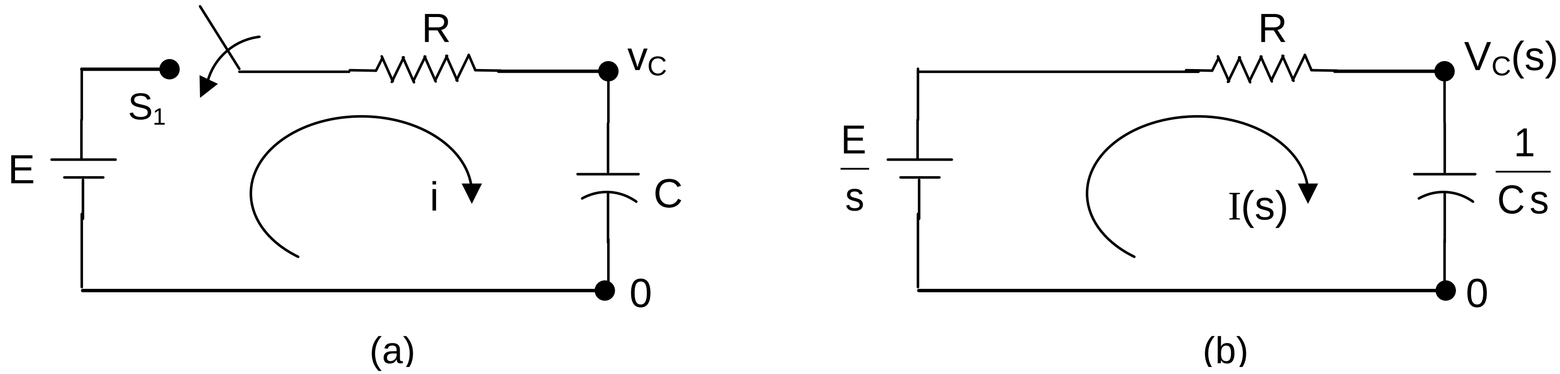


Fig. 7.68 Time domain and s domain - RC circuit.

The transform circuit for time $t > 0$ is shown in Fig. 7.68 (b). From this

$$I(s) = \frac{E/s}{R + \frac{1}{Cs}} = \frac{EC}{RC\,s + 1} = \frac{E/R}{s + \frac{1}{RC}} \tag{7.85}$$

Taking inverse LT $\quad i(t) = \frac{E}{R}\, e^{-\frac{1}{RC}t}$ (7.86)

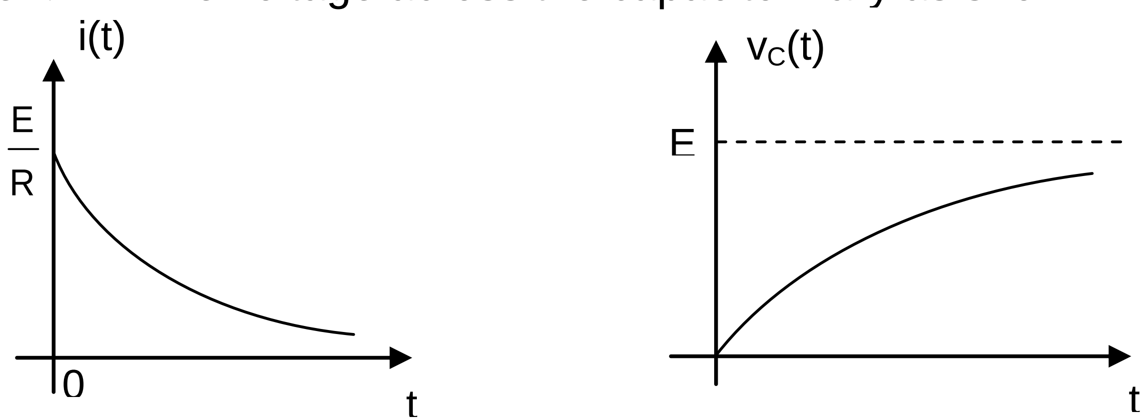
Voltage across the capacitor is $V_C(s) = \frac{1}{Cs}\, I(s) = \frac{E/RC}{s\,(s + \frac{1}{RC})} = E\left(\frac{1}{s} - \frac{1}{s + \frac{1}{RC}}\right)$ (7.87)

Voltage across the capacitor is $V_C(s) = \frac{1}{Cs} I(s) = \frac{E/RC}{s\left(s + \frac{1}{RC}\right)} = E\left(\frac{1}{s} - \frac{1}{s + \frac{1}{RC}}\right)$ (7.87)

Taking inverse LT, we get the capacitor voltage as

$v_C(t) = E\left(1 - e^{-\frac{1}{RC}t}\right)$ (7.88)

The circuit current and the voltage across the capacitor vary as shown in Fig. below.

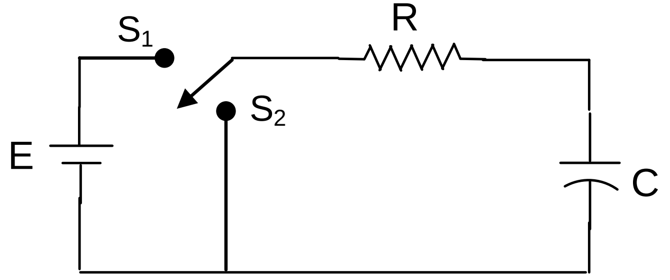


(a) (b)

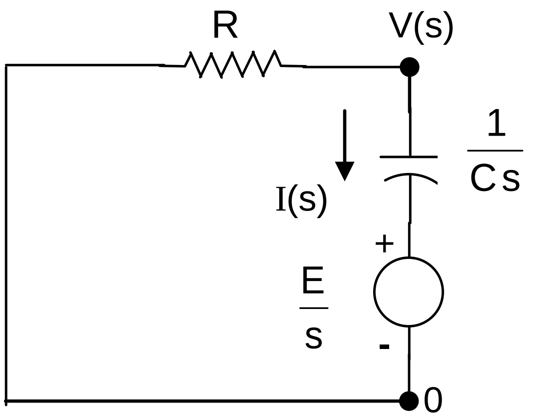
Now, consider the circuit shown in Fig. (a).The switch was in position S_1 for sufficiently long time to establish steady state condition. At time t = 0, it is moved to position S_2.

Before the switch is moved to position S_2, the capacitor gets charged to voltage E. Since the voltage across the capacitor maintains continuity,

$v_C(0^+) = v_C(0^-) = E$



(a)



(b)

The transform circuit for time t > 0 is shown in Fig. (b). From this

$$I(s) = -\frac{E/s}{R + \frac{1}{Cs}} = -\frac{EC}{RCs+1} = -\frac{E/R}{s + \frac{1}{RC}} \tag{7.89}$$

Taking inverse LT $\quad i(t) = -\frac{E}{R}\, e^{-\frac{1}{RC}t}$ (7.90)

It is to be seen that $R\,I(s) + V_C(s) = 0$

Thus $$V_C(s) = -R\,I(s) = \frac{E}{s + \frac{1}{RC}} \qquad (7.91)$$

Taking inverse LT $$v_C(t) = E\, e^{-\frac{1}{RC}t} \qquad (7.92)$$

The wave form of circuit current and the capacitor voltage are shown in Fig. 7.71.

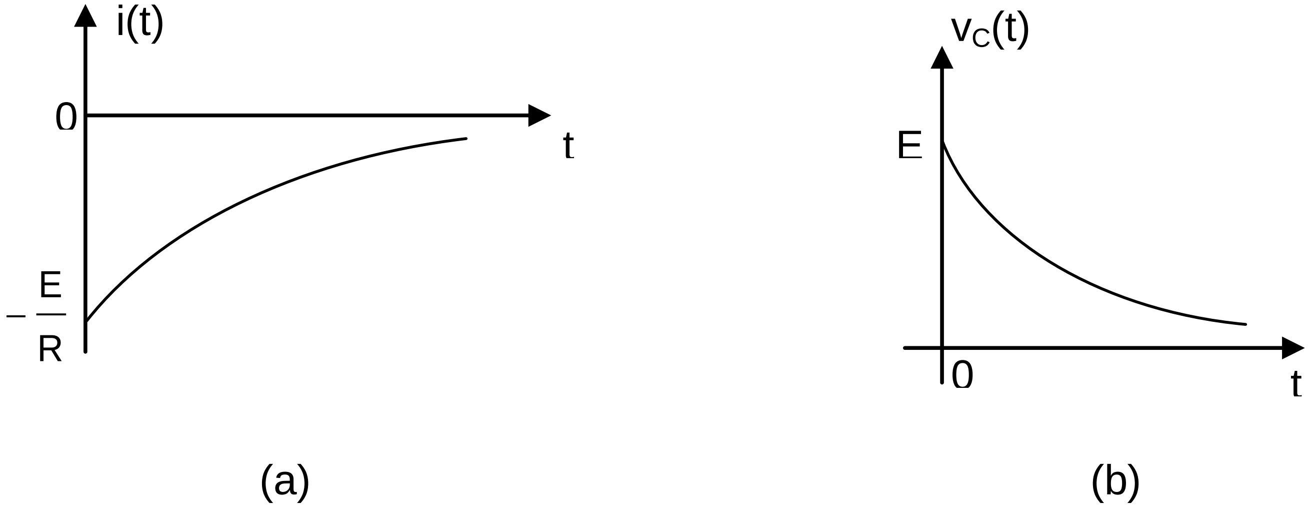
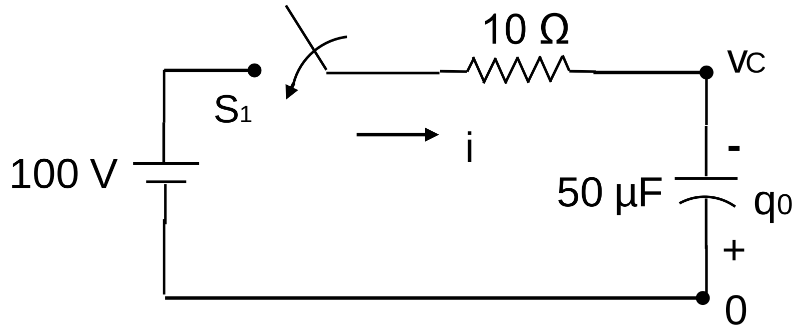


Fig. 7.71 Plot of i(t) and $v_C(t)$ as given by Eq. (7.90) and (7.92).

Example 7.28 In the RC circuit shown below, the capacitor has an initial charge q_0 = 2500 μC. At time t = 0, the switch is closed. Find the circuit current for time t > 0.



Solution

$$v_C(0) = -\frac{q_0}{C} = -\frac{2500 \text{ X } 10^{-6}}{50 \text{ X } 10^{-6}} = -50 \text{ V}$$

Transform circuit for time t > 0 is shown in Fig. 7.73.

Referring to Fig. 7.73,

$$I(s) = \frac{\frac{100}{s} + \frac{50}{s}}{10 + \frac{20000}{s}} = \frac{150}{10s + 20000} = \frac{15}{s + 2000}$$

Taking inverse LT, current $i(t) = 15 \; e^{-2000t}$ A

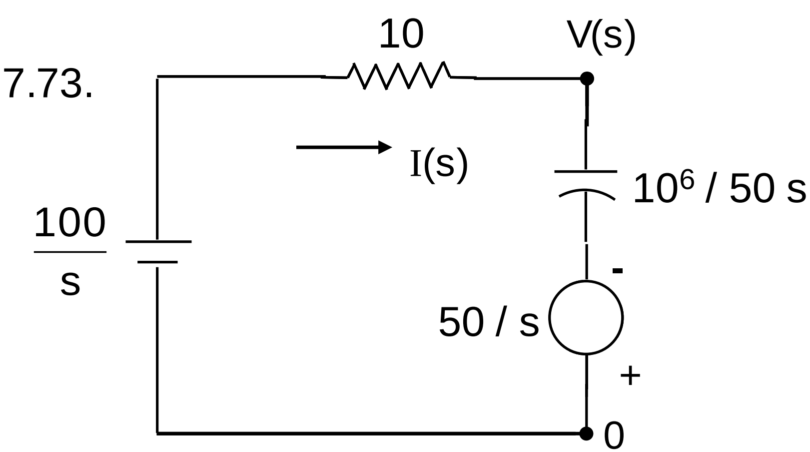
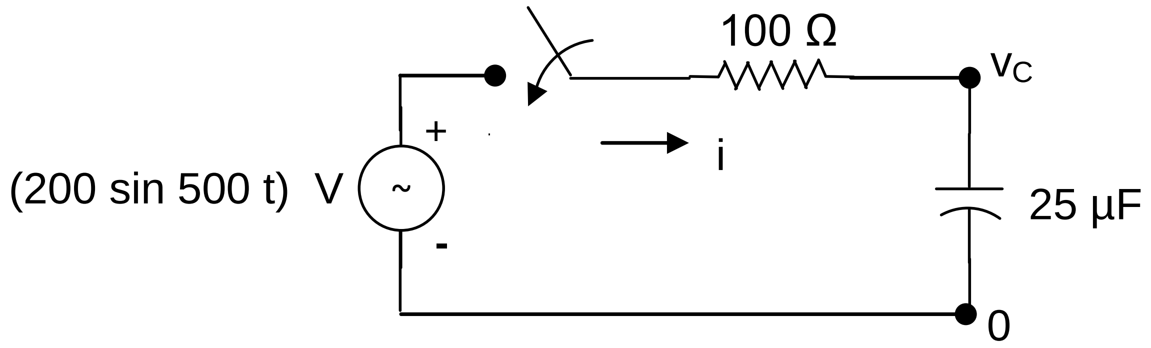


Fig. 7.73 Circuit - Example 7.28.

Example 7.29 For the circuit shown below, find the transient current, assuming that the initial charge on the capacitor as zero, when the switch is closed at time t = 0.



Solution $E(s) = \dfrac{200 \times 500}{s^2 + 250000}$; $\dfrac{1}{Cs} = \dfrac{10^6}{25s}$

$$\text{Therefore, } I(s) = \frac{\dfrac{10^5}{s^2+250000}}{100 + \dfrac{4 \times 10^4}{s}} = \frac{10^5\, s}{(s^2+250000)(100s + 4 \times 10^4)}$$

$$= \frac{1000\, s}{(s^2+250000)(s+400)} = \frac{K_1 s + K_2}{s^2 + 250000} + \frac{K_3}{s+400}$$

$$K_3 = \left.\frac{1000\,s}{s^2 + 250000}\right|_{s = -400} = -0.9756$$

Further, $1000\,s = (K_1\,s + K_2)\,(s + 400) - 0.9756\,(s^2 + 250000)$

$$= (K_1 - 0.9756)\,s^2 + (400\,K_1 + K_2)\,s + (400\,K_2 - 0.9756 \times 250000)$$

Comparing the coefficients, in LHS and RHS we have

$K_1 - 0.9756 = 0$ and $400\,K_1 + K_2 = 1000$

On solving, $K_1 = 0.9756$; $K_2 = 609.76$

Thus, $$I(s) = \frac{0.9756\,s}{s^2 + 250000} + \frac{609.76}{s^2 + 250000} - \frac{0.9756}{s + 400}$$

$$= 0.9756\,\frac{s}{s^2 + 500^2} + 1.2195\,\frac{500}{s^2 + 500^2} - \frac{0.9756}{s + 400}$$

Taking inverse LT $i(t) = 0.9756 \cos 500\,t + 1.2195 \sin 500\,t - 0.9756\,e^{-400t}$ A

Knowing that $\sqrt{(0.9756)^2 + (1.2195)^2} = 1.5617$ and $\tan^{-1}(0.9756 / 1.2195) = 38.66^0$

current $i(t) = 1.5617 \sin(500\,t + 38.66^0) - 0.9756\,e^{-400t}$ A

7.8.3 RLC CIRCUIT

Consider the RLC series circuit shown in Fig. 7.75 (a). Assume that there is no initial charge on the capacitor and there is no initial current through the inductor. The switch is closed at time t = 0. Transform circuit for time t > 0 is shown in Fig. 7.75 (b).

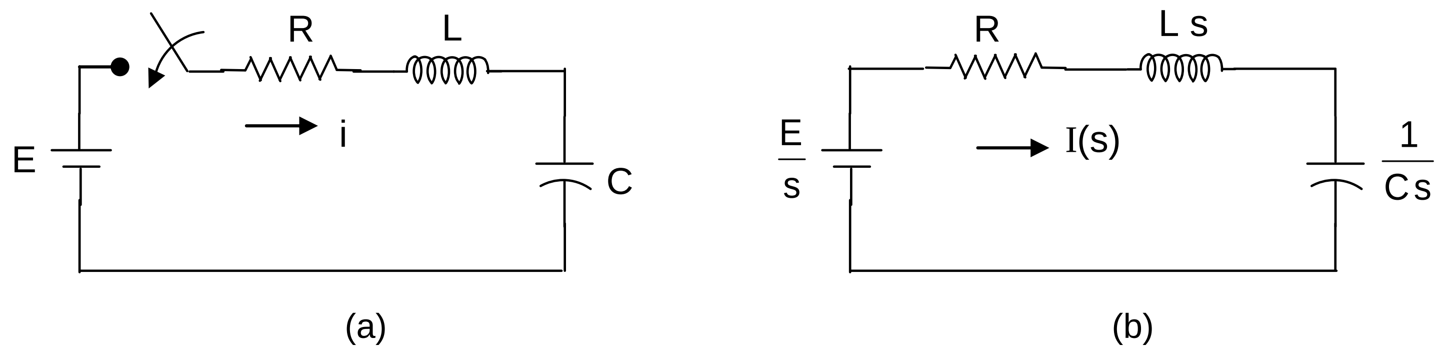


Fig. 7.75 Time domain and s domain - RLC circuit.

Using the transform circuit, expression for the current is obtained as

$$I(s) = \frac{E/s}{R + Ls + \frac{1}{Cs}} = \frac{EC}{RCs + LCs^2 + 1} = \frac{E/L}{s^2 + \frac{R}{L}s + \frac{1}{LC}} \tag{7.93}$$

The roots of the denominator polynomial are

$$s_1, s_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} = \alpha \pm \beta \tag{7.94}$$

$$\text{where } \alpha = -\frac{R}{2L} \text{ and } \beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \tag{7.95}$$

Depending on whether $(\frac{R}{2L})^2 > \frac{1}{LC}$, $(\frac{R}{2L})^2 = \frac{1}{LC}$ or $(\frac{R}{2L})^2 < \frac{1}{LC}$ the discriminant value will be positive, zero or negative and three different cases of solutions are possible.

The value of R, for which the discriminant is zero, is called the critical resistance, R_C.

Then $\frac{R_C^2}{4L^2} = \frac{1}{LC}$;

$$\text{Thus } R_C = 2\sqrt{\frac{L}{C}} \tag{7.96}$$

If the circuit resistance $R > R_C$, then $(\frac{R}{2L})^2 > \frac{1}{LC}$.

If the circuit resistance $R < R_C$, then $(\frac{R}{2L})^2 < \frac{1}{LC}$.

Case 1

$$\left(\frac{R}{2L}\right)^2 > \frac{1}{LC} \quad \text{i.e. } R > R_C \tag{7.97}$$

The two roots s_1 and s_2 are real and distinct. $s_1 = \alpha + \beta$ and $s_2 = \alpha - \beta$ (7.98)

$$\text{Then, } I(s) = \frac{K_1}{s - (\alpha + \beta)} + \frac{K_2}{s - (\alpha - \beta)} \tag{7.99}$$

Taking inverse LT, we get

$$i(t) = K_1\, e^{(\alpha + \beta)\, t} + K_2\, e^{(\alpha - \beta)\, t} = e^{\alpha t}\, [K_1\, e^{\beta t} + K_2\, e^{-\beta t}] \tag{7.100}$$

Its plot is shown in Fig. 7.76. In this case the current is said to be over-damped.

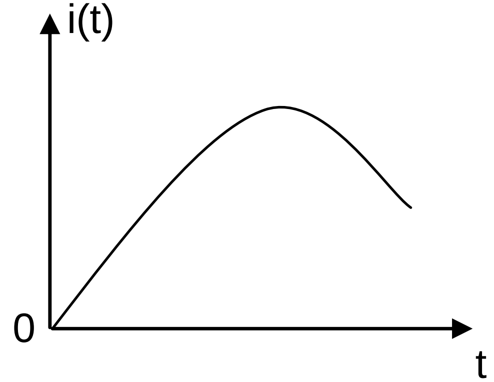


Fig. 7.76 RLC circuit over-damped response.

Case 2

$$\left(\frac{R}{2L}\right)^2 = \frac{1}{LC} \quad \text{i.e. } R = R_C \tag{7.101}$$

Then, $\beta = 0$ and hence the roots are $s_1 = s_2 = \alpha$ (7.102)

Thus, $$I(s) = \frac{E/L}{(s-\alpha)^2} = \frac{K}{(s-\alpha)^2} \tag{7.103}$$

Taking inverse LT, we get $i(t) = K\,t\,e^{\alpha t}$ (7.104)

The plot of this current transient is shown in Fig. 7.77. In this case, the current is said to be critically damped.

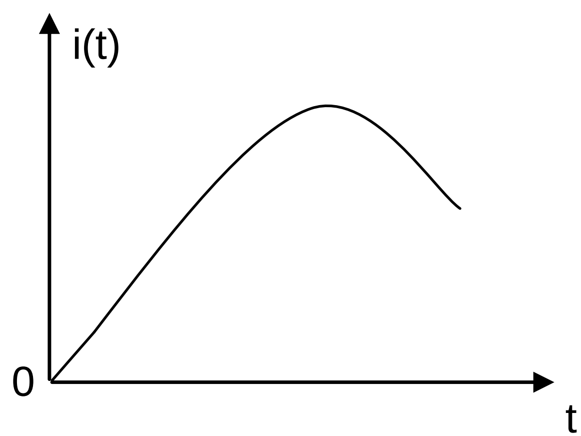


Fig. 7.77 RLC circuit critically-damped response.

<u>Case 3</u> $\quad (\frac{R}{2L})^2 < \frac{1}{LC}$ i.e. $R < R_C$ (7.105)

For this case, the roots are complex conjugate, $s_1 = \alpha + j\,\beta$ and $s_2 = \alpha - j\,\beta$ (7.106)

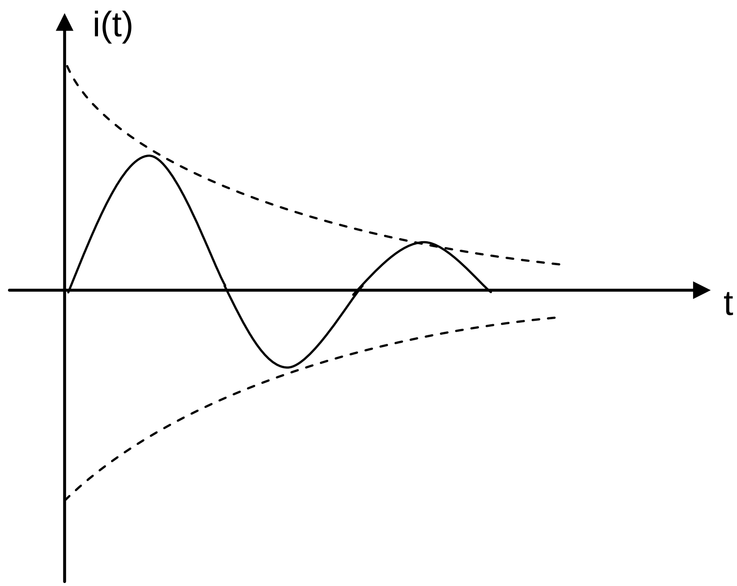
Then, $I(s) = \dfrac{E/L}{(s-\alpha-j\,\beta)\,(s-\alpha+j\beta)} = \dfrac{E/L}{(s-\alpha)^2+\beta^2} = \dfrac{E}{L\beta}\,\dfrac{\beta}{(s-\alpha)^2+\beta^2}$ (7.107)

$$= A\,\frac{\beta}{(s-\alpha)^2+\beta^2} \tag{7.108}$$

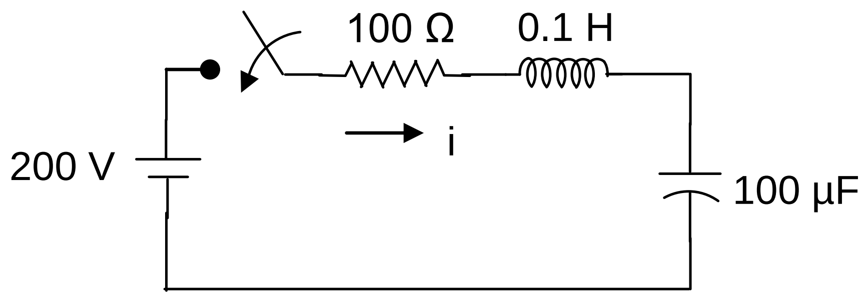
Taking inverse LT, we get $\quad i(t) = A\ e^{\alpha t}\ \sin\beta t$ (7.109)

As seen in Equation 7.95, α will be a negative number. Thus, for this **under damped** case, the current is oscillatory and at the same time it decays.

Waveform shown is a exponentially decaying sinusoidal wave



Example 7.30 For the RLC circuit shown, find the expression for the transient current when the switch is closed at time t = 0. Assume initially relaxed circuit conditions.



Solution The transform circuit is shown in Fig. 7.80.

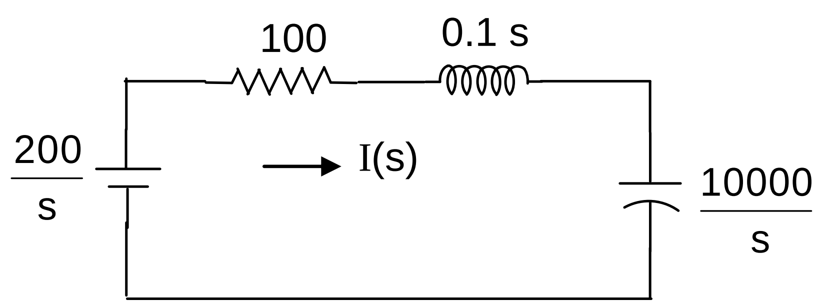


Fig. 7.80 Transform circuit - Example 7.30.

$$\text{Current } I(s) = \frac{200/s}{100 + 0.1s + \dfrac{10000}{s}} = \frac{200}{0.1s^2 + 100s + 10000} = \frac{2000}{s^2 + 1000s + 100000}$$

Current $I(s) = \dfrac{200/s}{100 + 0.1s + \dfrac{10000}{s}} = \dfrac{200}{0.1s^2 + 100s + 10000} = \dfrac{2000}{s^2 + 1000s + 100000}$

The roots of the denominator polynomial are

$$s_1, s_2 = \frac{-10^3 \pm \sqrt{10^6 - 0.4 \text{ X } 10^6}}{2} = -1127 \text{ and } -887.3$$

$$\text{Therefore, } I(s) = \frac{2000}{(s + 1127)\,(s + 887.3)} = \frac{K_1}{s + 1127} + \frac{K_2}{s + 887.3}$$

$$K_1 = \left.\frac{2000}{s + 887.3}\right|_{s = -112.7} = 2.582$$

$$K_2 = \left.\frac{2000}{s + 112.7}\right|_{s = -887.3} = -2.582$$

$$\text{Thus, } I(s) = 2.582 \left[\frac{1}{s + 112.7} - \frac{1}{s + 887.3}\right]$$

Taking inverse LT, we get current $i(t) = 2.582\,(e^{-112.7t} - e^{-887.3t})$ A

This is an example for over-damped.

Example 7.31 Taking the initial conditions as zero, find the transient current in the circuit shown in Fig. 7.81 when the switch is closed at time t = 0.

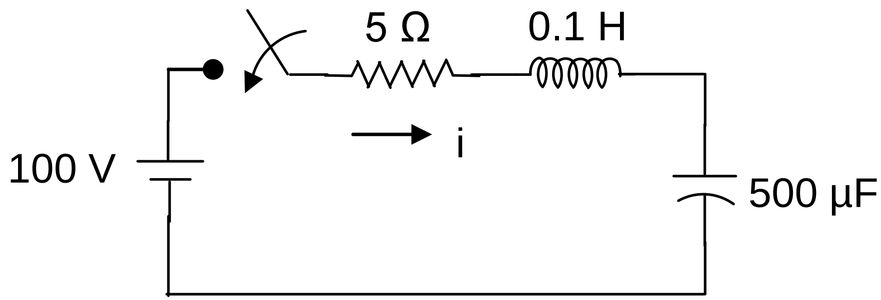


Fig. 7.81 Circuit for Example 7.31.

Solution The transform circuit is shown in Fig. 7.82.

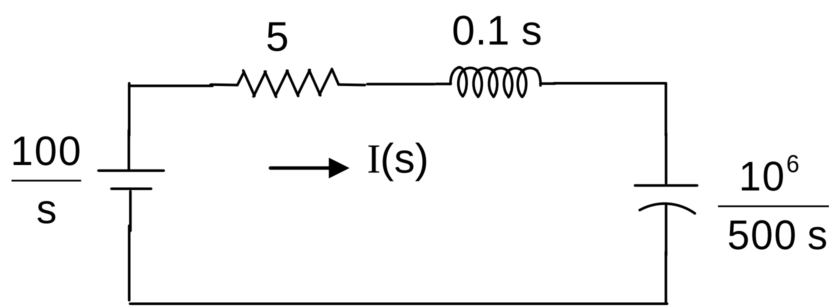


Fig. 7.82 Transform circuit - Example 7.31.

$$\text{Current } I(s) = \frac{100/s}{5 + 0.1s + \dfrac{10^6}{500\,s}} = \frac{100}{0.1s^2 + 5s + 2000} = \frac{1000}{s^2 + 50s + 20000}$$

Current $I(s) = \dfrac{100/s}{5 + 0.1s + \dfrac{10^6}{500\,s}} = \dfrac{100}{0.1s^2 + 5s + 2000} = \dfrac{1000}{s^2 + 50s + 20000}$

The roots of the denominator polynomial are

$$s_1,\, s_2 = \frac{-50 \pm \sqrt{2500 - 80000}}{2} = -25 \pm j139.1941$$

It can be seen that

$$s^2 + 50\,s + 20000 = (s + 25)^2 + (139.1941)^2$$

Thus, $I(s) = \dfrac{1000}{(s+25)^2 + (139.1941)^2} = 7.1842\,\dfrac{139.1941}{(s+25)^2 + (139.1941)^2}$

Taking inverse LT, we get $\quad i(t) = 7.1842\ e^{-25t}\ \sin(139.1941\ t)$ A

This is an example for under-damped.

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DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

CIRCUIT THEORY – SEEA1201

UNIT-V- RESONANCE AND COUPLED CIRCUITS

In the last lesson, the following points were described:

1. How to compute the total impedance in parallel and series-parallel circuits?
2. How to solve for the current(s) in parallel and series-parallel circuits, fed from single phase ac supply, and then draw complete phasor diagram?
3. How to find the power consumed in the circuits and also the different components, and the power factor (lag/lead)?

In this lesson, the phenomenon of the resonance in series and parallel circuits, fed from single phase variable frequency supply, is presented. Firstly, the conditions necessary for resonance in the above circuits are derived. Then, the terms, such as bandwidth and half power frequency, are described in detail. Some examples of the resonance conditions in series and parallel circuits are presented in detail, along with the respective phasor diagrams.

Keywords: Resonance, bandwidth, half power frequency, series and parallel circuits,

After going through this lesson, the students will be able to answer the following questions;

1. How to derive the conditions for resonance in the series and parallel circuits, fed from a single phase variable frequency supply?
2. How to compute the bandwidth and half power frequency, including power and power factor under resonance condition, of the above circuits?
3. How to draw the complete phasor diagram under the resonance condition of the above circuits, showing the currents and voltage drops in the different components?

Resonance in Series and Parallel Circuits

Series circuit

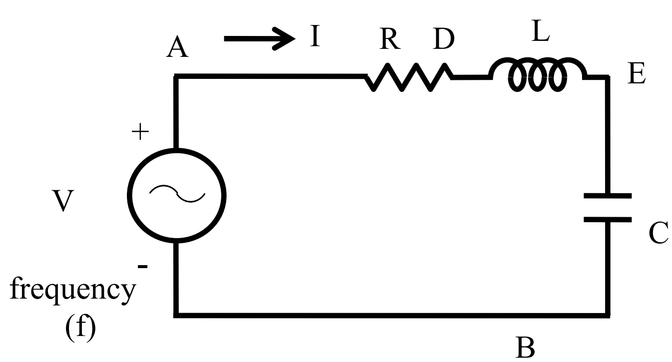

Fig. 17.1 (a) Circuit diagram.

The circuit, with resistance R, inductance L, and a capacitor, C in series (Fig. 17.1a) is connected to a single phase variable frequency (f) supply.

The total impedance of the circuit is

$$Z\angle\phi = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

where,

$$Z = \sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]} \quad ; \quad \phi = \tan^{-1}\frac{(\omega L - 1/\omega C)}{R} \quad ; \quad \omega = 2\pi f$$

The current is

$$I\angle-\phi = \frac{V\angle 0°}{Z\angle\phi} = (V/Z)\angle-\phi$$

where $I = \dfrac{V}{\left[R^2 + (\omega L - (1/\omega C))^2\right]^{\frac{1}{2}}}$

The current in the circuit is maximum, if $\omega L = \dfrac{1}{\omega C}$.

The frequency under the above condition is

$$f_o = \frac{\omega_o}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

This condition under the magnitude of the current is maximum, or the magnitude of the impedance is minimum, is called resonance. The frequency under this condition with the constant values of inductance L, and capacitance C, is called resonant frequency. If the capacitance is variable, and the frequency, f is kept constant, the value of the capacitance needed to produce this condition is

$$C = \frac{1}{\omega^2 L} = \frac{1}{(2\pi f)^2 L}$$

The magnitude of the impedance under the above condition is $|Z| = R$, with the reactance $X = 0$, as the inductive reactance $X_l = \omega L$ is equal to capacitive reactance $X_C = 1/\omega C$. The phase angle is $\phi = 0°$, and the power factor is unity ($\cos\phi = 1$), which means that the current is in phase with the input (supply) voltage.. So, the magnitude of the current ($|(V/R)|$) in the circuit is only limited by resistance, R. The phasor diagram is shown in Fig. 17.1b.

The magnitude of the voltage drop in the inductance L/capacitance C (both are equal, as the reactance are equal) is $I\cdot\omega_o L = I\cdot(1/\omega_o C)$.

The magnification of the voltage drop as a ratio of the input (supply) voltage is

$$Q = \frac{\omega_o L}{R} = \frac{2\pi f_o L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

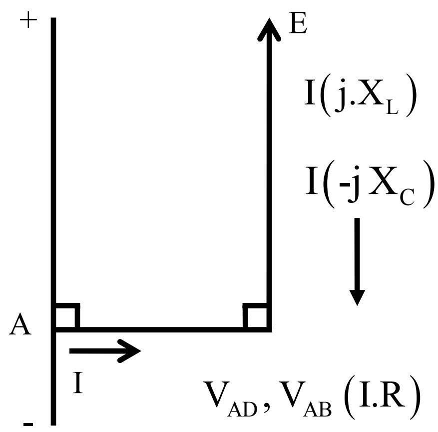

Fig. 17.1 (b) Phasor Diagram

It is termed as Quality (Q) factor of the coil.

The impedance of the circuit with the constant values of inductance L, and capacitance C is minimum at resonant frequency (f_o), and increases as the frequency is changed, i.e. increased or decreased, from the above frequency. The current is maximum at $f = f_o$, and decreases as frequency is changed ($f > f_o$, or $f < f_o$), i.e. $f \neq f_o$. The variation of current in the circuit having a known value of capacitance with a variable frequency supply is shown in Fig. 17.2.

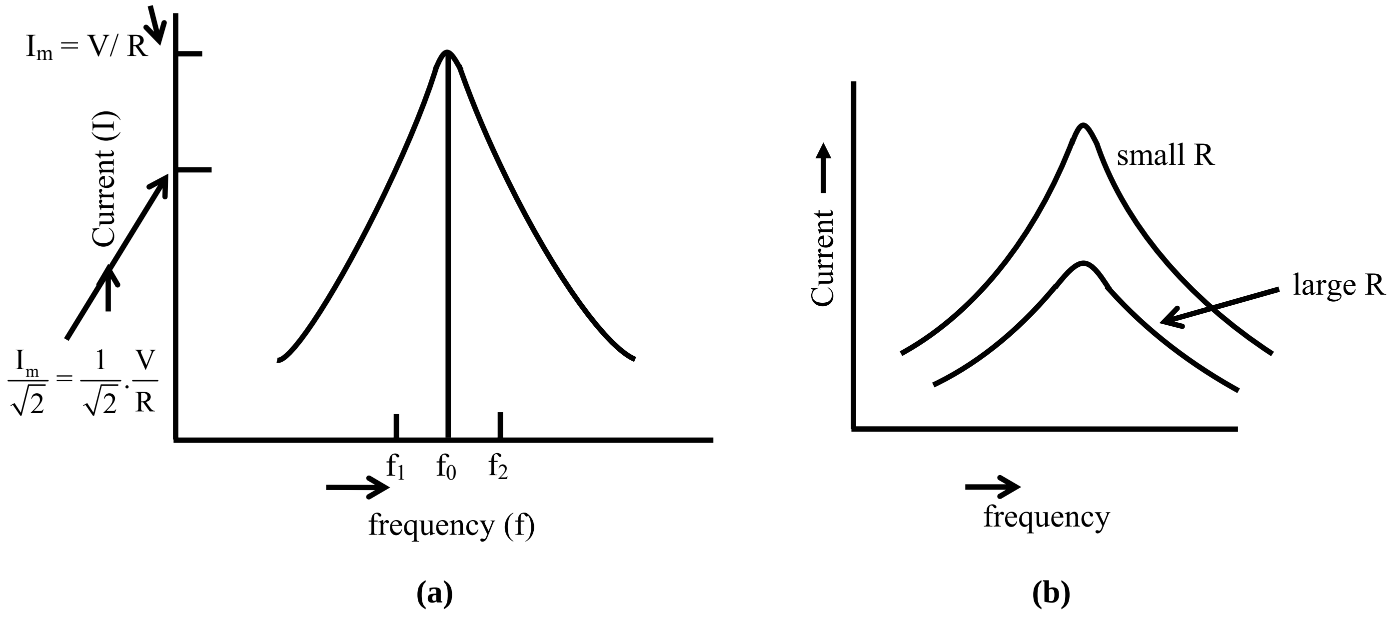

Fig. 17.2 Variation of current under variable frequency supply

The maximum value of the current is (V/R). If the magnitude of the current is reduced to $(1/\sqrt{2})$ of its maximum value, the power consumed in R will be half of that with the maximum current, as power is I^2R. So, these points are termed as half power points. If the two frequencies are taken as f_1 and f_2, where $f_1 = f_0 - \Delta f/2$ and $f_2 = f_0 + \Delta f/2$, the band width being given by $\Delta f = f_2 - f_1$.

The magnitude of the impedance with the two frequencies is

$$Z = \left[R^2 + \left(2\pi(f_0 \pm \Delta f/2)L - \frac{1}{2\pi(f_0 \pm \Delta f/2)C} \right)^2 \right]^{\frac{1}{2}}$$

As $(2\pi f_0 L = 1/2\pi f_0 C)$ and the ratio $(\Delta f / 2 f_0)$ is small, the magnitude of the reactance of the circuit at these frequencies is $X = X_{L0}(\Delta f / f_0)$. As the current is $(1/\sqrt{2})$ of its maximum value, the magnitude of the impedance is $(\sqrt{2})$ of its minimum value (R) at resonant frequency.

So, $Z = \sqrt{2} \cdot R = \left[R^2 + \left(X_{L0}(\Delta f / f_0) \right)^2 \right]^{\frac{1}{2}}$

From the above, it can be obtained that $(\Delta f / f_0) X_{L0} = R$

or $\Delta f = f_2 - f_1 = \dfrac{R f_0}{X_{L0}} = \dfrac{R f_0}{2\pi f_0 L} = \dfrac{R}{2\pi L}$

The band width is given by $\Delta f = f_2 - f_1 = R/(2\pi L)$

It can be observed that, to improve the quality factor (Q) of a coil, it must be designed to have its resistance, R as low as possible. This also results in reduction of band width and losses (for same value of current). But if the resistance, R cannot be decreased, then Q will decrease, and also both band width and losses will increase.

Example 17.1

A constant voltage of frequency, 1 MHz is applied to a lossy inductor (r in series with L), in series with a variable capacitor, C (Fig. 17.3). The current drawn is maximum, when C = 400 pF; while current is reduced to $(1/\sqrt{2})$ of the above value, when C = 450 pF. Find the values of r and L. Calculate also the quality factor of the coil, and the bandwidth.

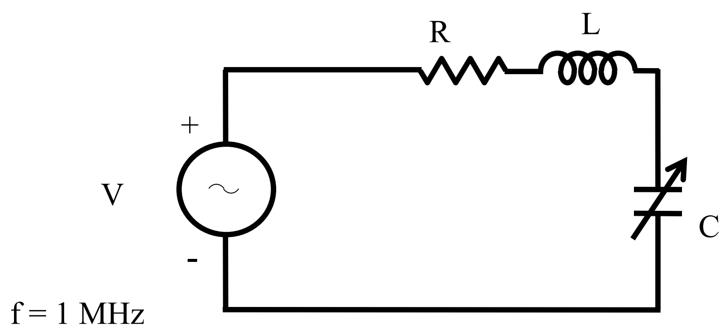

Fig. 17.3 Circuit diagram

Solution

$f = 1 \text{ MHz} = 10^6 \text{ Hz}$ $\omega = 2\pi f$ $C = 400\, pF = 400 \cdot 10^{-12}\, F$

$I_{max} = V / r$ as $X_L = X_C$ $X_c = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi \cdot 10^6 \times 400 \cdot 10^{-12}} = 398\ \Omega$

$X_L = X_C = 2\pi f L = 398\ \Omega$ $L = \dfrac{398.0}{2\pi \cdot 10^6} = 63.34\ \mu H$

$C_1 = 450\ pF$ $X_{C1} == \dfrac{1}{2\pi \cdot 10^6 \times 450 \cdot 10^{-12}} = 353.7\ \Omega$

$Z\angle\phi = r + j\left(X_L - X_{C1}\right) = r + j\,(398.0 - 353.7) = (r + j\,44.3)\ \Omega$

$I = \dfrac{I_{max}}{\sqrt{2}} = \dfrac{V}{\sqrt{2} \cdot r} = \dfrac{V}{Z} = \dfrac{V}{\sqrt{r^2 + (44.3)^2}}$

From above, $\sqrt{2} \cdot r = \sqrt{r^2 + (44.3)^2}$ or $2r^2 = r^2 + (44.3)^2$

or $r = 44.3\ \Omega$

The quality factor of the coil is $Q = \dfrac{X_L}{r} = \dfrac{398.0}{44.3} = 8.984$

The band with is

$\Delta f = f_2 - f_1 = \dfrac{r}{2\pi L} = \dfrac{44.3}{2\pi \times 63.34 \cdot 10^{-6}} = \dfrac{44.3}{398 \cdot 10^{-6}} = 0.1113 \cdot 10^6 = 0.1113\ MHz$

$= 111.3 \cdot 10^3 = 111.3\ kHz$

Parallel circuit

The circuit, with resistance R, inductance L, and a capacitor, C in parallel (Fig. 17.4a) is connected to a single phase variable frequency (f) supply.

The total admittance of the circuit is

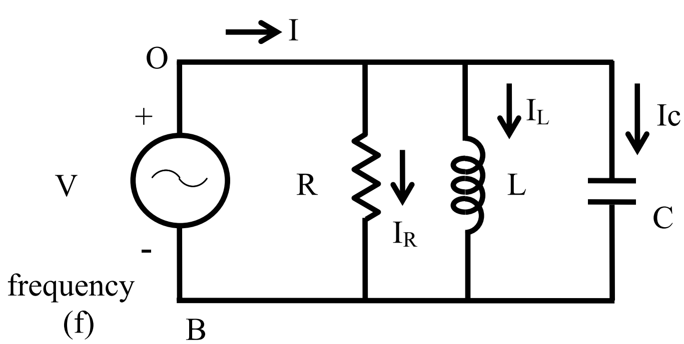

Fig. 17.4 (a) Circuit diagram.

$$Y\angle\phi = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

where,

$$Y = \sqrt{\left[\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2\right]} \; ; \quad \phi = \tan^{-1}\left[R\left(\omega C - \frac{1}{\omega L}\right)\right] \; ; \quad \omega = 2\pi f$$

The impedance is $Z \angle -\phi = 1/Y \angle \phi$

The current is

$I \angle \phi = V \angle 0° \cdot Y \angle \phi = (V \cdot Y) \angle \phi = V \angle 0° / Z \angle -\phi = (V/Z) \angle \phi$

where, $I = V \sqrt{\left[\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2\right]}$

The current in the circuit is minimum, if $\omega C = \dfrac{1}{\omega L}$

The frequency under the above condition is

$$f_o = \frac{\omega_o}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

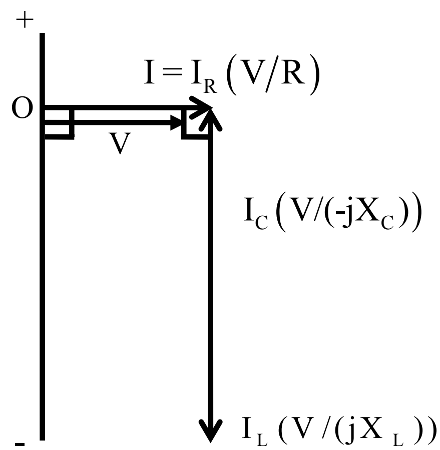

Fig. 17.4 (b) Phasor Diagram

This condition under which the magnitude of the total (supply) current is minimum, or the magnitude of the admittance is minimum (which means that the impedance is maximum), is called resonance. It may be noted that, for parallel circuit, the current or admittance is minimum (the impedance being maximum), while for series circuit, the current is maximum (the impedance being minimum). The frequency under this condition with the constant values of inductance L, and capacitance C, is called resonant frequency. If the capacitance is variable, and the frequency, f is kept constant, the value of the capacitance needed to produce this condition is

$$C = \frac{1}{\omega^2 L} = \frac{1}{(2\pi f)^2 L}$$

The magnitude of the impedance under the above condition is ($|Z| = R$), while the magnitude of the admittance is ($|Y| = G = (1/R)$). The reactive part of the admittance is

$B = 0$, as the susceptance (inductive) $B_L = (1/\omega L)$ is equal to the susceptance (capacitive) $B_C = \omega C$. The phase angle is $\phi = 0°$, and the power factor is unity ($\cos\phi = 1$). The total (supply) current is phase with the input voltage. So, the magnitude of the total current ($|(V/R)|$) in the circuit is only limited by resistance R. The phasor diagram is shown in Fig. 17.4b.

The magnitude of the current in the inductance, L / capacitance, C (both are equal, as the reactance are equal), is $V(1/\omega_o L) = V \cdot \omega_o C$. This may be termed as the circulating current in the circuit with only inductance and capacitance, the magnitude of which is

$$|I_L| = |I_C| = V\sqrt{\frac{C}{L}}$$

substituting the value of $\omega_o = 2\pi f_o$. This circulating current is smaller in magnitude than the input current or the current in the resistance as $\omega_o C = (1/\omega_o L) > R$.

The input current increases as the frequency is changed, i.e. increased or decreased from the resonant frequency ($f > f_o$, or $f < f_o$), i.e. $f \neq f_o$.

In the two cases of series and parallel circuits described earlier, all components, including the inductance, are assumed to be ideal, which means that the inductance is lossless, having no resistance. But, in actual case, specially with an iron-cored choke coil, normally a resistance r is assumed to be in series with the inductance L, to take care of the winding resistance and also the iron loss in the core. In an air-cored coil, the winding resistance may be small and no loss occurs in the air core.

An iron-cored choke coil is connected in parallel to capacitance, and the combination is fed to an ac supply (Fig. 17.5a).

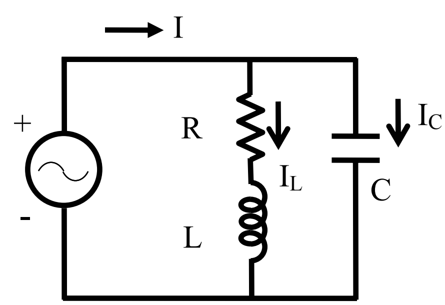

Fig. 17.5 (a) Circuit diagram.

The total admittance of the circuit is

$$Y = Y_1 + Y_2 = \frac{1}{r + j\omega L} + j\omega C = \frac{r - j\omega L}{r^2 + \omega^2 L^2} + j\omega C$$

If the magnitude of the admittance is to be minimum, then

$$\omega C = \frac{\omega L}{r^2 + \omega^2 L^2} \quad \text{or} \quad C = \frac{L}{r^2 + \omega^2 L^2} .$$

The frequency is

$$f_o = \frac{\omega_o}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{L}{C} - r^2}$$

This is the resonant frequency. The total admittance is $Y\angle 0° = \dfrac{r}{r^2 + \omega^2 L^2}$

The total impedance is $Z\angle 0° = \dfrac{r^2 + \omega^2 L^2}{r}$

The total (input) current is

$$I\angle 0° = \frac{V\angle 0°}{Z\angle 0°} = V\angle 0° \cdot Y\angle 0° = \left(\frac{V}{Z}\right)\angle 0° = (V \cdot Y)\angle 0° = \frac{V \cdot r}{r^2 + \omega^2 L^2}$$

This current is at unity power factor with $\phi = 0°$. The total current can be written as

$$I\angle 0° = I + j\,0 = I_L \angle -\phi_L + j\,I_C = I_L \cos\phi_L + j\left(I_L \sin\phi_L - I_C\right)$$

So, the condition is $I_C = I_L \sin\phi_l$

where $I_C = \dfrac{V}{X_C} = V \cdot \omega C$; $I_L = \dfrac{V}{\sqrt{r^2 + \omega^2 L^2}}$; $\sin\phi_L = \dfrac{\omega L}{\sqrt{r^2 + \omega^2 L^2}}$

From the above, the condition, as given earlier, can be obtained.

The total current is $I = I_L \cos\phi_L$

The value, as given here, can be easily obtained. The phasor diagram is shown in Fig. 17.5b. It may also be noted that the magnitude of the total current is minimum, while the magnitude of the impedance is maximum.

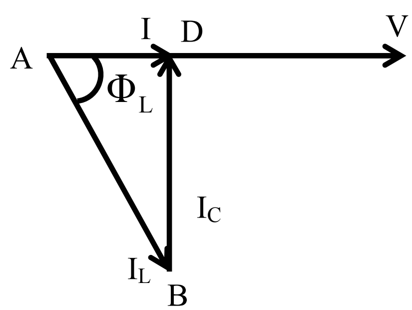

Fig. 17.5 (b) Phasor Diagram

Example 17.2

A coil, having a resistance of 15 Ω and an inductance of 0.75 H, is connected in series with a capacitor (Fig. 17.6a. The circuit draws maximum current, when a voltage of 200 V at 50 Hz is applied. A second capacitor is then connected in parallel to the circuit (Fig. 17.6b). What should be its value, such that the combination acts like a non-inductive resistance, with the same voltage (200 V) at 100 Hz? Calculate also the current drawn by the two circuits.

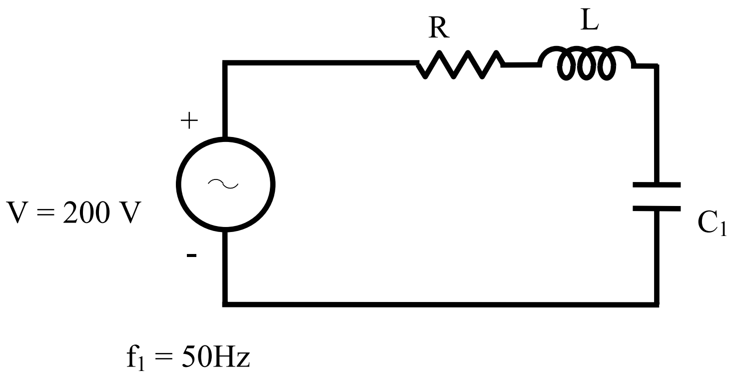

Fig. 17.6 (a) Circuit diagram

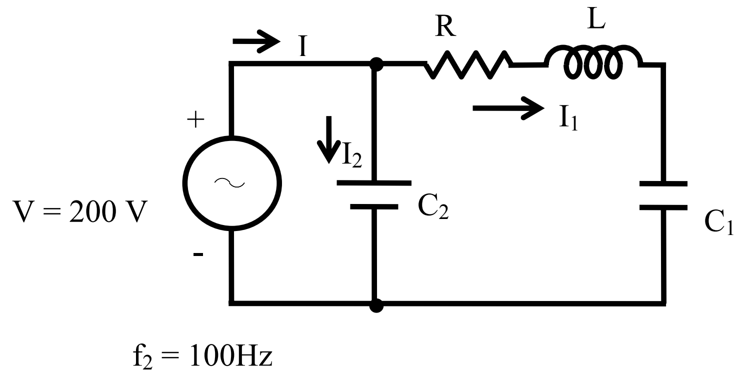

Fig. 17.6 (b) Circuit diagram

Solution

$f_1 = 50$ Hz V = 200 V R = 15 Ω L = 0.75 H

From the condition of resonance at 50 Hz in the series circuit,

$$X_{L1} = \omega_1 L = 2\pi\, f_1\, L = X_{C1} = \frac{1}{\omega_1 C_1} = \frac{1}{2\pi\, f_1\, C_1}$$

So, $C_1 = \dfrac{1}{(2\pi\, f_1)^2 L} = \dfrac{1}{(2\pi\cdot 50)^2 \times 0.75} = 13.5\cdot 10^{-6} = 13.5\,\mu F$

The maximum current drawn from the supply is, $I_{\max} = V/R = 200/15 = 13.33\,A$

$f_2 = 100$ Hz $\omega_2 = 2\pi\, f_2 = 2\pi\cdot 100 = 628.3\ rad/s$

$X_{L2} = 2\pi\, f_2 L = 2\pi\cdot 100\cdot 0.75 = 471.24\ \Omega$

$$X_{C2} = \frac{1}{2\pi\, f_2 C_1} = \frac{1}{2\pi\cdot 100\cdot 13.5\cdot 10^{-6}} = 117.8.\ \Omega$$

$Z_1\angle\phi_1 = R + j\left(X_{L2} - X_{C2}\right) = 15 + j(471.24 - 117.8) = 15 + j\,353.44$

$= 353.75\angle 87.57°\ \Omega$

$$Y_1\angle -\phi_1 = \frac{1}{Z_1\angle\phi_1} = \frac{1}{15 + j\,353.44} = \frac{1}{353.75\angle 87.57°} = 2.827\cdot 10^{-3}\ \angle -87.57°$$

$= (0.12 - j\,2.824)\cdot 10^{-3}\ \Omega^{-1}$

$Y_2 = 1/Z_2 = j\left(\omega_2\, C_2\right)$

As the combination is resistive in nature, the total admittance is

$Y\angle 0° = Y + j\,0 = Y_1 + Y_2 = (0.12 - j\,2.824)\cdot 10^{-3} + j\,\omega_2\, C_2$

From the above expression, $\omega_2\, C_2 = 628.3\cdot C_2 = 2.824\cdot 10^{-3}$

or, $C_2 = \dfrac{2.824\cdot 10^{-3}}{628.3} = 4.5\cdot 10^{-6} = 4.5\,\mu F$

The total admittance is $Y = 0.12\cdot 10^{-3}\ \Omega^{-1}$

The total impedance is $Z = 1/Y = 1/\left(0.12\cdot 10^{-3}\right) = 8.33\cdot 10^{3}\ \Omega = 8.33\ k\Omega$

The total current drawn from the supply is

$I = V \cdot Y = V / Z = 200 \times 0.12 \cdot 10^{-3} = 0.024\ A = 24 \cdot 10^{-3} = 24\ mA$

The phasor diagram for the circuit (Fig. 17.6b) is shown in Fig. 17.6c.

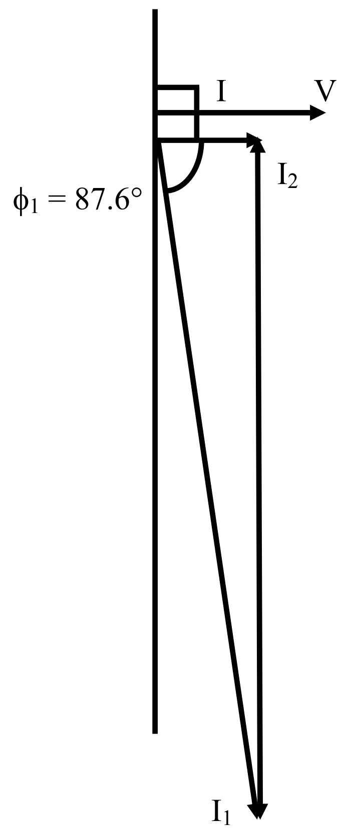

Fig. 17.6 (c) Phasor diagram

The condition for resonance in both series and parallel circuits fed from single phase ac supply is described. It is shown that the current drawn from the supply is at unity power factor (upf) in both cases. The value of the capacitor needed for resonant condition with a constant frequency supply, and the resonant frequency with constant value of capacitance, have been derived. Also taken up is the case of a lossy inductance coil in parallel with a capacitor under variable frequency supply, where the total current will be at upf. The quality factor of the coil and the bandwidth of the series circuit with known value of capacitance have been determined. This is the final lesson in this module of single phase ac circuits. In the next module, the circuits fed from three phase ac supply will be described.

Problems

17.1 A coil having a resistance of 20 Ω and inductance of 20 mH, in series with a capacitor is fed from a constant voltage variable frequency supply. The maximum current is 10 A at 100 Hz. Find the two cut-off frequencies, when the current is 0.71 A.

17.2 With the ac voltage source in the circuit shown in Fig. 17.7 operating a frequency of f, it was found that $I = 1.0\angle 0°$ A. When the source frequency was doubled (2f), the current became $I = 0.707\angle -45°$ A. Find:

a) The frequency f, and
b) The inductance L, and also the reactances, X_L and X_C at 2f

17.3 For the circuit shown in Fig. 17.8,

a) Find the resonant frequency f_0, if R = 250 Ω, and also calculate Q_0 (quality factor), BW (band width) in Hz, and lower and upper cut-off frequencies (f_l and f_2) of the circuit.
b) Suppose it was desired to increase the selectivity, so that BW was 65 Hz. What value of R would accomplish this?

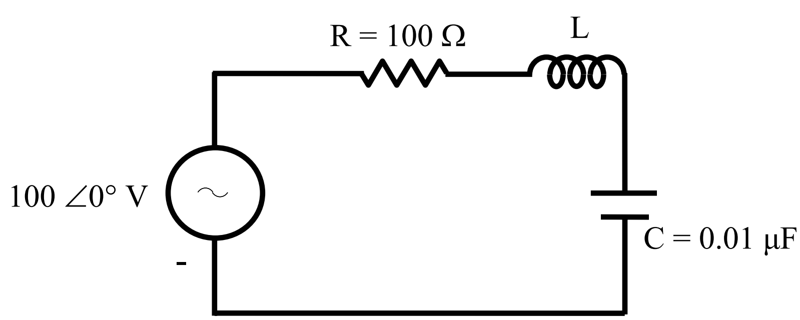

Fig. 17.7

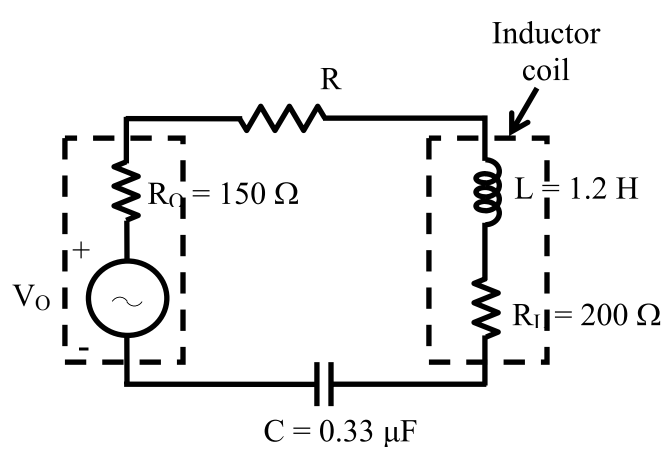

Fig. 17.8

17.4 (a) For the circuit shown in Fig. 17.9, show that the circulating current is given by $V.\sqrt{C/L}$, if R is small and V is the input voltage.

(b) Find the total current at

(i) resonant frequency, f_0, and

(ii) at a frequency, $f_1 = 0.9\ f_0$.

17.5 The circuit components of a parallel circuit shown in Fig. 17.10 are $R = 60\ k\Omega$, $L = 5mH$, and $C = 50\ pF$. Find

a) the resonant frequency, f_0,
b) the quality factor, Q_0, and
c) the bandwidth.

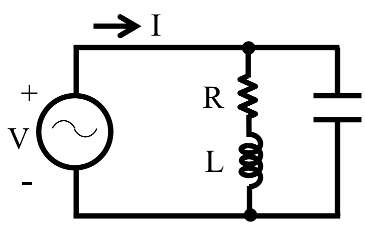

Fig. 17.9

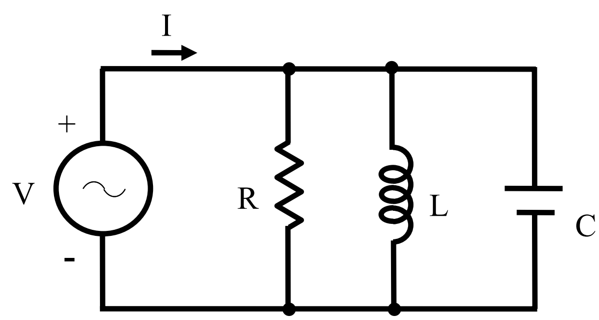

Fig. 17.10

List of Figures