Physics 243 Lecture Notes

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Contents

Introduction to Modern Physics

0.1 1900's Unresolved Experiments

Michaelson and Morley Experiment

Blackbody Radiation Spectrum

0.2 Special Theory of Relativity

For fast moving particles, their energy, mass, and momentum are different from those in the classical physics. New laws also govern how motion is added (velocity addition).

Any particle that moves faster than 0.1 c must be treated relativistically.

Particles that move slower than 0.01 c can be treated classically.

0.3 Quantum Mechanics

Important Experiments

0.4 Review

0.4.1 Waves

$$v = \lambda f$$

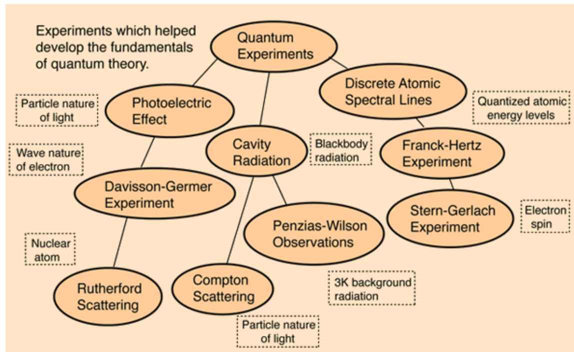


Figure 1: Modern Physics Experiments

0.4.2 Electric Potential Energy

$$U = qV$$

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

Chapter 38

Photons: Light Waves Behaving As Particles

38.1 Light Absorption: The Photoelectric Effect

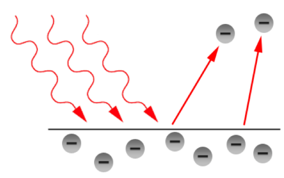


Figure 38.1: Photoelectric Effect

Photoelectric Effect from HyperPhysics
http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html

Photon Energy

$$E = hf = \frac{hc}{\lambda}$$

$$h = 6.626 \times 10^{-34} \text{ J.s}$$

Ex. 38.1 What is the Planck's constant in eV.s?

Photon Momentum

$$p = \frac{E}{c} = \frac{[E = hf]}{[c = \lambda f]} = \frac{h}{\lambda}$$

Ex. 38.2 Black-and-white photographic film (with some exceptions) is less sensitive to red light than blue light and has almost no sensitivity to infrared. How can these properties be understood on the basis of photons?

Ex. 38.3 Human skin is relatively insensitive to visible light, but ultraviolet radiation can cause severe burns. How can you explain this in the context of photon energies?

Ex. 38.4 A laser pointer with a power output of 5.00 mW emits red light with wavelength $\lambda = 650$ nm. What is the magnitude of the momentum of each photon, and how many photons does the laser pointer emit each second?

Ans. 1.02×10^{-27} kg.m/s; 1.63×10^{16}

Work Function The minimum energy required to remove an electron from the surface of a material.

Energy Balance:

$$\text{photon energy} = \text{energy to remove an electron} + \text{kinetic energy}$$

The most energetic emitted electrons are those that were bound to the material with minimum energy, ϕ:

$$\text{photon energy} = \text{work function} + \text{max. kinetic energy}$$

$$hf = \phi + K_{\text{max}}$$

Ex. 38.5 The material called *phosphors* that coat the inside of a fluorescent lamp convert ultraviolet radiation (from the mercury-vapor discharge) into visible light. Could one also make a phosphor that converts visible light to ultraviolet?

Ex. 38.6 If the reverse potential required to stop the electrons ejected in the Photoelectric effect is 1.25 V, find (a) the maximum kinetic energy, and (b) the maximum speed of the emitted photoelectrons.

Ans. 1.25 eV; 6.63×10^5 m/s

Photoelectric Function for Selected Materials
http://hyperphysics.phy-astr.gsu.edu/hbase/tables/photoelec.html

Determining ϕ and h experimentally

$$hf = \phi + K_{\text{max}}$$
$$hf = \phi + q_e V_{\text{stopping}}$$
$$V = \frac{h}{q_e} f - \frac{\phi}{q_e}$$

The Stopping Potential vs Frequency Graph. The slope is $\frac{h}{q_e}$, the vertical intercept is $-\frac{\phi}{q_e}$.

Ex. 38.7 For a particular cathode material in a photoelectric-effect experiment, you measure stopping potentials 1.0 V, 2.0 V, and 3.0 V for light of wavelength of 600 nm, 400 nm, and 300 nm, respectively. Determine the work function ϕ and the implied value of Planck's constant, h.

Ans. 1.0 eV; 6.4×10^{-34} J.s

38.2 Light Emitted: X-Ray Production and Bremsstrahlung

Electrons are emitted from a heated cathode, then are accelerated by the anode-cathode potential difference V_{AC} , and eventually collide with the anode.

Energy Balance

kinetic energy of electrons = energy of emitted light+energy losses in the anode

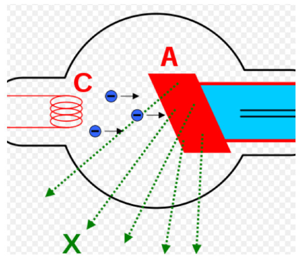


Figure 38.2: X-Ray emission during slowing down of fast electrons.

For the most energetic photons, all the kinetic energy has been converted into light energy without losses:

$$q_e V_{AC} = hf_{\text{max}} = \frac{hc}{\lambda_{\text{mn}}} \tag{38.1}$$

Ex. 38.8 Electrons in an x-ray tube accelerate through a potential difference of 10.0 kV before striking a target. If an electron produces one photon on impact with the target, what is the minimum wavelength of the resulting x rays?

Ans. 0.124 nm

38.3 Light Scattering: Compton Scattering

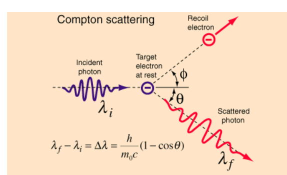


Figure 38.3: Light scatters off electrons: Compton scattering.

Elastic Collisions: Energy and Momentum are conserved, but we must use the exact formulae for the energy and momentum (the relativistic).

Note. If particles move at speeds 0.1c or above, they must be treated relativistically! Classic treatment does not lead to significant errors for speeds less than 0.01c.

$$\lambda' - \lambda = \frac{h}{mc}\left(1 - \cos\theta\right)$$

Wavelength shift
http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/compton.html#c1

Derivation

$$E = \gamma mc^2 \quad \vec{p} = \gamma m\vec{v} \quad \text{where } \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

38.4 Wave-Particle Duality: The Uncertainty Principle

There are fundamental limitations on the precision with which we know simultatenously the position and momentum of a particle.

Heisenberg's Principle: The more precisely the position is determined, the less precisely the momentum is known in this instant, and vice versa.

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Planck's contant: $\hbar = \frac{h}{2\pi} = 1.0546 \times 10^{-34}$ J.s

$$\Delta\left(x = ct\right)\Delta\left(p = \frac{E}{c}\right) = \Delta E \Delta t \geq \frac{\hbar}{2}$$

The history of Heisenberg's Principle
http://aip.org/history/heisenberg/p08.htm

Ex. 38.9 A tellurium-sapphire laser can produce light at a wavelength of 800 nm in ultrashort pulses that last only 4.00×10^{-15} s. The energy in a single pulse produced by one such laser is 2.00μJ, and the pulses propagate in the positive x-direction. Find (a) the frequency of the light; (b) the energy and minimum energy uncertainty of a single photon in the pulse; (c) the minimum frequency uncertainty of the light i the pulse; (d) the spatial length of the pulse, as multiple of the wavelength; (e) the momentum and minimum momentum uncertainty of a single photon in the pulse; and (f) the approximate number of photons in the pulse

Solution:

(a) **Ans.** 3.75×10^{14} Hz

(b) **Ans.** 2.48×10^{-19} J; 1.32×10^{-20} J

(c) **Ans.** 1.99×10^{13} Hz

(d) **Ans.** 1.50 wavelengths

(e) **Ans.** 4.40×10^{-29} kg.m/s

(f) **Ans.** 8.06×10^{12} photons

38.5 Formulae and Constants

$$\text{Speed of light:} \quad c = 3.0 \times 10^8 \text{ m/s}$$

$$\text{Planck's Constant} \quad h = 6.626 \times 10^{-34} \text{ J.s} = 4.14 \times 10^{-15} \text{ eV.s}$$

$$\hbar = 1.0546 \times 10^{-34} \text{ J.s}$$

$$\text{The electron charge and mass:} \quad q_e = 1.6 \times 10^{-19} \text{ C} \quad m_e = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Photon Energy and Momentum:} \quad E = hf = \frac{hc}{\lambda} \quad p = \frac{E}{c} = \frac{h}{2\pi}$$

$$\text{Photoelectric Effect:} \quad hf = \phi + K_{\text{max}} = \phi + qV_{\text{stopping}}$$

$$\text{X-Ray Emission:} \quad qV = hf_{\text{max}} = \frac{hc}{\lambda_{\text{min}}}$$

$$\text{Compton Scattering:} \quad \lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta)$$

$$\text{Uncertainty Principle:} \quad \Delta x \Delta p \geq \frac{\hbar}{2} \quad \Delta E \Delta t \geq \frac{\hbar}{2}$$

38.6 Problems

P38.1 The human eye is most sensitive to green light of wavelength 505 nm. When people are kept in the dark room until their eyes adapt to the darkness, a single photon of green light will trigger receptor cells in the rods of the retina. (a) What is the frequency of this photon? (b) How much energy in eV does it deliver to the receptor cells? (c) How fast a typical bacterium of mass 9.5×10^{-12} g would move if it had that much energy?

Ans. (a) 5.94×10^{14} Hz; (b) 2.46 eV; (c) 9.1×10^{-3} m/s.

P38.2 What would the minimum work fuction for a metal have to be for visible light (380 - 750 nm0 to eject photoelectrons?

Ans. 1.656 eV

P38.3 The photoelectric work function of calcium is 2.9 eV. If light having a wavelength of 240 nm falls on calcium, find (a) the kinetic energy in eV of the most energetic electrons, (b) the stopping potential, and (c) the speed of those electrons.

Ans. (a) 2.28 eV; (b) 2.28 V; (c) 8.95×10^{5} m/s.

P38.4 The graph in Fig. 38.4 shows the stopping potential in Volts as a function of the frequency of the incident light falling on a metal surface in Hertz. (a) Find the photoelectrc work function for this metal. (b) What value of Planck's constant does the graph yield?

Ans. (a) 4.445 eV; (b) 5.69×10^{-34} Hz;

P38.5 The cathode-ray tubes that generated pictures in early color television sets were sources of x rays. If the acceleration volage in a television tube is 15.0 kV, what are the shortest-wavelength x rays produced by the television?

Ans. 0.0829 nm

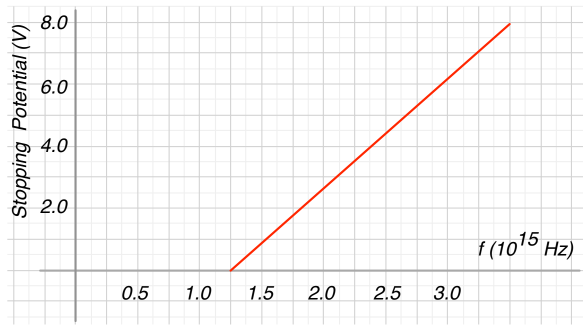


Figure 38.4: Stopping Potential vs. Frequency

P38.6 A photon scatters in the backward direction ($\phi = 180°$) from a free proton that is intiially at rest. What must the wavelength of the incident photon be if it is to undergo a 10.0% change in wavelength as a result of the scattering?

Ans. 2.65×10^{-14} m

P38.7 A photon with wavelength $\lambda = 0.1385$ nm scatters from an electron that is intiially at rest. What must be the angle between the direction of propagation of the incident and scattered photons if the speed of the electron immediately after the collision is 8.90×10^6 m/s?

Ans. 118°

P38.8 A laser pulse produces light of wavelength 625 nm in an ultrashort pulse. What is the minimum duration of the pulse if the minimum uncertainty in the energy of the photons is 1.0%

Ans. 16.6 fs

38.7 Review

38.7.1 Diffraction Limited Resolution

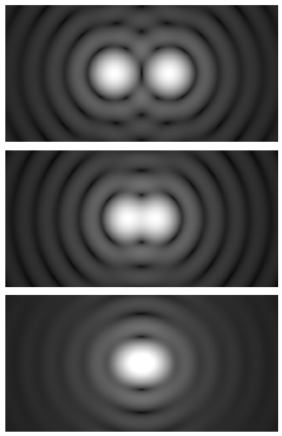


Figure 38.5: Airy diffraction patterns generated by light from two points passing through a circular aperture. Source: http://en.wikipedia.org/wiki/Angular_resolution

Rayleigh Criterion The angular resolution is given in radians.

$$\theta \approx 1.22\frac{\lambda}{a}$$

Spacial Resolution:

$$\Delta l = \theta \times l \approx 1.22\frac{\lambda l}{a}$$

Example: Microscope. The spacial resolution depends on diameter and focal length of the lense, on the refractive index of the medium, wavelength. It is usually on the order of magnitude of the wavelengths used by the microscope. Thus, for a visible-light microscope, the resolution is approx. 200 nm.

38.7.2 Diffraction Grating

$$d \sin \Theta = m\lambda$$

In lab: $\lambda = 632.8$ nm, $d = 0.25$ mm

$$\frac{d}{\lambda} \approx 400$$

38.7.3 Rotational Motion: centripetal force, acceleration, angular momentum.

$$F = ma = m\frac{v^2}{r}$$

$$L = I\omega = mr^2\omega = mr\,(r\omega) = mr(v) = m(v)r = pr$$
$$\vec{L} = \vec{r} \times \vec{p}$$

38.7.4 Standing Wave Patterns

Standing Wave Pattern on a String Integer number of half-wavelengths fit the distance.

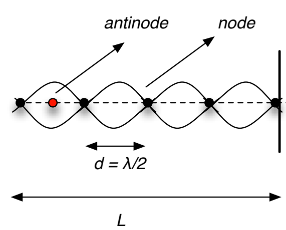


Figure 38.6: Standing Waves on a String

Standing Wave Patterns in a circle: Integer number of wavelengths fit the circumference.

http://en.wikipedia.org/wiki/Angular_momentum#mediaviewer/File:Circular_Standing_Wave.gif

Example: Four wavelengths fit a circle

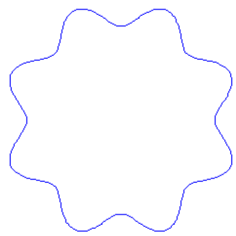


Figure 38.7: Standing Waves on a Circle

Chapter 39

Electrons Behaving As Waves

39.1 Electron Waves

De Broglie Wavelength

$$p = \frac{h}{\lambda}$$

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

When the particles are accelerated through applying a potential difference, V, then

$$\text{K.E.} = \frac{p^2}{2m} = qV$$

$$p = \sqrt{2mqV}$$

Ex. 39.1 In an electron-diffraction experiment using an accelerating voltage of 54 V, an intensity maximum occurs for $\theta = 50°$. X-ray diffraction indicates that the atomic spacing in the target is $d = 2.18 \times 10^{-10}$ m.The electrons have negligible kinetic energy before being accelerated. Find the electron wavelength.

Solution:

$$\lambda = \frac{h}{\sqrt{2mqV}} = 1.7 \times 10^{-10} \text{ m}$$

Solution:

$$d \sin\theta = \lambda \approx 1.7 \times 10^{-10} \text{ m}$$

Note. Diffraction technique can study objects that are comparable in size to the wavelength of the used light. For the visible light - we can see no smaller than 200 nm. Using electron diffractions (electron microscopy), we can resolve objects about 0.2 nm.

Ex. 39.2 Find the speed and kinetic energy of a neutron ($m = 1.675 \times 10^{-27}$ kg) with de Broglie wavelength $\lambda = 0.200$ nm (a typical interatomic spacing in crystals). Compare this energy wtih the average translational kinetic energy of an ideal-gas molecule at room temperature (293 K).

Review of Equipartition Theorem: On average, each degree of freedom contributes $\frac{1}{2}kT$ to the total energy of the gas.

Translational Kinetic Energy: $\frac{3}{2}kT$

Boltzmann's Constant: 1.38×10^{-23} J/K

39.2 Experimental Evidence of the Structure of the Atom

39.2.1 Atomic Spectra

Discreete vs. Continuus Spectra

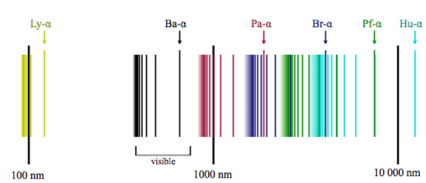


Figure 39.1: Hydrogen Spectrum

39.2.2 The Rutherford's Gold Foil Experiment, 1910-1911

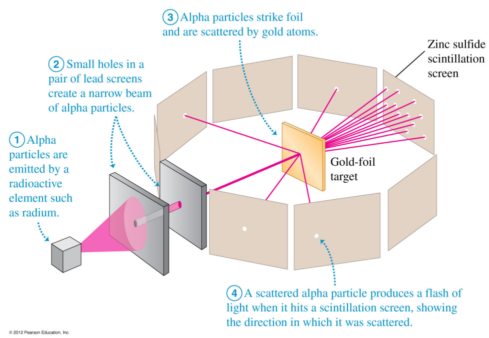


Figure 39.2: Rutherford's Gold Foil Experiment

Question If the electrons were orbitting the nucleus, they must have a (centripetal) acceleration. Therefore, they must emit raditation (any accelerating particle does!). Then, they must lose energy, and if the loose energy, their angular momentum must be changing and they must be "spiraling" down onto the nucleus!?!?!

Ex. 39.3 An alpha article (+2e) is aimed directly at a gold nucleus (79e) . What minimum initial kinetic energy must the alpha particle have to approach within 5.0×10^{-14} m of the center of the gold nucleus before reversing direction.? Assume gold nucleus is at rest and has approx. 50 times the mass of the alpha particle.

Ans. 4.6 MeV

Note. ^{226}Ra (Radium) emits naturally alpha particles of 4.78 MeV.

39.3 The Bohr Model of the Atom

39.3.1 Energy Levels

- The energy of an atom can have only certain particular values.

- The atom can change from one level to another, but never to intermediate values.

$$hf = \frac{hc}{\lambda} = E_1 - E_2$$

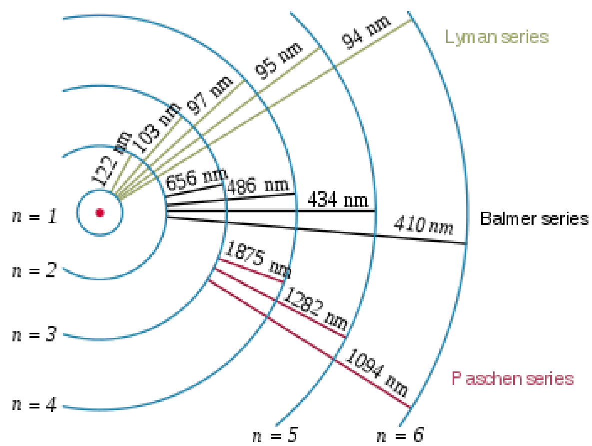


Figure 39.3: Energy levels of the sodium atom relative to the ground level.

39.3.2 The Franck-Hertz Experiment, 1914

Experimental confirmation of the atomic energy levels.

Ex. 39.4 Mercury atoms hava an excited energy level of 4.9 eV above ground. Light with what wavelength can raise the atoms into the first excited state?

$$\Delta E = hf = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{E} = \frac{(4.136 \times 10^{-15}\ \text{eV.s})\,(3.0 \times 10^{8}\ \text{m/s})}{4.9\ \text{eV}} = 250\ \text{nm}$$

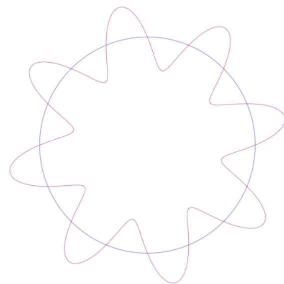


Figure 39.4: Circular Standing Wave Patterns. Source: Wolfram Mathematica http://demonstrations.wolfram.com/ElectronWavesInBohrAtom/

39.3.3 Quantization of the Angular Momentum

Quantization:

$$n\lambda = 2\pi r_n$$

DeBroglie Wavelength:

$$\lambda = \frac{h}{p} = \frac{h}{mv_n}$$

Quantization of the angular momentum: $L = mv_n r_n$.

$$\frac{2\pi r_n}{n} = \frac{h}{mv_n}$$

$$L_n = mv_n r_n = n\frac{h}{2\pi} = n\hbar \tag{39.1}$$

39.3.4 The Bohr Model for the Hydrogen Atom

$$F = ma$$

$$k\frac{e^2}{r_n^2} = m_e\frac{v_n^2}{r_n}$$

$$ke^2 = m_e v_n^2 r_n = L_n v_n = n\hbar v_n$$

$$v_n = \frac{ke^2}{n\hbar} = \frac{1}{4\pi\epsilon_o}\frac{e^2}{nh}(2\pi) = \frac{1}{\epsilon_o}\frac{e^2}{2nh}$$

$$r_n = \epsilon_o \frac{n^2h^2}{\pi m_e e^2}$$

Ex. 39.5 What is the radius for the smallest Bohr orbit

Solution:

$$a_o = \frac{\left(8.854\times10^{-12}\mathrm{C}^2/\mathrm{N.m}^2\right)\left(6.626\times10^{-34}\mathrm{J.s}\right)^2}{\pi\left(9.109\times10^{-31}\mathrm{kg}\right)\left(1.602\times10^{-19}\mathrm{C}\right)^2} = 5.29\times10^{-11}\ \mathrm{m}$$

Hydrogen Energy Levels

$$K_n = \frac{1}{2}mv_n^2 = \frac{1}{\epsilon_o^2}\frac{m_e e^4}{8n^2h^2}$$

$$U_n = -\frac{1}{4\pi\epsilon_o}\frac{e^2}{r_n} = -\frac{1}{\epsilon_o^2}\frac{m_e e^4}{4n^2h^2}$$

$$E_n = -\frac{1}{\epsilon_o^2}\frac{m_e e^4}{8n^2h^2}$$

Ionization Energy: The energy difference between the $n \to \infty$ level and the $n = 1$ level.

Rydberg constant

$$R = \frac{m_e e^4}{8\epsilon_o^2 h^3 c} = 1.097\times10^7\ 1/\mathrm{m}$$

$$E_n = -\frac{hcR}{n^2} = -\frac{13.6\ \mathrm{eV}}{n^2}$$

Ex. 39.6 Determine the wavelength of the light emitted for n_1 to n_2 transition in the Hydrogen atom *Solution:*

$$hf = \frac{hc}{\lambda} = \Delta E = E_{n_2} - E_{n_1} = -\frac{1}{\epsilon_o^2}\frac{m_e e^4}{8n_2^2h^2} + \frac{1}{\epsilon_o^2}\frac{m_e e^4}{8n_1^2h^2}$$

$$\frac{hc}{\lambda} = \Delta E = E_{n_2} - E_{n_1} = -hcR\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$
$$\frac{1}{\lambda} = -R\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

Hydrogen Atom Series: Lyman Series (into n=1, UV); Balmer (into n=2, visible); Paschen (into n=3, infrared)

Ex. 39.7 Determine the wavelength of the 3 to 2 transition?

Ans. 656.3 nm

Note. Technically, we must use the center of mass of the electron-proton system. For a two-body system, we can use the reduced mass

$$m_r = \frac{m_1 m_2}{m_1 + m_2}$$

Hydrogenlike Atoms: Singly ionized He^+; doubly-ionized Li^2+, etc. The nuclear charge is Ze and must be factored into the total energy:

$$E_n = -\frac{Z^2}{\epsilon_o^2}\frac{m_e e^4}{8n^2h^2}$$

39.4 Continuous Spectra

39.4.1 Stefan-Boltzmann Law for Blackbody

$$I = \sigma T^4 \qquad \sigma = 5.6704 \times 10^{-8}\,\frac{\mathrm{W}}{\mathrm{m^2K^4}}$$

39.4.2 Wien's Displacement Law

$$\lambda_{\max} T = 2.90 \times 10^{-3}\ \mathrm{m.K}$$

Ex. 39.8 The peak-intensity wavelength of red dwarf stars, which have surface temperatures around 3,000 K, is about 1,000 nm. So why are we able to see these stars, and why do they appear red?

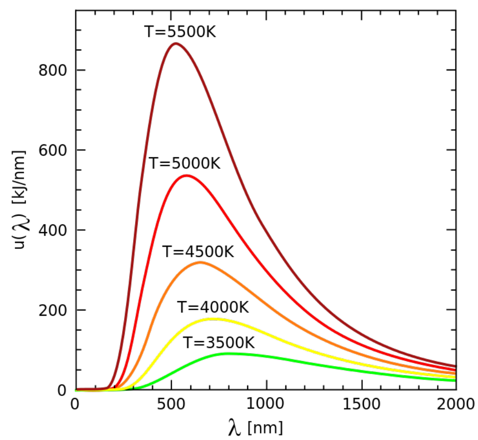


Figure 39.5: Emission of Light from a Blackbody. Source: Wikipedia

39.4.3 Planck's Quantum Hypothesis

$$I(\lambda) = \frac{2\pi\hbar c^2}{\lambda^5\left(e^{\frac{\hbar c}{\lambda kT}} - 1\right)}$$

39.5 Problems

P39.1 For crystal diffraction experiments, wavelengths on the order of 0.20 nm are often approrpirate. Find the energy in electron volts for a particle with this wavelength if the particle is (a) photon; (b) an electron; and (c) and alpha particle ($m = 6.64 \times 10^{-27}$ kg)

Ans. 6.2 kEv; 38 eV; 0.021 eV

P39.2 An alpha particle emitted in the radioactive decay of ^{238}U (Uranium-238) has an energy of 4.20 MeV. What is its deBroglie wavelength?

Ans. 7.02×10^{-15} m

P39.3 Calculate the de Broglie wavelength of a typical person walking through a doorway. Will you exhibit a wave-like behaviour when walking through the "single slit" of a doorway?

Ans. 8.8×10^{-36} m

P39.4 Through what potential difference must electrons be accelerated so they will have the same wavelength as an x ray of 0.150 nm?

Ans. 66.9 V

P39.5 A hydrogen atom initially in the ground level absorbs a photon, which excites it to the $n = 4$ level. Determine the wavelength and frequency of the photon.

Ans. 97.3 nm; 3.08×10^{15} Hz

P39.6 A triply-ionized beryllium ion, Be^{3+}, behaves very much like a hydrogen atom except that the nuclear charge is four times as great. What is the ground-level energy of Be^{3+}? What is the ionization energy of Be^{3+}?

Ans. -218 eV; 218 eV

P39.7 Find the longest and the shortest wavelengths for the Lyman series for the hydrogen.

Ans. 121.5 nm; 91.16 nm

P39.8 A typical blue supergiant star (the type that explodes and leaves behind a black hole) has a surface temperature of 30,000 K and a visual luminosity 100,000 times that of our sun. Our sun radiates at the rate of 3.86×10^{26} W. Assuming that this star behaves as line an ideal blackbody, what is the principle wavelength in which it radiates?

Ans. 97 nm

P39.9 The brightest star in the sky, Sirius, is actually a binary system. Spectral analysis of the smaller one, a white dwarf called Sirius B, has a surface temperature 24,000 K and radiates energy at a total rate of 1.0×10^{25} W. Assuming it behaves line an ideal blackbody, (a) what is the total radiated intensity of Sirius B, and (b) what is the peak-intensity wavelength?

Ans. 1.9×10^{10} W/m^2 , 20 nm

P39.10 The uncertainty in the y-component of a proton's position is 2.0×10^{-12} m. Waht is the minimum uncertainty in a simultaneous measurement of the y-component of the proton's velocity?

Ans. 1.6×10^4 m/s

P39.11 Show that in the Bohr model, the frequency of revolution of the an electron in its circular orbit around a stationary hydrogen nucleus is $f = \frac{m_e e^4}{4\epsilon_o^2 n^3 h^3}$.

39.6 Formulae and Constants

$$\text{Speed of light:} \quad c = 3.0 \times 10^{8} \text{ m/s}$$

$$\text{Planck's Constant} \quad h = 6.626 \times 10^{-34} \text{ J.s} = 4.14 \times 10^{-15} \text{ eV.s}$$

$$\hbar = 1.0546 \times 10^{-34} \text{ J.s}$$

$$\text{The electron charge and mass:} \quad q_e = 1.6 \times 10^{-19} \text{ C} \quad m_e = 9.1 \times 10^{-31} \text{ kg}$$

$$\text{Photon Energy and Momentum:} \quad E = hf = \frac{hc}{\lambda} \quad p = \frac{E}{c} = \frac{h}{2\pi}$$

$$\text{Photoelectric Effect:} \quad hf = \phi + K_{\text{max}} = \phi + qV_{\text{stopping}}$$

$$\text{X-Ray Emission:} \quad qV = hf_{\text{max}} = \frac{hc}{\lambda_{\text{min}}}$$

$$\text{Compton Scattering:} \quad \lambda' - \lambda = \frac{h}{mc}\left(1 - \cos\theta\right)$$

$$\text{Uncertainty Principle:} \quad \Delta x \Delta p \geq \frac{\hbar}{2} \quad \Delta E \Delta t \geq \frac{\hbar}{2}$$

$$\text{DeBroglie Wavelength:} \quad \lambda = \frac{h}{p} = \frac{h}{mv}$$

$$\text{Hydrogen Energy Levels:} \quad E_n = -\frac{hcR}{n^2} = -\frac{13.60 \text{ eV}}{n^2}$$

$$\text{Rydberg Constant:} \quad R = \frac{m_e e^4}{8\epsilon_o^2 h^3 c} = 1.097 \times 10^{7} \text{ 1/m}$$

$$\text{Stefan-Boltzmann:} \quad I = \sigma T^4 \quad \sigma = 5.6704 \times 10^{-8} \frac{\text{W}}{\text{m}^2\text{K}^4}$$

$$\text{Wien's Displacement Law:} \quad \lambda_{\text{max}} T = 2.90 \times 10^{-3} \text{ m.K}$$

$$\text{Planck's Radiation Law:} \quad I(\lambda) = \frac{2\pi\hbar c^2}{\lambda^5 \left(e^{\frac{hc}{\lambda kT}} - 1\right)}$$

39.7 Planck's Radiation Law

Planck's Law

$$I(\lambda) = \frac{2\pi hc^2}{\lambda^5 \left(e^{\frac{hc}{\lambda kT}} - 1\right)}$$

$$\frac{1}{x} = \frac{hc}{\lambda kT} \rightarrow x = \lambda \frac{kT}{hc}$$

$$k = 1.38 \times 10^{-23} \text{ J/K} \quad h = 6.26 \times 10^{-34} \text{ J.s} \quad c = 3.0 \times 10^8 \text{ m/s}$$

$$I(\lambda) = \frac{2\pi hc^2 \left(\frac{kT}{hc}\right)^5}{x^5 \left(e^{\frac{1}{x}} - 1\right)}$$

$$f(x) = \frac{1}{x^5 \left(e^{\frac{1}{x}} - 1\right)} \tag{39.2}$$

$$f'(x) = \frac{1}{x^7} \frac{1}{\left(e^{\frac{1}{x}} - 1\right)^2} \left(-5x\left(e^{\frac{1}{x}} - 1\right) + e^{\frac{1}{x}}\right) \tag{39.3}$$

$$f'(x) = 0 \quad \rightarrow \quad 5x\left(e^{\frac{1}{x}} - 1\right) = e^{\frac{1}{x}}$$

$$5x = \frac{e^{\frac{1}{x}}}{e^{\frac{1}{x}} - 1} \quad \rightarrow \quad 5x = \frac{1}{1 - e^{-\frac{1}{x}}}$$

$$5x - 5xe^{-\frac{1}{x}} = 1 \qquad x = 0.201405$$

Wien's Displacement Law

$$\lambda_{\max} = \frac{2.901 \times 10^{-3}}{T}$$

Planck's Law

$$I(\lambda) = \frac{2\pi hc^2}{\lambda^5 \left(e^{\frac{hc}{\lambda kT}} - 1\right)}$$

$$x = \frac{hc}{\lambda kT} \quad \rightarrow \quad \lambda = \frac{hc}{\lambda kT} \quad d\lambda = -\frac{hc}{kT}\frac{dx}{x^2}$$

$$\int_o^\infty I(\lambda) d\lambda = \int_\infty^o \frac{2\pi hc^2}{\left(\frac{hc}{xkt}\right)^5 (e^x - 1)} \left(-\frac{hc}{x^2 kT} dx\right)$$

$$-\int_\infty^o \frac{2\pi hc^2}{x^2} \frac{hc}{kT} dx \;\; x^5 \left(\frac{kT}{hc}\right)^5 \frac{1}{e^x - 1}$$

Switching the limits and canceling some constants:

$$I = \int_o^\infty I(\lambda) d\lambda = +\frac{2\pi h^2 c^3}{h^5 c^5} (kT)^4 \int_o^\infty \frac{x^3}{e^x - 1} dx \tag{39.4}$$

$$I = \int_o^\infty I(\lambda) d\lambda = +\frac{2\pi h^2 c^3}{h^5 c^5} (kT)^4 \int_o^\infty \frac{x^3}{e^x - 1} dx = \frac{2\pi k^4}{h^3 c^2} (T)^4 \times \frac{\pi^4}{15}$$

$$\int_o^\infty I(\lambda) d\lambda = \frac{2\pi^5 k^4}{h^3 c^2} T^4$$

Stefan-Boltzmann Law for Blackbody Radiation

$$I = \sigma T^4 \qquad \sigma = 5.6704 \times 10^{-8} \frac{\mathrm{W}}{\mathrm{m^2 K^4}}$$

39.8 Review

39.8.1 The Wave Equation

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y(x,t)}{\partial t^2} \tag{39.5}$$

$$y(x,t) = A\cos(kx - \omega t) + B\sin(kx - \omega t) \tag{39.6}$$

$$k^2 = \frac{\omega^2}{v^2} \rightarrow \omega = vk \tag{39.7}$$

$$2\pi f = v\frac{2\pi}{\lambda} \rightarrow v = \lambda f$$

39.8.2 Complex Numbers

$$i = \sqrt{-1} \quad i^0 = 1 \quad i^1 = i \quad i^2 = -1 \quad i^3 = -i \quad i^4 = +1 \quad i^5 = i$$

$$z = x + iy \quad |z| = \sqrt{\left(\mathrm{Re}^2 + \mathrm{Im}^2\right)} = \sqrt{x^2 + y^2}$$

39.8.3 The Exp, Sin, and Cos series

Euler's Formula

$$e^{ix} = \cos x + i \sin x \tag{39.8}$$

39.8.4 The Harmonic Oscillator

$$\omega = \sqrt{\frac{k}{m}}$$

$$U = \frac{1}{2}kx^2$$

Chapter 40

Quantum Mechanics

40.1 The Wave Function and the Schrödinger Equation

If particles can exhibit wave nature, they should be described by a function that satisfies a "wave equation".

For classical waves:

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2}$$

$$f(x,t) = A\cos(kx - \omega t) + B\sin(kx - \omega t) \qquad \omega = vk$$

For free-particle waves:

$$E = \frac{p^2}{2m} \qquad E = hf = \hbar\omega \qquad p = \frac{h}{\lambda} = \hbar k$$

$$\hbar\omega = \frac{\hbar^2 k^2}{2m}$$

The Schrödinger Equation for a free particle

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} = i\hbar\frac{\partial \Psi(x,t)}{\partial t}$$

The Wave Function

$$\Psi(x,t) = A\cos(kx-\omega t) + iA\sin(kx-\omega t) = Ae^{i(kx-\omega t)}$$

The Meaning of the Wave Function: Probability that a particle is located within $(x, x+dx)$ interval

$$|\Psi(x,t)|^2 dx$$

Note. The square of the absolute value of the wave function alone is called Probability Distribution.

Normalization of the total probability to find a particle.

$$\int_{-\infty}^{+\infty} |\Psi(x,t)|^2 dx \equiv 1 \tag{40.1}$$

Ex. 40.1 The wave function

$$\Psi(x,t) = Ae^{i(k_1 x-\omega_1 t)} + Ae^{i(k_2 x-\omega_2 t)}$$

is a superposition of two free-particle wave functions. Both k_1 and k_2 are positive. Find the probability distribution function for $\Psi(x,t)$.

Ans. $2|A|^2\left(1+\cos\left((k_1-k_2)x-(\omega_1-\omega_2)t\right)\right)$

Note. The probability to find the particle is different for different locations. There are locations that are more probable (particle is somewhat localized!)

Note. The wave velocity

$$y = y_o\cos(kx-\omega t) \longrightarrow v_{\text{wave}} : \text{phase} = kx-\omega t = const.$$

$$k\frac{dx}{dt} - \omega = 0 \longrightarrow \frac{dx}{dt} = v_{\text{av}} = \frac{\omega}{k} = \lambda f$$

Average velocity of the "wave function"

$$((k_1-k_2)x-(\omega_1-\omega_2)t) = const. \longrightarrow v_{\text{av}} = \frac{\omega_1-\omega_2}{k_1-k_2}$$

For a free particle for which we know its momentum ($p = \hbar k$), the probability distribution is independent of the position, and the total probability cannot be normalized! Heisenberg's principle requires that the $\Delta x \to \infty$.

Wave Packet In order to "localize" a particle, we need to allow for some uncertainty in its momentum Δp so that its position uncertainty becomes finite ($\Delta x \Delta p \geq \frac{\hbar}{2}$). Its wave function is constructed as a sum (superposition) of wavefunctions of different k.

$$\Psi(x,t) = \int_{-\infty}^{+\infty} A(k) e^{i(kx-\omega t)} dk$$

Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} + U(x)\Psi(x) = i\hbar \frac{\partial \Psi(x,t)}{\partial t}$$

Note. The potential energy is a function of x and not of t. So, it will affect the spacial part of $\Psi(x,t)$ but not the time.

Stationary state: A state with a definite energy. It does NOT mean stationary particle, but rather the probability does not change with time. For such states, x and t can be separated:

$$\Psi(x,t) = \psi(x) e^{-i\frac{Et}{\hbar}}$$

Time-independent Schrödinger equation.

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x)}{\partial x^2} + U(x)\psi(x) = E\psi(x) \tag{40.2}$$

Ex. 40.2 Prove that the probability distribution function is time-independent for a stationary state.

$$|\Psi(x,t)|^2 = \Psi^*(x,t)\Psi(x,t) = |\psi(x)|^2$$

Ex. 40.3 Given is a wave function

$$\psi(x) = A_1 e^{ikx} + A_2 e^{-ikx}$$

Is this a valid time-independent wave function for a free particle? (If yes, it must satisfy the time-independent Schrödinger equation.)

40.2 Particle in a Box without potential energy.

Boundary Conditions:

1. Probability to find the particle outside the box must be zero. Hence, $\psi(x) = 0$ for $x = 0$ and $x = L$.

2. The first derivative must go to zero, $\frac{\psi(x)}{x} = 0$ for $x = 0$ and $x = L$.

Schrödinger Equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} = E\psi(x)$$

Solution:

$$\psi(x) = A_1 e^{ikx} + A_2 e^{-ikx}$$

$$\psi(x) = 2iA_1 \sin kx = C \sin kx \quad \text{with} \quad k = \frac{n\pi}{L} \rightarrow \lambda = \frac{2L}{n}$$

Compare! Standing wave pattern on a string is formed only when $\lambda_n = \frac{2L}{n}$!!!

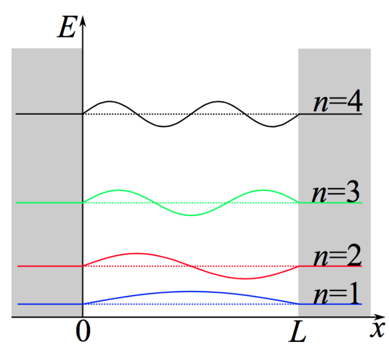


Figure 40.1: Particle in a Box. Source: Wikipedia https://commons.wikimedia.org/wiki/File:Particle_in_a_box_wavefunctions.svg

Normalization Gives us the value of the remaining constant, C.

$$\int_o^L C^2 \sin^2 \frac{n\pi x}{L} dx = 1$$

$$\psi(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} \tag{40.3}$$

Example: $\psi(x) = Ax + B$ is a solution of the Schrödinger equation, but cannot be the wave function describing the particle inside a box because the bondary conditions lead to $A = B = 0$ (the trivial solution).

Energy

$$p_n = \hbar k_n = \frac{nh}{2L}$$

$$E_n = \frac{p_n^2}{2m} = \frac{n^2 h^2}{8mL^2} = \frac{n^2\pi^2\hbar^2}{2mL^2} = n^2 E_1 \tag{40.4}$$

Note. We cannot have $n = 0$, as the we need to have non-zero wave function, the first solution corresponds to $n = 1$.

Ex. 40.4 Find the first two energy leves for an electron confined to a one-dimensional box 5.0×10^{-10} m. **Ans.** 1.5 eV, 6.0 eV

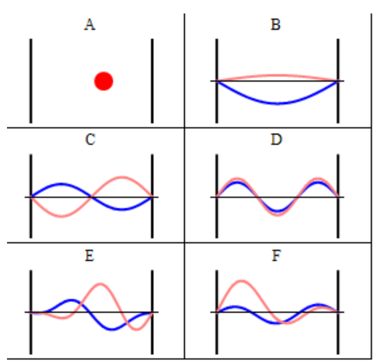


Figure 40.2: Particle in a Box. Source: Wikipedia http://en.wikipedia.org/wiki/Particle_in_a_box

40.3 Potential Wells

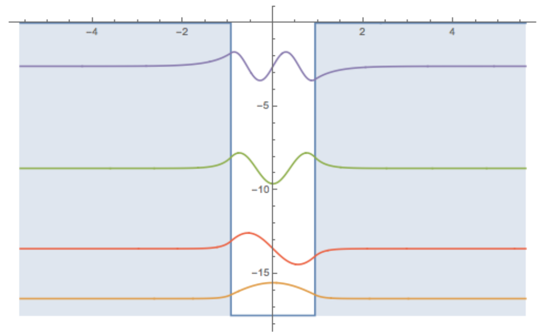


Figure 40.3: A square potential well. The potential energy is zero within the well, and $U(x)$ outside. Source: Wolfram Demonstrations Project http://demonstrations.wolfram.com/BoundStatesInASquarePotentialWell/

A square-well potential

$$U(x) = \begin{cases} U_o & \text{if } x < 0; \\ 0 & x \in [0, L]; \\ U_o & \text{if } x < 0. \end{cases}$$

Bound States $E < U_o$

Inside the Well: Same as the particle in a box:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} = E\psi(x)$$

$$\psi(x) = A\cos(kx) + B\sin(kx)$$

$$E = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m} \longrightarrow k = \sqrt{\frac{2mE}{\hbar^2}}$$

Outside the Well:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + U_o\psi(x) = E\psi(x)$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} = (E - U_o)\,\psi(x)$$

$$\frac{\partial^2\psi(x)}{\partial x^2} = +\frac{2m}{\hbar^2}\,(U_o - E)\,\psi(x)$$

We know which function will satisfy the equation: the exp. Cos or sin will not do in this case, as $U_o - E$ is positive. We need a combination of two exponents to make sure that $\psi(x)$ doesn't got to ∞ at either end of the well.

$$\psi(x) = Ce^{kx} + De^{-kx}$$

Inside the well: sin/cos function, outside: exponentially decaying functions. Note that the probability to find the particle outside is not zero.

Matching Boundary Conditions Only a discrete and limited set of energy levels satisfy the boundary conditions.

40.4 Barriers and Tunneling

Example: How much energy does a 60-kg person need to be able to leave the earth's gravitational potential field?

Potential Energy Barrier

$$U(x) = \begin{cases} 0 & \text{if } x < 0; \\ U_o & x \in [0, L]; \\ 0 & \text{if } x < 0. \end{cases}$$

Tunneling probability Probability that the particle gets through the barrier is proportional to the square of the ratio for the amplitudes of the wave functions on the two sides of the barrier.

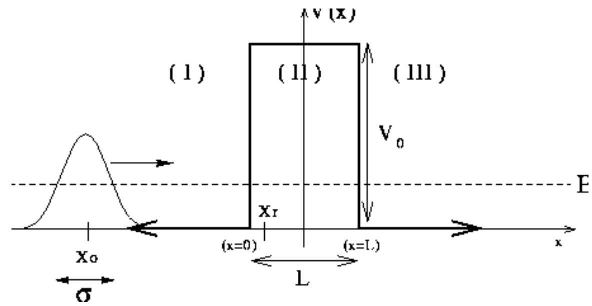


Figure 40.4: Potential Barrier

$$T = Ge^{-2\kappa L} \quad G = 16\frac{E}{U_o}\left(1 - \frac{E}{U_o}\right) \quad \kappa = \frac{\sqrt{2m\,(U_o - E)}}{\hbar} \tag{40.5}$$

Ex. 40.5 A 2.0-eV electron encounters a barrier 5.0 eV high. What is the probability that it will tunnel through the barrier if the barrier width is 1. 1.00 nm and 2. 0.5 nm?

Ans. 7.1×10^{-8}, 5.2×10^{-4}

40.5 The Harmonic Oscillator

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + \frac{1}{2}k'\psi(x) = E\psi(x)$$

Note. k' ($F = -k'x$) is the force constant of the harmonic oscillator and has nothing to do with the wave number k ($p = \hbar k$).

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} = \left(E - \frac{1}{2}k'\psi(x)\right)\psi(x)$$

Energy Levels in the Harmonic Oscillator

$$E_n = \left(n + \frac{1}{2}\right)\hbar\sqrt{\frac{k'}{m}} = \left(n + \frac{1}{2}\right)\hbar\omega \quad n = 0, 1, 2, ... \tag{40.6}$$

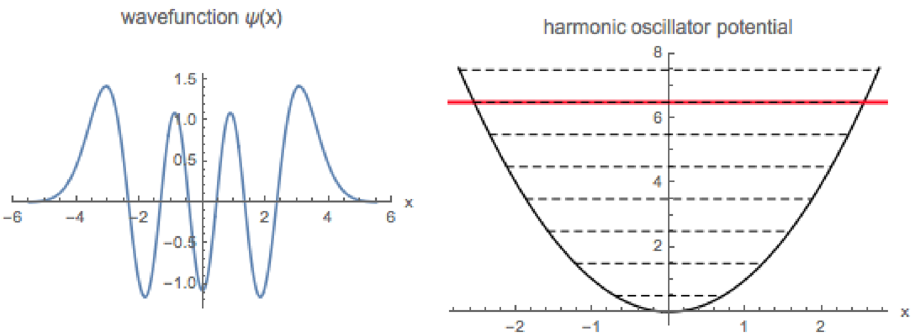


Figure 40.5: The Harmonic Oscillator. Source:Wolfram Demonstration Projects http://demonstrations.wolfram.com/QuantizedSolutionsOfThe1DSchroedingerEquationForAHarmonicOsc/

Ex. 40.6 A sodium atom of mass 3.82×10^{-26} kg vibrates within a crstal. The potential energy increases by 0.0075 eV when the atom is displaced 0.014 nm from its equilibrium. Assuming harmonic oscillator, 1. Find the angular frequency ω according to Newtonian mechanics. 2. Find the spacing (in eV) of adjacent vibratory energy levels according to quantum mechanics. 3. What is the wavelength of a photon emitted in a transition between two adjacent levels?

Ans. 1.79×10^{13} rad/s, 0.0118 eV, 105μm

The Wave Function for the $n = 0$ state:

$$\psi(x) = Ce^{-\frac{\sqrt{mk'}x^2}{2\hbar}}$$

The value of C is determined through normalization of the probability.

40.6 Problems

P40.1 A free particle moving in one dimension has wave function

$$\Psi(x,t) = A\left(e^{i(kx-\omega t)} - e^{i(2kx-4\omega t)}\right)$$

where k and ω are positive real constants. (a) At $t = 0$ what are the two smallest positive values of x for which the probability distribution function $|\Psi(x,t)|^2$ is a maximum? (b) Calculate v_{av} as the distance the maxima have moved divided by the lapsed time.

Ans. $\left(x = \frac{1}{2}\lambda\ ,\ \frac{3}{2}\lambda\right)$; $\left(v_{\text{av}} = \frac{3\omega}{k}\right)$

P40.2 Compute $|\Psi|^2$ for $\Psi = \psi \sin\omega t$, where ψ is time independent and ω is a real constant. Is this a wave function for a stationary state?

Ans. No

P40.3 A particle moving in one dimension (the x axis) is described by the wave function:

$$\psi(x) = \begin{cases} Ae^{-bx} & \text{if } x \geq 0; \\ Ae^{bx} & \text{if } x < 0. \end{cases}$$

whre $b = 2.00\text{m}^{-1}$, $A > 0$, and the $+x$ axis points toward the right. If the wave function is normalized, (a) determine the value of A, and (b) the probability of finding this particle within 50.0 cm of the origin.

Ans. $\sqrt{b}$, 0.865

P40.4 A proton is in a box of width L. What must the width of the bo be for the ground-level energy to be 5.0 MeV, a typical value for the energy with which the particles in a nucleus are bound? (compare with nucleus size $\approx 10^{-14}$ m.)

Ans. 6.4×10^{-15} m

P40.5 Find the width L of a one-dimensional box for which the ground-state energy of an electron in the box equals the absolute value of the ground state of a hydrogen atom.

Ans. 1.66×10^{-10} m

P40.6 Calculate the wavelength of the absorbed photon in the $n = 1$ to $n = 4$ transition for an electron confined to a box with width of 0.125 nm.

Ans. 3.44 nm

P40.7 The penetrating distance η in a finite potential well is the distance at which the wave function has decreased to $\frac{1}{e}$ of the wave function at the classical turning point. It can be shown to be:

$$\eta = \frac{\hbar}{\sqrt{2m\left(U_o - E\right)}}$$

Find η for an electron having a kinetic energy of 13 eV in a potential well with $U_o = 20$ eV.

Ans. 7.4×10^{-11} m

P40.8 An electron with initial kinetic energy 5.0 eV encounters a barrier with height U_o and width 0.60 nm. What is the transmission coefficient if $U_o = 7.0$ eV?

Ans. 5.5×10^{-4}

P40.9 The ground-state energy of a harmonic oscillator is 5.60 eV. If the oscillator undergoes a transition from its $n = 3$ to $n = 2$ level by emitting a photon, what is the wavelength of the photon?

Ans. 111 nm

P40.10 A wooden block with mass 0.250 kg is oscillating on the end of a spring with a force constant 110 N/m. Calculate the ground-level energy and the energy separation between adjacent levels. (Are the quantum effects important?)

Ans. 6.93×10^{-15} eV, 1.39×10^{-14} eV

40.7 Review

40.7.1 Magnetic Moment of an Orbiting Electron

Question What happens in magnets placed in magnetic field? How do they align?

In a uniform B, the magnet just aligns (non-zero torque, but zero force). In a non-uniform B, the magnet will experience a force and will be deflected! Thus a beam of atoms through non-uniform B, will experience force towards the stronger fields.

$$I = \frac{dq}{dt} = \frac{ev}{2\pi r} \quad ; \qquad \vec{\mu} = I\vec{A} \ ; \qquad U = -\vec{\mu}\cdot\vec{B} \tag{40.7}$$

$$\mu = IA = \frac{ev}{2\pi r}\pi r^2 = \frac{evr}{2} \qquad\qquad L = pr = mvr$$

$$\mu = \frac{e}{2m}L \tag{40.8}$$

In the Schrödinger formulation, Eq. 40.8 still holds, that is $\frac{\mu}{L} = \frac{e}{2m}$

Quantization of Angular Momentum Section 39.3.3, Equation 39.1

$$L = \frac{nh}{2\pi} = n\hbar \longrightarrow \mu = \frac{ne\hbar}{2m}$$

Bohr Magneton Natural unit for magnetic moment.

$$\mu_{\mathrm{B}} = \frac{e\hbar}{2m}$$

40.8 Formulae and Constants

Speed of light: $c = 3.0 \times 10^8$ m/s

Planck's Constant $h = 6.626 \times 10^{-34}$ J.s $= 4.14 \times 10^{-15}$ eV.s

$\hbar = 1.0546 \times 10^{-34}$ J.s

The electron charge and mass: $q_e = 1.6 \times 10^{-19}$ C $\quad m_e = 9.1 \times 10^{-31}$ kg

Photon Energy and Momentum: $E = hf = \dfrac{hc}{\lambda} \quad p = \dfrac{E}{c} = \dfrac{h}{2\pi}$

Photoelectric Effect: $hf = \phi + K_{\text{max}} = \phi + qV_{\text{stopping}}$

X-Ray Emission: $qV = hf_{\text{max}} = \dfrac{hc}{\lambda_{\text{min}}}$

Compton Scattering: $\lambda' - \lambda = \dfrac{h}{mc}(1 - \cos\theta)$

Uncertainty Principle: $\Delta x \Delta p \geq \dfrac{\hbar}{2} \quad \Delta E \Delta t \geq \dfrac{\hbar}{2}$

DeBroglie Wavelength: $\lambda = \dfrac{h}{p} = \dfrac{h}{mv}$

Hydrogen Energy Levels: $E_n = -\dfrac{hcR}{n^2} = -\dfrac{13.60 \text{ eV}}{n^2}$

Rydberg Constant: $R = \dfrac{m_e e^4}{8\epsilon_o^2 h^3 c} = 1.097 \times 10^7$ 1/m

Stefan-Boltzmann: $I = \sigma T^4 \quad \sigma = 5.6704 \times 10^{-8} \dfrac{\text{W}}{\text{m}^2\text{K}^4}$

Wien's Displacement Law: $\lambda_{\text{max}} T = 2.90 \times 10^{-3}$ m.K

Planck's Radiation Law: $I(\lambda) = \dfrac{2\pi\hbar c^2}{\lambda^5 \left(e^{\frac{hc}{\lambda kT}} - 1\right)}$

Schrödinger Equation $$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}+U(x)\psi(x)=E\psi(x)$$

Normalization of the Wave Function $$\int_{-\infty}^{+\infty}|\Psi(x,t)|^2dx\equiv 1$$

Particle in a Box: $$\psi(x)=\sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}\ \ ;\ \ E_n=\frac{p_n^2}{2m}=\frac{n^2h^2}{8mL^2}$$

Tunneling Probability $$T=Ge^{-2\kappa L}\quad G=16\frac{E}{U_o}\left(1-\frac{E}{U_o}\right)\quad \kappa=\frac{\sqrt{2m\left(U_o-E\right)}}{\hbar}$$

Harmonic Oscilator $$E_n=\left(n+\frac{1}{2}\right)\hbar\sqrt{\frac{k'}{m}}=\left(n+\frac{1}{2}\right)\hbar\omega\quad n=0,1,2,...$$

Chapter 41

Atomic Structure

41.1 The Schrödinger Equation in 3D

41.2 Particle in a Three-Dimensional Box

$$E_x = \frac{n_x^2 \pi^2 \hbar^2}{2mL_x^2} \qquad E_y = \frac{n_y^2 \pi^2 \hbar^2}{2mL_y^2} \qquad E_z = \frac{n_z^2 \pi^2 \hbar^2}{2mL_z^2}$$

$$\psi_{n_x,n_y,n_z}(x,y,z) = \left(\frac{2^{3/2}}{L_x L_y L_z}\right) \sin\frac{n_x \pi x}{L_x} \sin\frac{n_y \pi y}{L_y} \sin\frac{n_z \pi z}{L_z}$$

Cubical Box $L_x = L_y = L_z = L$

$$\psi_{n_x,n_y,n_z}(x,y,z) = \left(\frac{2}{L}\right)^{3/2} \sin\frac{n_x \pi x}{L} \sin\frac{n_y \pi y}{L} \sin\frac{n_z \pi z}{L} \tag{41.1}$$

Question Is this wavefunction normalized?

$$E = E_x + E_y + E_z = \left(\frac{n_x^2}{L^2} + \frac{n_y^2}{L^2} + \frac{n_z^2}{L^2}\right) \frac{\pi^2 \hbar^2}{2m}$$

$$E_{n_x,n_y,n_z} = \left(n_x^2 + n_y^2 + n_z^2\right) \frac{\pi^2 \hbar^2}{2mL^2} \tag{41.2}$$

Degenerate States Different states that have the same energy

Ex. 41.1 Consider the following two states:

$$(n_x, n_y, n_z) = (2, 3, 2) \quad \text{and} \quad (n_x, n_y, n_z) = (2, 2, 3)$$

a Are the two states different?

b Are they degenerate?

41.3 The Hydrogen Atom

$$U = -\frac{1}{4\pi\epsilon_o}\frac{e^2}{r}$$

$$E_n == \frac{1}{4\pi\epsilon_o}\frac{m_r e^4}{2n^2\hbar^2} = -\frac{13.6 \text{ eV}}{n^2}$$

Note. m_r is the reduced mass discussed in the Bohr Model of the atom (Sec. 39.3.4, p. 25)

41.3.1 Quantization of Orbital Angular Momentum

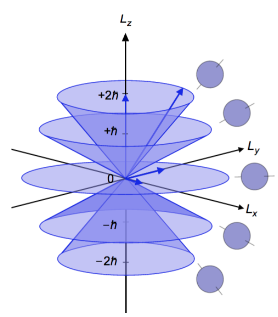


Figure 41.1: Quantization of the orbital angular momentum along a given axis (z). $l = 2$, $L = \sqrt{6}\hbar$, and $L_z = \{-2, -1, 0, +1, +2\}\hbar$.

The Orbital Angular Momentum values are quantized (See 39.3.3, 25):

$$L = \sqrt{l(l+1)}\hbar \qquad l = 0, 1, 2, \ldots, n-1 \tag{41.3}$$

We cannot know L without uncertainty. Only, we can know its one component (e.g. L_z), the other two components remain unknown due to the uncertainty principle.

z-component of L

$$L_z = m_l\hbar \qquad m_l = 0, \pm 1, \pm 2, \ldots, \pm l \tag{41.4}$$

Quantum Number Notation

Table 41.1: Shell and State Labels

n=1	K shell	l=0	s states
n=2	L shell	l=1	p states
n=3	M shell	l=2	d states
n=4	N shell	l=3	f states

Table 41.2: Hydrogen Atom Quantum States

n	l	m_l	Spectroscopy	Shell
1	0	0	1s	K
2	0	0	2s	L
	1	-1, 0, +1	2p	L
3	0	0	3s	M
	1	-1, 0, +1	3p	M
	2	-2,-1,0,+1,+2	3d	M
4	0	0	4s	

41.3.2 Electron Spin

Quantum Spin Number $s = \frac{1}{2}$

Spin Angular Momentum $\vec{S}$

$$S = \sqrt{s(s+1)}\hbar$$

Z-component of the Spin Angular Momentum:

$$\vec{S}_z = \pm\frac{\hbar}{2}$$

Quantum number rules for a given n:

$$0 \le l \le n–1 \quad , \quad |m| \le l \quad , \quad m_s = \pm\frac{1}{2} \tag{41.5}$$

Ex. 41.2 Consider an electron in the N shell.

(a) What is the smallest orbital angular momentum it could have?

(b) What is the largest orbital angular momentum it could have?

(c) What is the largest orbital angular momentum it could have in any direction?

(d) What is the the largest spin angular momentum it could have in any chosen direction?

(e) What is the ratio of its spin angular momentum in the z-direction to its maximum orbital angular momentum in the z-direction?

Ans. 0, $2\sqrt{3}\hbar$, $3\hbar$, $\frac{\hbar}{2}$, $\frac{1}{6}$

Ex. 41.3 The orbital angular momentum of an electron has a magnitude of 4.716×10^{-34} kg.m^2/s. What is the angular momentum quantum number l for this electron?

Ans. 4

Electron Probability Distribution

Ex. 41.4 The ground-state wave function for hydrogen ($1 - s$ state) is

$$\psi_{1s}(r) = \frac{1}{\sqrt{\pi a^3}} e^{-r/a} \tag{41.6}$$

Is this function normalized? What is the probability that the electron will be found at a distance less than a from the nucleus?

Ans. 0.323

41.4 The Zeeman Effect

Energy levels with different quantum numbers of same energy will split in the presence of a magnetic field. The magnetic energy $U = -\mu_z B$ is added to the overall energy, thus the degeneracy is removed.

$$I = \frac{dq}{dt} = \frac{ev}{2\pi r} \quad ; \qquad \vec{\mu} = I\vec{A} = -\frac{e}{2m}\vec{L}$$

$$\text{along z:} \;\; \mu_z = -\frac{e}{2m} m_l \hbar \quad (m_l = 0, \pm 1, \pm 2, \ldots, \pm l)$$

Bohr magneton

$$\mu_{\mathrm{B}} = \frac{e\hbar}{2m} = 5.788 \times 10^{-5}\ \mathrm{eV/T} = 9.274 \times 10^{-24}\ \mathrm{J/T or\ A.m^2}$$

Once the B field is switched on, the energy shift will be due to the additional magnetic energy:

$$U = -\vec{\mu} \cdot \vec{B}$$

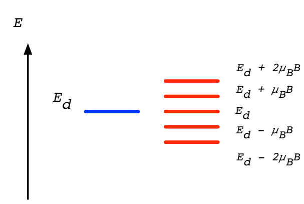


Figure 41.2: Splitting of the energy levels for a d state ($l = 2$).

Ex. 41.5 An atom in a state with $l = 1$ emits a photon with wavelength 600.00 nm as it decays to a state with $l = 0$. If the atom is placed in a magnetic field with magnitude $B = 2.00T$, what are the shifts in the energy levels and in the wavelength that result from the interaction between the atom's orbital magnetic moment and the magnetic field?

Ans. $m_l\,(1.16 \times 10^{-4}\ \mathrm{eV})$ 0.034 nm

Note. The relative shifts are very small!
Note. Not all transitions are allowed!

Selection Rules: The photon carries away $1\hbar$ of angular momentum. Hence, in transitions, $\Delta l = \pm 1$, and $\Delta m_l = 0, \pm 1$ (conservation of angular momentum).

41.5 Many-Electron Atoms and the Exclusion Principle

41.5.1 Pauli's Exclusion Principle

NO TWO ELECTRONS CAN OCCUPY THE SAME QUANTUM-MECHANICL STATE IN A GIVEN SYSTEM. Or, no two electrons can have the same values for all four quantum numbers.

$$\text{Argon } Z = 18 \;:\quad 1s^2 2s^2 2p^6 3s^2 3p^6$$

$$\text{Potassium } Z = 19 \;:\quad 1s^2 2s^2 2p^6 3s^2 3p^6 4s$$

Note. In Ka, 4s has slightly lower energy that 3d!

41.5.2 The Central-Field Approximation

For atoms when one electron is screened from the nucleus by other electrons, we can use the Central-Field Approximation. (Not to be confused with the "hydrogen-like" atoms in the Bohr model!)

Energy Levels with screening

$$E_n = -\frac{Z_{\text{eff}}^2}{n^2}(13.6\text{eV}) \tag{41.7}$$

11. The measured energy of a 3s state of sodium is -5.138 eV. Calculate the value of Z_{eff}.

Ans. 1.84

Electrons in the outer shells are more likely to qualify for the Central-Field Approximation.

12. The valence electron in potassium has a 4s ground state. Calculate the approximate energy of the $n = 4$ state having the smallest Z_{eff} (the one with the largest orbital angular momentum). *Solution:*

$$n = 4 \ \ \text{N Shell} \quad l = 3 \ \ \text{f} \longrightarrow E_4 = -\frac{Z_{\text{eff}}^2}{2^2}(13.6\text{eV}) = -.85 \text{ eV}$$

41.6 X-Ray spectra

Review of Sec. 38.2 Bremsstrahlung and X-Ray production

$$q_e V_{AC} = hf_{\max} = \frac{hc}{\lambda_{\text{mn}}}$$

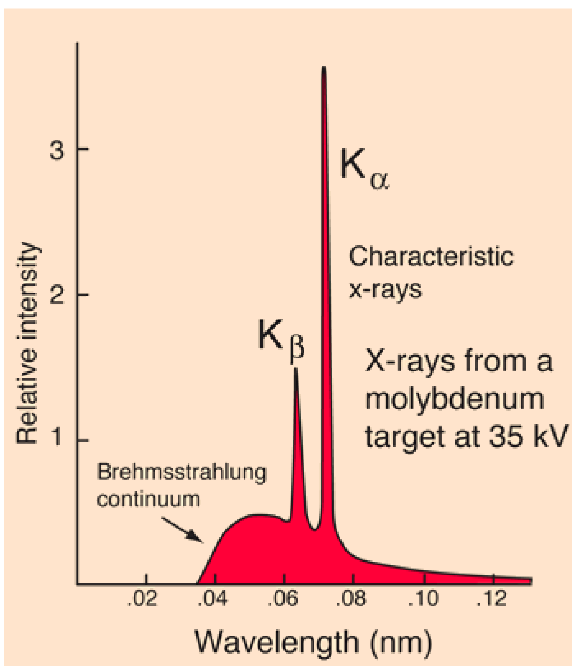


Figure 41.3: Bremsstrahlung spectrum. X-Rays produced with an accelerating voltage of 35 kV and a molybdenum target. Source: http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/xrayc.html

http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/xrayc.html

In the X-ray production, some inner shell electrons may be excited to higher states. Their vacancies will be filled in by electrons from the outer

shells. The highest-intensity line, K_α, corresponds to drop from the L (n=2) to K (n=1).
Assume $Z_{\text{eff}} = Z - 1$

$$E_i \approx -\frac{(Z-1)^2}{2^2}(13.6 \text{ eV})$$

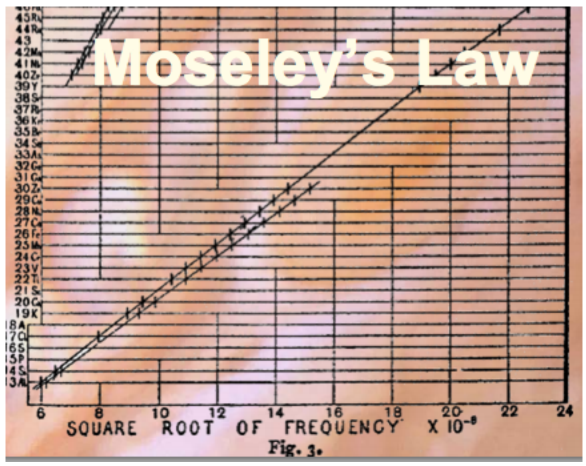


Figure 41.4: Moseley's Law. Source: http://isnap.nd.edu/Lectures/Laboratory/05_X-Ray-Spec.pdf

Moseley's law: The frequency for transitions depends on $(Z-1)^2$.

$$f = \left(2.48 \times 10^{15} \text{ Hz}\right)(Z-1)^2 \tag{41.8}$$

13. You measure the K_α wavelength for an unknown element, obtaining the value of 0.0709 nm. What is the most likely element?

    Solution:

    $$f = \frac{c}{\lambda} = 4.23 \times 10^{18} \text{ Hz}$$

    $$f = \left(2.48 \times 10^{15} \text{ Hz}\right)(Z-1)^2 \longrightarrow Z = 42.3$$

    Molybdenum is $Z = 42$.

41.7 Problems

P41.1 For a particle in a three-dimensional box, what is the degeneracy (number of different quantum states with the same energy) of the following energy levels: (a) $\frac{3\pi^2\hbar^2}{2mL^2}$ (b) $\frac{9\pi^2\hbar^2}{2mL^2}$

Ans. 1, 3

P41.2 What is the the enegy difference between the two lowest energy levels for a proton in a cubical box with side length 1.00×10^{-14} m, the approximate diameter of the nucleus?

Ans. 6.15 MeV

P41.3 Consider states with angular-momentum quantum number $l = 2$. In units of $\hbar$, what is the largest possible value of L_z and of L?

Ans. $2\hbar$, $2.45\hbar$

P41.4 *For a hydrogen atom in $1s$ state, at what value of r does $P(r)$ have its maximum value?

Ans. $r = a$, where a is the Bohr radius

P41.5 For germanium (Ge, $Z = 32$), make a list of the number of electrons in each subshell (1s, 2s, 2p, ...) Use the allowed values of the quantum numbers along with the exclusion principle.

P41.6 Write the ground state configuration for carbon (C). Compare with that for silicon (Si). Where is Si in the periodic table relative to C?

P41.7 The energy of an electron in the 4s state of sodium is -1.947 eV. What is the effective net charge of the nucleus "seen" by this electron?

Ans. 9.49

P41.8 For an outer electron in the 4p states of potassium, on the average 17.2 inner electrons screen the nucleus. What is the effective net charge of the nucleus "seen" by this outer electron? Find the energy of this outer electron?

Ans. $Z_{\text{eff}} = 1.8$; -2.75 eV

P41.9 A K_α X-ray emitted from a sample has an energy of 7.46 keV. Of which element is the sample made?

Ans. Nickel (Ni)

P41.10 The 5s electron in rubidium (Rb) sees an effective charge of $2.77e$. Calculate the ionization energy of this electron.

Ans. -4.18 eV

41.8 Formulae and Constants

Speed of light: $c = 3.0 \times 10^8$ m/s

Planck's Constant $h = 6.626 \times 10^{-34}$ J.s $= 4.14 \times 10^{-15}$ eV.s

$\hbar = 1.0546 \times 10^{-34}$ J.s

The electron charge and mass: $q_e = 1.6 \times 10^{-19}$ C $m_e = 9.1 \times 10^{-31}$ kg

Photon Energy and Momentum: $E = hf = \dfrac{hc}{\lambda} \quad p = \dfrac{E}{c} = \dfrac{h}{2\pi}$

Photoelectric Effect: $hf = \phi + K_{\text{max}} = \phi + qV_{\text{stopping}}$

X-Ray Emission: $qV = hf_{\text{max}} = \dfrac{hc}{\lambda_{\text{min}}}$

Compton Scattering: $\lambda' - \lambda = \dfrac{h}{mc}(1 - \cos\theta)$

Uncertainty Principle: $\Delta x \Delta p \geq \dfrac{\hbar}{2} \quad \Delta E \Delta t \geq \dfrac{\hbar}{2}$

DeBroglie Wavelength: $\lambda = \dfrac{h}{p} = \dfrac{h}{mv}$

Hydrogen Energy Levels: $E_n = -\dfrac{hcR}{n^2} = -\dfrac{13.60\text{ eV}}{n^2}$

Rydberg Constant: $R = \dfrac{m_e e^4}{8\epsilon_o^2 h^3 c} = 1.097 \times 10^7$ 1/m

Stefan-Boltzmann: $I = \sigma T^4 \quad \sigma = 5.6704 \times 10^{-8} \dfrac{\text{W}}{\text{m}^2\text{K}^4}$

Wien's Displacement Law: $\lambda_{\text{max}} T = 2.90 \times 10^{-3}$ m.K

Planck's Radiation Law: $I(\lambda) = \dfrac{2\pi\hbar c^2}{\lambda^5 \left(e^{\frac{hc}{\lambda k T}} - 1\right)}$

Schrödinger Equation $-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}+U(x)\psi(x)=E\psi(x)$

Normalization of the Wave Function $\int_{-\infty}^{+\infty}|\Psi(x,t)|^2dx\equiv 1$

Particle in a Box: $\psi(x)=\sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$; $E_n=\frac{p_n^2}{2m}=\frac{n^2h^2}{8mL^2}$

Tunneling Probability $T=Ge^{-2\kappa L}$ $G=16\frac{E}{U_o}\left(1-\frac{E}{U_o}\right)$ $\kappa=\frac{\sqrt{2m\left(U_o-E\right)}}{\hbar}$

Harmonic Oscilator $E_n=\left(n+\frac{1}{2}\right)\hbar\sqrt{\frac{k'}{m}}=\left(n+\frac{1}{2}\right)\hbar\omega \quad n=0,1,2,...$

Hydrogen Atom $E_n==\frac{1}{4\pi\epsilon_o}\frac{m_re^4}{2n^2\hbar^2}=-\frac{13.6\text{ eV}}{n^2}$

Orbital Angular Momentum $L=\sqrt{l(l+1)}\hbar \quad l=0,1,2,\ldots,n-1$

Magnetic Quantum Number $L_z=m_l\hbar \quad m_l=0,\pm1,\pm2,\ldots,\pm l$

Spin $S=\sqrt{s(s+1)}\hbar \;\; s=\pm\frac{1}{2}$

Bohr magneton $\mu_{\mathrm{B}}=\frac{e\hbar}{2m}=5.788\times10^{-5}\text{ eV/T}=9.274\times10^{-24}\text{ J/Tor A.m}^2$

Magnetic Energy: $U=-\vec{\mu}\cdot\vec{B}$

The Central-Field Approximation $E_n=-\frac{Z_{\text{eff}}^2}{n^2}(13.6\text{eV})$

Moseley's Law $f=\left(2.48\times10^{15}\text{ Hz}\right)(Z-1)^2$

Chapter 42

Molecules and Solids

42.1 Molecular Bonds

Ionic

Covalent

Van der Waals

Hydrogen

42.2 Molecular Spectra

42.2.1 Rotational Energy Levels

$$E = \frac{L^2}{2I} \tag{42.1}$$

$$L = \sqrt{l(l+1)}\hbar \quad I = m_r \times r_o^2 \quad m_r = \frac{m_1 m_2}{m_1 + m_2}$$

Selection Rules $\Delta l = \pm 1$ Conservation of Angular Momentum.

Ex. 42.1 C and O are separated by 0.1128 nm in the CO molecule. Determine the three lowest rotational levels of CO

Ans. 0; 0.479 meV; 1.437 meV

Ex. 42.2 Find the wavelength emitted in the $l = 2$ to $l = 1$ transition

Ans. 1.29 mm

42.2.2 Vibrational Energy Levels

$$E = \left(n + \frac{1}{2}\right)\hbar\omega = \left(n + \frac{1}{2}\right)\hbar\sqrt{\frac{k'}{m_r}} \tag{42.2}$$

$$n = 1, 2, 3, \ldots \quad m_r = \frac{m_1 m_2}{m_1 + m_2} \quad k' : \text{force constant}$$

Selection Rules $\Delta n = \pm 1$

42.3 Structure of Solids

42.3.1 Crystal Lattices

42.3.2 Bonding in solids

Ionic crystals

Covalent crystals

Metallic crystals

42.4 Energy Bands

Ex. 42.3 The measured energy of a 3s state of sodium is -5.138 eV. Calculate the value of Z_{eff}.

Ans. 1.84

Electrons in the outer shells are more likely to qualify for the Central-Field Approximation.

Ex. 42.4 The valence electron in potassium has a 4s ground state. Calculate the approximate energy of the $n = 4$ state having the smallest Z_{eff} (the one with the largest orbital angular momentum). *Solution:*

$$n = 4 \ \text{ N Shell} \quad l = 3 \ \text{ f} \longrightarrow E_4 = -\frac{Z_{\text{eff}}^2}{2^2}(13.6\text{eV}) = -.85 \text{ eV}$$

Chapter 43

Nuclear Physics

43.1 Nuclei

43.1.1 Properties

Radius of Nucleus

$$R = R_o A^{1/3} \quad R_o = 1.2 \text{ fm} \;\; ; \; A - \text{atomic number} \tag{43.1}$$

Ex. 43.1 The most common kind of iron nucleus has mass number $A = 56$. Find the radius, approximate mass, and approximate density of the nucleus.

Ans. 4.6 fm; 9.3×10^{-26} kg; $2.3 \times 10^{17}\ \frac{\text{kg}}{\text{m}^3}$

43.1.2 Isotopes

Isotopes Same atomic number, different mass. Or, same number of protons, different number of neutrons.

43.1.3 Magnetic Moments

Spin

$$S = \sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)}\hbar = \sqrt{\frac{3}{4}}\hbar \quad ; \; S_z = \pm\hbar \tag{43.2}$$

Table 43.1: Nuclear masses

	u	kg ($\times 10^{-27}$)	MeV/c^2
	1	1.66053878	934.05
Proton	1.007276	1.672622	940.85
Neutron	1.008665	1.674927	942.15
Electron	$\approx 1/1836m_p$		
	0.00054858	9.10938×10^{-31}	0.512
Hydrogen ($^1_1$H)	1.007825		
Deuteron ($^2_1$H)	2.014102		
Tritium ($^3_1$H)	3.016049		

Total Angualr Momentum of Nucleus $\vec{J}$

$$J = \sqrt{j(j+1)}\hbar \quad ; \; J_z = m_j\hbar \quad (m_j = -j, -j+1, ...j-1, j) \tag{43.3}$$

Nuclear magneton

$$\mu_n = \frac{e\hbar}{2m_p} = 5.05078 \times 10^{-27} \text{ J/T} = 3.15245 \times 10^{-8} \text{ eV/T} \tag{43.4}$$

Table 43.2: Nuclear Magnetic Moments z-Components

μ_{sz}	μ_n	J/T	eV/T
proton	2.7928		
neutron	1.9130		

Ex. 43.2 Protons are placed in a 2.30-T magnetic field that points in the positive z-direction. What is the energy difference between states with the z-component f proton spin angular momentum parallel and antiparallel to the field? Find the frequency and wavelength of the emitted photon in the transition.

Ans. 4.05×10^{-7} eV; 97.9 MHz and 3.06 m

43.2 Nuclear Structure

43.2.1 Nuclear Binding Energy

$$E_B = \left(ZM_H + Nm_n -_Z^A M\right) c^2 \tag{43.5}$$

Ex. 43.3 Find the binding energy of a deuteron, $m(^2_1\mathrm{H}) = 2.014102u$

Solution:

$$1.007825u + 1.008665u - 2.014102u = 2.224 \text{ MeV}$$

The most tightly bound nuclei

- Hyper Physics, http://hyperphysics.phy-astr.gsu.edu/hbase/nucene/nucbin2.html#c1
- AJP, **63**, 653 (1995), http://scitation.aip.org/content/aapt/journal/ajp/63/7/10.1119/1.17828

Ex. 43.4 Find the binding energy per nucleon of $^{62}_{28}\mathrm{Ni}$ ($m = 61.928349u$), which has the highest binding energy per nucleon

Ans. 545.3 MeV

43.2.2 The Nuclear Force and Nuclear Models

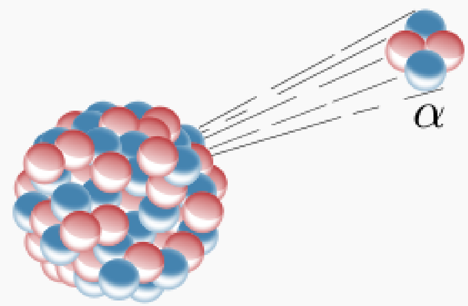
43.3 Radioactivity

43.3.1 Alpha Decay

α He Nuclei, $4_2\mathrm{He}$

$$^Z_A\mathrm{X} \longrightarrow ^{Z-2}_{A-4}\mathrm{Y} +^4_2\mathrm{He}$$

$$^{226}_{88}\mathrm{Ra} \longrightarrow ^{222}_{86}\mathrm{Ra} +^4_2\mathrm{He}$$



	Alpha decay
Composition	2 protons, 2 neutrons
Statistics	Bosonic
Symbol	α, α^{2+}, He^{2+}
Mass	$6.64465675(29) \times 10^{-27}$ kg[1] 4.001506179125(62) u 3.727379240(82) GeV/c^2
Electric charge	2 e
Spin	0[2]

Figure 43.1: α decay

Ex. 43.5 Radium is a product of the Uranium decay. It itself is radioactive and decays into Radon via an α particle. Calculate the kinetic energy of the emitted α particle in the α-decay of $^{226}_{88}$Ra into $^{222}_{86}$Rn.

Solution:

$$226.025403u-(222.01757u+4.002603u) = +0.005229u = 4.871\text{MeV/c}^2$$

43.3.2 Beta Decay

β^- Electrons (Not to be confused with the atomic electrons!)

β^- **Decay:** When too many neutrons for the nucleus to be stable, a neutron flips to a proton. The average decay time for a free neutron is 15 minutes!

$$p \longrightarrow n + \beta^+ + \nu_e$$

β^+ **Decay:** When too few neutrons for the nucleus to be stable, a proton turns into neutron.

$$p \longrightarrow n + \beta^- + \bar{\nu}_e$$

Electron Capture: An orbital (usually K) electron can be captured by a proton to transform into a neutron.

$$p + \beta^- \longrightarrow n + \nu_e$$

43.3.3 Gamma Decay

γ **Decay:** High-energy EM radiation, or high-energy photons (gamma-ray photons)

Ex. 43.6 During the α decay of radium ^{228}Ra the product ^{222} Rn is left in an excited state. It subsequently drops to its ground state by emitting a gamma-ray (high-energy photon). If the difference between the two states is 0.186 MeV, determine the wavelength and frequency of the gamma-ray photon.

43.3.4 Natural Radioactivity

^{14}**C** Carbon from the atmosphere, the air we breathe.

^{40}**K** Potassium from

^{238}**U** The most abundant radioactive nuclide. It undergoes 14 decays to end as a stable isotope of lead, ^{206} Pb.

43.4 Activities and Half-Lives

Activity: Decay Rate, $\frac{dN(t)}{dt}$ The number of decays per unit time. Si Unit: Bequerel (Bq). 1 decay/s is 1 Bq. 1 Ci = 3.7×10^{10} Bq)

$$-\frac{dN(t)}{dt} = \lambda N(t)$$

λ: Decay constant

$$N(t) = N_o e^{-\lambda t} \tag{43.6}$$

Half Life $\tau_{1/2}$

$$\tau_{1/2} = \frac{\ln 2}{\lambda} \tag{43.7}$$

Lifetime: The mean lifetime

$$T_{\text{mean}} = \frac{1}{\lambda} = \frac{\tau_{1/2}}{\ln 2} \tag{43.8}$$

Ex. 43.7 The isotope ^{57}Co decays by electron capture to ^{57}Fe with aa half-life of 272 d. The ^{57}Fe nucleus is produced in an excited state, and it almost instantaneously emts gamma rays that we can detect. Find the mean lifetime and decay constant for ^{57}Co. If the activity of a ^{57}Co radiation source is now 2.00 μCi, how many nuclei does the source contain?

Ans. 392 d, 2.51×10^{12} nuclei

Ex. 43.8 Before 1900 the activity per unit mass of atmospheric carbon due to the presence of ^{14}C averaged about 0.255 Bq per gram of carbon. What fraction of carbon were ^{14}C? **Ans.** 1.33×10^{-12}

Ex. 43.9 An archaelogical specimen containing 500 mg of carbon, you observe 174 decays in one hour. What is the age of the specimen, assuming that its activity per unit mass of carbon when it died was that average value of the air? **Ans.** 8020 yr

43.5 Biological Effects of Radiation

Absorbed Dose of Radiation

Unit	Symbol	J/kg
Gray	GY	1
rad	rad	0.01

Equal energies of different kinds of radiation cause different extenst of biological effect.

Relative biological effectiveness (RBE) Quality factor (QF)

Radiation	RBE (rem/rad)
X rays and γ rays	1
Electrons	1.0-1.5
Slow neutrons	3-5
Protons	10
alpha particles	20
heavy ions	20

$$\text{Equivalent dose (Sv)} \;=\; \text{RBE} \;\times\; \text{Absorbed dose (Gy)}$$

$$\text{Equivalent dose (rem)} \;=\; \text{RBE} \;\times\; \text{Absorbed dose (rad)}$$

Background

43.6 Problems

P43.1 Hydrogen atoms are placed in an external 1.65-T magnetic field. (a) The *protons* can make transitions between states where the nuclear spin component is parallel and antiparallel to the field by absorbing or emitting a photon. Which state has lower energy: the state with the nuclear spin component parallel or antiparallel to the field? (b) The *electrons* can make transitions between states where the electron spin component is prallel and antiparallel to the field by absorbing or emitting a photon. Which state has lower energy: the state with the electron spin component parallel or antiparallel to the field.

P43.2 Hydrogen atoms are placed in an external magnetic field. The protons can make transitions between states in which the nuclear spin component is prallel and antiparallel to the field by absorbing or emitting a photon. What magnetic field magnitude is required for this transition to be induced by photons with frequency 22.7 MHz?

Ans. 0.533 T

P43.3 The most common isotopoe of uranium, $^{238}_{92}$U, has atomic mass 238.050783u. Calculate (a) the mass defect; (b) the binding energy (in MeV); (c) the binding energy per nucleon.

Ans. 1.93 u; 1.80×10^3 MeV; 7.56 MeV

P43.4 What nuclide is produced in the following radioactive decays: (a) α radioactive decay of $^{239}_{94}$Pu; (b) β^- radioactive decay of $^{24}_{11}$Na; (c) β^+ radioactive decay of $^{15}_{8}$O?

Ans. $^{235}_{92}$U; $^{24}_{12}$Mg; $^{15}_{7}$N

P43.5 $^{238}_{92}$U decays spontaneously by α emission to ^{234}Th. Calculate the total energy released by this process and the recoil velocity of the $^{234}_{90}$Th nucleus. The atomic masses are 238.050788u for $^{238}_{92}$U and 234.043601u for $^{234}_{90}$Th. Assume the initial $^{238}_{92}$U is at rest.

Ans. 4.270 MeV; 0.071176 MeV

P43.6 If a 6.13-g sample of an isotope having a mass number of 124 decays at a rate of 0.350 Ci, what is its half-life?

Ans. 5.01×10^4 yr

P43.7 A 12.0-g sample of carbon from living matter decays at the rate of 180.0 decays/min due to the radioactive ^{14}C in it. What will be the decay rate of this sample in 1000 yr and in 50,000 yr?

Ans. 159 decays/min; 0.43 decays/min

P43.8 A sample from timbers at an archeological site containing 500 g of carbon provides 3070 decays/min. What is the age of the sample?

Ans. 7573 yr

P43.9 It has become popular for some people to have yearly whole-body scans using x rays, just to see if they detect anything suspicious. Typically, one such scan gives a dose of 12 mSv applied to the whole body. By contrast, a chest x ray typically administers 0.20 mSv to only 5.0 kg of tissue. How many chest x rays would deliver the same total amount of energy to the body of a 75-kg person as one whole-body scan?

Ans. 900

Chapter 37

Special Theory of Relativity

37.1 Invariance and Relativity

Inertial Frames Frames that move relative to each other at a constant velocity.

Galilean coordinate transformation

$$x = x' + ut \longrightarrow \frac{dx}{dt} = \frac{dx'}{dt} + u \longrightarrow v_x = v'_x + u$$

Einstein I The laws of physics are the same in every inertial frame of reference.

Einstein II The speed of light in vacuum is the same in all inertial frames of reference and is independent of the motion of the source.

Consequence: It is impossible for an inertial observer to travel at the speed of light in vacuum.

Simultaneity: Two events happen at the same time and location. **This depends on the coordinate system**

Question Are Newton's Laws invariant? That is, do they remain the same in any reference frame?

Question Are Maxwell's Equations invariant?

37.2 Lorentz' transformation

$$x' = \frac{x - ut}{\sqrt{1 - \frac{u^2}{c^2}}}$$

$$t' = \frac{t - ux/c^2}{\sqrt{1 - \frac{u^2}{c^2}}}$$

Michaelson and Morley Experiment

37.2.1 Time Dilation and Length Contraction

Length Contraction

$$L_{\parallel} = L_o \left(1 - \frac{u^2}{c^2}\right) \tag{37.1}$$

Time Dilation

$$\Delta t = \frac{\Delta t_o}{\sqrt{1 - \frac{u^2}{c^2}}} \tag{37.2}$$

Ex. 37.1 Mu-mesons (muons) distintegrate spontaneously after an average lifetime of 2.2×10^{-6} sec. They come to earth in cosmic racys and can also be produced artificially in lab. The muons are actually created at the top of the atmosphere at 10 km or so, but they are easily detected in physical laboratories on the surface of the earth. 1. Using the classical physics laws to determine how far they travel if they move at the speed of light? 2. From our perspective how long does a muon moving at 99%c live? 3. How much distance can it travel in that time?? 4. From the muon perspective, how long is the 5-km distance?

Ans. 600 m; 1.6×10^{-5} s; 4.7 km; 700 m

37.2.2 Velocity Addition

$$v_x = \frac{v'_x + u}{1 + uv'_x/c^2} \tag{37.3}$$

Ex. 37.2 Imagine you walk at 5 mph along the airport walkway that also moves at 5mph with respect to ground. How much is your speed if you walk along the direction of the walkway, and against the direction?

Ex. 37.3 What if you walked at speed of light, and the walkway moved at the speed of light?

Ex. 37.4 What if you walked at $0.3c$ while the walkway moved at $0.5c$?

Ans. $0.7c$; $-0.23c$

37.3 Doppler Effect and Electromagnetic Waves

EM waves approaching observer

$$f = \sqrt{\frac{c+u}{c-u}} f_o \tag{37.4}$$

EM waves receding from observer

$$f = \sqrt{\frac{c-u}{c+u}} f_o \tag{37.5}$$

37.4 Relativistic Dynamics

Gamma

$$\gamma = \frac{1}{1 - \frac{u^2}{c^2}}$$

Relativistic Momentum

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{d}{dt}\left(\frac{m}{\sqrt{1 - \frac{v^2}{c^2}}} v\right) = \gamma m \vec{v}$$

$$m_{\text{rel}} = \frac{m}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Newton Second Law Still valid!

$$\vec{F} = \frac{d\vec{p}}{dt}$$

Ex. 37.5 An electron with a rest mass $m_o = 9.1 \times 10^{-31}$ kg is moving opposite to an electric field of magnitude $E = 5.00 \times 10^5$ V/m. All other forces are negligible in comparison to the electric-field force. Find the magnitudes of the relativistic mass and momentum at the instants when $v = 0.010c$, $v = 0.10c$, and $v = 0.90c$.

Ans. $1.00005m_o$; $1.005m_o$; $2.29m_o$;

Ex. 37.6 What is the acceleration in those cases?

Solution:

For $\vec{F}$ and $\vec{v}$ along the same line:

$$F = \frac{d}{dt}(\gamma m_o v) = \gamma^3 m_o a$$

Ans. $8.8 \times 10^{16}\ \frac{\mathrm{m}}{\mathrm{s}^2}$; $2.5 \times 10^{14}\ \frac{\mathrm{m}}{\mathrm{s}^2}$

37.5 Work and Energy

Total Energy

$$E = K + mc^2 = \gamma mc^2 \tag{37.6}$$

Kinetic Energy

$$K = (\gamma - 1)mc^2 \tag{37.7}$$

Ex. 37.7 Show that in the limit of $v \ll c$, the new kinetic energy form reduces to the classic form.

Energy-momentum

$$E^2 = \left(mc^2\right)^2 + (pc)^2 \tag{37.8}$$

Ex. 37.8 Compton Scattering. Light with wavelength λ is scattered off an electron initially at rest. If the scattered light has a wavelength of λ' and emergest at an angle ϕ with respect to the incident direction, derive the Compton scattering formula

$$\lambda' - \lambda = \frac{h}{mc}\,(1 - \cos\phi)$$

37.6 Problems

P37.1 The negative pion (π^-) is an unstable particle with an average lifetime of 2.6×10^{-8} s (measured in the rest fram of the pion). (a) Calculate the speed of the pion if its average lifetime measured in the laboratory is 4.20×10^{-7} s. (b) What distance, measured in the lab, does the pion travel during its average lifetime?

Ans. 0.998c; 126 m

P37.2 A spacecraft of the Trade Federation flies past the planet Coruscant at a speed of 0.600c. A scientist on Coruscant measures the length of the moving spacecraft to be 74.0 m. What is the length of the spacecraft from the perspective of the passengers on the spacecraft?

Ans. 92.5 m

P37.3 Two particles are created in a high-energy accelerator and move off in oposite directions. The speed of one, as measured in the laboratory, is 0.650c, and the speed of each particle relative to the to the other s 0.950c. What is the speed of the scond particle, as measured in the laboratory?

Ans. 0.784c

P37.4 *Two particles in a high-energy accelerator experiment are approaching each other head-on with a relative speed of 0.89c. Both particles travel at the same speed as measured in the laboratory. What is the speed of each particle, as measured in the lab?

Ans. 0.611c

P37.5 How fast must you be approaching a red traffic light ($\lambda = 675$ nm) for it to appear yellow ($\lambda = 575$ nm)?

Ans. 0.159c

P37.6 A soruce of electromagnetic radiation ismoving in a radial direction relative to you. The frequency you measure is 1.25 times the frequency measured in the rest frame of the source. What is the speed of the source relative to you? Is the source moving away or toward?

Ans. 0.22c toward

37.7 Formulae and Constants

$$\text{Speed of light:} \quad c = 3.0 \times 10^{8}\ \text{m/s}$$

$$\text{Planck's Constant} \quad h = 6.626 \times 10^{-34}\ \text{J.s} = 4.14 \times 10^{-15}\ \text{eV.s}$$

$$\hbar = 1.0546 \times 10^{-34}\ \text{J.s}$$

$$\text{The electron charge and mass:} \quad q_e = 1.6 \times 10^{-19}\ \text{C} \quad m_e = 9.1 \times 10^{-31}\ \text{kg}$$

$$\text{Photon Energy and Momentum:} \quad E = hf = \frac{hc}{\lambda} \quad p = \frac{E}{c} = \frac{h}{2\pi}$$

$$\text{Compton Scattering:} \quad \lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta)$$

$$L_{\|} = L_o\left(1 - \frac{u^2}{c^2}\right) \qquad \Delta t = \frac{\Delta t_o}{\sqrt{1 - \frac{u^2}{c^2}}}$$

$$\text{Velocity Addition} \quad v_x = \frac{v'_x + u}{1 + uv'_x/c^2}$$

$$\text{Doppler Effect} \quad f = \sqrt{\frac{c+u}{c-u}} f_o \quad f = \sqrt{\frac{c-u}{c+u}} f_o$$

$$\vec{p} = \gamma m \vec{v} \quad m_{\text{rel}} = \gamma m \quad \vec{F} = \frac{d\vec{p}}{dt}$$

$$E_{\text{rest}} = mc^2 \quad E_{\text{total}} = K + mc^2 \quad K = (\gamma - 1)mc^2$$

$$E^2 = \left(mc^2\right)^2 + (pc)^2$$
