Fluid Mechanics and Pumps

INDEX

Lesson 1

UNITS AND DIMENSIONS, PROPERTIES OF FLUIDS

1.1 Introduction

Fluid is a substance which has no definite shape and will continuously deform or flow whenever an external force is applied to it e.g. water, milk, steam, gas, etc. It cannot preserve its shape unless it is restricted into a particular form depending upon the shape of its surroundings.

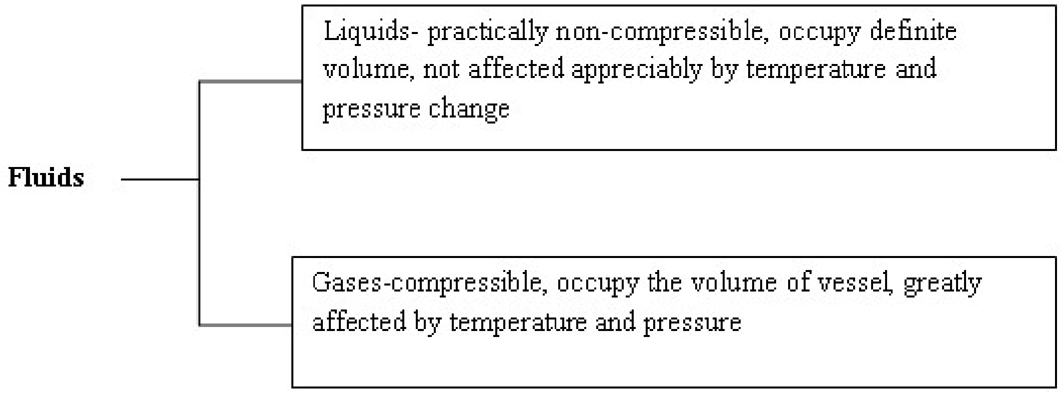


Fig. 1.1 Fluids classified as liquids and gases

Fluid Mechanics is the study of fluids either in motion (fluid dynamics/kinematics) or at rest (fluid statics). Gases and liquids (e.g. air, water) come under the category of fluid.

One of the areas of modern fluid mechanics is Computational Fluid Mechanics which deals with numerical solutions using computers. Fluid mechanics comprises of the following subjects:

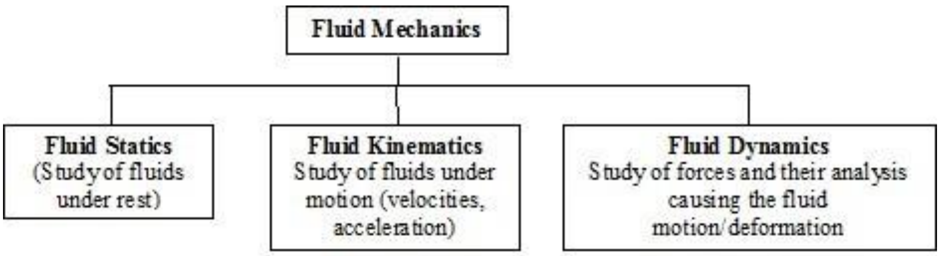


Fig. 1.2 Subjects covered under fluid mechanics

1.2 Why to Study Fluid Mechanics?

Fluid mechanics is one of the basic courses in Engineering. It is a bridge course between what you have already studied in physics and core B.Tech. courses which you will be studying after 1st year of your degree programme. Dairy plants handle various types of fluids such as milk, water, air, refrigerants, steam etc. It is very important to learn the behaviour of fluid under various conditions in order to design the system for handling of such fluids in dairy plants. Fluid mechanics is a branch of Engineering Science, the knowledge of which is needed in the design of:

- Water supply and treatment system
- Pumps used for handling of different fluids

- Ships, submarines, aeroplanes, Automobiles
- Storage tanks (milk silo, tankers, feed tanks, balance tanks etc.)
- Piping systems for various utilities, pipefitting & valves, flow meters etc.
- Measuring instrument
- Cleaning-In-Place (CIP) systems for optimum performance
- Heat transfer behaviour in processing equipments (such as HTST pasteurizers, spray dryers etc.)

1.3 Units and Dimensions

Solution to numerical and engineering problems becomes meaningless without units. One of the space projects of NASA, Mars pathfinder long back in 1999 crashed because the Jet Propulsion Laboratory engineers assumed that a measurement was in meters, but the supplying company's engineers had actually made the measurement in feet (Fox et al., 2004). This incident truly represents the importance of units. A unit of measurement is a definite magnitude of a physical quantity. The different systems of unit are:

1. SI system: It is the International System of Units (abbreviated SI from the French Le Système International d'Unités.

2. CGS system: It is a system of physical units based on centimetre as the unit of length, gram as a unit of mass, and second as a unit of time.

3. MKS system: It is a metric system of physical units based on meter as the unit of length, kilogram as a unit of mass, and second as a unit of time.

4. FPS system The foot-pound-second system or FPS system is a system of units built on the three fundamental units foot for length, pound for either mass or force and second for time.

Table 1.1 Commonly used units in CGS, MKS, FPS and SI

Dimension	CGS units	MKS units	FPS Unit	SI units
Length (L)	Centimeter (cm)	Meter (m)	Foot (ft)	meter, (m)
Mass (M)	Gram (g)	Kilogram (kg)	Pound (lb_M)	kilogram (kg)
Time (T)	Second (sec)	Second (sec)	Second (sec)	Second (s)
Force (F)	Dyne (Dyn)	Kilogram-force (kgf)	Pound-force (lb_F)	Newton (N) (=kg m/s^2)
Temperature (θ)				
· Absolute	Rankine (R)	Celsius (°C)	Kelvin (K)	Kelvin (K)
· Ordinary	Fahrenheit (°F)	Celsius (°C)	Kelvin (K)	Celsius (°C)

Non-standard abbreviations:

fps = feet per second

gpm = gallons per minute

cfs or cusecs = cubic feet per second

cumecs = cubic meters per second

Note: Cusec and cumecs are non-standard abbreviations as it is widely used to measure large water flows.

- 1 British or imperial gallon = 1.2 U.S. Gallon (±0.1%)
- 1 U.S. gallon = 3.78 Litres

When not specified, assume U.S. gallons

Table 1.2 Unit prefixes in SI system

Factor	Prefix	Symbol
10^9	Giga	G
10^6	Mega	M
10^3	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n

Table 1.3 Quantities, dimensions and units

Quantity	**Dimensions** (M LT)	**Preferred units** (SI)
Length (L)	L	m
Time (T)	T	s
Mass (M)	M	kg
Area (A)	L^2	m^2
Volume (Vol)	L^3	m^3
Velocity (V)	LT^{-1}	m/s
Acceleration (a)	LT^{-2}	m/s^2
Discharge (Q)	$L^3\ T^{-1}$	m^3/s
Force (F)	MLT^{-2}	N
Pressure (p)	$ML^{-1}T^{-2}$	Pa
Shear stress (τ)	$ML^{-1}T^{-2}$	N/m^2
Density (ρ)	ML^{-3}	kg/m^3
Specific weight (ω)	$ML^{-2}T^{-2}$	N/m^3
Energy/Work/Heat (E)	ML^2T^{-2}	J
Power (P)	ML^2T^{-3}	W
Dynamic viscosity (μ)	$ML^{-1}T^1$	N s/m^2 or Pa.s
Kinematic viscosity (υ)	$L^2\ T^{-1}$	m^2/s

Some important units and conversions

Dyne = g cm/s^2

1 dyne = 10^{-5} N

1 pound = 0.453 kg

Pressure: 1 atm = 101.325 kPa, 1 bar = 10^5 Pa

1 m = 3.28 ft

1 m = 100 cm

1 feet = 30.5 cm

1 feet = 12 inch

1 inch = 2.54 cm

1 km = 0.621 miles

1 ha = 2.47 acre

1 acre = 4 046.85 m^2

1 litre = 0.264 gallon

°C = (5/9) * (°F – 32)

Table 1.4 Pressure conversion table

Pressure units						
	Pascal (Pa)	**Bar (bar)**	**Technical atmosphere (at)**	**Atmosphere (atm)**	**Torr (Torr)**	**Pound-force per square inch (psi)**
1 Pa	≡ 1 N/m^2	10^{-5}	1.0197×10^{-5}	9.8692×10^{-6}	7.5006×10^{-3}	145.04×10^{-6}
1 bar	100,000	≡ 10^6 dyn/cm^2	1.0197	0.98692	750.06	14.5037744
1 at	98,066.5	0.980665	≡ 1 kgf/cm^2	0.96784	735.56	14.223
1 atm	101,325	1.01325	1.0332	≡ 1 atm	760	14.696
1 torr	133.322	1.3332×10^{-3}	1.3595×10^{-3}	1.3158×10^{-3}	≡ 1 Torr; ≈ 1 mmHg	19.337×10^{-3}
1 psi	6.894×10^3	68.948×10^{-3}	70.307×10^{-3}	68.046×10^{-3}	51.715	≡ 1 lbf/in^2

Example reading: 1 Pa = 1 N/m^2 = 10^{-5} bar = 10.197×10^{-6} at = 9.8692×10^{-6} atm = 7.5006×10^{-3} torr = 145.04×10^{-6} psi

1.4 Properties of Fluid

1. *Mass density* (ρ): Mass of fluid per unit of its volume is called mass density.

$$\rho = \frac{mass}{volume}$$

Unit: kg/m^3

Dimension: ML^{-3}

2. ***Weight Density (ω)***: Weight of fluid per unit of its volume is called weight density.

$$\frac{Weight\,of\;\;fluid}{Volume} = \rho . g$$

3. ***Specific Gravity***: Ratio of density of a substance to the density of pure water at 4°C is called specific gravity.

$$\text{Specific gravity} = \frac{\text{Density of substance}}{\text{Density of water at } 4°C} \quad (\text{Dimension } M°L°T°)$$

4. ***Specific Volume (v)***: Volume of substance per unit of its mass is called specific volume.

$$v = \frac{1}{\rho} = \frac{volume}{mass}$$

Unit: m^3/ kg

Dimension: M^{-1}L^3

5. Vapour Pressure

Liquid exhibits a tendency to vaporise or evaporate. This process takes place at free surface of liquid where liquid molecules continuously escape to atmosphere. The liquid molecules which escape from free surface of liquid are in gaseous state, exert their own partial pressure on surface of the liquid. This pressure produced by the molecules ejected is known as the vapour pressure. Higher the vapour pressure of free liquid surface, higher will be the rate at which molecules will escape into the atmosphere.

- Molecular activity increases with increase in temperature resulting in increase in vapour pressure. Boiling occurs when vapour pressure of the free liquid surface is equal to saturation vapour pressure.
- Saturation vapour pressure is attained when the space above the liquid is saturated with vapours of the liquid. Equilibrium is established between the liquid vapour interface in which rate of molecules escaping from the liquid surface is equal to the rate at which molecules return to the liquid. For a liquid saturation vapour pressure is the maximum attainable vapour pressure at a given temperature. Mercury has a very low vapour pressure (1.2 μmHg at 20°C). Therefore, it is used in barometers and thermometers for accurate results.

6. ***Cohesion***: It is intermolecular force of attraction between similar types of molecules.

7. ***Adhesion*****:** It is the force of attraction between molecules of:

a) Two different liquids which do not mix or

b) Between liquids and solid containing liquid.

8. Compressibility:

i) Compressible fluids: The fluids which undergoes a change in volume or density when pressure is applied.

ii) Incompressible fluids: The fluid which does not show a change in volume or density when pressure is applied.

Compressibility is the property of the fluid due to which there will be a change in volume when the fluid is subjected to an external pressure and is reciprocal of Bulk Modulus of Elasticity (k).

$$\text{Compressibility} = \frac{1}{\textit{Bulk Moldulus of Elasticity}(k)}$$

$$\text{Bulk Modulus of Elasticity (k)} = \frac{dP}{\Delta v / V} = \frac{\textit{Changein pressure}}{\textit{Volumetric Strain}}$$

9. *Viscosity*: It is property of liquid which provides resistance to flow. For example, flowability of honey is poor as compared to milk. Honey is highly viscous. The viscosity of honey is much higher than milk.

1.5 Classification of Fluids

Types of Fluids:

Ideal Fluid: Ideal fluid is one which has no property other than density. Such fluids have no viscosity, no surface tension and are incompressible. When such fluid flows, no resistance is encountered. Ideal fluid is imaginary fluid as all the fluids have some viscosity.

Real Fluid: The fluids which have viscosity, surface tension in addition to density. All the fluids have these properties whether large or small. The fluids can also be classified in the following manner:

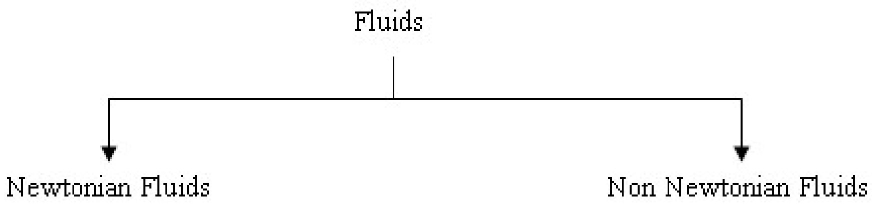
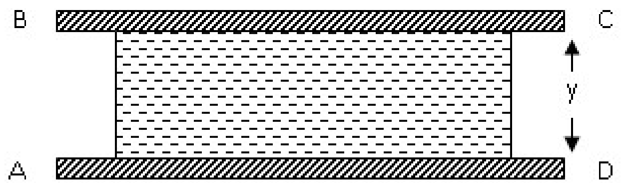


Fig. 1.3 Newtonian and non-Newtonian fluids

- Newtonion Fluids: Fluids which follow Newton’s Law of viscosity are called Newtonian fluid.
- Non-Newtonion Fluids: Fluids which do not obey Newton’s law of viscosity are called non-Newtonian fluids.

1.6 Newton’s Law of Viscosity

Consider a fluid contained between two parallel plates as shown in the figure 1.4:



[Fig. 1.4 Fluid contained between two parallel plates (Click fpr Animation)]

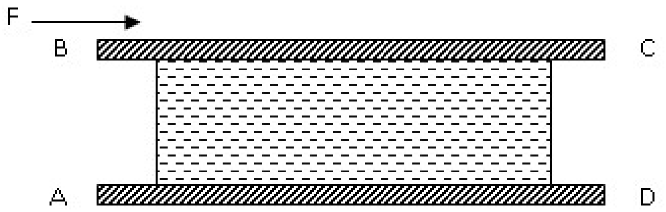


Fig. 1.5 Shear force is applied on the upper plate

Plate AD is the stationary plate where as BC is the moving plate and distance between the plates is y units. Initially BC is at rest. The area of the plate is A. Suppose a shear force is applied to top plate at point B. By shear force we mean a force that is applied tangentially and parallel to a surface. It can be seen in figure 1.5:

The upper plate starts moving and attains a velocity say u m/s. Now the position changes from ABCD to AB'C'D as shown in figure 1.6.

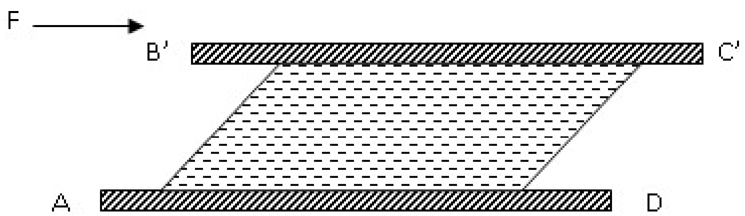


Fig. 1.6 Upper plate starts moving

As we know, fluid molecules are arranged in layers. When the upper plate starts moving, the fluid layer just touching the top plate starts moving the same velocity. Then the next layer starts moving and so on. It can be seen in figure 1.7.

[Fig. 1.7 Velocity profile(Click for Animation)]

The distribution of fluid velocity from the top plate to the bottom is known as velocity gradient or velocity profile and is given as:

Velocity gradient = $\frac{du}{dy}$

Shear stress τ = (shear force)/Area

τ is pronounced as Tau and is the symbol of shear stress.

Note: shear stress is similar to pressure but here shear force is involved

Here, shear stress is proportional to velocity gradient:

$$\tau \propto \frac{du}{dy}$$

Or,

$$\tau = \mu\frac{du}{dy}$$

Here, μ is known as coefficient of viscosity or dynamic viscosity. The SI unit of dynamic viscosity is Ns/m^2.

CGS units of dynamic viscosity is poise:

1 poise = 0.1 Ns/m^2

1 Centi poise (CP) = 0.01 poise

Kinematic Viscosity:

$$Kinematic\ viscosity = \frac{Dynamic\ viscosity}{density\ of\ substance}$$

SI Units: m^2/s

CGS units = Stoke

1.7 Numerical

Q.1	**A plate moves at 2 m/s with a shearing force of 3.5 N/m^2. The distance between moving plate and fixed plate is 0.08 mm. Determine viscosity of fluid between plates.** **Solution:** Velocity of plate (u) = 2 m/s $\frac{shearing\ force}{area}\ \frac{(F)}{(A)} = 3.5\frac{N}{m^2}$ $\frac{F}{A} = \mu\frac{du}{dy}$ As dy is very small $\frac{dv}{dy} \approx \frac{u}{y}$ $u = \frac{f \times y}{a \times u}$ $= \frac{3.5 \times 8 \times 10^{-3}}{2}$ $= 1.4\times10^{-2}\ Ns/m^2$
Q. 2	**Two rectangular flat plates of dimensions 850 mm×600 mm are placed such that distance between plates is 20 mm. The space between them is filled with**

	oil of specific gravity 0.92. The lower plate is fixed. The upper plate moves at 4.5 m/s and requires a force of 200 N to maintain this state. Determine dynamic and kinematic viscosity of oil. **Solution:** Velocity of plate (u) = 4.5 m/s Force (F) = 200 N Area of plate (A) = 850 mm × 600 mm Distance between plates (y) = 20 mm Density of subs = sp. gr × density of H_2O at 4°C =0.92×1000 = 920 kg/m^3 $Dynamic\ viscosity\ u = \frac{F \times y}{A \times u}$ $= \frac{200 \times 20 \times 10^{-3}}{850 \times 10^{-3} \times 600 \times 10^{-3} \times 4.5}$ = 1.7429 Ns/m^2 $Kinematic\ viscosity = \frac{Dynamic\ viscosity}{density\ of\ subs}$ $= \frac{1.7429}{920}$ = 0.0018945 m^2/s
Q. 3	**A liquid has specific gravity of oil is 1.85 and kinematic viscosity of 8 stokes. What is its dynamic viscosity?** **Solution:** Kinematic viscosity = 8 stokes = 8×10^{-4} m^2/s Density of substance = sp. gr × density of H_2O at 4°C = 1.85×1000 = 1850 kg/m^3 Dynamic viscosity = kinetic viscosity× density of subs $= \frac{1850 \times 8}{10000}$ = 1.48 Ns/m^2
Q. 4	**The space between two parallel plates 5 mm apart is filled with crude oil. A force of 2 N is required to drag the upper plate at constant velocity of 0.8 m/s. The lower plate is stationary. The area of upper plate is 0.09 m^2.**

Determine dynamic viscosity and kinematic viscosity if specific gravity of oil is 0.9.

Solution:

Distance between two plates (dy) = 5×10^{-3}

Force required (F) = 2 N

Area of plate (A) = 0.09 m^2

Velocity of plate (v) = 0.08 m/s

$$Dynamic\ viscosity = \frac{F \times dy}{A \times dv}$$

$$= \frac{2 \times 5 \times 10^{-3}}{0.08 \times 0.09}$$

= 1.38 Ns/m^2

Specific gravity (sp. gr.) = 0.9

$$Kinematic\ viscosity = \frac{Dynamic\ viscosity}{density\ of\ fluid}$$

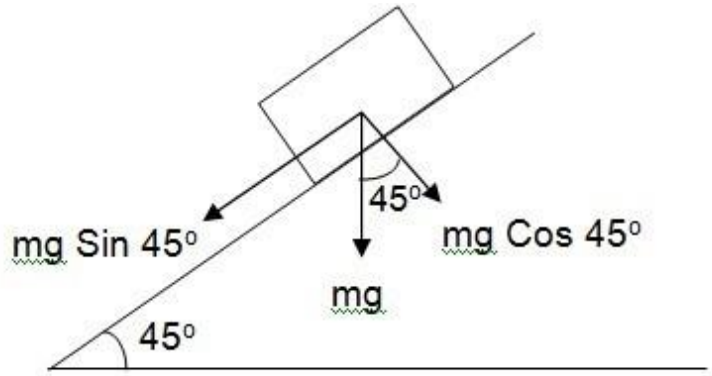
$$= \frac{dynamic\ viscosity}{sp.gr \times density\ of\ H_2O\ at\ 4^{\circ}C}$$

$$= \frac{1.38}{0.9 \times 1000}$$

= 1.53×10^{-3} m^2/s

Q. 5 **A plate has an area of 1m^2, it slides down on inclined plane having angle of inclination 45° to the horizontal with a velocity of 0.5 m/s. The thickness of the oil filled between the plate and the plane is 1 mm. Find the viscosity of fluid if the weight of plate is 70.72 N.**

Solution: A = 1 m^2, v = 0.5 m/s, y = 1 mm = 10^{-3} m, F = 70.72 N



$$\frac{F}{A} = \mu \frac{du}{dy}$$

$$\frac{mgSin\,45^o}{A} = \mu \frac{du}{dy}$$

$$\frac{70.72\,Sin45°}{1} = \frac{\mu * 0.5}{10^{-3}}$$

$$\mu = 0.1 \text{ Ns/m}^2 = 1 \text{ poise}$$

Q. 6	**The distance between two parallel plates is 10 mm, the space in between is filled with oil of viscosity 0.831 Ns/m^2. A flat thin plate of dimensions 1.5 m * 0.8 m moves through the oil, calculate the force required to drag the thin plate at the velocity of 1.2 m/s when:** a) Plate is moving at the centre b) The thin plate is at the distance of 3 mm from one of the plane surfaces. **Solution:** v = 1.2 m/s, A = 1.5 X 0.8 m^2, μ = 0.831 Ns/m^2 a) $F_{total} = F_1 + F_2$ $$F_T = A.\mu.\frac{dv}{dy} + A.\mu.\frac{dv}{dy}$$ $$F_T = 2A.\mu.\frac{dv}{dy} = \frac{2X1.5X0.8X0.831X1.2}{5X10^{-3}}$$ = 478.656 N b) $F_{total} = F_1 + F_2$

$$F_T = A.\mu.\frac{dv}{dy1} + A.\mu.\frac{dv}{dy2}$$

$$F_T = 1.5X0.8X0.831X\frac{1.2}{3X10^{-3}} + 1.5X0.8X0.831X\frac{1.2}{7X10^{-3}}$$

= 398.88 + 170.95

= 569.83 N

Numerical Exercise

1. A liquid has a specific gravity of 1.9 and kinematic viscosity of 1 stoke. What is its dynamic viscosity? **[Ans: 0.19 Ns/m^2]**
2. The space between two parallel plates 5 mm apart is filled with crude oil. A force of 1 N is required to drag the upper plate at a constant velocity of 0.8 m/s. The lower plate is stationary. The area of the upper plate is 0.09 m^2. Determine: (i) The dynamic viscosity, and (ii) the kinematic viscosity of the oil in stokes if the specific gravity of oil is 0.9. **[Ans: 0.07 Ns/m^2]**
3. A plate has an area of 1 m^2. It slides down an inclined plane, having angle of inclination 45° to the horizontal, with a velocity of 0.5 m/s. The thickness of the oil film between the plane and the plate is 1 mm. Find the viscosity of the fluid if the weight of the plate is 71.72 N. **[Ans: 0.000077 Ns/m^2]**
4. A flat plate weighing 1 N has a surface area of 0.1 m^2. It slides down an inclined plane at 30° to the horizontal, at a constant speed of 3 m/s. If the inclined plane is lubricated with an oil of viscosity 0.1 N.s/m^2, find the thickness of the oil film. **[Ans: 0.101 m]**
5. A plate moves at 2 m/s with shearing force of 3.5 N/m^2. The distance between the moving plate and fixed plate is 1 mm. Determine the viscosity of the fluid between the plates. **[Ans: 0.060 Ns/m^2]**
6. Two rectangular flat plates of dimensions 850mm X 600 mm are placed such that the distance between the parallel plates is 20 mm. The space in-between is filled with oil of specific gravity 0.92. The lower plate is fixed. The upper plate moves at 4.5 m/s and requires a force of 201 N to maintain this speed. Determine the dynamic and kinematic viscosity of oil. **[Ans: 0.002 Ns/m^2, 2.17*10^{-6} m^2/s]**
7. Determine the dynamic viscosity of a liquid having specific gravity of 1.85 and kinematic viscosity of 8 stokes. **[Ans: 1.752 Ns/m^2]**

8. A flat thin plate is dragged at a constant velocity of 1.2 m/s applying a force of 20 N. The lower plate is stationary and the area of upper plate is 0.1 m^2. Distance between the plates is 10 mm. If the specific gravity of oil filled between the plates is 0.92, calculate the dynamic and kinematic viscosity of oil. **[Ans: 1.666 Ns/m^2, 1.811 * 10^{-3} m^2/s]**

9. Two large parallel flat plates are positioned 40 mm apart. A 2.5 mm thick plate of 0.5 m^2 area is being towed in lubricating oil filled between the plates with a constant force of 200 N. Calculate the towing speed of plate when it remains equidistant from the two parallel plates. The viscosity of the oil is 0.946 N.s/m^2. **[Ans: 3.9 m/s]**

10. The distance between two parallel planes is 10 mm. The space in between is filled with oil of viscosity 0.831 N.s/m^2. A flat thin plate of dimensions 1.5 m * 0.8 m moves through the oil. Calculate the force required to drag the thin plate at a velocity of 1.2 m/s when:

    a. the plate is moving at the center. **[Ans: 478.65 N]**

    b. The thin plate is at a distance of 3 mm from one of the plane surfaces. **[Ans: 569.82 N]**

11. A steel shaft of 60 mm diameter slides smoothly through a pipe of 65 mm internal diameter and length 120 mm. A vertical force of 250 N is required to pull the shaft back to the original position at the same constant speed. Determine the viscosity of oil filled between the shaft and pipe. Take velocity as 4.5 m/s. **[Ans: 5.67 Ns/m^2]**

12. A cubical block weighing 20 kg and having a 20 cm edge is allowed to slide down an inclined plane making an angle of 20° with the horizontal on which there is a thin film of oil having viscosity 0.22 N.s/m^2. What velocity will be attained by the block if the film thickness is estimated to be 0.025 mm. **[Ans: 0.19 m/s]**

13. A flat plate weighing 350 N is sliding on a smooth inclined plate at 25° to the horizontal. The flat plate is moving with a velocity of 1.35 m/s over the inclined surface. The inclined plate is lubricated such that the thickness of lubricant applied between the plane and plate is 2 mm. Determine the dynamic and kinematic viscosity of fluid if the area of the moving plate is 0.8 m^2. The density of the lubricant is 1785 kg/m^3. **[Ans: 0.221 Ns/m^2, 1.23*10^{-4} m^2/s]**

14. In a stream of glycerin in motion, at a certain point the velocity gradient is 0.25 s^{-1}. The mass density of fluid is 1289 kg/m^3 and kinematic viscosity is 5.2X10^{-3} m^2/s. Calculate the shear stress at the point. **[Ans: 1.6757 N/m^2]**

Lesson 2
STATIC PRESSURE OF LIQUIDS, HYDRAULIC PRESSURE, ABSOLUTE AND GAUGE PRESSURE

2.1 Introduction

Fluid has property that it exerts force on all the sides, top and bottom. Pressure exerted by fluid is given as force per unit area which is as follows:

$$\text{Pressure} = \frac{\text{Force Applied}}{\text{Area Exposed}}$$

The SI unit of pressure is newton per square metre (N/m^2). This is also known as pascal (Pa). The values of standard atmospheric pressure are as follows:

- 760 mm of mercury column
- 10.3 m of water column
- 101.3 kN/m^2
- 101.3 kPa
- 1 bar

2.2 Hydrostatic Law

Pressure in a continuously distributed uniform static fluid varies only with vertical distance and is independent of the shape of the container. The pressure is the same at all points on a given horizontal plane in the fluid. The pressure increases with depth in the fluid.

2.2.1 Pressure head of liquid

Consider a fluid of density ρ, having fluid element of area ΔA at a depth distance ‘h’ from the top surface of liquid (Fig. 2.1).

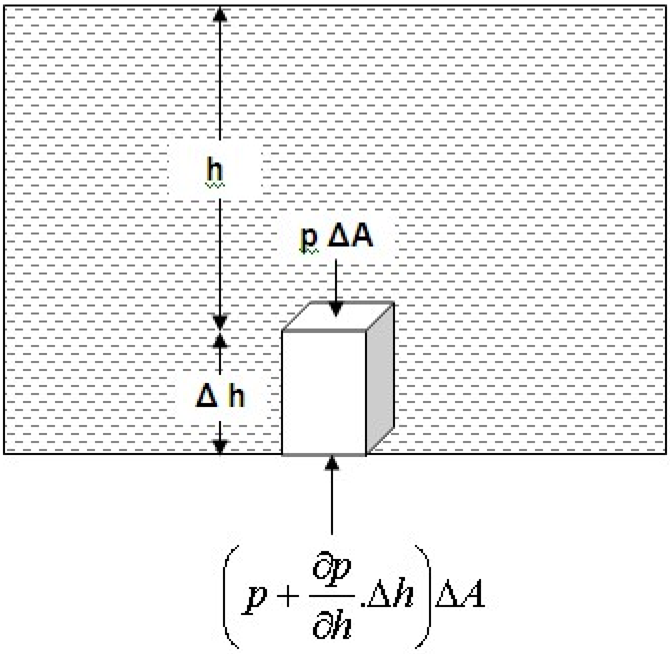


Fig. 2.1 Forces acting on fluid element of height Δh

Height of fluid element = Δh

Pressure at top of fluid element = p

Force on the top of fluid element = p ΔA

Weight of fluid element = ΔA. Δh. ρ g

Upward force acting at bottom of the fluid element = $\left(p+\frac{\partial p}{\partial h}.\Delta h\right)\Delta A$

Under equilibrium conditions (downward force = upward force),

$$\mathbf{p.\Delta A + \Delta A.\Delta h.\rho g} = \left(p+\frac{\partial p}{\partial h}.\Delta h\right)\Delta A$$

$$\mathbf{\Delta h.\rho g} = \frac{\partial p}{\partial h}.\Delta h$$

$$\frac{\partial p}{\partial h} = \rho g$$

$$\frac{\partial p}{\partial h} = \varpi$$

Where, $\rho g = \varpi$ (Weight Density or Specific Weight)

Pressure variation in any static fluid is described by the above basic pressure height relationship. This equation describes hydrostatic law and indicates that rate of increase of pressure in a vertical downward direction is equal to the weight density (also known as specific weight) of fluid at that point.

Integrating,

$$\int \frac{\partial p}{\partial h} = \rho g$$
$$P = \rho g h$$

Here h is known as the pressure head and its unit is in metres (m).

2.3 Liquid Paradox

The pressure in a liquid is not a function of shape or size of the container. Pressure is only the function of density and height or depth 'h' inside the liquid at which pressure has to be calculated.

Click for Animation (Fig. 2.2 Pressure for vessel of various shapes)

Q. 1: Calculate the pressure at 5 m, 10 m and 15 m in a tank filled with water.

Solution: P_1 at 5 m = $\rho g h$ = 1000 × 9.8 ×5 = 49 000 Pa

P_2 at 10 m = $\rho g h$ = 1000 × 9.8 ×10 = 98 000 Pa

P_3 at 15 m = $\rho g h$ = 1000 × 9.8 ×15 = 1 47 000 Pa

2.4 Gauge Pressure and Absolute Pressure

Gauge Pressure

It is convenient to measure pressure in terms of taking atmospheric pressure as reference datum. Pressure measured above atmospheric pressure is known as gauge pressure. The atmospheric pressure on the scale is marked as zero.

Absolute Pressure

Since, atmospheric pressure changes with atmospheric condition, a perfect vacuum is taken as an absolute standard of pressure. Pressure measured above perfect vacuum are called absolute pressure. The figure 2.3 explains the concept of gauge and absolute pressure.

Absolute pressure = atmospheric pressure + gauge pressure

$$P_{abs} = P_{atm} + P_{gauge}$$

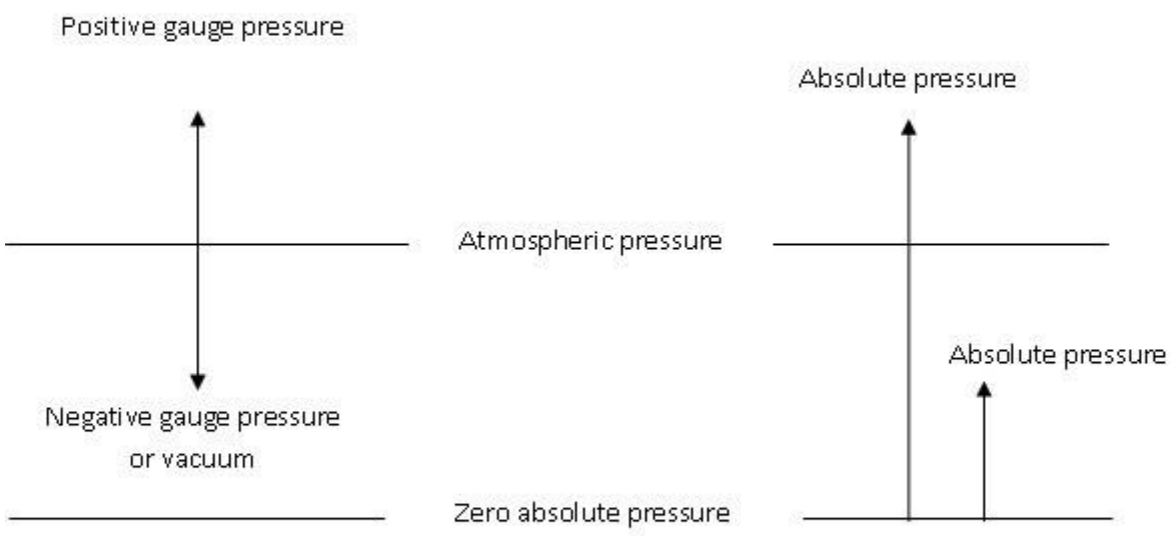


Fig. 2.3 Concept of gauge and absolute pressure

2.5 Pascal's Law

It states that the intensity of pressure at any point in a liquid at rest is same in all direction. We consider a fluid element of dimensions as shown in the figure 2.4. P_s is the pressure exerted on the inclined surface, δ is the linear dimension of the surfaces x, y and z.

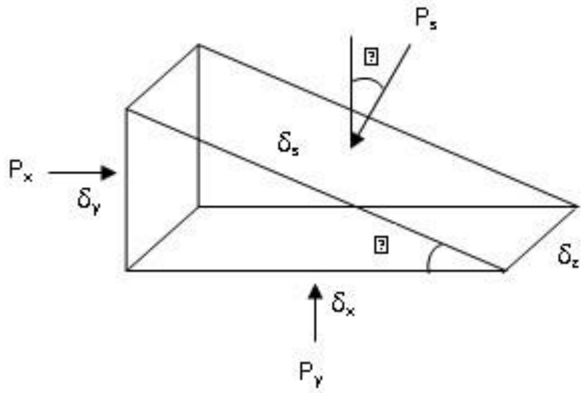


Fig. 2.4 Fluid element

Click for Animation (Fig. 2.5 Pressure acting on fluid element)

Please see Figure 2.5

Equation for horizontal forces:

$$P_x.\delta_y.\delta_z = P_s Sin\Theta.\delta_z.\delta_s$$

$$P_x.\delta_y.\delta_z = \delta_z.\delta_s.P_s.\frac{\delta_y}{\delta s}$$

$$P_x = P_s \text{ -----------------(i)}$$

***Equation for vertical forces*:**

$$P_y.\delta_x.\delta_z = P_s.\text{Cos}\theta.\delta_s.\delta_z + \frac{1}{2}.\delta_y.\delta_x.\delta_z.\rho g$$

$$P_y.\delta_x.\delta_z = P_s.\frac{\delta_x}{\delta s}\,\delta_s.\delta_z + \underbrace{\frac{1}{2}.\delta_y.\delta_x}_{\text{Can be neglected as } \delta \text{ is very small}}.\delta_z.0\rho g$$

$$P_y = P_s \qquad \text{(ii)}$$

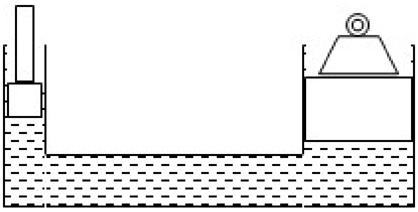
From equation (i) and (ii) we have,

$$P_x = P_y = P_s$$

2.6 Numerical

Q1. A hydraulic press has a diameter ratio between the two piston of 8:1. The diameter of the larger piston is 600 mm and it is required to support a mass of 3500 kg. The press filled with hydraulic fluid of sp.gravity 0.8. Calculate the force required on the smaller piston to provide the required force: a) When the two pistons are at the same level. B) When the smaller piston is 2.6 m below the larger piston.

Solution.



a) $P_1 = \dfrac{F}{A} = \dfrac{3500X9.81}{\dfrac{\pi(600)^2}{4(1000)^2} \rightarrow A_1}$

$P_2 = \dfrac{F_2}{\dfrac{\pi(75)^2}{4(1000)^2} \rightarrow A_2}$

$P_1 = P_2$

$$\frac{3500X9.81}{\frac{\pi}{4}X\left(\frac{600}{1000}\right)^2} = \frac{F_2}{\frac{\pi}{4}X\left(\frac{75}{1000}\right)^2}$$

$F_2 = \dfrac{3500X9.81X75X75}{600X600}$

$= 536.48$ N

b) $P_A = P_1 + \rho gh$

$$= \frac{3500X9.8}{\frac{\pi}{4}X\left(\frac{600}{1000}\right)^2} + 800X9.8X2.6$$

$P_2 = \dfrac{F_2}{A_2} = \dfrac{F_2}{\frac{\pi}{4}X\left(\frac{75}{1000}\right)^2}$

$P_A = P_2$

$$\frac{3500X9.8}{\frac{\pi}{4}X\left(\frac{600}{1000}\right)^2} + 800X9.8X2.6 = \frac{F_2}{\frac{\pi}{4}X\left(\frac{75}{1000}\right)^2}$$

$$\frac{34300}{0.2826} + 20384 = \frac{F_2}{0.0044156}$$

$$\frac{400060.518}{0.2826} = \frac{F_2}{0.0044156}$$

$F_2 = 625.94$N

Q2. The diameter of the piston of a hydraulic jack is 6 times greater than the diameter of the plunger which is 200 mm. They are both at same height and made of same material. The piston weighs 100 N and supports a mass of 50 kg. The jack is filled with oil of density 900 kg/m^3. Calculate the force required at the plunger to support the piston and mass if carries at a level 10 cm above that of plunger.

Solution:

$$A_1 = \frac{\pi}{4} X \frac{(1200)^2}{(1000)^2}; \quad A_2 = \frac{\pi}{4} X \frac{(200)^2}{(1000)^2};$$

$$P_1 = \frac{50 X 9.81}{\frac{\pi}{4} X \left(\frac{1200}{1000}\right)^2} + \frac{100}{\frac{\pi}{4} X \left(\frac{1200}{1000}\right)^2} + \rho g X \frac{10}{100}$$

$$P_2 = \frac{F_2}{\frac{\pi}{4} X \left(\frac{200}{1000}\right)^2}$$

Therefore, $P_1 = P_2$ (According to Pascal's Law)

$$\frac{50 X 9.8}{\frac{\pi}{4} X \left(\frac{1200}{1000}\right)^2} + \frac{100}{\frac{\pi}{4} X \left(\frac{1200}{1000}\right)^2} + \frac{900 X 9.8 X 10}{100} = \frac{F_2}{\frac{\pi}{4} X \left(\frac{200}{1000}\right)^2}$$

$$\frac{490}{1.1304} + \frac{100}{1.1304} + 882 = \frac{F_2}{0.0314}$$

$$\frac{490 + 100 + 997.0128}{1.1304} = \frac{F_2}{0.0314}$$

$$\frac{1587.0128}{1.1304} = \frac{F_2}{0.0314}$$

$$F_2 = \frac{0.0314 X 1587.0128}{1.1304} = 44.08\ N$$

Q3. The ratio of the diameter of plunger and ram is 1:5, the diameter of plunger is 40 mm; find the weight lifted by the hydraulic press when the force is applied at plunger is 500 N.

Solution. $P_1 = P_2$

$$A_1 = \frac{\pi}{4} X \frac{(200)^2}{(1000)^2}$$

$$A_2 = \frac{\pi}{4} X \frac{(40)^2}{(1000)^2}$$

$$P_1 = \frac{F_1}{\frac{\pi}{4} X \left(\frac{200}{1000}\right)^2} \; ; \; P_2 = \frac{500}{\frac{\pi}{4} X \left(\frac{40}{1000}\right)^2}$$

$$\frac{F_1}{\frac{\pi}{4} X \left(\frac{200}{1000}\right)^2} = \frac{500}{\frac{\pi}{4} X \left(\frac{40}{1000}\right)^2}$$

$$F_1 = \frac{200 X 200 X 500}{40 X 40} = 62500 N$$

Lesson 3
PRESSURE HEAD OF A LIQUID, PRESSURE ON VERTICAL RECTANGULAR SURFACE

3.1 Pressure Force Acting on a Vertical Rectangular Surface

When the fluid is in static condition, there will not be any relative motion between adjacent fluid layers. The velocity gradient as well as shear stress will be zero. The forces acting on fluid particles will be due to pressure of fluid normal to the surface and due to gravity.

Consider a plane surface of arbitrary shape immersed vertically in a static mass of fluid as shown in Fig. 3.1.

Let,

C = Centre of gravity

P= Centre of pressure

$\bar{x}$ = depth of centre of gravity from free liquid surface 1-1'

$\bar{h}$ = depth of centre of pressure from free surface of liquid 1-1'

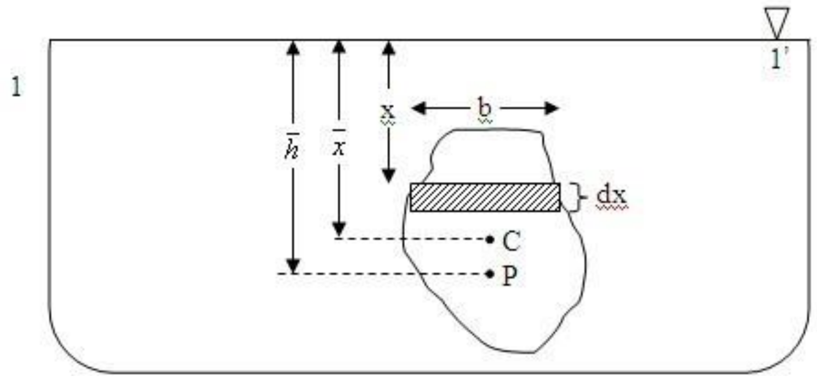


Fig. 3.1 A plane surface of arbitrary shape immersed vertically in a static fluid

1'

The distance of centre of gravity from free surface is $\bar{x}$. Let P be the centre of pressure at which the resultant pressure on the rectangular plate acts.

Consider an elementary strip of 'dx' thickness and width b at the distance x from the free surface of liquid. Let pressure acting on the strip is p. If density of the liquid is ρ, then

Then total pressure force (F) acting on the elementary strip = $p\, b\, dx$

$$= \rho g x\ b\, dx$$

Total pressure force acting on whole area = $\rho g \int x\ b\, dx$

Since $\int x \, b \, dx$ = moment of surface area about the free liquid surface, we can take its value surface as $\bar{x}.A$

Then total pressure force, F = $\rho g \bar{x}.A$.
(where 'A' is area of plate)
At point C pressure = $\rho g \bar{x}$

Total pressure force is equal to total area multiplied by the pressure at the centre of gravity of the plate surface immersed in the liquid.

3.2 Location of Centre of Pressure

The pressure on the immersed surface increases with depth. As shown in the following figure the pressure will be minimum at the top and maximum at the bottom of immersed plane.

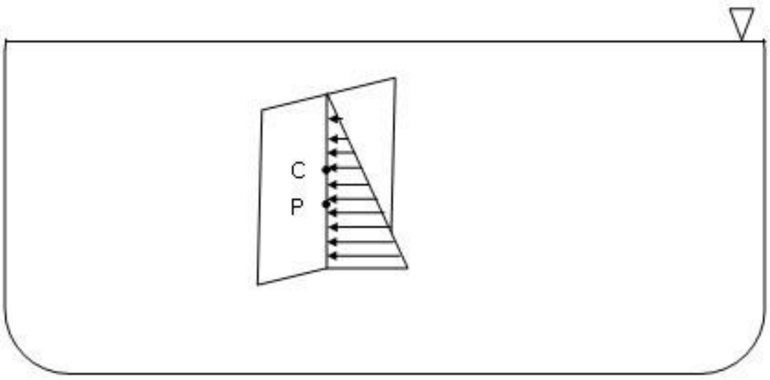


Fig. 3.2 Location of centre of pressure

Suppose centre of pressure (P) is $\bar{h}$ from the free surface of the liquid;

Thus the resultant pressure will act at a point P much below the centre of gravity. Point P is known as the "centre of pressure" at which the resultant pressure acts on the immersed surface.

Total pressure force acting on the elementary strip = p b dx
$= \rho g x \, b \, dx$

Moment of pressure force about free liquid surface = $\rho g x^2 \, b \, dx$

Total moment of pressure force for entire area = $\rho g \int x^2 \, b \, dx$

Since $\int x^2 \, b \, dx$ = moment of inertia of entire surface about free surface 1-1' = I_o

Total moment of pressure force for entire area = $\rho g I_o$----------------- (i)

The sum of moment of pressure force = F$\bar{h}$ --------------- (ii)

Equating equation (i) & (ii):

$F\bar{h} = \rho g I_o$

$\rho g A\bar{x}\ \bar{h} = \rho g I_o$

$$\bar{h} = \frac{I_o}{A\bar{x}} \qquad \text{.........................(iii)}$$

From theorem of parallel axis for moment of inertia we have,

$$I_o = I_c + A\bar{x}^2 \qquad \text{.........................(iv)}$$

Here,

I_c = moment of inertia of area about an axis passing through centre of gravity.

$\bar{x}$ = distance of centre of gravity from free liquid surface

Placing value of I_o into (iii),

$$\bar{h} = \frac{I_c}{A\bar{x}} + \bar{x}$$

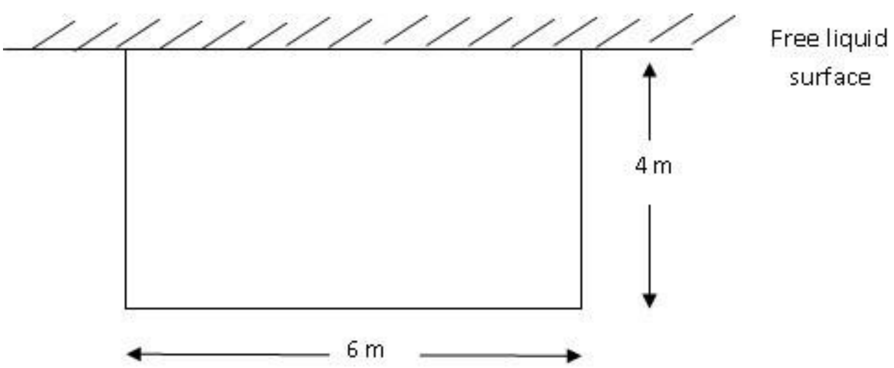
Where, I_c is moment of inertia of the immersed figure;

For rectangular surface $I_c = \frac{bd^3}{12}$ (b= base of rectangle, d= height or depth)

3.3 Numerical

Q1. A rectangular plate 4 m * 6 m is vertically immersed in water such that 6 m side is parallel to free surface. Calculate, the total pressure force and centre of pressure if the top of rectangular plate is: a) touching the free liquid surface, b) 3 m below the surface of water.

Solution.



a) $\bar{x}$ = 2 m; Area = 24 m^2; ρ = 1000 kg/m^3; g = 9.81 m/s

Total pressure force = $\rho g \bar{x}.A$

$= 1000 \times 9.81 \times 2 \times 24$

$= 470880$ N

$= 470.88$ kN

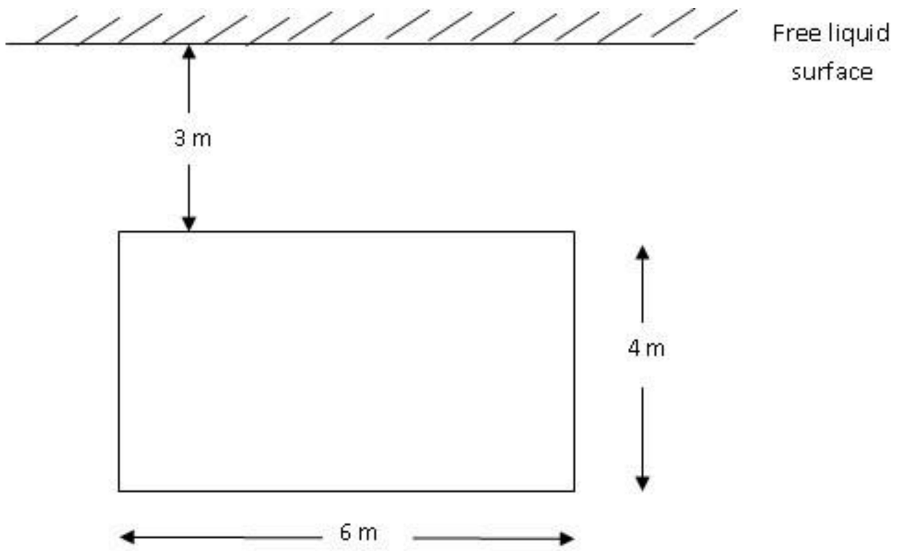
Centre of pressure $(\bar{h}) = \frac{I_c}{A\bar{x}} + \bar{x}$

$$I_c = \frac{bd^3}{12} = \frac{6 X 64}{12} = 32$$

$$\bar{h} = \frac{32}{24 X 2} + 2 = \frac{8}{3}$$

Therefore, $\bar{h} = 2.67$ m

b) $\bar{x} = 5$m; Area = 24 m^2; $\rho = 1000$ kg/m^3; g = 9.81 m/s^2



Total pressure force = $\rho g \bar{x}.A$

$= 1000 \times 9.81 \times 5 \times 24$

$= 1177200$N

$= 1.1772$MN

Centre of pressure $(\bar{h}) = \frac{I_c}{A\bar{x}} + \bar{x}$

$$= \frac{32}{24 X 5} + 5 = \frac{79}{15}$$

$= 5.26$ m

Q2. A rectangular plate 2 m * 3 m is placed vertically in oil bath such that 3 m side is parallel to the free surface of liquid. Calculate the hydrostatic force and centre of pressure if the top of rectangular plate is 2.5 m below the oil surface. Specific gravity of oil = 0.1

Solution.

$\bar{x}$ = 3.5 m; A = 6m^2; g = 9.8 m/s^2

ρ = 0.1 * 1000 = 100 kg/m^3

Total hydrostatic force,

F = ρgA$\bar{x}$

= 100 * 9.8 * 6 * 3.5

= 20580 N = 20.5 kN

Centre of pressure ($\bar{h}$) = $\frac{I_C}{A\bar{x}} + \bar{x}$

$$I_c = \frac{bd^3}{12} = \frac{3X8}{12} = 2\,m$$

Put the value of I_c, A and $\bar{x}$ in relation of $\bar{h}$

$$= \frac{1 + 36.75}{10.5} = \frac{37.75}{10.5}$$

= 3.59 m

Lesson 4

COMPRESSIBLE AND NON COMPRESSIBLE FLUIDS, SURFACE TENSION AND CAPILLARITY

4.1 Compressible and Non Compressible Fluids

1. Compressible fluids: The fluids which undergoes a change in volume or density when subjected to external forces.
2. Incompressible fluids: The fluid which does not show a change in volume or density when subjected to external forces.

Compressibility

It is the property of the liquid or fluid due to which there will be a change in volume/density when the fluid is subjected to an external force and is represented by Bulk Modulus of Elasticity (k). Compressibility is the reciprocal of Bulk Modulus of Elasticity.

$$Compressibility = \frac{1}{Bulk\ Moldulus\ of\ Elasticity(k)}$$

$$Bulk\ Modulus\ of\ Elasticity(k) = \frac{dP}{\Delta v/V} = \frac{Change\ in\ pressure}{Volumetric\ Strain}$$

Ideal Fluids are incompressible.
Real Fluids are compressible.

4.1.1 Numericals

Q.1. Change in pressure for a liquid is 2 MN/m^2. The volumetric strain induced in the liquid was 0.05. Determine the bulk modulus of elasticity for the liquid.

$$\Delta p = 2MN/m^2$$

$$Volumetric\ strain\ = \frac{\Delta v}{v} = 0.05$$

$$Bulk\ modules = \frac{\Delta p}{\Delta v/v} = \frac{2 \times 10^6}{5} \times 100$$

$$= 4 \times 10^7 N/m^2$$

Q.2. A cylinder having 10 litre volume is completely filled with oil. When a pressure of 9.2 MN/m^2 is applied the volume decreases by 500 ml. Find the bulk modulus of oil.

Pressure (P) = 9.2 MN/m^2 = 9.2 × 10^6 N/m^2

Change in volume = $\Delta_{\text{v = 500 ml}} = 0.5\ L$

Original volume v = 10 L

$$\text{Bulk modules} = \frac{\text{p}}{\Delta\text{v/v}} = \frac{9.2 \times 10^6}{5} \times 10 \times 10$$

$$= 1.84 \times 10^8\ N/m^2$$

Q.3. Bulk modulus of water is k = 2.07 × 10^6 kN/m^2. Determine the pressure required to compress 10 litre of water by 0.1 litre.

Bulk modules =

$$k = 2.07 \times 10^6 kN/m^2$$

Change in volume v= 0.1 L

$$\Delta v/v = \frac{0.1}{10}$$

$$k = \frac{P}{\Delta v/v}$$

$$P = 2.07 \times 10^6 \times 10^3 \times \frac{1}{10 \times 10}$$

$$= 2.07 \times 10^7 \ N/m^2$$

4.2 Surface Tension

The cohesive force of attraction between molecules is responsible for the phenomenon of surface tension. The molecules beneath the free liquid surface are surrounded by other molecules creating an equilibrium. The force of attraction (cohesion) acting on the molecules will be balanced but at the free liquid surface there are no molecules above the liquid surface, therefore a net force pulls the molecules in inward direction. This force causes the molecules at the surface to cohere (stick) more strongly with the molecules beneath them. Thus a surface film is formed which acts as an elastic or stretched membrane.

Fig. 4.1 Forces of attraction on a liquid molecule (Click for Animation)

Surface tension depends on the following factors:

i) Nature of the liquid.
ii) Nature of Surroundings (gas, liquid or solid)
iii) Kinetic Energy of molecules
iv) Temperature.

Unit of Surface Tension: N/m

Surface tension is denoted by σ = Force per unit length having units N/m

With increase in temperature, the surface tension decreases.

Phenomena and examples of surface tension in daily life:

1. Walking of insect on the surface of water
2. Fully filled glass of water does not over flow immediately
3. Rise of liquid in capillary
4. Detergents are also called surfactant. It reduces the surface tension of water and water enters into cloth layer which enables proper cleaning
5. Emulsion: It is a liquid-liquid dispersed phase. Oil separated from water due to surface tension

   Collection of dust on the water surface
6. Liquid drops take a spherical shape

4.2.1 Different cases for surface tension

Fig. 4.2 Water droplet and its sectional view (Click for Animation)

Case 1: Water droplet

Pressure force $f = \mathrm{p}.\frac{\pi}{4}\mathrm{d}^2$

Surface tension force acting around the circumference = $f' = \sigma \times \pi d$

Where, $\sigma =$ Surface tension

Under equilibrium

$$\mathrm{p}.\frac{\pi}{4}\mathrm{d}^2 = \sigma \times \pi d$$

$$\mathrm{p} = \frac{4\sigma}{\mathrm{d}}$$

Case 2: Soap bubble

Pressure force $f = \mathrm{p}.\frac{\pi}{4}\mathrm{d}^2$

Surface tension force acting around the circumference = $f' = 2\,\sigma \times \pi d$

Where, $\sigma =$ Surface tension

Under equilibrium

$$\mathrm{p}.\frac{\pi}{4}\mathrm{d}^2 = 2\,\sigma \times \pi d$$

$$\mathrm{p} = \frac{8\sigma}{\mathrm{d}}$$

Case 3: Liquid Jet

Fig. 4.3 Sectional view of water jet (Click for Animation)

Pressure force $f = \mathrm{p.L.d}$

Surface tension force acting around the circumference = $f' = \sigma \times 2L$

Under equilibrium

$$\mathrm{p.L.d} = \sigma \times 2\,L$$

$$\mathrm{p} = \frac{2\sigma}{\mathrm{d}}$$

4.2.2 Capillarity

It is a phenomenon in which liquid in a glass tube capillary rises or falls from its original level due to the combined effect of cohesion and adhesion.

Fig. 4.4 Capillary Rise (Click for Animation)

Let glass tube of diameter = d

Density of liquid = ρ

Rise in liquid level = h

Angle of contact between water surface and glass = θ

Force in upward direction = $\sigma.\pi.d.\text{Cos}\theta$ ------------(i)

$$Force\ of\ gravity\ downward = \frac{\pi}{4}d^2 \times h \times \rho g \quad \ldots\ldots\ldots\ldots\ldots..(ii)$$

Equation (i) = (ii)

$$\frac{\pi}{4}d^2.h.\rho g = \sigma.\pi.d.cos\theta$$

$$H = \frac{4\sigma Cos\theta}{\rho g}$$

Note

a) For laboratory purpose the diameter of the glass tube should be greater than 8 mm.

b) For glass tube with length more than 12 cm; the effect of capillarity becomes negligible.

c) Capillary rise occurs in the case of water. But in other case like in mercury there is capillary depression.

4.2.3 Non-wetting and wetting liquid

According to the surface wetting ability liquids can be classified as

Non-wetting liquid: Angle of contact $\theta > \pi/2$

e.g. Mercury with density 13600 kg/m^3

Wetting liquid: Angle of contact $\theta < \pi/2$

e.g. Water with density 1000 kg/m^3

[Fig. 4.5 Wetting and non-wetting liquid (Click for Animation)]

Table 4.1 Condition for capillary rise and depression

Sl. No.	Condition	Capillary action	Type of meniscus formed	Angle of contact between liquid surface and glass
1	When adhesion is greater than cohesion	Rise in capillarity	Concave	$\theta < \pi/2$
2	When cohesion is greater than adhesion	Fall in capillarity	Convex	$\theta > \pi/2$ (for mercury 130° - 150°)

Lesson 5
SIMPLE MANOMETER, PIEZOMETER, U-TUBE MANOMETER

5.1 Introduction

There are many techniques for the measurement of pressure and vacuum. Instruments used to measure pressure are called pressure gauges. Manometers are used for the measurement of very low pressures as well as vacuum especially in hydraulic laboratories.

5.2 Manometers

Manometers are used for measuring pressures by balancing the fluid column of fluid against another column of fluid of known specific gravity. Manometers can be classified as:

1. Simple manometers
   a. Piezometer
   b. U-tube manometer
2. Micro manometers (or single column manometers)
   a. Vertical column micro manometer
   b. Inclined column micro manometer
3. Differential manometers
   a. Upright U-tube differential manometer
   b. Inverted U-tube differential manometer

1. Simple manometers

It consists of a glass tube with one end open to the atmosphere and other end connected to a point at which pressure is to be measured.

a. Piezometer

It consists of glass tube connected to a vessel or pipe at which static pressure is to be measured. It is the simplest of all the manometers (Fig. 5.1). It is used to measure very low pressures.

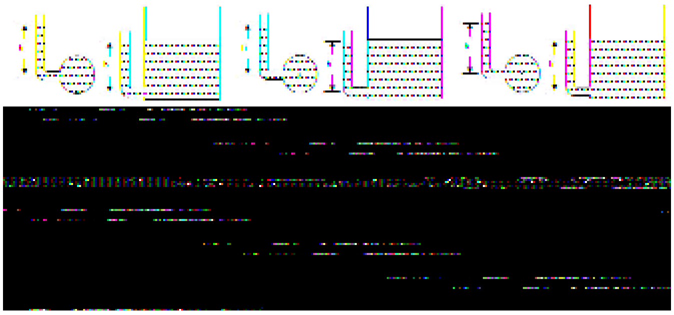


Fig. 5.1 Piezometer

The pressure in piezometer is given by the following equation.

$$p = \rho g h$$

Where,

ρ = density of liquid

h = height of liquid in the piezometer from the centre of the pipe.

g = acceleration due to gravity.

b. U-tube manometer

The manometer is named so because it consists of a glass tube having the shape of alphabet 'U'. One end is open to the atmosphere and other end connected to a point at which pressure is to be measured.

Let ρ_1 = density of liquid for which pressure has to be determined

ρ_2 = density of manometer liquid (assume mercury)

ω_1 = weight density of liquid for which pressure has to be determined

ω_2 = weight density of manometer liquid

S_1 = Specific gravity of liquid for which pressure has to be determined

S_2 = Specific gravity of manometer liquid

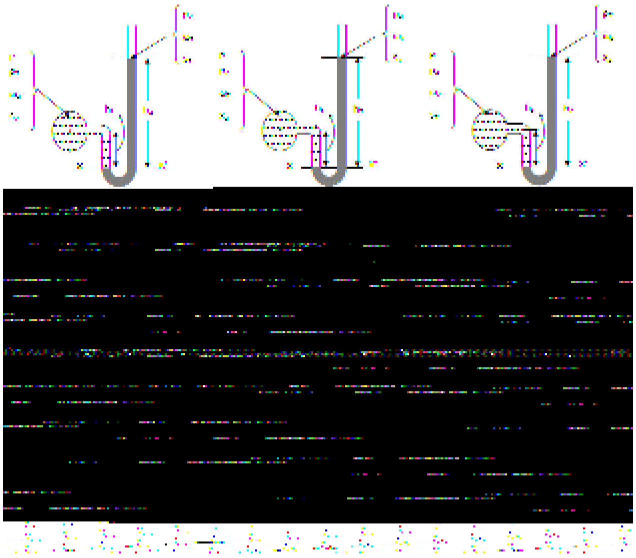


Fig. 5.2 U-tube manometer

Let 'h' be the pressure in terms of height of fluid in the pipe.

'h_1' is the distance from the datum line XX' to the centre of pipe

'h_2' is the height of manometer liquid from the datum line XX' in the right limb

Pressure in the left limb at XX' $= P + \rho_1 g h_1 = P + \omega_1 h_1$

Pressure in the right limb at XX' $= \rho_2 g h_2 = \omega_2 h_2$

According to Pascal's law, at datum line pressure will be equal

$P + \omega_1 h_1 = \omega_2 h_2$

$P = \omega_2 h_2 - \omega_1 h_1$ -------- (i)

$P = \rho_2 g h_2 - \rho_1 g h_1$ ---------- (ii)

On dividing equation (ii) by ρg where ρ is the density of the water:

$$\frac{P}{\rho g} = \frac{\rho_2 g h_2}{\rho g} - \frac{\rho_1 g h_1}{\rho g} \quad \text{\_\_\_\_\_\_(iii)}$$

$$\text{Sp. Gravity (s)} = \frac{\text{Density of substance}}{\text{Density of Water}}$$

$$\frac{P}{\rho g} = S_2 h_2 - S_1 h_1 \quad \text{\_\_\_\_\_\_\_\_(iv)}$$

Since $P = \rho g h$ where 'h' is head of water

$h = S_2 h_2 - S_1 h_1$ ----------------(v)

5.3 Rules for Writing Equations for Manometers

Step 1: Draw a neat diagram of a manometer

Step 2: Consider a suitable datum line XX'. It should be in such a manner so that manometer liquid touching the datum line in the two limbs is the same. At figure 5.3, point A and B are on the datum line, liquids is same in both the columns i.e. left and right limbs.

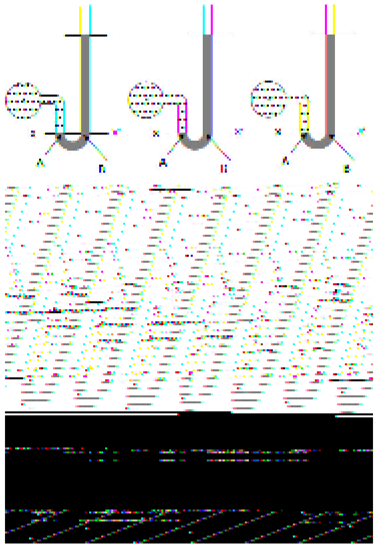


Fig. 5.3 Selection of datum line XX'

Step 3: Mark the distances of centre of pipe and the liquid level in the vertical column from the datum line X-X'

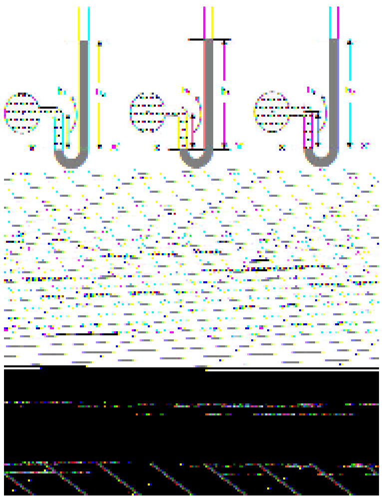


Fig. 5.4 Marking liquid levels

Step 4: Let h in meters is the pressure head in the centre of pipe.

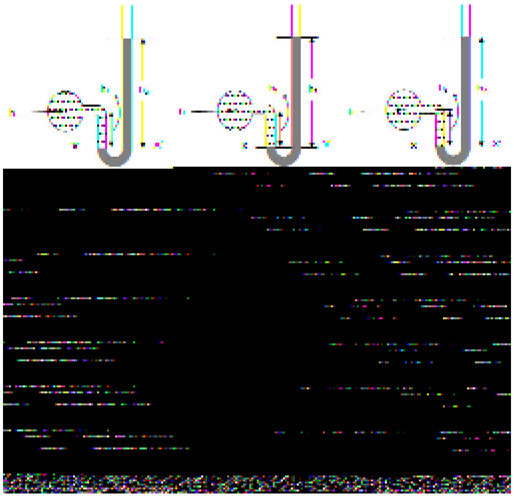


Fig. 5.4 Pressure head h at the centre of pipe

Step 5: Write the equation for pressure head in the left limb starting from center of pipe.

Pressure head in left limb at X-X' = $h + h_1S_1$

Where,

S_1 = Specific gravity of liquid for which pressure has to be determined

S_2 = Specific gravity of manometer liquid (assume mercury)

Let 'h' be the pressure in terms of height of fluid in the pipe.

'h_1' is the distance from the datum line XX' to the centre of pipe

'h_2' is the height of heavy liquid from the datum line XX' in the right limb

Step 6: Write the equation for pressure head in the right limb starting from center of pipe.

Pressure head in right limb at X-X' = h_2S_2

Step 7: Equate the pressure heads in the two limbs (left and right) to get the value of h.

Equating pressure head at X-X' as the pressure at datum line would be equal.

$$h + h_1S_1 = h_2S_2$$

or,

$$h = h_2S_2 - h_1S_1$$

Note

1. Moving downward from a point in a manometer, all the pressure heads will be added.
2. Moving upward from a point in a manometer, all the pressure heads will be subtracted.

Lesson 6
MICRO MANOMETERS AND INCLINED MANOMETER

6.1 Introduction

Micro-manometers and inclined manometers are modified forms of simple U-tube manometer.

6.2 Micro-Manometer

Micro-manometer is also known as single column manometers. The construction of a micro-manometer is as follows:

a. One limb of manometer is a tank of large cross sectional area as compared to the cross sectional area of the other limb.
b. This tank acts as a reservoir to hold the manometer fluid.
c. The cross sectional area of the tank is 100 times greater than that of the other limb.
d. When there is change in the pressure in the pipe, there is negligible change in the level of fluid in tank. This change can be neglected and pressure can be measured as height of liquid in the other column.

It is of two types:

i. Vertical column micro-manometer
ii. Inclined column micro-manometer

6.3 Vertical Column Micro-manometer

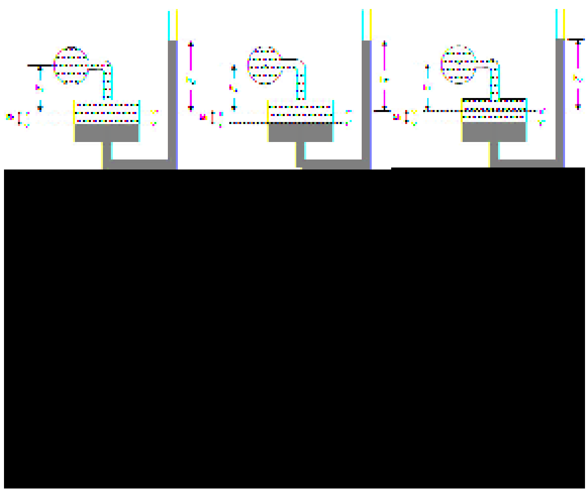


Fig. 6.1 Vertical column micro-manometer

Initially, when there is no fluid flowing in the pipe the level of manometer liquid is at XX'. But due to liquid pressure in the pipe the level of manometer liquid moves down from XX' to YY' in the tank and rises in the right limb to the distance h_2 from XX'.

Let,

ρ_1 = density of liquid for which pressure has to be determined

ρ_2 = density of manometer liquid (assume mercury)

S_1 = Specific gravity of liquid for which pressure has to be determined

S_2 = Specific gravity of manometer liquid

δh = Fall in the level of liquid in the tank

A = Area of cross-section of the tank

a = Area of cross-section of the right limb

h = Pressure head of fluid in the pipe (as head of water)

Let 'h' be the pressure in terms of height of fluid in the pipe.

'h_1' is the distance from the datum line XX' to the centre of pipe

'h_2' is the height of heavy liquid from the datum line XX' in the right limb

The rise in the manometer fluid in the right limb will be equal to the fall of level in the tank.

Therefore:

$$\delta h A = a.h_2$$

$$\delta h = \frac{a.h_2}{A} \quad \text{...} \quad \text{(i)}$$

Pressure in the left limb at YY' = $h + (h_1 + \delta h)S_1$

Pressure in the right limb at YY'= $(h_2 + \delta h)S_2$

According to Pascal's law,

Leftlimb right limb

$$h + (h_1 + \delta h)S_1 = (h_2 + \delta h)S_2$$

$$h + h_1 S_1 + \delta h.S_1 = h_2 S_2 + \delta h.S_2$$

$$h = \delta h(S_2 - S_1) + (h_2 S_2 - h_1 S_1) \text{.......................................(ii)}$$

From equation (i) and (ii)

$$h = \frac{ah_2}{A}(S_2 - S_1) + (h_2 S_2 - h_1 S_1)$$

If the cross sectional area of the tank A is very large compared to cross sectional area of the right limb then i.e. A >> a, then,

Ratio of a/A will be zero and the above equation can be re-written as:

$$h = (h_2 S_2 - h_1 S_1)$$

6.4 Inclined column Micro-manometer

Inclined manometers are more sensitive than vertical column manometers. Initially, when there is no fluid flowing in the pipe the level of manometer liquid is at XX'. But due to liquid pressure in the pipe the level of manometer liquid moves down from XX' to YY' in the tank and rises in the right limb to the distance h_2 from XX'.

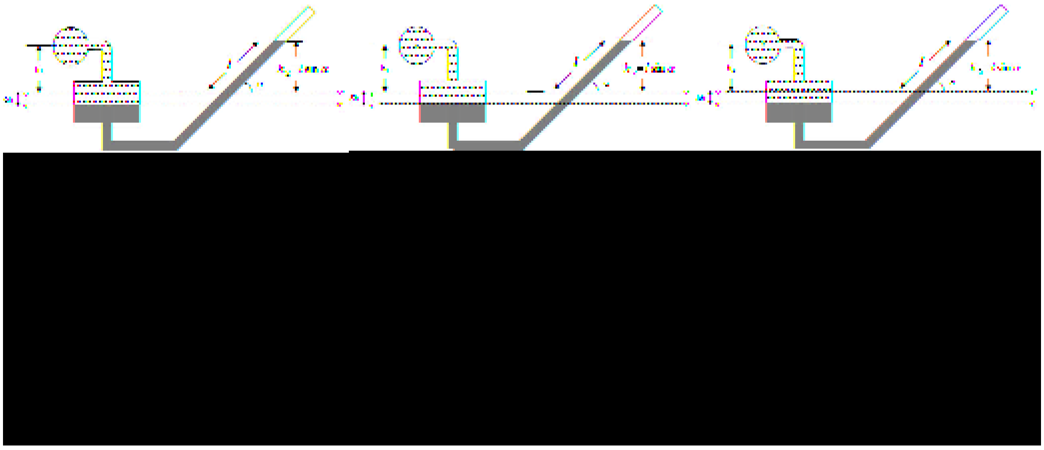


Fig. 6.2 Inclined column micro-manometer

Let,

S_1 = Specific gravity of liquid for which pressure has to be determined

S_2 = Specific gravity of manometer liquid

δh = Fall in the level of liquid in the tank

A = Area of cross-section of the tank

a = Area of cross-section of the right limb

h = Pressure head of fluid in the pipe (as head of water)

'h_1' is the distance from the datum line XX' to the centre of pipe

'h_2' is the height of heavy liquid from the datum line XX' in the right limb

l = length of manometer fluid along the right limb

The rise in the manometer fluid in the right limb will be equal to the fall of level in the tank.

From equation for vertical column micro-manometer we have:

$$h = \frac{ah_2}{A}(S_2 - S_1) + h_2 S_2 - h_1 S_1$$

$$\text{if } \mathrm{A} >> \mathrm{a} \text{ such that } \frac{\mathrm{a}}{\mathrm{A}} \text{ is negligible}$$

$$\therefore h = h_2 S_2 - h_1 S_1$$

And since $h_2 = l \sin \alpha$

$$h = l \sin \alpha S_2 - h_1 S_1$$

Lesson 7

DIFFERENTIAL MANOMETERS

7.1 Introduction

Differential manometers are used to measure the difference of pressures between two points in a pipe or in two different pipes. There are two types of differential manometers.

1. U-tube upright differential manometer
2. U-tube inverted differential manometer

7.2 U-tube Upright Differential Manometer

It is used to measure pressure difference at two points in a pipe or between two pipes at different levels.

Case 1 - U-tube upright differential manometer connected at two points in a pipe at same level

The construction and arrangement of a manometer connected at two different points, A and B, of a pipe is shown in figure 7.1.

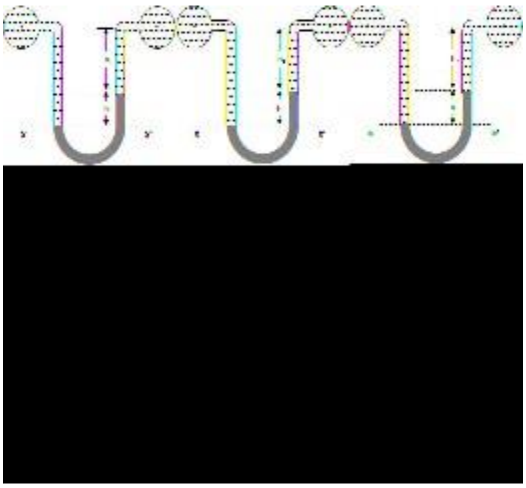


Fig. 7.1 U-tube upright differential manometer

Let,

ρ_1 = density of liquid flowing in the pipeline

ρ_2 = density of manometer liquid (assume mercury)

S = Specific gravity of liquid for which pressure has to be determined

S_1 = Specific gravity of manometer liquid

h_A be the pressure in terms of height of fluid in the pipe at point A

h_B be the pressure in terms of height of fluid in the pipe at point B

h is the distance of mercury level in the right limb from the datum line XX'

h_1 is the height of manometer liquid level in the right limb from the centre of pipe at point B.

Pressure difference at two points in a pipe.

Left limb eq. $h_A + (h+h_1)S$......................(i)

Right limb eq. $h_B + h_1S + hS_1$......................(ii)

* Pressure is same at the datum line :

$$h_A + (h_1 + h)S = h_B + h_1 + hS_1$$

$$h_A - h_B = -h_1S - hS + -h_1S + hS_1$$

$$h_A - h_B = h(S_1 - S)$$

Case 2 - U-tube upright differential manometer connected between two pipes at different levels and carrying different fluids

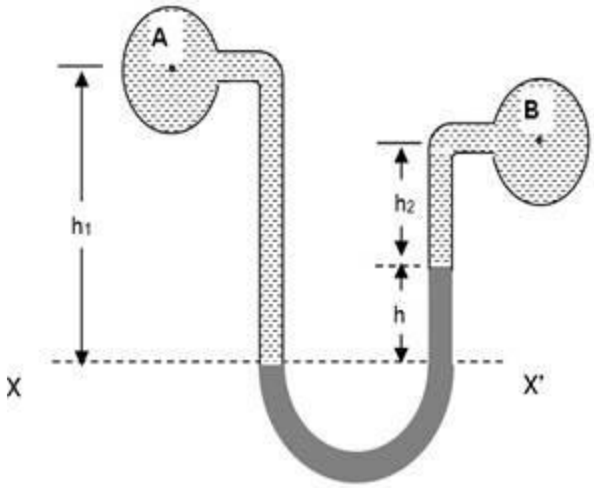


Fig. 7.2 Vertical differential manometer (pressure difference between two pipes)

Let,

S_1 = Specific gravity of liquid in pipe A

S_2 = Specific gravity of liquid in pipe B

S = Specific gravity of manometer liquid

h_A be the pressure head in terms of height of fluid in the pipe at point A

h_B be the pressure head in terms of height of fluid in the pipe at point B

h is the distance of mercury level in the right limb from the datum line XX'

h_1 is the height of manometer liquid level in the left limb from the from the datum line XX'

h_2 is the height of manometer liquid level in the right limb from the from the centre of pipe at point B.

$$\text{Left limb eq}: h_A + h_1 S_1$$

$$\text{Right limb eq}: h_B + h_2 S_2 + hS$$

$$\text{* Pressure is same at the datum line :}$$

$$h_A + h_1 S_1 = h_B + h_2 S_2 + hS$$

$$h_A - h_B = h_2 S_2 - h_1 S_1 + hS$$

7.2 U-tube Inverted Differential Manometer

In such types of manometers light fluids for e.g. oil is used as manometer fluid. In the previous derivation, the term (h*S) is added, but here in the left and right limb equations, it is necessary to subtract (h*S) term.

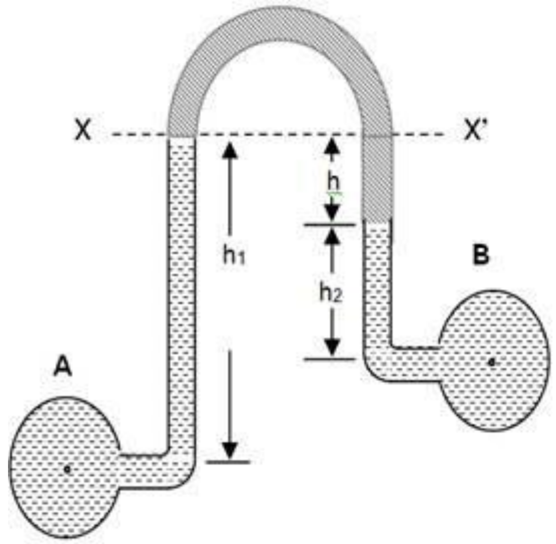


Fig. 7.3 Inverted differential manometer

Let,

S_1 = Specific gravity of liquid in pipe A

S_2 = Specific gravity of liquid in pipe B

S = Specific gravity of manometer liquid

h_A be the pressure head in terms of height of fluid in the pipe at point A

h_B be the pressure head in terms of height of fluid in the pipe at point B

h is the distance of manometer liquid level in the right limb from the datum line XX'

h_1 is the height of manometer liquid level in the left limb from the from the datum line XX'

h_2 is the height of manometer liquid level in the right limb from the from the centre of pipe at point B

$$\text{Left limb eq}: h_A - h_1S_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$
$$\text{Right limb eq}: h_B - h_2S_2 - hS \ldots\ldots\ldots\ldots (ii)$$
$$\text{* Pressure is same at the datum line :}$$
$$h_A + h_1S_1 = h_B - h_2S_2 - hS$$
$$h_A - h_B = h_1S_1 - h_2S_2 - hS$$

7.4 Numerical

Q1. A U-tube monometer is used to measure the pressure of oil (specific gravity 0.85) flowing in a pipe line. It's left end connected to pipe and right limb is open to the atmosphere. The centre of pipe is 100 mm below the level of mercury in the right limb. If the difference of mercury level in the two line is 160 mm, then determine the head and pressure.

Solution:

$h = 13.6 * 160 * 10^{-3} – 0.85 * 60 * 10^{-3}$

$= 2176 * 10^{-3} – 51.00 * 10^{-3}$

$= (2176\text{-}51) * 10^{-3}$

$= 2125 * 10^{-3}$

$= 2.125$ m

$P = \rho gh$

$= 1000 * 9.81 * 2.125$

$= 20846.25$ N/m^2

Q2. A U-tube monometer containing mercury was used to find the negative pressure in the pipe. The right limb was open to the atmosphere; find the vacuum pressure in pipe if the difference of mercury level in two pipes is 100 mm and height of water in the left limb from the centre of pipe was found to 40 mm.

Solution:

$h = - (h_1S_1 + h_2S_2)$

$= - (40*1+100*13.6) * 10^{-3}$

$= - (40+1360) * 10^{-3}$

$= -1400 * 10^{-3}$

$= -1.4$ m

$P = \rho gh$

$= 1000 * 9.81 * 1.4$

$= 13734$ N/m^2

Q3. A simple U-tube manometer containing mercury is connected to a pipe in which an oil of specific gravity 0.80 is flowing. The pressure in the pipe is vacuum. The other end of the manometer is open to atmosphere, find the vacuum pressure in pipe if the difference of mercury level in two limbs is 200 mm and height of oil in left end from the centre of pipe is 150 mm below.

Solution.

$h = h_2S_2 - h_1S_1$

$= (200 * 13.6 – 150 * 0.8) * 10^{-3}$

$= (2720-120) * 10^{-3}$

$= 2600 * 10^{-3}$

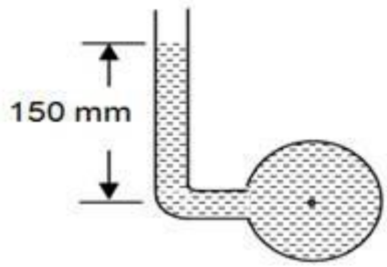
$= 2.6$ m

$P = \rho gh$

$= 800 * 9.81 * 2.6$

$= 20404.8$ N/m^2

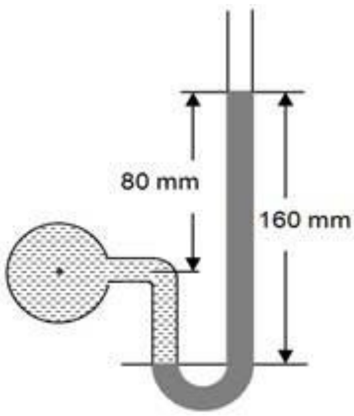
Q 1: A piezometer was connected to a pipe to measure the pressure of water. The rise of water level in piezometer was 150 mm. Calculate the pressure of water. [Ans = 1471.5 N/m^2]



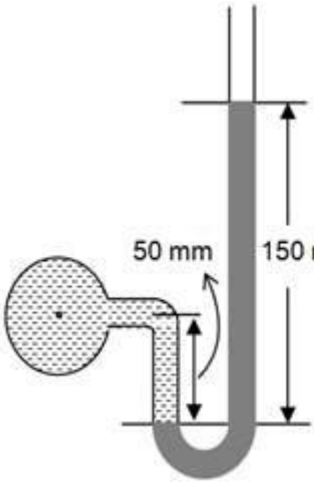
Q 2: Pressure in a pipe carrying oil (Specific Gravity 0.92) is 20 kN/m^2. A piezometer is connected to the pipe. Find the rise of liquid level in the piezometer. [Ans = 2.21 m]

Q 3: A U-tube manometer is connected to a pipe for measuring the pressure of oil (Specific gravity 0.92) flowing in the pipeline. Mercury (Specific gravity 13.6) is used as manometer liquid and the difference of mercury level in the two limbs is 200 mm. The centre of pipe is 80 mm below the level of mercury in the right limb. Determine the pressure in the pipeline and the head.

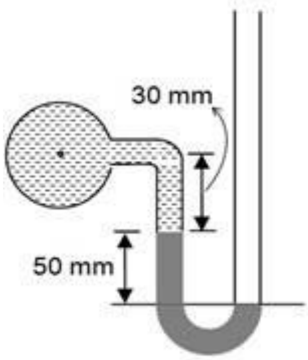
[Ans: 23.55 kN/m^2]



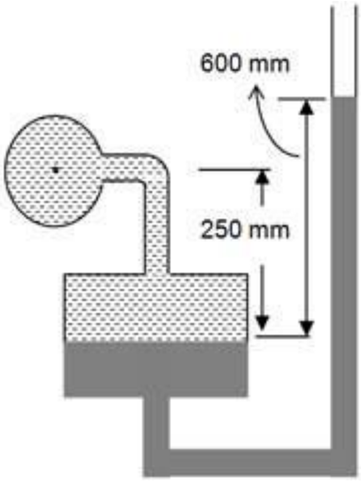
Q 4: The pressure of water in a domestic supply line was measured using manometer shown in the figure. Derive the equation for head and determine the pressure. [Ans: h= 1.99 m, p = 19.52 kN/m^2]



Q 5: Negative pressure in a water pipe was measured using U-tube manometer containing mercury (Specific gravity 13.6). The left limb was connected to the water pipe and right limb was exposed to the atmosphere. Height of water upto centre of pipe in the left limb was 30 mm. Difference between the mercury levels in the two limbs was 50 mm. Determine the vacuum pressure in the pipe and also derive the equation used. [Ans: h= -0.710 m, p = -6.965 kN/m^2]



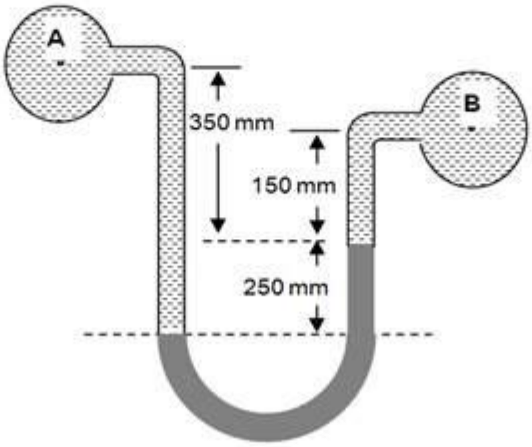
Q 6: A vertical column micro manometer is connected to a pipe containing oil of specific gravity 0.92. The ratio of area of reservoir to that of vertical column is 150. Calculate the oil pressure in the pipe. [Ans: h= 7.98 m, p = 78.28 kN/m^2]



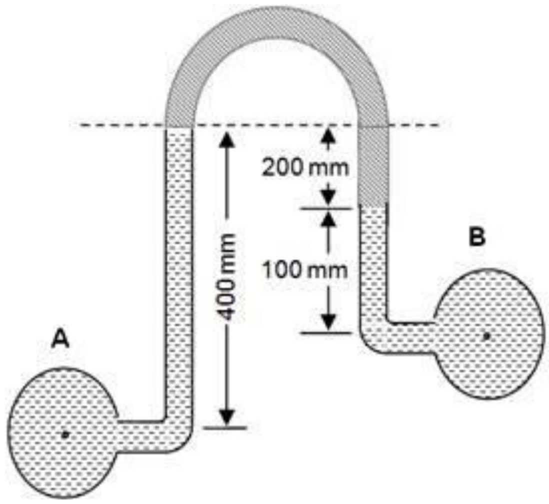
Q 7: Difference in pressure in pipe at two points A and B was measured using a differential manometer. The specific gravity of oil in the pipe is 0.85. If the difference in the mercury level in the two limbs is 200 mm, calculate the pressure difference. [Ans: h= 2.55 m, p = 25.01 kN/m^2]

Q 8: Pressure difference in a water pipe at two points A and B was measured using a differential manometer. If the difference in the mercury level in the two limbs is 50 mm, calculate the pressure difference between two points. [Ans: h= 0.63 m, p = 6.18 kN/m^2]

Q 9: A U-tube differential manometer is connected to two pipes at A and B. Pipe A Contains oil of Specific Gravity 0.92 and pipe B is carrying water. If the pressure at point A is 125 kN/m^2 find the pressure at point B. [Ans: h= -3.098 m, p = 30.39 kN/m^2]



Q 10: An inverted tube differential manometer having an oil of specific gravity 0.9 is connected to two different pipes carrying water under pressure. Determine the pressure in the pipe B. The pressure in pipe A is 2 m of water. [Ans: h= 1.88 m, p = 18.44 kN/m^2]


200 mm
100 mm
400 mm
A
B

Lesson 8

MECHANICAL GAUGES

8.1 Introduction

- Manometers are suitable for lower pressure i.e. near to atmosphere pressures.
- For measuring medium and high pressure elastic pressure gauge such as tubes, diaphragms, bellows etc. are used.
- Elastic deformation in these elements shows the effect of pressure.
- Since, there elements deform within elastic limit therefore these gauges are also called elastic gauges.
- Mechanical gauges are called secondary instruments because they have to be calibrated with help of primary instrument such as manometer.

8.2 Simple Mercury Barometer

- Measures the absolute atmospheric pressure
- Pressure is given as p = $\rho g h$ where ρ = density of mercury, g = 9.81 m/s^2, h = height of mercury in the barometer.
- Major disadvantage: fragile, mercury is harmful and may spill.

Simple mercury barometer and the photograph showing the coloumn of mercury rise in the barometer are shown in figure 8.1 and 8.2 respectively.

Fig. 8.1 Simple mercury barometer (Click for Animation)

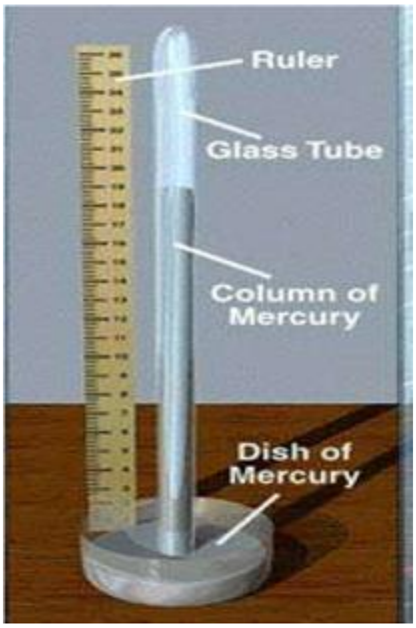


Fig. 8.2 Photograph of simple mercury barometer

8.3 Aneroid barometer

Aneroid barometer uses elastic diaphragm to measure atmospheric pressure.

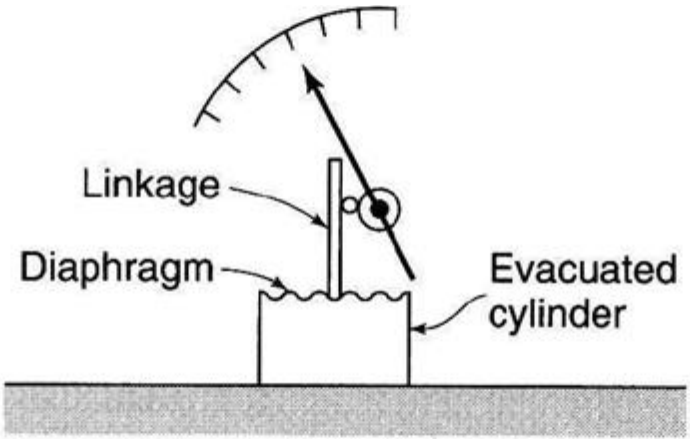


Fig. 8.3 Working principle of Aneroid barometer

Fig. 8.4 Aneroid barometer

8.4 Bourdon Tube Pressure Gauges

- It is used to measure high as well as low pressure.
- Pressure element consists of a metal tube of elliptical cross section.
- This tube is bend in a form of segment of circle and responds by bending inward due to increase in pressure
- When one end of tube is connected to source of pressure, the pressure inside the Bourdon tube causes the tube to expand and bend inward.
- A simple pinion and sector arrangement is provided to convert the linear movement of the tube into angular movement of the pointer.
- The pressure is indicated by the pointer over dial which can be graduated on a suitable scale.

Fig. 8.5 Bourdon tube pressure gauge

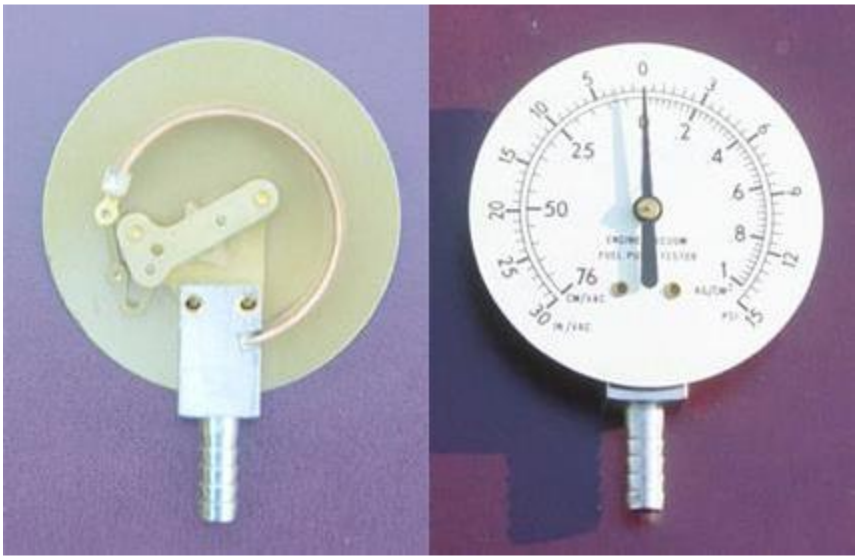


Fig. 8.6 Bourdon tube pressure gauge

8.5 Diaphragm Gauge

- Consists of metallic disc or diaphragm for actuating the pointer.
- When pressure is applied on lower side of diaphragm, it gets deflected upward.
- The movement of diaphragm is transmitted to a strain gauge or transducer which converts the pressure signal into electrical signal. In analogue devices, a rack and pinion system is provided which moves the pointer.

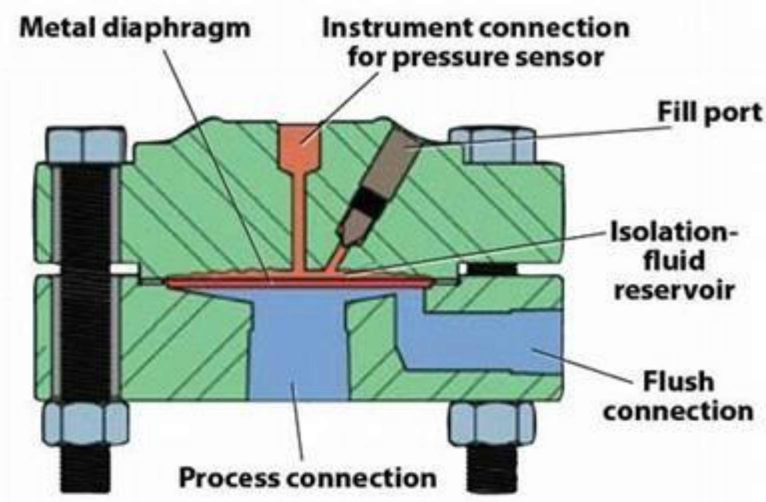


Fig. 8.7 Diaphragm gauge

8.6 Vacuum gauge

(a) Bourdon gauge can be used to measure vacuum by bending the tube inward instead of outward pressure in pressure gauge.
(b) Vacuum gauge is graduated in mm of Hg below atmospheric pressure.

8.7 Pressure Transducers

Transducers are instruments which convert one form of signal into another form of signal. Electronic pressure transducers sense the signals and convert into electronic/electrical signals which can be further processed. Few of them are strain gauge, piezoelectric, capacitive, magnetic etc.

Lesson 9
ARCHIMEDES PRINCIPLE, STABILITY OF FLOATING BODIES

9.1 Introduction

Archimedes principle can be used in dairy industry for designing equipments to measure density, floating devices for measuring liquid level etc. The stability of conveying packages or milk cans is based on the location of centre of gravity. Some common phenomena encountered in daily life are depicted below:

- Why do a ship weighing tonnes easily floats in sea where as a hammer sinks in water?
- Why a person does not sink in Dead Sea?
- How a life jacket does help in time of emergency?

9.2 Buoyancy

When a body is submerged in fluid, it experiences an upward thrust due to the fluid pressure. This vertical upward force is known as buoyant force (F_B). It is shown in Fig. 9.1.The tendency of anybody to be lifted upward in a fluid against the force of gravity is known as buoyancy.

9.3 Centre of Buoyancy

The point through which the buoyant force acts is called the centre of buoyancy. It is at the centre of gravity of the volume of displaced fluid. Here in Fig. 9.1 point B is known as the centre of buoyancy.

Click for Animation (Fig. 9.1 A wooden block floating in water)

9.4 Archimedes Principle

Archimedes Principle states that the buoyant force acting on the body immersed in fluid is equal to the weight of fluid displaced by the body.

This principle explains the loss of weight in a body immersed in fluid, which is equal to the weight of fluid displaced by it. The volume of fluid displaced by the floating body is just enough to balance its weight.

9.5 Numericals

Q. 1. The dimensions of a wooden block floating in the water is 4 m * 2 m * 1 m (length * width * depth). The density of block is 700 kg/m^3. Determine

(i) Volume of water displaced

(ii) Position of centre of buoyancy

Solution:

(i) $W = (4 * 2 * 1) * 700 * 9.81$

$F_B = V_f * 1000 * 9.81$

$W = F_B$ (according to Archimedes Principle)

Therefore, $4 * 2 * 1 * 700 * 9.81 = V_f * 1000 * 9.81$

$V_f = 5.6\ m^3$

(ii) $h * 2 * 4 = 5.6$

$h = 0.7$ m

Position of centre of buoyancy = h/2 = 0.35 m

Q2. The dimensions of a wooden block floating in the water is 4 m * 2 m * 1 m having density of block 700 kg/m^3. Calculate the volume of concrete block of specific gravity = 3.5 that may be placed on the block which will:

(a) Completely on the immerse the block into water

(b) Completely immerse the wooden + concrete block together.

Solution:

$$\rho_{concret} = 3500 kg/m^3$$

$$(a)\ W = (4\times2\times1)\times700\times9.81 + V_c\times3500\times9.81$$

$$F_B = (4\times2\times1)\times1000\times9.81$$

$$2\times1\times4\times1000\times9.81 = 2\times1\times4\times700\times9.81 + 3500\times9.81\times V_c$$

$$V_c\times3500 = (1000-700)\times(2\times1\times4)$$

$$V_c = \frac{300\times8}{3500} = \frac{24}{35} m^3 \quad Ans.$$

$$(b)\ W = (2\times1\times4)\times700\times9.81 + V_C\times3500\times9.81)$$

$$F_B = 2\times1\times4\times1000\times9.81 + V_C\times1000\times9.81$$

$$= 2\times1\times4\times700\times9.81 + V_C\times3500\times9.81 = 2\times1\times4\times1000\times9.81 + V_C\times1000\times9.81$$

$$= V_C(3500-1000) = (1000-700)\times2\times1\times4$$

$$V_C = \frac{300\times8}{2500} = \frac{24}{25} = 0.96\ m^3 \quad \text{Ans.}$$

Q3. A metallic cube 30 cm side and weighing 450 N is lowered into a tank containing two fluid layers of water and mercury. Determine the position of block at mercury and water interface when it has reached at equilibrium.

Solution:

$$W = 450\,N$$

$$450 = F_B = h_1\times0.3\times0.3\times13600\times9.81 + h_2\times0.3\times0.3\times1000\times9.81 \ldots\ldots\ldots.(i)$$

$$12007.44h_1 + 882.9h_2 = 450 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(i)$$

$$h_1 + h_2 = 0.3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots(ii)$$

$$\begin{array}{r} h_1 + h_2 = 0.3 \\ \hline 12007.44h_1 + 882.9h_2 = 450 \\ -12007.44h_1 \pm 12007.44h_2 = 3602.232 \\ \hline 11124.54h_2 = 3152.232 \end{array}$$

$$h_2 = 0.2833\ m$$

$$\mathrm{h}_1 + \mathrm{h}_2 = 0.3$$

$$h_1 = 0.3 - h_2$$

$$= 0.3 - 0.2833$$

$$= 0.0167\ m$$

9.6 Stability of Floating Body

Stability of a body can be understood by keeping a solid cone on a table. The different positions are shown in Fig. 9.2. Here the stability of the cone depends on how close the centre of gravity to the plane on which any object is resting. But in case of immersed and floating body the stability or equilibrium is determined by the position of centre of buoyancy. This will be discussed in the lesson 10.

A = Stable B = Unstable C = Neutral

Fig. 9.2 Stability of a solid cone

Lesson 10
EQUILIBRIUM OF FLOATING BODIES, METACENTRIC HEIGHT

10.1 Introduction

In the previous lesson, the position (i.e. center of gravity) of a solid cone affecting its stability was discussed. This lesson will cover the equilibrium states of the floating bodies and the factors affecting it.

10.2 Equilibrium of Floating Body

1. Stable equilibrium
2. Unstable equilibrium
3. Neutral equilibrium

10.2.1 Stable equilibrium

When a body is given a small angular displacement, i.e. it is tilted slightly by some external force and then it returns back to original position due to internal forces. Such equilibrium is called stable equilibrium.

10.2.2 Unstable equilibrium

If a body does not return to its original position from the slightly displaced angular position and moves father away when give a small angular displacement such equilibrium is called an unstable equilibrium.

10.2.3 Neutral equilibrium

The body remains at rest in any position to which it may be displaced, no net force tends to return the body to its original state or to drive it further away from the original position, is called neutral equilibrium. Fig. 10.1 shows the case of neutral equilibrium.

Click for Animation (Fig. 10.1 Case of neutral equilibrium of a ball floating on water)

10.3 Metacentre

When a small angular displacement is given to a body floating in a liquid, it starts oscillation about some point M. This point about which the body starts oscillating is called the metacentre (Fig. 10.2).

Metacentre (M) may be defined as the point of intersection of the axis of body passing through centre of gravity (G) and original centre of buoyancy (B) and a vertical line passing through the new centre of buoyancy (B') of the titled position of the body.

Click for Animation (Fig. 10.2 Metacentre of a wooden block floating on water)

10.4 Metacentric height

Metacentric height: The distance between the centre of gravity of a floating body and the metacentre, i.e. distance GM is called meta-centric height (Fig. 10.2). Relation between centre of gravity and metacentre in different three types of equilibrium:

(a) Stable equilibrium

In this, position of metacentre (M) remains higher than centre of gravity of body.

(b) Unstable equilibrium

In this position of metacentre (M) remains lower than centre of gravity of body.

(c) Neutral equilibrium

The position of metacentre (M) coincides with centre of gravity of body.

10.5 Conditions of Stability in Air Balloon

Case 1: When G is lower than B

The balloon having G (centre of gravity) below B (centre of buoyancy) is a condition of stable equilibrium as shown in Fig. 10.3. Under equilibrium condition the balloon will return back to its original position if it is tilted a bit due to wind etc.

Click for Animation (Fig. 10.3 Condition of stability of air balloon)

Case 2: When G is above B

Fig. 10.4 shows a air balloon that will be highly unstable when centre of gravity (G) is above centre of buoyancy (B).

[Click for Animation (Fig. 10.4 Unstable Condition of air balloon)]

10.6 Stability of ship

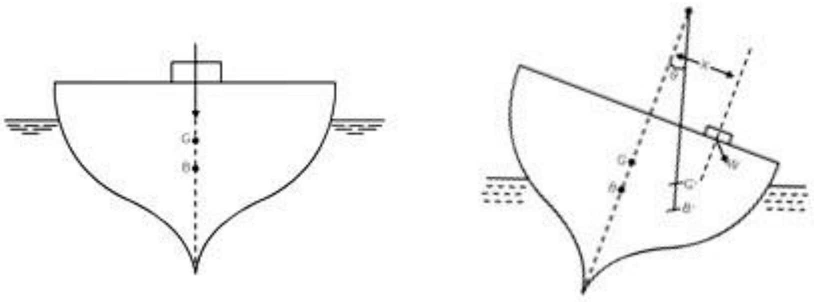


Fig. 10.5 Condition of stability of ship

The above cases were of submerged body. For floating body particularly ship the construction of ship body plays an important role in its stability. It is such so as to withstand high wave and tide (Fig. 10.5).

10.7 Calculation of Metacentric Height

In the Fig. 10.6:

G = Centre of gravity

O = The point at which line BM and top liquid surface intersect

M = Metacentre

B = Centre of buoyancy

BM = Distance between centre of buoyancy and metacentre

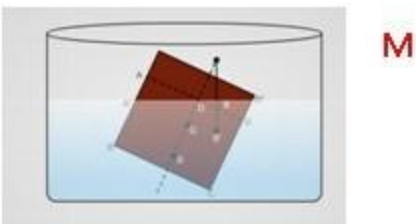


Fig. 10.6 Metacentric height

$$BM = \frac{I}{V}$$

Where I = area moment of inertia of the cross sectional area at the surface of fluid (Fig. 10.7).

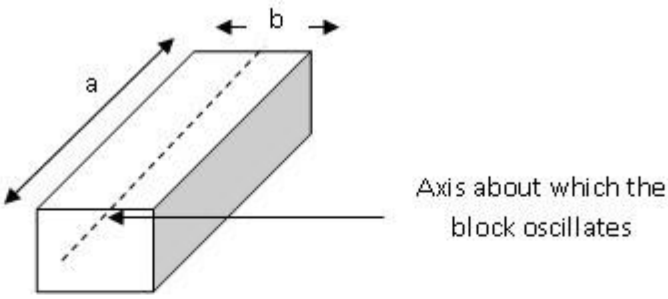


Fig. 10.7 a*b is the cross sectional area at the surface of liquid

Area moment of Inertia $I = \frac{ab^3}{12}$

V= volume of the displaced fluid.

Metacentric Height GM = BM - BG
If GM is +ve then G is above point B.
If GM is –ve then G is below point B.

10.8 Numericals

Q1. A wooden block of specific gravity = 0.75 floats in water if the size of the block is 1.2 m * 0.6 m * 0.5 m. Find its metacentric height i.e. GM.

Solution:

BM = I/V

$$\frac{1.2X(0.6)^3}{12} = 0.0216$$

V * 1000 * 9.81 = (1.2 * 0.6 * 0.5) * 750 * 9.81

V = 0.27 m^3

OG = 0.5/2 = 0.25 m

h * 0.6 * 1.2 = 0.27

$$h = \frac{0.27}{0.6X1.2} = 0.375$$

OB = h/2 = 0.1875 m

BG = OG - OB = 0.25 - 0.1875 = 0.0625 m

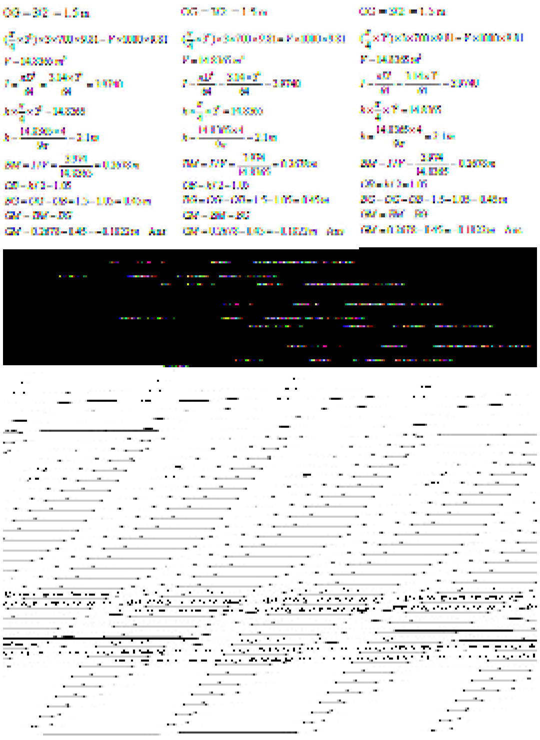
GM = BM - BG

$$BM = \frac{0.0216}{0.27} = 0.08\ m$$

GM = 0.08 - 0.0625 = 0.0175 m = 1.75 cm.

Q2. A solid cylinder 3 m in diameter and 3 m height is floating in water with its axis vertical. If the specific gravity of cylinder is 0.7 then find its meta centric height, i.e. GM.

Solution:



Q3. A wooden cube side 1 m is floating in water. Specific gravity of cube is 0.7; Find its meta centric height.

Solution:

1 * 1 * 1 * 700 * 9.81 = V * 1000 * 9.81

V = 0.7 m^3

h * 1 * 1 = 0.7

h = 0.7 m

OB = h/2 = 0.35 m

OG = 1/2 = 0.5 m

BG = OG - OB = 0.5 - 0.35 = 0.15 m

BM = I/V

$$I = \frac{ab^3}{12} = \frac{1X1X1X1}{12} = \frac{1}{12}$$

$$V = 0.7\ m^3$$

$$BM = \frac{I}{V} = \frac{\frac{1}{12}}{0.7}$$

$$= 0.119\ m$$

$$\begin{aligned} GM &= BM - BG \\ &= 0.119 - 0.15 \\ &= -0.031\ m \\ &= -3.1\ cm \end{aligned}$$

Lesson 11
CLASSIFICATION, STEADY, UNIFORM AND NON UNIFORM FLOW, LAMINAR AND TURBULENT

11.1 Introduction

In Fluid Mechanics, the knowledge of flow behavior is important as the analysis and calculations depends on the flow conditions.

11.2 Types of flow

1. ***Steady flow***

In steady flow fluid parameters such as velocity, density, pressure, acceleration etc. at a point do not change with time.

$$\frac{\partial v}{\partial t} = 0; \quad \frac{\partial \rho}{\partial t} = 0; \quad \frac{\partial P}{\partial t} = 0; \quad \frac{\partial a}{\partial t} = 0;$$

Where v: velocity; P: Pressure; ρ: Density; a: acceleration; t: time

2. ***Unsteady flow***

In unsteady flow fluid parameters such as velocity, density, pressure, acceleration etc. at a point changes with time.

$$\frac{\partial v}{\partial t} \neq 0; \quad \frac{\partial \rho}{\partial t} \neq 0; \quad \frac{\partial P}{\partial t} \neq 0; \quad \frac{\partial a}{\partial t} \neq 0;$$

3. ***Uniform flow***

In uniform flow if the velocity at a given instant of time is same in both magnitude and direction at all points in the flow, the flow is said to be uniform flow.

4. ***Non-uniform flow***

When the velocity changes from point to point in a flow at any given instant of time, the flow is described as non-uniform flow.

5. ***Compressible flow***

The flow in which density of the fluid varies during the flow is called compressible fluid flow. (i. e. $\rho \neq constant$). This is applicable in gas flow.

6. ***Incompressible flow***

In case of in compressible fluid flow, the density of the fluid remains constant during the flow. (i. e. $\rho = constant$). Practically, all liquids are treated as incompressible.

7. ***Pressurized flow***

Flow under pressure. e..g. liquid flowing in pipes with pressure.

8. ***Gravity flow***

Flow of fluid due to gravity.

9. ***One, two and three dimensional flow***

a. One Dimensional: When the flow properties (e.g. velocity, density pressure etc) vary only in one direction.
b. Two Dimensional flows: When the flow properties (e.g. velocity, density pressure etc) vary in only two directions.
c. Three Dimensional flows: When the flow properties (e.g. velocity, density pressure etc) vary in all the three directions.

10. **Rotational and irrotational flows:**

Rotational flow: The fluid particles while flowing also rotate about their own axis.
Irrotational flow: The fluid particles while flowing do not rotate about their own axis.

11. Laminar flow

In this type of fluid flow, particles move along well defied paths or steam lines. The fluid layers moves smoothly over the adjacent layer. The fluid particles move in a definite path and their paths do not cross each other (Fig. 11.1).

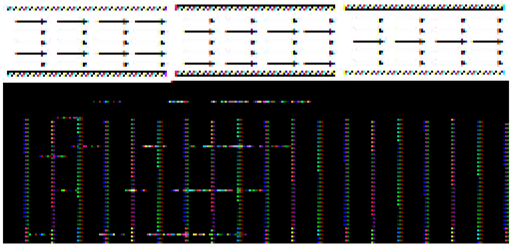


Fig. 11.1 Laminar flow

12. Turbulent Flow

In turbulent fluid flow, fluid particles move in a random and zigzag way (Fig. 11.2). Turbulence is characterized by the formation of eddies.

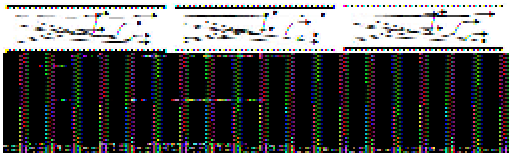


Fig. 11.2 Turbulent flow

The type of flow is determined by Reynold's Number.

11.3 Reynold's Number

It is defined as the ratio of inertia force of the flowing fluid to the viscosity force of the fluid. In case of pipe flow, it is determined by using the following equation.

$$Re = \frac{\rho V D}{\mu}$$

Where, Re=Reynold's Number

ρ= Density of fluid

V= Velocity of fluid

D= Diameter of pipe

μ= Viscosity of fluid

Reynold's Number (Re)	Flow type
Re < 2100	Laminar flow
2100 < Re < 4000	Transitional (flow can be laminar or turbulent)
Re > 4000	Turbulent

11.4 Streamline, Path Line and Streakline

Streamline: Is an imaginary line and velocity vector at any point on a stream line is tangent to the streamline (Fig. 11.3).

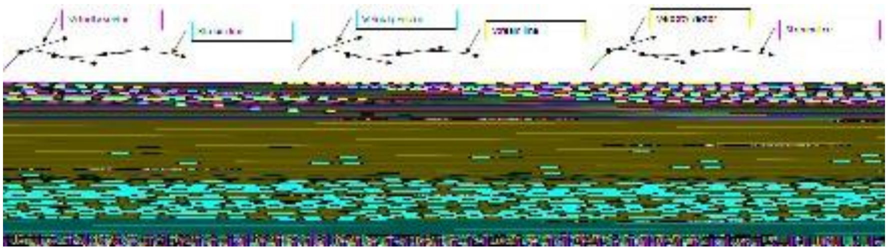


Fig. 11.3 Streamline

Path line is the path traced by a fluid particles.

Streaklines are obtained by joining the locus of points of all the fluid particles that have passed continuously through a fixed point during time t. Dye steadily injected into the fluid at a fixed point extends along a streakline.

11.5 Numericals

Q.1 Predict whether the flow would be laminar or turbulent in a pipe of diameter 5 cm. Consider density of liquid to be 950 kg/m^3, viscosity 0.2 Ns/m^2 and flow velocity 20 m/s.

Q.2 Calculate the Reynlod's number if pipe diameter is 4 cm, liquid density 900 kg/cm^3, viscosity 0.5 Ns/m^2 and flow velocity 10 m/s.

Lesson 12
CONTINUITY EQUATION

12.1 Introduction

Continuity equation is one of the widely used formulae in Fluid Mechanics. The equation based on the conservation of mass is called continuity equation. When fluid flows in any pipeline, the rate of fluid flowing at every section remains constant.

12.2 Continuity Equation

It is based on principle of conservation of mass and is expressed by the following relation.

Consider a pipe of varying diameter as shown in (Fig 12.1). The fluid is compressible and the density at section XX' is ρ_1 and at YY' is ρ_2.

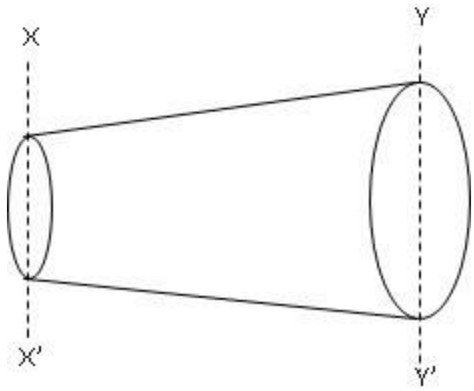


Fig. 12.1 Pipe of increasing diameter

At section XX':

Area of pipe = A_1

Velocity of fluid = V_1

Density of fluid = ρ_1

At YY':

Area = A_2

Velocity=V_2

Density = ρ_2

Volume flowing = A_1V_1 at section XX'

Mass of fluid flowing per second at section XX' = $\rho_1A_1V_1$ ....(i)

Mass of fluid flowing/sec at section YY'= $\rho_2A_2V_2$.............................(ii)

Mass flow rate into the system = Mass flow rate out of the system

For compressible fluid applying conservation of mass,

$\rho_1A_1V_1 = \rho_2A_2V_2$ (Mass flow rate remains constant)

For incompressible fluid

ρ = constant i.e. $\rho_1=\rho_2= \rho$

$\therefore A_1V_1 = A_2V_2$

Discharge, Q = AV

At XX, Discharge (m^3/s) $Q_1 = A_1V_1$

At YY, Discharge (m^3/s) $Q_2 = A_2V_2$

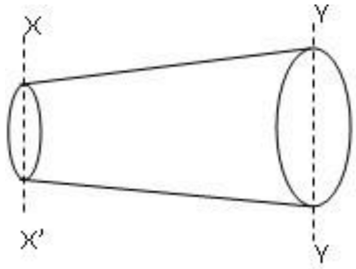
According to continuity equation,

$Q_1 = Q_2$

Thus, continuity equation relates flow velocity with area of the section, if area of flow at any section is decreased there is an increase in flow velocity.

12.3 Numericals

Q 1. The area of a pipe at section XX' is 315 cm^2. The area of pipe at section YY' is twice the area of section XX'. The velocity at section XX' is 4.5 m/s. Find the velocity at section YY'. Also determine the flow rate through the pipe.



Solution:

$A_1V_1 = A_2V_2$

$315 * 10^{-4} * 4.5 = 630 * V * 10^{-4}$

$V = \frac{315 X 4.5}{630} = 2.25 m/s$

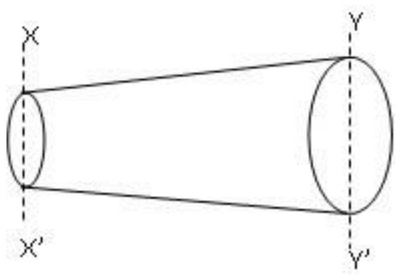
Discharge $= A_1V_1$

$= 315 * 10^{-4} * 4.5$

$= 1417.5 * 10^{-4}$

$= 0.14175\ m^3/s$

Q 2. A compressible liquid is flowing in a pipe. The diameter of section XX' and YY' is 150 mm and 250 mm respectively. The density at XX' is 998 kg/m³ and at YY' is 994 kg/m³. If the velocity of liquid at section XX' is 3.5 m/s, find the velocity at section YY'.



Solution:

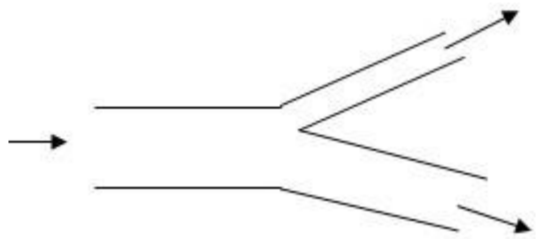
$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

$998 \times \pi/4 * 150^2 * 10^{-6} * 3.5 = 994 * \pi/4 * 250^2 * 10^{-6} * V_2$

$$V_2 = \frac{998 X 150 X 150 X 3.5}{994 X 250 X 250} = \frac{7859250}{621250} = 12.65 m/s$$

Q 3. A pipe 500 mm in diameter bifurcates into two pipes of diameter 200 mm and 350 mm respectively. If the flow is incompressible and the velocity of flow in 500 mm and 200 mm pipe is 4.0 m/s and 3.0 m/s respectively. Find the (a) Discharge through 500 mm & 350 mm diameter pipe.

Solution:



Discharge through 500 mm = $(\pi/4) * 500^2 * 10^{-6} * 4\ m^3/s$

$$Q_1 = Q_2 + Q_3$$

Or,

$$A_1V_1 = A_2V_2 + A_3V_3$$

$$\frac{\pi}{4}\times 500^2 \times 10^{-6} \times 4 = \frac{\pi}{4}\times 200^2 \times 10^{-6} \times 3 + \frac{\pi}{4}\times 350^2 \times 10^{-6} \times V_3$$

$$25000 \times 4 = 40000 \times 3 + 350 \times 350 \times V_3$$

$$1000000 - 120000 = 350x \times 350 \times V_3$$

$$880000 = 350 \times 350 \times V_3$$

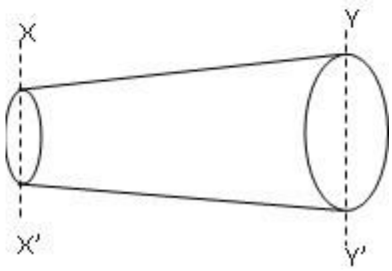
$$V_3 = \frac{880000}{350 \times 350} = 7.183 m/s \ \ Ans$$

$$\text{Discharge through 35mm pipe} = \frac{\pi}{4}(350)^2 \times 10^{-6} \times V_3$$

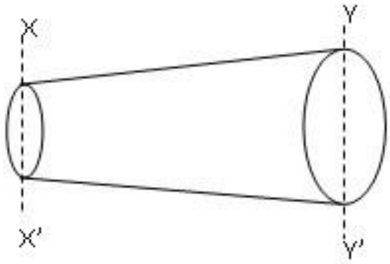
$$= 0.785 \times (350)^2 \times 7.18 \times 10^{-6} = 690446.75 \times 10^{-6} = 0.69\ m^3/s\ \text{Ans}$$

12.4 Numericals

Q.1. A incompressible liquid is flowing in a pipe. The diameter of section XX' and YY' is 170 mm and 200 mm respectively. If the velocity of liquid at section XX' is 5.5 m/s, find the velocity at section YY'.



Q. 2. A compressible liquid is flowing in a pipe. The diameter of section XX' and YY' is 50 mm and 100 mm respectively. The density at XX' is 995 kg/m^3 and at YY' is 990 kg/m^3. If the velocity of liquid at section XX' is 5.5 m/s, find the velocity at section YY'.



Lesson 13

BERNOULLI'S THEOREM

13.1 Introduction

Bernoulli's Theorem is based on the conservation of energy in fluid flow. There are three types of energy namely potential energy, kinetic energy and pressure energy possessed by the liquid. The theorem explains how these energies change from one form to another form. Many instruments such as Pitot tube, Venturimeter etc. are working on the principle of Bernouilli's theorem for the measurement of fluid flow.

13.2 Bernoulli's Theorem

Bernoulli's Theorem states that "in a steady ideal flow of in compressible fluid flow, the sum of pressure energy, kinetic energy and potential energy remains constant at every section provided no energy is added or taken out by an external source"

It is based on principle of conservation of energy and is expressed by the following relation:

Pressure energy + Kinetic energy + Potential energy = constant

Assumption for Bernoulli's equation

Bernoulli's theorem holds well under the following assumptions:

(1) The flow is along stream line.

(2) The flow is steady and continuous.

(3) The fluid is non-viscous (ideal fluid) and incompressible (ρ = constant).

(4) Flow is irrotational.

$$\text{Pressure head} = \frac{P}{\rho g} \quad \text{...} \quad \text{(i)}$$

$$\text{Velocity head} = \frac{V^2}{2g} \quad \text{...} \quad \text{(ii)}$$

$$\text{Potential head} = Z \quad \text{...} \quad \text{(iii)}$$

According to Bernoulli's theorem,

$$\frac{P}{\rho g} + \frac{v^2}{2g} + Z = \text{constant}$$

Consider a pipe of varying diameter as shown in (Fig 13.1).

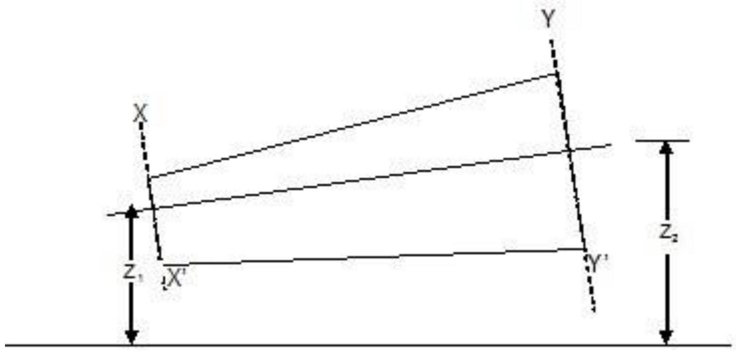


Fig. 13.1 Pipe of varying diameter

At section XX’

Cross sectional area of pipe = A_1

Velocity of fluid = V_1

Pressure of fluid = P_1

At YY’

Cross sectional area of pipe = A_2

Velocity of fluid = V_2

Pressure of fluid = P_2

Applying Bernoulli’s theorem at section XX’ and YY’

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + Z_2 = \text{constant}$$

The above equation holds true for an ideal fluid (i.e. when there is no loss of head between two points). However, in practice some energy is lost due to friction and it is denoted by H_L then

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + Z_2 + H_L$$

13.3 Total Energy Line (TEL) and Hydraulic Grade Line (HGL)

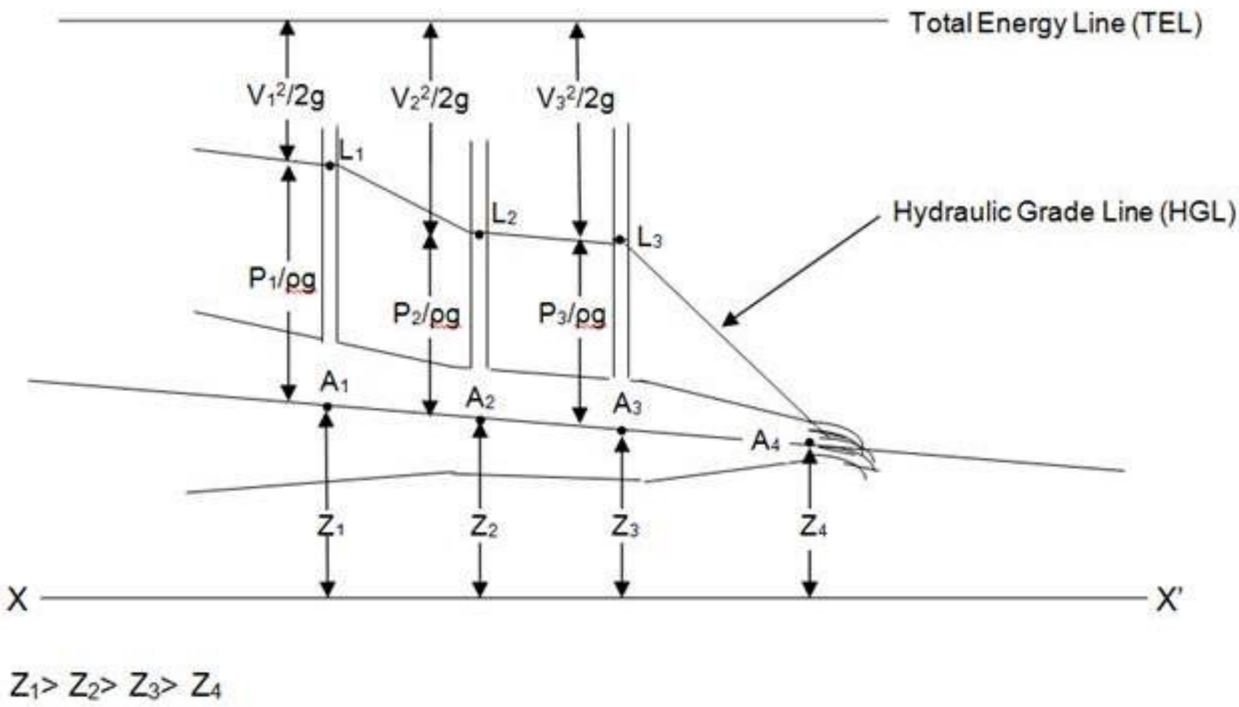


Fig. 13.2 Total Energy Line (TEL) and Hydraulic Grade Line (HGL)

(1) At points A_1, A_2 and A_3, piezometer tube measures the pressure head (P/ρg) by rise of liquid in the peizometer tube h_1, h_2 and h_3 respectively.

(2) Points A_1, A_2, A_3 and A_4 are located at a height of Z_1, Z_2, Z_3 and Z_4 above datum (reference line XX'). Z_1, Z_2, Z_3 and Z_4 are known as potential heads.

(3) Points L_1, L_2, and L_3 represent the water level in the peizometer and are located at a height of (Z_1+h_1), (Z_2+h_2) and (Z_3+h_3).

(4) Point L_4 is located at tip of nozzle is at height Z_4 above datum line.

(5) The line joining points L_1, L_2, L_3 and L_4 is known as HGL and (P/ρg + Z) is known as hydraulic gradient.

(6) Total energy line is line joining points denoting total head is represented as

$$\frac{P}{\rho g} + \frac{v^2}{2g} + Z$$

(7) At tip of nozzle (L_4) the liquid is exposed to atmosphere as a result pressure head becomes zero.

(8) For flow in converging pipe there is a decrease in the pressure head (P/ρg) and consecutively there is a rise in velocity energy.

(9) At tip of nozzle pressure head and entire pressure energy gets converted into K.E. causing the flow velocity to increase.

13.4 Numericals

Q 1. Water is flowing through an inclined pipeline of diameter 20 cm & 40 cm at section A & B respectively. Section A& B are located at height of 2 m & 2.5 m respectively from ground level. The discharge through pipe is 30 l/s. If the pressure at A is 20 kPa, find the pressure at point B.

Solution:

$1\ P_a = 1\ N/m^2$

$P_A = 20\ kPa, P_B = ?$

$1000\ l = 1\ m^3$

$1\ l = 10^{-3}\ m^3$

$Q = 30\ l/s = 0.03\ m^3/s$

$A_1V_1 = A_2V_2 = Q$

$A_1 = \frac{\pi}{4}(20)^2 \times 10^{-4} = 0.0314\ m^2$

$A_2 = \frac{\pi}{4}(40)^2 \times 10^{-4} = 0.1256\ m^2$

$V_1 = \frac{0.03}{A_1} = 0.955414012\ m/s$

$V_2 = \frac{0.03}{A_2} = 0.2388\ m/s$

$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$

$\frac{20000}{1000 \times 9.81} + \frac{0.912}{2 \times 9.81} + 2 = \frac{P_2}{1000 \times 9.81} + \frac{0.057}{2 \times 9.81} + 2$

$2.038 + 0.046 + 2 = \frac{P_2}{9810} + 0.0002 + 2.5 = 1.582 = \frac{P_2}{9810}$

$P_2 = 15519.42\ \text{Pa} \qquad P_2 = 15.519\ \text{kPa}$

Q 2. A horizontal pipe of diameter 250 mm carries oil (sp. gravity = 0.89) at the rate of 125 l/s and a pressure of 32 kPa. The pipe converges to a 100 mm diameter at a section located in down stream. Determine the pressure at down stream.

Solution:

$$P_1 = 32\,\text{kPa}\,;\,\text{P}_2 = ?$$
$$\text{Q} = 125\,\text{l/s} = 0.125\,\text{m}^3/s$$
$$0.049\,m^2 = A_1 = \frac{\pi}{4}\times(250)^2\times10^{-6}$$
$$0.0078\,m^2 = A_2 = \frac{\pi}{4}\times(100)^2\times10^{-6}$$
$$A_1V_1 = A_2V_2 = Q$$
$$V_1 = \frac{Q}{A_1} = \frac{0.125}{A_1} = 2.55\,m/s$$
$$V_2 = \frac{0.125}{A_2} = 16.02\,m/s$$
$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1 = \frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2$$
$$\frac{32000}{890\times9.81}+\frac{(2.55)^2}{2\times9.81} = \frac{P_2}{890\times9.81}+\frac{(16.02)^2}{2\times9.81}$$
$$3.66+0.33 = \frac{P_2}{8730.9}+13.08 = -79363.88 = -79.36\,kP\text{a Ans}$$

Q 3. The area of cross-section at point A in a converging pipe reduces from 0.45 m^2 to 0.22 m^2 at point B. The velocity & pressure at point A are 2.5 m/s and 200 kN/m^2 respectively. Neglecting the frictional head loss in pipe, calculate the pressure at point B which is 6 m above the level of A.

Solution:

$$P_A = 200\,\text{kN/m}^2\,;\,\text{P}_\text{B} = ?$$
$$V_A = 2.5\,m/s;\; V_B = ?$$
$$A_1V_1 = A_2V_2 = 0.45\times2.5 = 0.22\times V_B$$
$$V_B = 5.11\,m/s$$
$$\frac{P_A}{\rho g}+\frac{V_A^2}{2g}+Z_1 = \frac{P_B}{\rho g}+\frac{V_B^2}{2g}+Z_2$$
$$\frac{200\times10^3}{1000\times9.81}+\frac{(2.5)^2}{2\times9.81}+0 = \frac{P_B}{1000\times9.81}+\frac{(5.11)^2}{2\times9.81}+=6$$
$$20.38+0.31 = \frac{P_B}{9810}+1.33+6$$
$$13.36 = \frac{P_B}{9810}$$
$$P_B = 131061.6 = 131.06\,kN/m^2\text{ Ans}$$

Lesson 14

APPLICATION OF BERNOULLI'S THEOREM

14.1 Introduction

Bernoulli's theorem has number of applications. The working of many flow measuring devices are based on the principle of Bernoulli's theorem. Some equations used in fluid mechanics are also derived using the concept of fluid mechanics. Some of the devices used for the measurement of fluid flow are as under.

1. Stagnation Tube
2. Stagnation Tube + piezometer
3. Pitot tube
4. Flow through nozzle from reservoir

14.2 Stagnation Tube

Stagnation tube is used to measure velocity in an open flow (Fig. 14.1).

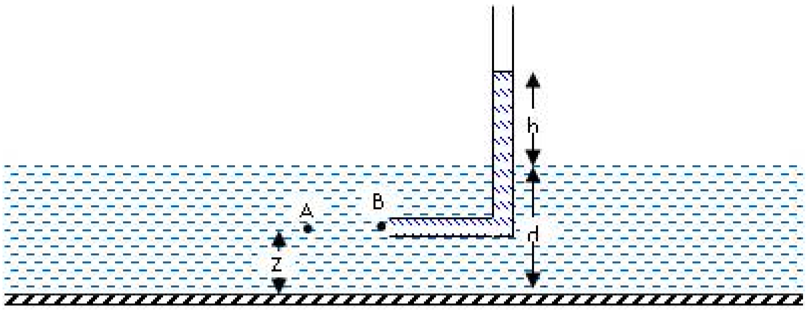


Fig. 14.1 Stagnation tube placed in an open channel flow

From Bernoulli's theorem,

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2$$

At point A :

Velocity $= V_1$

Pessure $= \rho g d$

Height $= Z$

At point B :

Velocity $= V_2 = 0$

Pressure $= \rho g d$

Height $= Z$

Applying Bernoulli's theorem,

$$d+\frac{V_1{}^2}{2g}+Z=(d+h)+0+Z$$
$$V_1^2=2gh$$
$$V_1=\sqrt{2gh}$$

14.3 Combination of Stagnation Tube and Piezometer

As in the above case stagnation tube can be used along with a piezometer to measure velocity in closed pipe flow (Fig. 14.2).

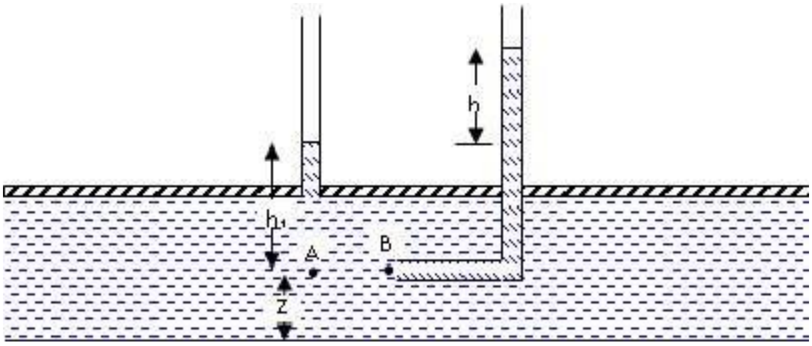


Fig. 14.2 Stagnation tube with a piezometer placed in a pipe (closed flow)

At point A :
Velocity. $=\mathrm{V}_1$
(P_1) Pressure $=\rho g h_1$
$Height=Z$

At point B :
Velocity $(\mathrm{V}_2)=0$
Pressure $(\mathrm{P}_2)=\rho g(h+h_1)$
$Height=Z$

$$\frac{P_1}{\rho g}+\frac{V_1{}^2}{2g}+Z_1=\frac{P_2}{\rho g}+\frac{V_2{}^2}{2g}+Z_2$$
$$h_1++\frac{V_1{}^2}{2g}\;Z=h+h_1+0+Z$$
$$V_1=\sqrt{2gh}$$

Note:

(1) The peizometer measures static pressure of fluid flowing in the pipe.

(2) The stagnation tube measures the stagnation pressure at any point in the fluid.

14.4 Pitot tube

Pitot tube is used for the measurement of fluid velocity. The construction of Pitot tube is shown in Fig. 14.3. Outer body of Pitot tube consists of ports at point A, for sensing the static pressure of fluid. At point B, fluid velocity becomes zero and inner tube is for sensing the stagnation pressure.

The two ends of Pitot tube are connected to U-tube manometer for measuring the pressure difference between the points A and B.

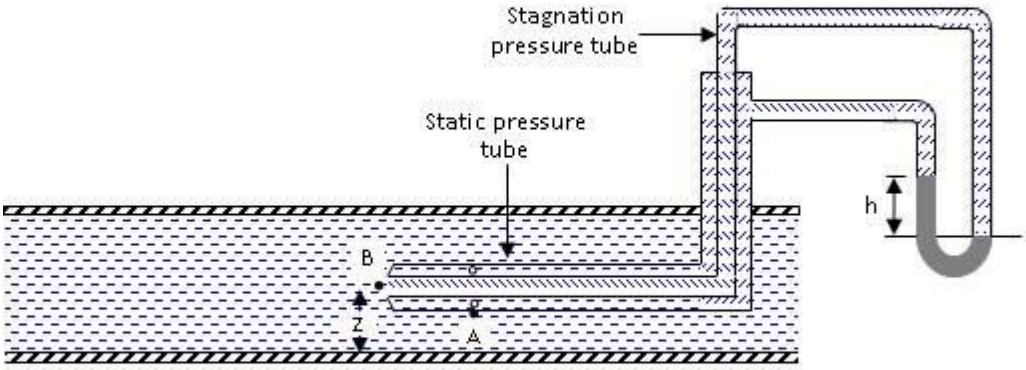


Fig. 14.3 Pitot tube

Note:

- Level of liquid in Pitot tube shows the position of total energy line (TEL). Pitot tube measures the total head and is therefore also known as total head tube.
- Level of liquid in piezometer represents the static pressure of the fluid and the position of hydraulic grade line (HGL).
- Pitot tube can be used to determine velocity distribution across the flow.

14.5 Flow Through Nozzle from Reservoir

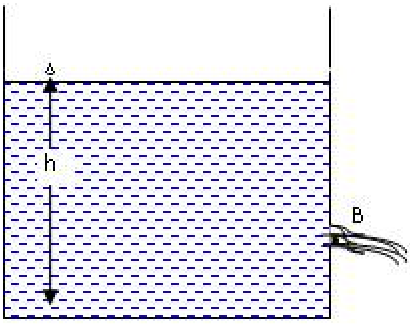


Fig. 14.4 Flow through nozzle from reservoir

Fig 14.4 shows a reservoir fitted with a nozzle on the side. Point A corresponds to free water surface and point B at the tip of nozzle of opening.

At point A

Velocity $V_1 = 0$ (since the top water level is moving very slow compared to velocity of water at B.

Pressure $(P_1) = 0$

Height of point A from datum line = h

At point B

Velocity = V_2

Pressure $(P_2) = 0$

Height of point B from datum line = 0

Apply Bernoulli's Theorem:

$0 + 0 + h = 0 + \frac{V_2^2}{2g} + 0$

$V_2^2 = 2gh$

$V_2 = \sqrt{2gh}$

Lesson 15

HEAD LOSS IN FLUID FLOW - MAJOR HEAD LOSS

15.1 Introduction

Liquids flow in a pipe due to pressure or gravity. In case of pressure flow, if a piezometer is connected to the pipe, the rise of liquid is suppose h meters. The pressure, P and head, h are related by the relation P = ρgh. In SI system of units the pressure is usually measured in terms of Pa (N/m^2) or kPa. In case of liquids it can also be expressed as head i.e. the rise of liquid in the piezometer. When a fluid flows through a pipe, it experiences some resistance due to which some of the energy of fluid is lost. The head loss in fluid flow is classified in two categories namely major head losses and minor head losses. In this module, reasons for such head losses and the method of estimating these head losses are discussed.

15.2 Flow through pipe

There are two cases of flow:

(i) Closed Conduit
(ii) Open Conduit

(i) ***Closed conduit***: It is a pipe or duct through which the fluid flows by completely filling the cross-section. Since, the fluid has no free surface; its pressure may be above or below the pressure of the atmosphere.

(ii) ***Open conduit***: It is a duct or open channel in which fluid flows with the free surface. If a closed pipe not running full, it may be treated as open channel.

15.3 Pressure/Head Loss

a. ***Major head losses in pipe flow:*** The major head losses in fluid flow are caused by friction of the conveying pipeline. The internal surface which comes in contact with the flowing fluid causes friction on the fluid layers. The extent of head loss depends on roughness of the pipeline as well as flow characteristics of the fluid. This head loss takes place continuously in the entire conveying pipeline/duct and it is characterized as major head loss.

b. ***Minor losses in pipe flow*****:** Minor head losses include head loss or pressure drop due to pipe fittings, valves, entrance and exit of pipe, sudden contraction or expansion etc. This head loss is relatively small in case of very long pipeline but it may be high in case of small pipe network involving many fittings and valves.

15.4 Major Losses in Pipe Flow (Friction)

The major head losses can be estimated by the following methods:

(a) Darcy's Formula (flow in pipe)

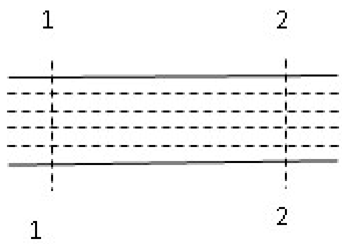


Fig. 15.1 Flow in pipe

Consider a uniform pipeline through which water is flowing at a uniform rate.

Let,

l = length of pipe

d = diameter of the pipe

v = velocity of water

f' = frictional resistance per unit wetted surface per unit velocity

h_f = loss of head due to friction

P_1 = pressure at 1 ---1

P_2 = pressure at 2--- 2

If there is no friction resistance, pressure P_1 and P_2 will be equal. Considering horizontal forces on the water at 1----1 and 2 ---2.

$$A = p_2 A + Frictional\ resistance$$

$$\frac{p_1 A - p_2 A}{\omega} = \frac{Frictional\ resistance}{\omega}$$

(Dividing by ω on both the sides)

$$\therefore \frac{p_1}{\omega} - \frac{p_2}{\omega} = \frac{Frictional\ resistance}{\omega\ A}$$

$$\therefore h_f = \frac{Frictional\ resistance}{\frac{\pi}{4} d^2 \times \omega}$$

It has been established from Froude's experiments that

∴Frictional resistance = Frictional resistance per unit area per unit nelocity × area × v^2

$$\therefore Frictional\ resistance = f' \times \pi dl \times v^2$$

$$\therefore h_f = \frac{f' \pi dl \times v^2}{\frac{\pi}{4} d^2 \times \omega} = \frac{4 f' l v^2}{\omega d}$$

Let us introduce a co-efficient f such that $f = \frac{f\omega}{2g}$

$$\therefore h_f = \frac{4}{\omega d} \times \frac{f\omega}{2g} \times l v^2 = \frac{4 f l v^2}{2gd}$$

This co-efficient is called Darcy's co-efficient or frictional co-efficient.

f is a function of Reynold's number (Re)

$$f = \frac{16}{\text{Re}} \text{ (for Re} < 2100)$$

$$= \frac{0.0791}{(\text{Re})^{\frac{1}{4}}} \text{ (for Re 4000 to} 10^6)$$

$$h_f = \frac{f_1 L V^2}{2gd} \text{ where } f_1 = 4f; f_1 = \text{friction factor}$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{VD}{\mu / \rho}$$

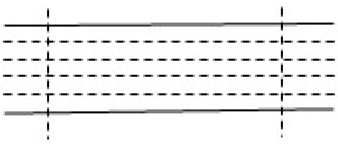
$$= \frac{VD}{\nu}$$

where,

ρ : *density*; V : velocity; D : Diameter of pipe; μ : viscosity of fluid; ν : Kinematic viscosity

(b) Chezy's Formula

Chezy's formula is used to calculate head loss for open conduit like canal, drain etc. Consider a long pipe through which water is flowing at a uniform rate as shown below:



Let, l = length of the pipe

d = diameter of pipe

A = $\pi/4\ d^2$

P = Perimeter, πd

V = velocity

f′ = frictional resistance per unit area per unit velocity

P_1 = pressure at 1 – 1

P_2 = pressure at 2 – 2

We know that $P_1A = P_2A$ + frictional resistance

$$\therefore \frac{P_1 A - P_2 A}{\omega} = \frac{\text{frictional resistance}}{\omega} \qquad [\because \textit{Dividing by } \omega]$$

$$\frac{P_1}{\omega} - \frac{P_2}{\omega} = \frac{\text{frictional resistance}}{\omega A}$$

i.e.

$$h_f = \frac{\text{frictional resistance}}{\pi/4\ d^2 \times \omega}$$

We also know that frictional resistance = $f' \times \pi dl \times V^2$

$$h_f = \frac{f' \times \pi dl \times V^2}{\omega A}$$
$$= \frac{f' \times P \times V^2}{\omega A} \qquad (\because \ \pi d = P)$$
$$= \frac{f' l v^2}{\omega} \times \frac{P}{A}$$
Substituting another term called hydraulic mean depth (also known as hydraulic radius) in the above equation, such that hydraulic mean depth
$$m = \frac{Area\ of\ flow}{Wetted\ perimeter} = \frac{A}{P}$$
$$\therefore \ \ h_f = \frac{f' l v^2}{\omega} \times \frac{1}{m}$$
$$\therefore \ \ V^2 = \frac{h_f . \omega . m}{f' . l} = \frac{\omega}{f'} \times m \times \frac{h_f}{l}$$
$$\therefore \ \ V = \sqrt{\frac{\omega}{f'} \times m \times \frac{h_f}{l}}$$
Now, Substituting two more terms in the above equation such that
$$\sqrt{\frac{\omega}{f'}} = C \ (A\ constant\ known\ as\ Chezy's\ constant)$$
$$and \ \ \frac{h_f}{l} = i \ \ (i.e.loss\ of\ head\ per\ unit\ length)$$
$$\therefore \ \ V = C.\sqrt{mi}$$

15.5 Numericals

Q 1. (a) Using Darcy’s formula calculate head loss due to friction in a pipe of diameter 150 mm and length 50 m. Velocity of water is 2 m/s and co-efficient of friction is 0.004. Kinematic viscosity of water is 0.013 stoke.

(b) In the above numerical calculate the head loss using Chezy’s formula considering C = 45.

Solution:

Given

L = 50 m

D = 0.150 m

V = 2.0 m/s

F = 0.004

v = 0.013 X 10^{-4} m^2/s

$$h_f = \frac{4fLV^2}{2gD}$$

$$= \frac{4 \times 0.004 \times 50 \times 2^2}{2 \times 9.81 \times 0.150} = 1.08732\ m$$

$$(b)\ \mathrm{V} = \mathrm{C}\sqrt{\mathrm{mi}}$$

$$m = D/4;$$

$$i = \frac{hf}{L}$$

$$2 = 45\sqrt{\frac{0.150}{4} \times \frac{h_f}{50}}$$

$$\left(\frac{2}{45}\right)^2 \times \frac{4 \times 50}{0.15} = h_f$$

$$h_f = 2.6337\ m$$

Q 2. In a piping system length & diameter of pipe is 100 m & 300 mm respectively, water is flowing at velocity of 4 m/s.

(a) Calculate head loss using Darcy's formula.
    Given Kinematic viscosity = 0.013 stokes.

(b) Find head loss if Chezy's constant C = 45.

Solution

$$L = 100\ m; D = 0.300\ m; V = 4.0\ m/s; v = 0.013 \times 10^{-4}\ m^2/s; C = 45$$

$$\mathrm{Re} = \frac{\mathrm{VD/v}}{v} = \frac{4 \times 0.3}{0.013 \times 10^{-4}} = 923076.9$$

$$f = \frac{16}{\mathrm{Re}} = \frac{16}{923076.92} = 1.73 \times 10^{-5}$$

$$(a)\ \mathrm{h_f} = \frac{4flv^2}{2gD} = \frac{4 \times 1.73 \times 10^{-5} \times 100 \times 4^2}{2 \times 9.81 \times 0.3} = 0.0188\ m = 1.88\ cm.$$

$$(b)\ \mathrm{V} = \mathrm{C}\sqrt{\mathrm{m.i}}$$

$$m = D/4; \mathrm{i} = \frac{\mathrm{h_f}}{L}$$

$$4 = 45\sqrt{\frac{0.3 \times \mathrm{h_f}}{4 \times 100}}$$

$$\left(\frac{4}{45}\right)^2 = \frac{0.3}{4} \times \frac{\mathrm{h_f}}{100}$$

$$\mathrm{h_f} = \frac{(4)^2 \times 4 \times 100}{(45)^2 \times 0.3} = 10.5349\ m$$

Q 3. Water is pumped from station A to station B in a pipe of length 60 m and diameter 250 mm, at a velocity of 3 m/s. Assuming Kinematic viscosity of water is 0.013 * 10^{-4} m^2/s. Find the frictional loss of head by using.

(a) Darcy's formula

(b) Chezy's formula where C = 60.

Solution:

$L = 60\,m; D = 0.25\,m; V = 3m/s; g = 9.81\,m/s^2;$

$v\,(\text{kinematic viscosity}) = 0.013 \times 10^{-4}\,m^2/s; C = 60.$

$$\text{Re} = \frac{VD}{v} = \frac{3 \times 0.25}{0.013 \times 10^{-4}} = 576923.076$$

$$f = \frac{16}{\text{Re}} = \frac{16}{576923.076}$$

$$(a)\ \text{h}_f = \frac{4fLV^2}{2gD} = \frac{4 \times 2.77 \times 10^{-5} \times 60 \times (3)^2}{2 \times 9.81 \times 0.250} = 0.0121\,m$$

$$(b)\ \text{V} = \text{C}\sqrt{\text{m.i}};\ \text{m} = \text{D/4};\ \text{i} = \text{h}_f / L$$

$$3 = 60\sqrt{\frac{0.25 \times h_f}{4 \times 60}}$$

$$\left(\frac{3}{60}\right)^2 = \frac{0.25}{4} \times \frac{h_f}{60}$$

$$h_f = \frac{(3)^2 \times 4 \times 60}{(60)^2 \times 0.25} = 2.4\,m$$

Lesson 16
HEAD LOSS IN FLUID FLOW: MINOR HEAD LOSS

16.1 Introduction

As discussed in the earlier lecture, minor losses include head loss or pressure drop due to pipe fittings, entrance and exit of pipe, sudden contraction or expansion etc. These are considered as minor because pressure drop is small compared to the major or frictional head loss. Minor Head losses are:

1. Head loss at the entrance of pipe
2. Head loss at the exit of pipe
3. Head loss due to obstruction in pipe
4. Head loss due to sudden contraction in pipe
5. Head loss due to sudden expansion in pipe
6. Head loss due to pipe fittings

16.2 Head Loss at the Entrance of Pipe (h_{en})

Point A shows the place where liquid is entering the piping system from the tank (Fig. 16.1).

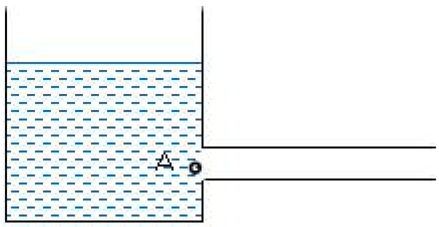
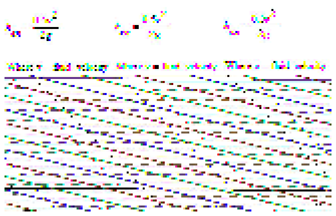


Fig. 16.1 Head loss at the entrance of pipe

The head loss due to entrance is given as:



16.3 Head Loss at Exit from Pipe (h_{ex})

Head loss at the pipe exit (point A) where liquid gets freely discharged into atmosphere is given as:

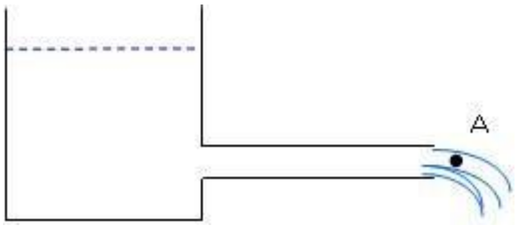


Fig. 16.2 Head loss at exit from pipe

$$h_{ex} = \frac{v^2}{2g}$$

Where,

V = velocity

16.4 Head Loss Due to Obstruction in Pipe (h_{obs})

If an irregular obstruction is placed in the pipe it will cause pressure drop. Figure 16.3 shows an irregular object placed in the path of flow.

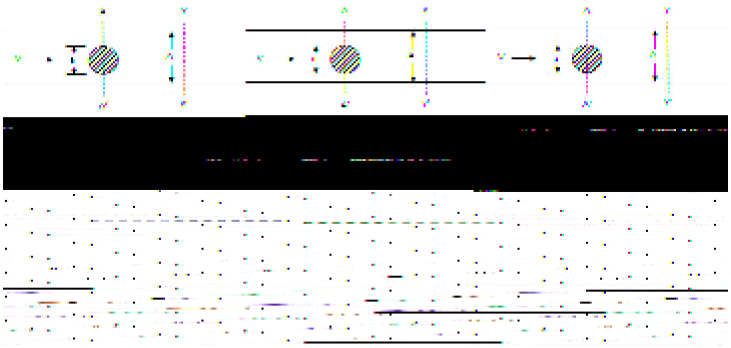
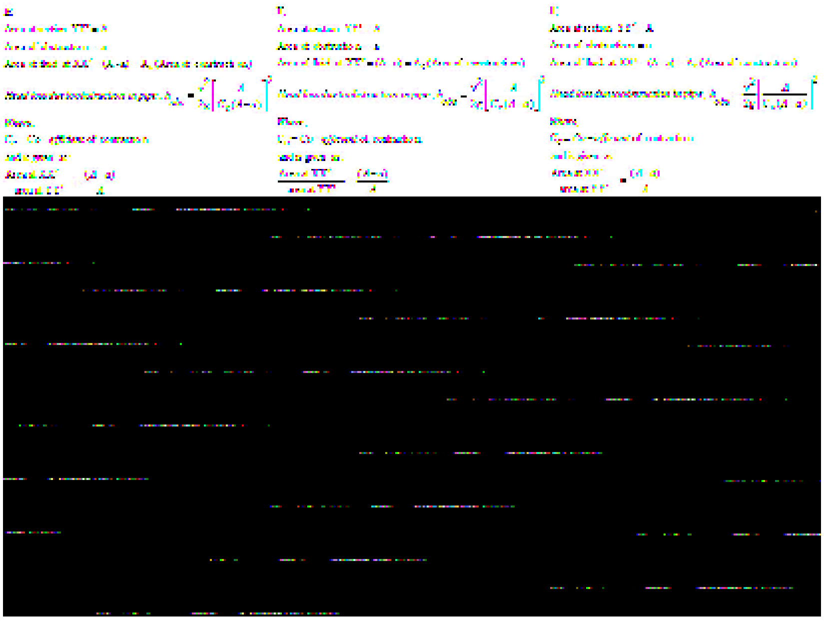


Fig. 16.3 Head loss due to obstruction in pipe



16.5 Head Loss Due to Sudden Contraction in Pipe (h_c)

After sudden contraction the streamlines contract to pass through a minimum cross sectional area (A_c) and the fluid stream widens to fill the pipe. This minimum cross section is known as vena contracta and A_c is the area of cross section at vena contracta.

	At Section 1-1´	At section 2-2´ vena contracta	At section 3-3´
Diameter of Pipe	$= d_1$	$= d_2$	$= d_2$
Area of pipe	$= A_1$	$= A_c$	$= A_2$
Flow velocities	$= V_1$	$= V_c$	$= V_2$

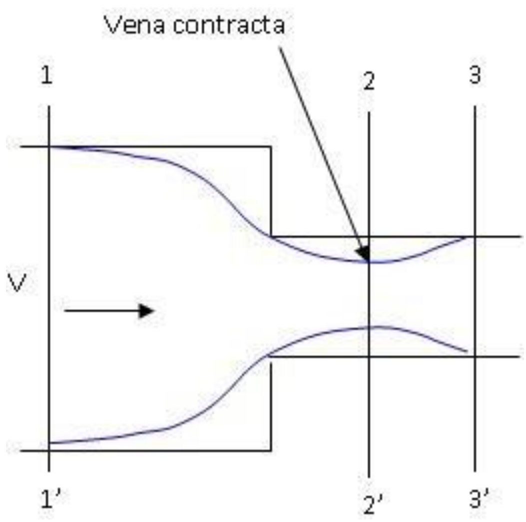


Fig. 16.4 Sudden contraction and formation of vena contracta

Formation of eddies takes place between the vena contract and pipe wall.

Head loss due to sudden contraction=head loss upto vena contracta + head loss due to sudden enlargement.

$$\mathrm{h}_c = 0 + \frac{(V_c - V_2)^2}{2g}$$

$$\mathrm{h}_c = \left(\frac{V_c - V_2}{2g}\right)^2 \qquad \dots (i)$$

Velocities of flow at vena contracta=V_C

Cross sectional area of vena contracta = A_c

Velocities of flow at section 2-2´ = V_2

Cross Sectional area at section 2-2´ = A_2

$$V_c A_c = V_2 A_2$$

$$V_c = \frac{V_2}{(A_c/A_2)} = \frac{V_2}{C_c} \qquad \dots (ii)$$

Where C_c= Coefficient of contraction

From equation (i) and (ii)

$$h_{sc} = \frac{\left(\frac{V_2}{C_c} - V_2\right)^2}{2g}$$

$$= \frac{V_2{}^2}{2g}\left(\frac{1}{C_c} - 1\right)^2$$

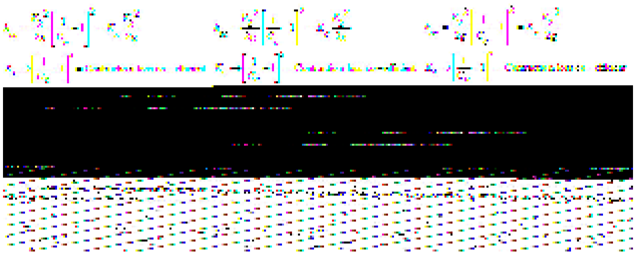
$$h_{sc} = K_c \frac{V_2{}^2}{2g}$$

Where k_c = Contraction loss coefficient

$$= \left(\frac{1}{C_c} - 1\right)^2$$

In general value of K_c may be taken as 0.5

$$h_c = 0.5 \frac{V_2{}^2}{2g}$$



16.6 Head Loss Due to Sudden Expansion in Pipe (h_e)

For fig 16.5 we have the following assumption:

	At Section 1-1´	At section 2-2´
Diameter of Pipe	$= d_1$	$= d_2$
Area of pipe	$= A_1$	$= A_2$
Flow velocities	$= V_1$	$= V_2$
Pressure	$= p_1$	$= P_2$

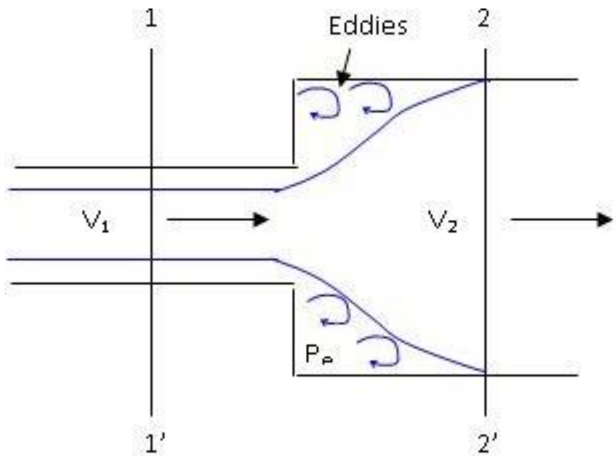


Fig. 16.5 Sudden enlargement

Due to sudden enlargement turbulent eddies are formed at the corner of the enlargement of the pipe section. The formation of eddies cause loss of energy in form of heat to the surrounding. Let P_e be the pressure of the eddying fluid h_e be the head loss due to enlargement.

Applying Bernoulli's theorem across section 1-1´ and 2-2´.

$$\frac{p_1}{\rho g} + \frac{V_1^{\ 2}}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{V_2^{\ 2}}{2g} + Z_2 + h_e$$

Since $Z_1 = Z_2$

$$h_e = \frac{p_1 - p_2}{\rho g}$$

$$+ \left(\frac{V_1^{\ 2}}{2g} - \frac{V_2^{\ 2}}{2g}\right) \qquad \ldots (i)$$

Net force acting between section 1-1´ and 2-2´

$$F = p_1 A_1 - P_2 A_2 + P_e (A_2 - A_1)$$

Since mean pressure due to the formation of eddies (P_e) is approximately equal to the inlet pressure (P_1),

$$p_e \cong p_1$$

$$F = P_1 A_1 - P_2 A_2 - P_1 (A_2 - A_1)$$

$$F = (P_1 - P_2)A_2 \qquad \dots(ii)$$

Let Q be the discharge m^3/sec through the pipe.

Change of momentum of liquid pressure

$$= \rho Q(V_2 - V_1)$$

from continuity equation $Q=A_2V_2$

$$= \rho A_2 V_2 (V_2 - V_1) \qquad \dots(iii)$$

Since net force = change of momentum of liquid per second

from equation (ii) and (iii)

$$(p_1 - p_2)A_2 = \rho A_2 V_2 (V_2 - V_1)$$

$$(p_1 - p_2) = \rho(V_2{}^2 - V_1 V_2) \qquad \dots(iv)$$

from equation (i) and (iv)

$$h_e = \frac{V_2{}^2}{g} - \frac{V_1 V_2}{g} + \frac{V_1{}^2}{g} - \frac{V_2{}^2}{2g}$$

$$= \frac{V_1{}^2 + V_2{}^2 - 2V_1 V_2}{2g}$$

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

16.7 Head Loss Due to Pipe Fittings (h_{pf})

Various pipe fittings like bend, elbow, tee etc also contribute to pressure drop. The head loss due to pipe fittings is given by the following equation:

$$h_{pf} = K_L \frac{v^2}{2g}$$

K_L= Friction loss co-efficient/ loss co-efficient, depend on the shape, size and type of pipe fittings.

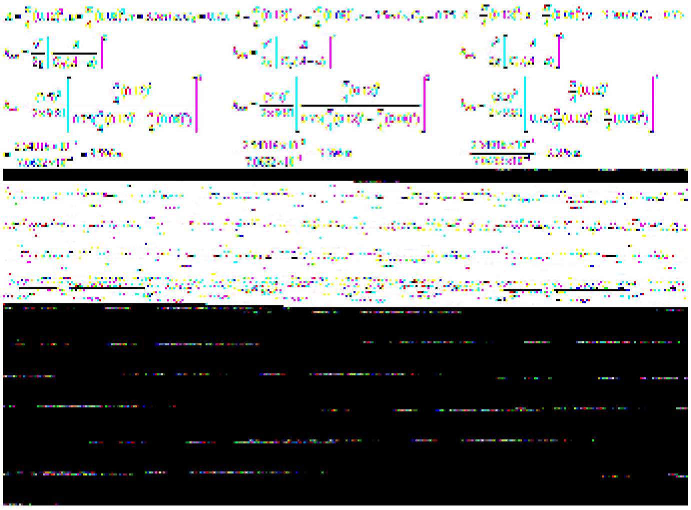
TYPE	K_L
Globe Valve	10
Gate valve(wide open)	0.2
45° Elbow	0.4

Bend	2.2

16.8 Numericals

Q 1. The velocity of water flowing through a 12 cm diameter pipe was found to be 3.5 m/s. The flow path in pipe is destructed by an iron plate of 8 cm diameter. Calculate, the loss of head due to obstruction if co-efficient of contraction $C_c = 0.75$.

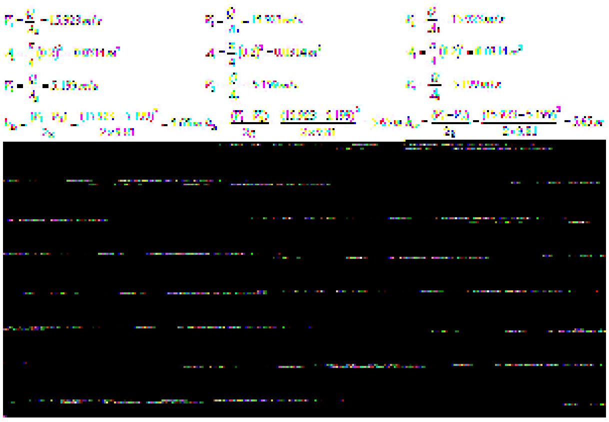
Solution:



Q 2. At a sudden enlargement of water line from diameter of 200 mm to 350 mm, volumetric flow rate 0.5 m^3 /s. Determine the head loss due to sudden enlargement.

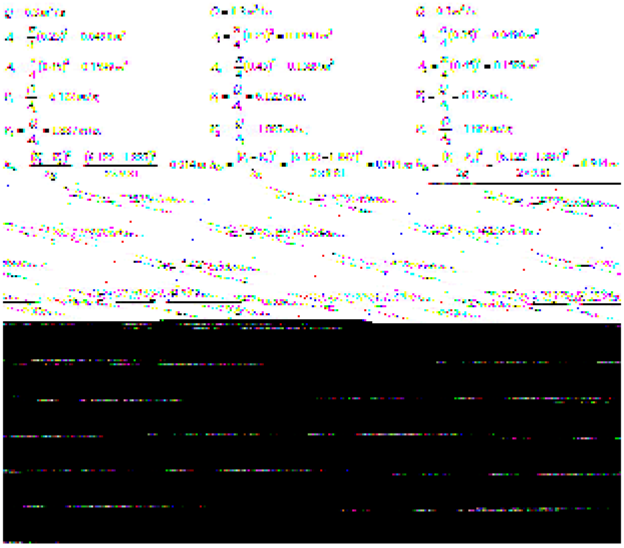
Solution:

D_1 = 200 mm; D_2 = 350 mm; Q = 0.5 m^3/s



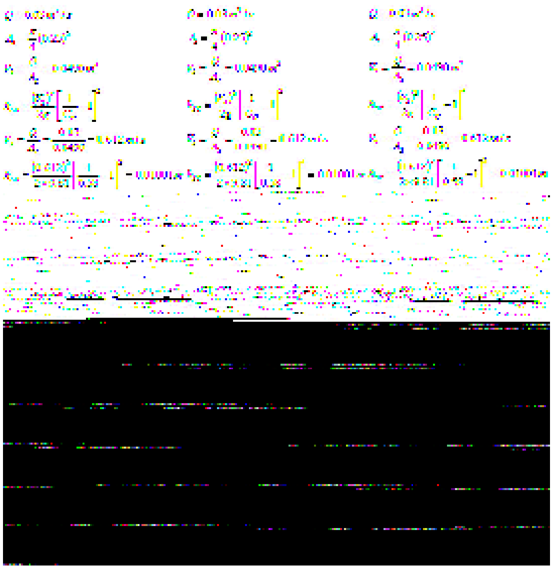
Q 3. A horizontal water pipe has an abrupt enlargement such that the diameters of small and large cross-section are 250 mm and 450 mm respectively. The volumetric flow rate in pipe is 0.3 m^3/s. Find the head loss due to abrupt enlargement.

Solution:



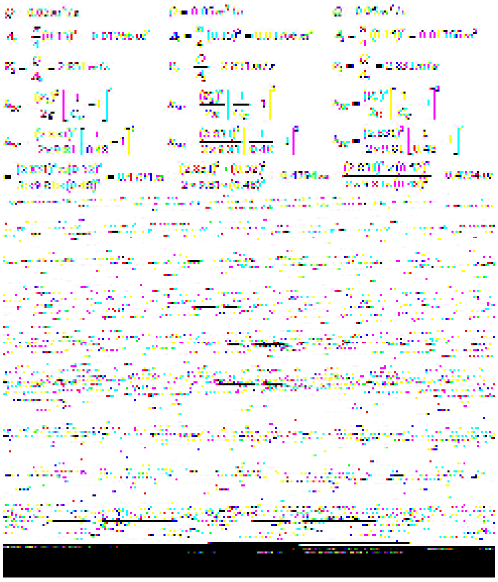
Q 4. At a sudden contraction of water line from diameter of 350 mm to 250 mm and volumetric flow rate 0.03 m^3/s. Determine the head loss due to contraction. Consider co-efficient of contraction as 0.58.

Solution:



Q 5. The dimension of 350 mm diameter horizontal pipe is suddenly reduced to diameter of 150 mm. If water is flowing at a rate of 0.05 m^3/s and co-efficient of contraction is 0.48 then determine head loss between two sections of the pipe.

Solution:



Lesson 17
PROBLEMS ON HEAD LOSS

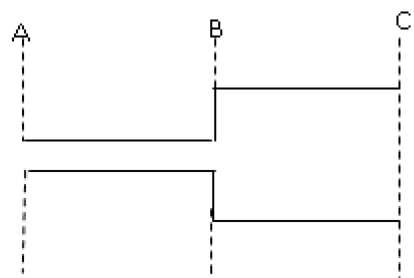
17.1 Numericals

Q 1. Discharge through a pipe line is 0.05 m3/s. The diameter of pipe at section AB is 10 cm which suddenly changes to 15 cm in section BC. The co-efficient of friction is 0.05 for both the pipes; determine the total head loss from A to C.

Solution:

$$Q = 0.05 m^3/s$$

Area	Section	Length	Diameter
$A_1 = \frac{\pi}{4}(0.1)^2$	AB	L_1	$D_1 = 0.10$ m
$A_2 = \frac{\pi}{4}(0.15)^2$	BC	L_2	$D_1 = 0.15$ m



$$\text{Total loss} = \text{Major loss} + \text{Minor loss}$$

$$V_1 = \frac{Q}{A_1} = 6.369\, m/s;\ V_2 = \frac{Q}{A_2} = 2.831\, m/s$$

$$h_f = \frac{4fLV^2}{2gD};\ h_{Se} = \frac{(V_1 - V_2)^2}{2g};\ f = 0.05$$

$$\text{Total head loss (h)} = \frac{4fL_1V_1^2}{2gD} + \frac{(V_1 - V_2)^2}{2g} + \frac{4fL_2V_2^2}{2gD_2}$$

$$h = \frac{4\times0.05\times5\times(6.36)^2}{2\times9.81\times0.1} + \frac{(6.36-2.83)^2}{2\times9.81} + \frac{4\times0.05\times10\times(2.83)^2}{22\times9.81\times0.15}$$

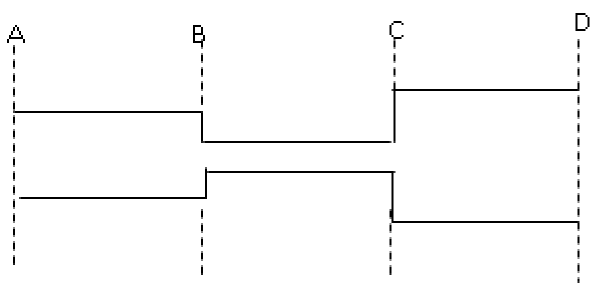
$$= 20.229 + 0.635 + 5.442$$

$$= 26.3066\, m$$

Q 2. Diameter and length of the different sections of a pipe line is given as follows:

Junctions B and C are cases of sudden contraction & enlargement respectively. If the flow rate is 0.01 m3/s, determine total head loss in entire length of pipe. Contraction loss co-efficient is 0.45.

Section	Length	Diameter	Co-efficient of friction
AB	$L_1 = 2.5$	$D_1 = 0.15$ m	0.02
BC	$L_2 = 5$	$D_2 = 0.10$ m	0.02
CD	$L_3 = 2.5$	$D_3 = 0.20$ m	0.02



$$A_1 = \frac{\pi}{4}(0.15)^2;\ A_2 = \frac{\pi}{4}(0.10)^2 \text{ and } A_3 = \frac{\pi}{4}(0.20)^2$$

$$V_1 = \frac{Q}{A_1}; V_2 = \frac{Q}{A_2}; V_3 = \frac{Q}{A_3}$$

$$\text{Total head loss (h)} = \frac{4fL_1V_1^2}{2gD_1} + K_C\frac{V_2^2}{2g} + \frac{4fL_2V_2^2}{2gD_2} + \frac{(V_2 - V_3)^2}{2g} + \frac{4fL_3V_3^2}{2gD_3}$$

$$h = \frac{4\times0.02\times2.5\times(0.56)^2}{2\times9.81\times0.15} + \frac{0.45\times(1.27)^2}{2\times9.81} + \frac{4\times0.02\times5\times(1.27)^2}{2\times9.81\times0.10} + \frac{(1.27)^2}{2\times9.81}$$

$$+\frac{4\times0.02\times2.5\times(0.32)^2}{2\times9.81\times0.20}$$

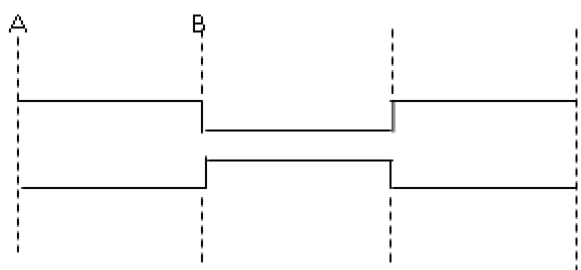
$$h = 0.021 + 0.036 + 0.328 + 0.045 + 0.005$$

$$h = 0.435\ m$$

Q 3. Diameter & length of different sections of pipe line connected to a large tank given as below. There is an abrupt expansion & contraction at junction B & C. Water is discharged directly into open at end of pipe at D. Determine the height of water surface above the discharge point to get a velocity of 2.5 m/s at the end of pipe. Take contraction loss co-efficient as 0.45.

Section	Length	Diameter	Co-efficient of friction
AB	$L_1 = 10$ m	$D_1 = 0.05$ m	0.005
BC	$L_2 = 20$ m	$D_2 = 0.10$ m	0.005
CD	$L_3 = 15$ m	$D_3 = 0.05$ m	0.005

Solution:



Section	Co-efficient	Area	Velocity
AB	0.005	$A_1=\frac{\pi}{4}(D_1)^2$	$V_1 = Q/A_1$
BC	0.005	$A_2=\frac{\pi}{4}(D_2)^2$	$V_2 = Q/A_2$
CD	0.005	$A_3=\frac{\pi}{4}(D_3)^2$	$V_3 = 2.5$ m/s

$$Q = V_3A_3 = 2.5 * \frac{\pi}{4}(0.05)^2$$

$$= 4.90 \text{ X } 10^{-3} \text{ m}^3/\text{s}$$

Height of water surface above discharge point = Total head loss.

Total head loss

$$= \frac{0.5V_1^2}{2g} + \frac{4fL_1V_1^2}{2gD_1} + \frac{(V_1 - V_2)^2}{2g} + \frac{4fL_2V_2^2}{2gD_2} + K_C\frac{V_3^2}{2g} + \frac{4fL_3V_3^2}{2gD_3} + \frac{V_3^2}{2g}$$

$$h = \frac{0.5\times(2.5)^2}{2\times9.81} + \frac{4\times0.005\times10\times(2.5)^2}{2\times9.81\times0.05} + \frac{(2.5-0.60)^2}{2\times9.81} + \frac{4\times0.005\times20\times(0.60)^2}{2\times9.81\times0.10}$$

$$+ 0.45\times\frac{(2.5)^2}{2\times9.81} + \frac{4\times0.005\times15\times(2.5)^2}{2\times9.81\times0.05} + \frac{(2.5)^2}{2\times9.81}$$

$$h = 0.159 + 1.274 + 0.184 + 0.073 + 0.143 + 1.911 + 0.318 == 4.062\ m$$

Lesson 18

DETERMINATION OF PIPE DIAMETER, DETERMINATION OF DISCHARGE, FRICTION FACTOR, CRITICAL VELOCITY

18.1 Piping Systems

a. Pipes connected in series

$$\text{Pipe in Series} : Q(\text{discharge}) = Q_1 = Q_2 = Q_3$$

$$\text{Head loss } (h) = h_{\text{Pipe }1} + h_{\text{Pipe }2} + h_{\text{Pipe }3}$$

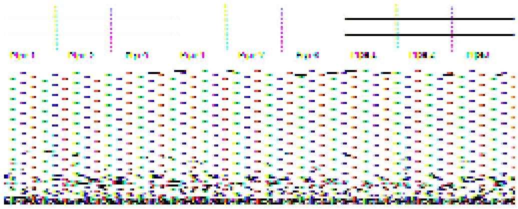


Fig. 18.1 Pipes connected in series

b. Pipes connected in Parallel

$$\text{Pipe in parallel} : Q = Q_1 + Q_2 + Q_3$$

If co-efficient of friction, diameter and length of pipes are same for pipe 1,2 and 3

$$h_{AB} = h_{\text{Pipe }1} = h_{\text{Pipe }2} = h_{\text{Pipe }3}$$

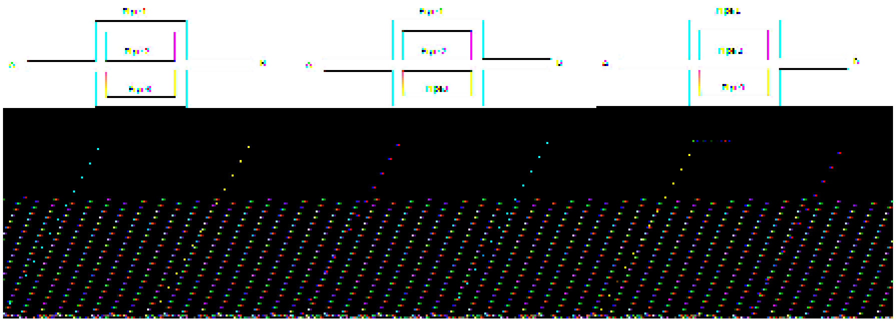


Fig. 18.2 Pipes connected in parallel

18.2 Dupit's Equation for Equivalent Pipe

Equivalent pipe

If several pipes of different lengths and diameter are connected in series, it can be replaced by single pipe called as equivalent pipe. This equivalent pipe of same diameter will have the same loss head and discharge which several pipes connected in series will have.

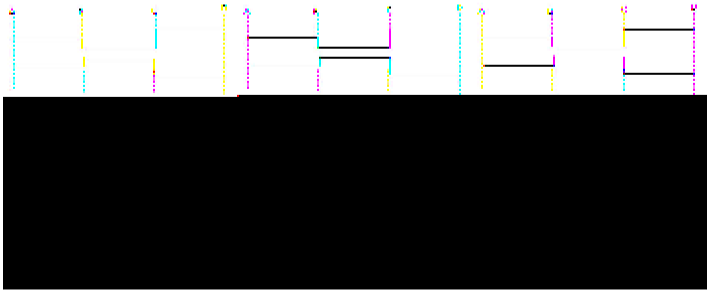


Fig. 18.3 Pipes of different lengths and diameters connected in series

Equivalent diameter

The uniform diameter of the equivalent pipe is the equivalent diameter series or compound pipe.

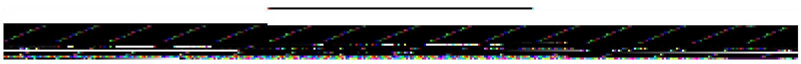


Fig. 18.4 Equivalent pipe

Equivalent length

The length of equivalent pipe which has the same head loss & discharge that of series or compound pipe.

Assumption for analysis

Neglect the minor losses (frictional loss) and consider only the major head losses (friction loss) since $h_{minor} << h_{major}$

$$h = \frac{C f L e Q^2}{D_{eq}^5} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (v)$$

On equating (iii) & (iv):

$$\frac{C f L e Q^2}{D_{eq}^5} = C f Q^2 \left[\frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} \right]$$

$$\frac{Le}{D_{eq}^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} + \ldots\ldots\ldots\ldots\ldots\ldots ()$$

Eq. (*v*) **is known as Dupit's eq. for equivalent pipe.**

Case1. Sometimes length of equivalent pipe is taken to be equal to the length of compound pipe i.e.

$$Le = L_1 + L_2 + L_3 + L_4 + \ldots\ldots\ldots$$

In such cases diameter of equivalent pipe D_{eq}, can be calculated from Dupit's equation.

Case2. Sometimes value of equivalent diameter is given & length of equivalent pipe requires replace compound pipe has to be determined by Dupit's equation.

18.3 Numericals

Q 1. Three pipes are connected in series

Pipe	Length (m)	Diameter (cm)
l_1	1000	50
l_2	750	40
l_3	500	30

(a) Calculate equivalent diameter considering the length equivalent pipe to be equal in that of compound pipe.

(b) Determine the length of equivalent pipe for an equivalent diameter of 40 cm.

Solution

(a)

$$l_e = l_1 + l_2 + l_3 = 1000 + 750 + 500 = 2250\ m$$

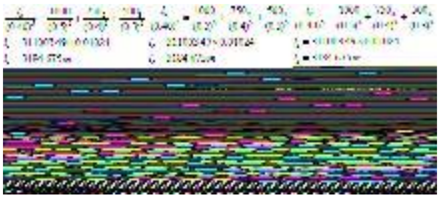
$$\frac{l_e}{D_{eq}{}^5} = \frac{l_1}{D_1^5} + \frac{l_2}{D_2^5} + \frac{l_3}{D_3^5}$$

$$\frac{2250}{D_{eq}{}^5} = \frac{1000}{(0.5)^5} + \frac{750}{(0.4)^5} + \frac{500}{(0.3)^5} = 205761.3169$$

$$D_{eq}{}^5 = 0.00723$$

$$D_{eq} = 0.610\ m$$

(b)



Q 2. Consider pipe 1, 2, 3 connected in series of length l_1, l_2, l_3 respectively. D_1, D_2, D_3 with a diameter of pipe 1, 2, 3.

Pipe	Length (m)	Diameter (cm)
l_1	1500	60
l_2	750	50
l_3	450	40

(a) Determine the eq. length for an eq. diameter of 50 cm.

(b) Determine the eq. diameter if the length of eq. pipe is equal to the length of compound pipe.

Solution

(a)

$$\frac{l_e}{D_{eq}^{\ 5}} = \frac{l_1}{D_1^{\ 5}} + \frac{l_2}{D_2^{\ 5}} + \frac{l_3}{D_3^{\ 5}}$$

$$\frac{l_e}{(0.5)^5} = \frac{1500}{(0.6)^5} + \frac{750}{(0.5)^5} + \frac{450}{(0.4)^5}$$

$$l_e = 87235.43 \times 0.03125 = 2726.107\ m$$

(b)

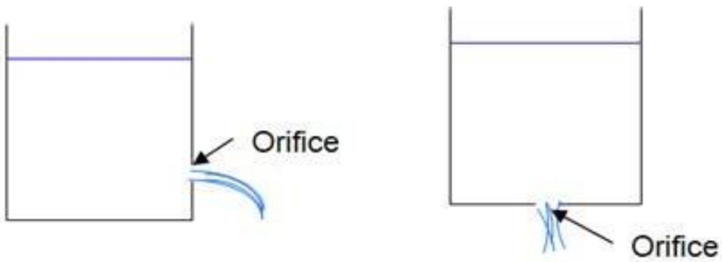
Lesson 19

ORIFICES, VENA CONTRACTA, HYDRAULIC COEFFICIENTS

19.1 Introduction

In engineering, in particular fluid dynamics and hydrometry, the volumetric flow rate, (also known as volume flow rate, rate of fluid flow or volume velocity) is the volume of fluid which passes through a given surface per unit time. The SI unit is m^3/s (cubic meters per second). It is usually represented by the symbol Q. Flow rate can be measured by orifice, notches, weirs etc. which will be discussed in this module.

19.2 Orifice



a. Orifice at the side of vessel wall b. Orifice at the bottom of vessel

Fig. 19.1 Orifice

An orifice is an opening in the side or bottom of a vessel/tank to measure the discharge. Fig. 19.1 shows an orifice.

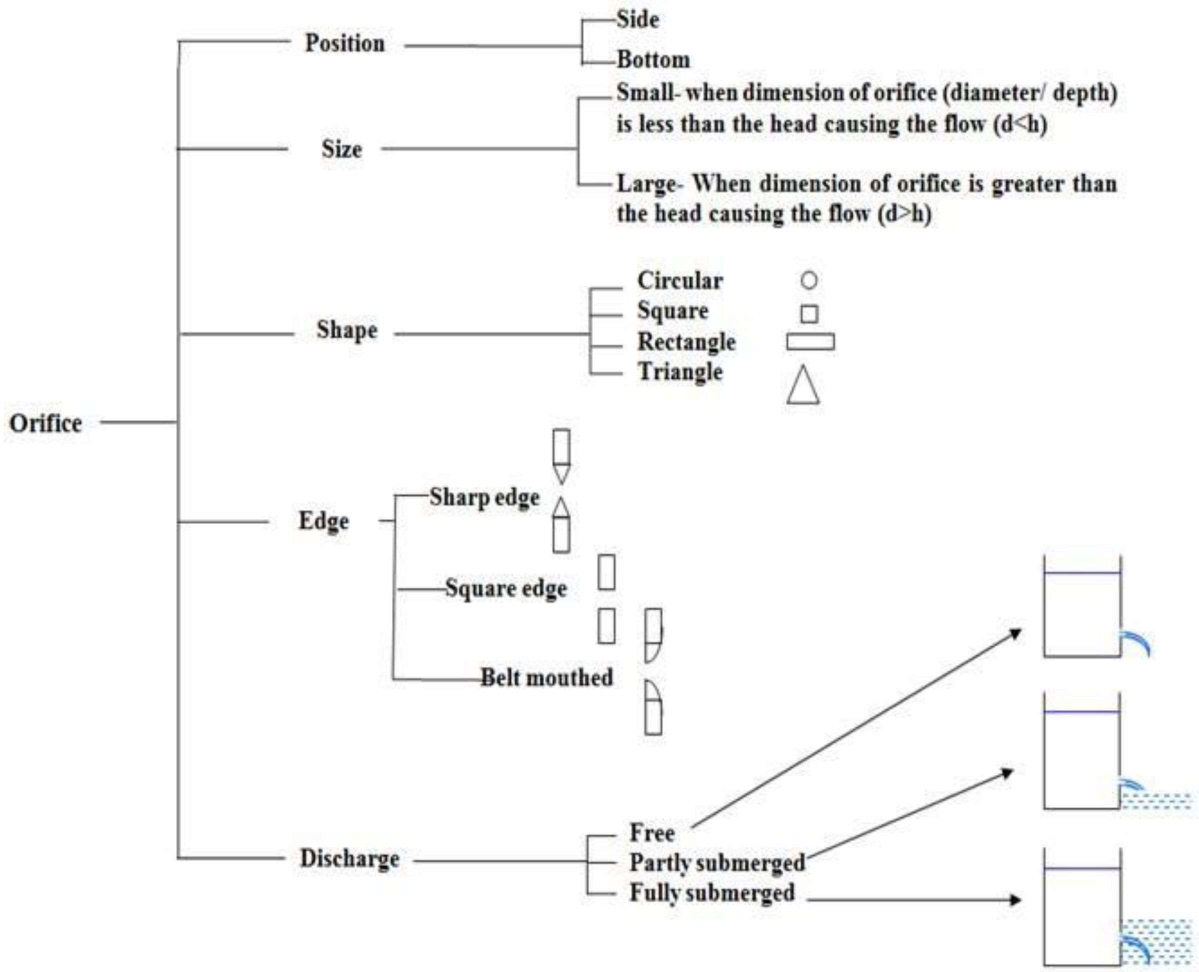


Fig. 19.2 Classification of orifice

19.3 Vena Contracta

Suppose a circular sharp edged orifice is in side of tank which discharges liquid directly into atmosphere (Fig. 19.3). Water jet from the orifice converges to minimum cross-section at XX' and then diverges again. The section at which the cross-sections area of jet is minimum & less than that of orifice is known as Vena Contracta.

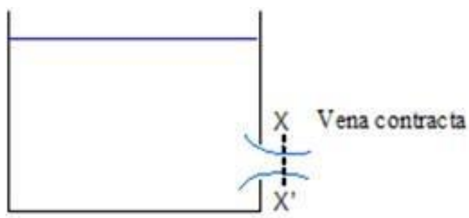


Fig. 19.3 Minimum cross section at XX' is known as vena contracta

19.4 Flow Velocity at Vena Contracta

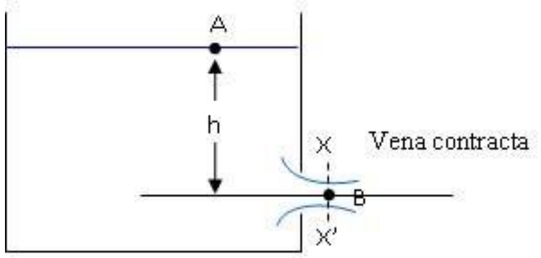


Fig 19.4 Vena contracta

At point A	At point B
$p_A = 0$	$p_B = 0$
$V_A = 0$	$V_B = V_B$
$Z_A = h$	$Z_B = 0$

From Bernoulli's theorem:

$$\frac{p_1}{\rho g} + \frac{{v_1}^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{{v_2}^2}{2g} + Z_2$$

Placing values at points A and B

$$\frac{{V_B}^2}{2g} = h$$

$$V_B = \sqrt{2gh}$$

Above equation is known as Torricelli's theorem.

19.5 Hydraulic Coefficient

1. Coefficient of velocity C_v
2. Coefficient of contraction C_c
3. Coefficient of discharge C_d
4. Coefficient of resistance C_r

$$C_v = \frac{actual\ velocity\ of\ jet}{Theoretical\ velocity} = \frac{V_a}{\sqrt{2gh}}$$

$$C_c = \frac{actual\ area\ at\ vena\ contracta}{Area\ of\ orifice} = \frac{A_c}{A}$$

$$C_d = \frac{actual\ discharge}{Theoretical\ discharge}$$

$$v = \sqrt{2gh}$$

Actual velocity < theoretical velocity.

So, value of all coefficients < 1.

$$C_r = \frac{head\ loss\ at\ orifice}{head\ at\ orifice}$$

Lesson 20

DISCHARGE LOSSES, TIME FOR EMPTYING A TANK

20.1 Discharge Losses

In an orifice actual discharge is always less than the theoretical discharge due to the following losses

a. Energy loss due to fluid viscosity

b. Formation of vena contracta, thus it reduces the area of flow.

Such losses are expressed using coefficient of discharge C_d. Coefficient of discharge for an orifice is the ratio of actual discharge to theoretical discharge.

$$C_d = \frac{actual\ discharge}{Theoretical\ discharge}$$

20.2 Time for Emptying a Tank

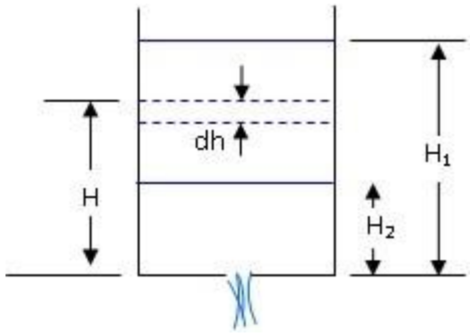


Fig. 20.1 Tank emptied from an orifice in the bottom

Area of orifice = a

Area of tank = A

Suppose the liquid level from H_1 falls to H_2 in time T seconds.

Suppose in time dt the fall in level is dh.

Volume of fluid discharged in time dt = (A dh)....................................(1)

Volume of fluid discharged from orifice = $\left(C_d\ a\sqrt{2gh}\right)dt$

Equating both equations

- (A dh) = $\left(C_d\ a\sqrt{2gh}\right)dt$

[-ve sign is from the fact that the head on the orifice decreases with increasing time]

$$\int_0^T dt = \int_{H_1}^{H_2} \frac{-(\mathrm{A\,dh})}{\left(C_d\ a\sqrt{2gh}\right)}$$

$$T = \frac{-(\mathrm{A})}{\left(C_d\ a\sqrt{2g}\right)} \int_{H_1}^{H_2} \frac{h^{\frac{-1}{2}}dh}{1/2}$$

$$T = \frac{-(A)}{\left(C_d\, a\sqrt{2g}\right)}\left[\frac{h^{\frac{-1}{2}}}{\frac{1}{2}}\right]_{H_1}^{H_2}$$

$$T = \frac{-2A}{\left(C_d\, a\sqrt{2g}\right)}\left[\sqrt{H_2} - \sqrt{H_1}\right]$$

$$T = \frac{2A}{C_d\, a\sqrt{2g}}\left[\sqrt{H_1} - \sqrt{H_2}\right]$$

When tank is empty $H_2 = 0$

$$T = \frac{2A}{C_d\, a\sqrt{2g}}\left[\sqrt{H_1}\right]$$

Note: The above equation is also applicable in case where the orifice is at the side wall of the tank. Head is measured from the centre of the orifice to the free surface of the liquid.

20.3 Time of Emptying a Circular Horizontal Tank

Consider a circular horizontal tank filled with liquid and having an orifice at its bottom.

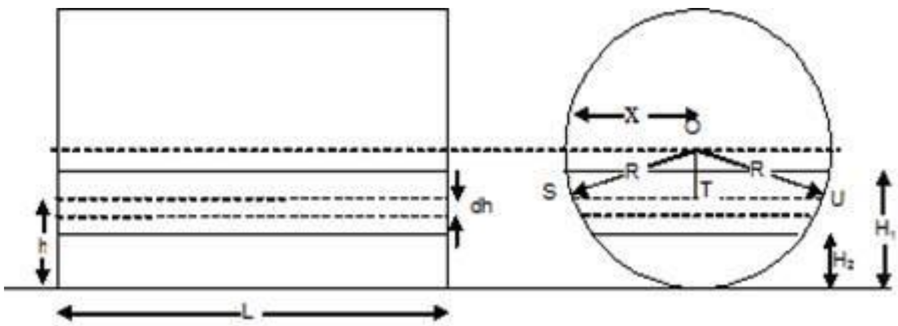


Fig. 20.2 Horizontal circular tank emptied from an orifice in the bottom

R = radius of the tank; L = length of the tank;

H_1 = initial height of the liquid; H_2 = final height of the liquid

a = area of orifice

T = time in seconds for the liquid to fall from height H_1 to H_2

Suppose at any instance height of the liquid in tank is h and there is decrease by dh distance in time dt. If x is the radius of liquid surface, at distance h from the orifice.
Volume of liquid leaving the tank in time dt.

$$= A\, dh = SU \times L \times dh$$

($\because$ *The distance* $SU = 2x$)

$$= 2x\, L\, dh \quad \text{.......................................(i)}$$

Flow velocity through the orifice = $\sqrt{2gh}$

Discharge through orifice in time dt,

$$= C_d \times \text{area of orifice} \times \text{flow velocity} \times dt$$

$$= C_d\, a\sqrt{2gh}\ dt \quad \text{.......................................(ii)}$$

Since volume of liquid leaving the tank = volume of liquid flowing through the orifice

Equating (i) & (ii)

$$-2xLdh = C_d a\sqrt{2gh}\, dt$$

Negative sign in above equation is because of decrease in head on the orifice with increasing time.

$$dt = \frac{-2xLdh}{C_d a\sqrt{2gh}} \quad \text{.......................................(iii)}$$

From the geometry of horizontal circular tank,

OS = R and OT = (R-h)

$$x = ST = \sqrt{(OS)^2 - (OT)^2} = \sqrt{R^2 - (R-h)^2 =} \sqrt{2Rh - h^2}$$

Substituting this value of x in (iii)

$$dt = \frac{-2\sqrt{2Rh - h^2}Ldh}{C_d a\sqrt{2gh}}$$

Total time T to empty tank from height H_1 to H_2 can be calculated by integrating above expression between H_1 and H_2.

$$\int_0^T dt = \frac{-2L}{C_d a\sqrt{2g}} \int_{H_1}^{H_2} \sqrt{2Rh - h^2}\, h^{-1/2}\, dh$$

$$T = \frac{-2L}{C_d a\sqrt{2g}} \int_{H_1}^{H_2} \sqrt{2R - h}\; dh$$

$$T = \frac{-2L}{C_d a\sqrt{2g}} \left[\frac{2(R-h)^{3/2}}{3/2} \times (-1)\right]_{H_1}^{H_2}$$

$$T = \frac{-2L}{C_d a\sqrt{2g}} \left[\frac{2}{2}(2R-h)^{3/2}\right]_{H_1}^{H_2}$$

$$T = \frac{4L}{3C_d a\sqrt{2g}}\left[(2R - H_2)^{3/2} - (2R - H_1)^{3/2}\right] \quad \text{................(iv)}$$

Case 1: Time for emptying the tank completely ($H_2 = 0$)

From (iv)

$$T = \frac{4L}{3C_d a\sqrt{2g}}\left[(2R)^{3/2} - (2R - H_1)^{3/2}\right]$$

Case 2: Tank is half-full at the beginning and is being emptied

$H_1 = R$ and $H_2 = 0$

From (iv)

$$T = \frac{4L}{3C_d a\sqrt{2g}}\left[(2R)^{3/2} - (R)^{3/2}\right] = \frac{4LR^{3/2}}{3C_d a\sqrt{2g}}(2\sqrt{2} - 1)$$

$$T = 0.55L\frac{R^{3/2}}{C_d a}$$

Lesson 21

EXTERNAL AND INTERNAL MOUTHPIECE

21.1 Mouthpiece

Mouthpiece is an extended form of orifice in which a tube or pipe is attached to the orifice. The length of pipe attached to the orifice is 2 to 3 times diameter of orifice (Fig. 21.1). A mouthpiece is used to measure discharge.

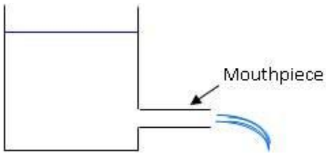


Fig. 21.1 Mouthpiece

21.2 Classification of Mouthpiece

Mouth piece can be classified as follows:

a. On basis of position

i. Internal: Pipe is fixed inside the tank/vessel
ii. External: Pipe is fixed and projected outside the tank walls

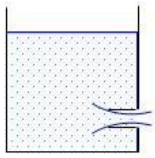
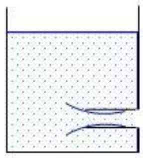
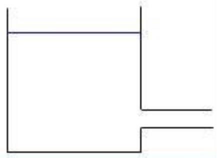
b. Flow pattern

i. Running free: Water jet after contraction in mouthpiece does not touches pipe internal walls.
ii. Running full: Water jet after contraction in mouthpiece touches pipe internal walls.

c. Shape of mouthpiece

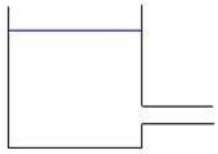
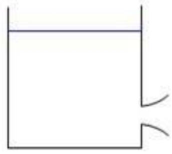
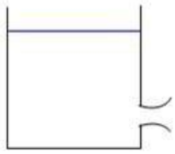
According to the shape of the mouthpiece cylindrical, converging, converging-diverging etc

Table 21.1 Classification of mouthpiece on basis of position

Classification	Type	Discharge	Diagram
1. Basis of position	a. Internal (Running free)	$Q = 0.5\ a\sqrt{2gh}$	
	b. Internal (Running full)	$Q = 0.707\ a\sqrt{2gh}$	
	c. External	$Q = 0.855\ a\sqrt{2gh}$	

***** a = Area of mouthpiece

Table 21.2 Classification of mouthpiece on basis of shape

Classification	Type	Discharge	Diagram
2. Basis of shape	a. Cylindrical	$Q = C_d\, a\sqrt{2gh}$	
	b. Convergent	$Q = C_d\, a\sqrt{2gh}$	
	c. Convergent-divergent	$Q = a_c\sqrt{2gh}$ Where $\frac{a_1}{a_c} = \sqrt{1 + \frac{H_a - H_c}{H}}$	

21.3 Flow Through an Internal Cylindrical Mouthpiece

Internal cylindrical mouthpiece is also known as Borda's mouthpiece.

21.3.1 Borda's mouthpiece running free

When the mouthpiece length is equal to its diameter, the liquid jet after contracting does not touch the sides of the tube. The mouth piece runs free i.e. without touching the tube.

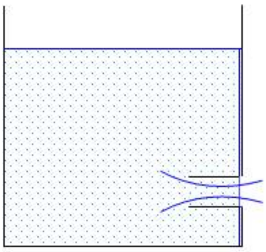


Fig. 21.2 Borda's mouthpiece running free

Pressure force on mouthpiece = pressure × area of mouthpiece = $\omega H \times \alpha$

Where,

ω = Weight density of liquid (ρg)

H = Height of liquid above the mouthpiece

α = Area of mouthpiece

α_c = Area at vena contracta

V = flow velocity

Mass of liquid flowing per sec = $\rho\, \alpha_c\, V$

We know,

Momentum = mass X velocity

Rate of change of momentum = mass of liquid flowing/sec × change of velocity

$= (\rho\ \alpha_c\ V) \times$ (final velocity−initial velocity)

Placing,

Initial Velocity = 0

Final Velocity = V

$= \rho\ \alpha_c V\ (V - 0)$

$= \rho\ \alpha_c\ V^2$

Since weight density $\omega = \rho g$

Rate of change of momentum $= \frac{\omega}{g} \alpha_c\ V^2$---------------------------(i)

Pressure force $= \omega\ H\ \alpha$ _______________________________________(ii)

Rate of change of momentum = Pressure force

Equating equation (i) and (ii)

$\omega\ H\ \alpha = \frac{\omega}{g} \alpha_c\ V^2$ ____(iii)

Torricelli's equation $V = C_v\sqrt{2gH}$ --------------------------(iv)

From equation (iii) and (iv)

$$\omega\ H\ \alpha = \frac{\omega}{g} \alpha_c \left(C_v\sqrt{2gH}\right)^2$$

Simplifying,

$$\frac{\alpha_c}{\alpha} = \frac{1}{2C_v^2}$$

In case of no loss of head $C_v = 1.0$ --------------------------------(v)

$$\frac{\alpha_c}{\alpha} = \frac{1}{2C_v^2} = 0.5$$

Since,

$$C_c = \frac{\alpha_c}{\alpha}$$

$C_c = 0.5$______________________(vi)

$C_d = C_c \times C_v$ (vii)

From equ (v), (vi) and (vii)

$C_d = C_c \times C_v = 0.5 \times 1 = 0.5$

Discharge $Q = C_d\ \alpha\ \sqrt{2gH}$

Or,

Discharge $Q = 0.5\ \alpha\ \sqrt{2gH}$

21.3.2 Borda's mouthpiece running full

Internal cylindrical mouthpiece is also known as Borda's mouthpiece. Mouthpiece tube is about 3 times its diameter, the liquid jet after contraction in the tube touches the internal walls of the tube. Such a condition is known as mouthpiece running full (Fig. 21.3).

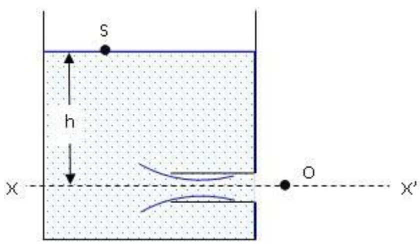


Fig. 21.3 Borda's mouthpiece running full

Considering XX' as datum line

	At point S	At point O
Potential head or height from datum line	h	0
Pressure	P_s	P_o
Velocity	V_s	V

Applying Bernoulli's theorem to the free liquid surface at point S and the outlet of mouthpiece O,

$$\frac{P_s}{w} + \text{h} + \frac{V_s^2}{2g} = \frac{P_o}{w} + 0 + \frac{V^2}{2g} + head\ loss \qquad \dots(i)$$

$V_s = 0$ as $V_s << V$ and V_s is very small $\qquad \dots(ii)$

$P_\alpha = P_o =$ atmospheric pressure $\qquad \dots(iii)$

$$Head\ loss = \frac{(V_c - V)^2}{2g} \qquad \dots(iv)$$

Placing values of (ii), (iii) & (iv) in equation (i)

$$h = \frac{V^2}{2g} + \frac{(V_c - V)^2}{2g}$$

$$h = \frac{V^2}{2g} + \frac{V^2}{2g}\left(\frac{V_c}{V} - 1\right)^2 \qquad \dots(v)$$

From equation of continuity

$V_c a_c = V a$

$\frac{V_c}{V} = \frac{\alpha}{\alpha_c}$

$\frac{\alpha}{\alpha_c} = \frac{1}{C_c} = \frac{1}{0.5}$

Thus,

$\frac{V_c}{V} = \frac{1}{0.5}$

From equ v

$$h = \frac{V^2}{2g} + \frac{V^2}{2g}\left(\frac{1}{0.5} - 1\right)^2 = 2\frac{V^2}{2g}$$

Or,

$h = 2\frac{V^2}{2g}$

Placing $V = C_v\sqrt{2gH}$ $f$rom Torricelli's theorem in the above equation,

$$h = 2 \times \frac{\left(C_v \times \sqrt{2gH}\right)^2}{2g}$$

$$h = 2C_v^2 h$$

$Cofficient\ of\ velocity\ C_v = \frac{1}{\sqrt{2}} = 0.707$

Coefficient of contraction $C_C = 1$

[Since the area of jet at exit equals the area of the mouthpiece]

$$C_d = C_c C_v = 1 \times 0.707 = 0.707$$

Discharge through a Borda's mouthpiece running full,

$$Q = C_d \alpha \sqrt{2gH}$$

$$Q = 0.707\alpha\sqrt{2gH}$$

Lesson 22

TYPES OF NOTCHES, RECTANGULAR AND TRIANGULAR NOTCHES, RECTANGULAR WEIRS

22.1 Flow Over Notches and Weirs

22.1.1 Notch

A notch may be defined as an obstruction over which the flow of liquid occurs. As the depth of flow above the base of the notch is related to the discharge, the notch forms a useful measuring device. In case of measuring tank or reservoir, the opening is provided at the side of the tank such that the liquid surface in the tank is below the top edge of the opening. In fact, this is a large opening which has no upper edge, so that it has a variable area depending upon the level of the free surface.

21.1.2 Weir

A weir is a notch on a large scale used for measuring the flow of a river, canal etc. It is a concrete or masonry structure of substantial breadth built across the river in the direction of flow. This allows the excess water to flow over its entire length to the downstream side. Thus a weir is similar to a small dam constructed across the river, with a difference that the excess water flows downstream only through a small portion called spillway and in case of weir, the excess water flows over its entire length.

21.1.3 Nappe and crest

The sheet of water flowing through a notch or over a weir is known as nappe or vein. The bottom edge of the notch or the top of a weir over which water flows is known as sill or crest. The height above the bottom of the tank or channel is known as crest height.

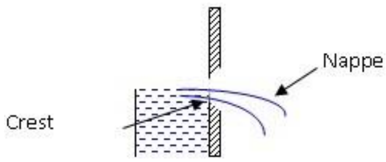


Fig. 21.1 Nappe and crest

Table 21.1 Difference between orifice and notch

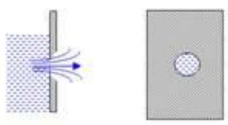
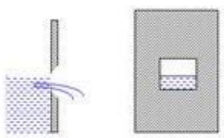
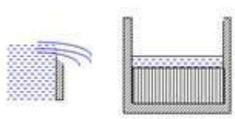
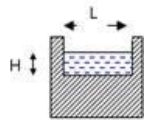
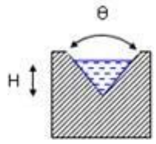
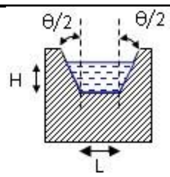
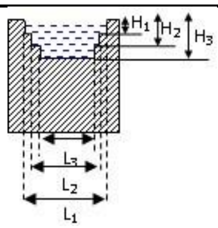
Orifice	Notch
An orifice may be defined as an opening provided in the side or bottom of tank or vessel such that the liquid flows through the entire orifice. 	A notch may be defined as an opening provided in the side of tank or vessel such that the liquid surface in tank is below the top edge of opening. 

Table 21.2 Difference between Notches and Weirs

Notch	Weir
A notch may be defined as an opening provided in the side of tank or vessel such that the liquid surface in tank is below the top edge of opening. 	A weir may be defined as any regular obstruction in open stream over which the flow takes place. 

Small structure	Large structure
Made of metallic plates.	Made of concrete/bricks.
Measure small flow rate.	Measure large flow rate.

Table 21.3 Types of notches

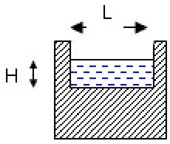
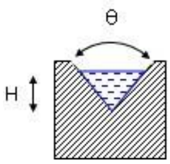
	Types of notches	Diagram	Discharge/flow rate
a.	Rectangular		$Q = \frac{2}{3} C_d \text{ L} \sqrt{2g}\, (H)^{3/2}$
b.	Triangular		$Q = \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2} (H)^{5/2}$
c.	Trapezoidal		Q = Q1 + Q2 $Q = \frac{2}{3} C_{d1} \text{ L} \sqrt{2g}\, (H)^{3/2} + \frac{8}{15} C_{d2} \sqrt{2g} \tan\frac{\theta}{2} (H)^{5/2}$
d.	Stepped		$Q_1 = \frac{2}{3} C_d \text{ L}_1 \sqrt{2g}\, (H_1)^{3/2}$ $Q_2 = \frac{2}{3} C_d \text{ L}_2 \sqrt{2g} \left[(H_2)^{3/2} - (H_1)^{3/2}\right]$ $Q_3 = \frac{2}{3} C_d \text{ L}_2 \sqrt{2g} \left[(H_3)^{3/2} - (H_2)^{3/2}\right]$ Q = Q1 + Q2 + Q3

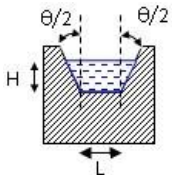
21.2 Types of Weir

1. Shape

- Rectangular
- Triangular
- Trapezoidal

Table 21.4 Types of weir on basis of shape

	Types of weir	Diagram	Discharge/flow rate
a.	Rectangular		$Q = \frac{2}{3} C_d \text{ L} \sqrt{2g}\, (H)^{3/2}$
b.	Triangular		$Q = \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2} (H)^{5/2}$

c.	Trapezoidal		$Q = Q_1 + Q_2$ $Q = \frac{2}{3} C_{d1}\, L \sqrt{2g}\, (H)^{3/2} + \frac{8}{15} C_{d2} \sqrt{2g} \tan\frac{\theta}{2} (H)^{5/2}$

Note: The discharge equation for rectangular, triangular and trapezoidal weir is same as of notch.

2. ***Nature of discharge***
    - Free: Liquid level on the downstream side is lower than the crest.

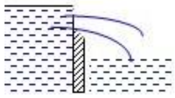


Fig. 21.2 Free flowing weir

- Drowned: Liquid level submerges the crest

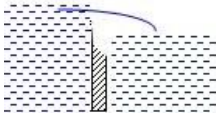


Fig. 21.3 Drowned weir

3. ***Width of crest***
    - Sharp: The crest is narrow

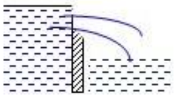


Fig. 21.4 Sharp crest weir

- Broad: The crest is broad

Fig. 21.5 Broad crest weir

Lesson 23
NUMERICALS ON ORIFICE, MOUTHPIECE, NOTCH AND WEIR

23.1 Numericals

1. Calculate the flow velocity of water at the opening of an orifice if the orifice is located 4 m below the water surface in a tank.

Solution

$$V_{13} = \sqrt{2gh}$$

$$= \sqrt{2 * g * 4}$$

$$= \sqrt{2 * 9.81 * 4}$$

= 8.85 m/s

2. Calculate the coefficient of velocity C_v if the actual velocity of jet is 5.1 m/s. Orifice is located 2m below the water surface in a tank.

Solution

$$Coefficient\ of\ velocity\ C_v = \frac{Actual\ velocity\ jet}{The\ orifice\ velocity}$$

$$\frac{V_a}{\sqrt{2gh}}$$

$$\frac{5.1}{\sqrt{2 * 9.81 * 2}}$$

$$\frac{5.1}{6.26}$$

= 0.814

3. Determine discharge through a internal mouthpiece (running free) if there are a y mouth piece is 0.09 m^2. The distance between the centre of mouthpiece and free water surface is 2.5 m.

Solution

$$Q = 0.5\ a \sqrt{2gh}$$

Where

a = area of mouthpiece

h = distance between the centre of mouthpiece and free water surface.

$Q = 0.5 * 0.09 \sqrt{2 * 9.81 * 2.5}$

= 0.315 m^3/s

4. Determine discharge through a internal mouthpiece (running free) if the are a of mouthpiece is 0.15 m^2. The distance between the centre of mouthpiece and free water surface is 3.0 m.

Solution

$Q = 0.707\ a \sqrt{2gh}$

$= 0.707 * 0.15 \sqrt{2 * 9.81 * 3}$

= 0.813 m^3/s

5. Determine flow rate through a rectangular hutch if L = 0.20 m and H = 0.15 m. Take value of c_d = 0.645

Solution

$$Q = \frac{2}{3} C_d\, L \sqrt{2g}\, (H)^{3/2}$$

$$= \frac{2}{3} * 0.645 * 0.2 * \sqrt{2 * 9.81} * (0.15)^{3/2}$$

= 0.022 m^3/s

6. Determine flow rate through a triangular hutch if H = 0.20 m and $\theta = 60°$. Take value of c_d = 0.85.

Solution

$$Q = \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2} (H)^{5/2}$$

$$= \frac{8}{15} * 0.85 * \sqrt{2 * g} * \tan\frac{60}{2} (0.20)^{5/2}$$

= 0.0207 m^3/s

7. Determine flow rate through a rectangular weir if L = 0.30 m and H = 0.25 m. Take value of c_d = 0.95

Solution

$$Q = \frac{2}{3} C_d \, L \sqrt{2g} \, (H)^{3/2}$$

$$= \frac{2}{3} * 0.95 * 0.3 * \sqrt{2 * 9.81} * (0.25)^{3/2}$$

$$= 0.105 \text{ m}^3/\text{s}$$

Lesson 24

VENTURIMETER AND PITOT TUBE

24.1 Introduction

Measuring the flow of liquids is a critical need in many industrial plants particularly dairy and food processing plants. Flow measurement is the quantification of bulk fluid movement. Flow can be measured in a variety of ways. Positive-displacement flow meters accumulate a fixed volume of fluid and then count the number of times the volume is filled to measure flow. Other flow measurement methods rely on forces produced by the flowing stream as it overcomes a known constriction, to indirectly calculate flow. Flow may be measured by measuring the velocity of fluid over a known area. A flowmeter is an instrument used to measure linear, nonlinear, mass or volumetric flow rate of a liquid or a gas. Both gas and liquid flow can be measured in volumetric or mass flow rates, such as liters per second or kilograms per second. These measurements can be converted between one another if the material's density is known.

24.2 Venturimeter

Venturimeter is used to the measurement of flow rate. It is generally used for large diameter pipes.

24.2.1 Construction

Fig. 24.1 Venturimeter

Fig. 24.2 Venturimeter (cut view)

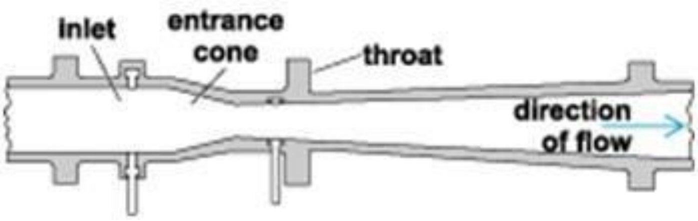


Fig. 24.3 Venturimeter section

The venturi tube has a entrance zone, converging conical inlet, a cylindrical throat, and a diverging recovery cone. The main sections of venturimeter are indicated in Fig. 24.3.

a) Entrance Section

It is a straight cylinder having length equal to 5 to 8 times the diameter of the pipe.

b) Convergence Section:

Here, the diameter of the tube gradually decreases. The angle of cone is a1 = 21 ± 5°. When liquid flows inside the venturimeter, the velocity of fluid increases and correspondingly the pressure falls.

c) Throat: At this section, the diameter of the venturemeter is minimum. Velocity is maximum and pressure is minimum. Throat diameter = 1/3 to 1/4th inlet diameter.

d) Diverging section: Again the diameter of the tube gradually increases. Here due to gradual divergence pressure is build up to the original inlet pressure. The cone angle is 5-7°.

Small size venturimeter are made of brass or, bronze and large venturimeters are made of cast iron or stainless steel.

24.2.2 Working principle

In the venturimeter the fluid is accelerated through a converging cone of angle 21 ± 2° and the pressure difference between the upstream side of the cone and the throat is measured and provides a signal for the rate of flow.

The fluid slows down in diverging cone with smaller angle (5 - 7°) where most of the kinetic energy is converted back to pressure energy. High pressure and energy recovery makes the venturimeter suitable where only small pressure heads are available.

Flow rate can be given as:

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2g \frac{p_1 - p_2}{w}}$$

Where,

Q = flow rate,

C_d = Coefficient of discharge. It is not constant and depends on pipe geometry, Reynolds number of the flow etc.

a_1 = area at the entrance of venturi

a_2 = area at the venturi throat

p_1 = Pressure at entrance of venture

p_2 = Pressure at venture throat

w = ρg

24.2.3 Advantages

1. The venturi tube is suitable for clean, dirty and viscous liquid and some slurry services.
2. Loss of head due to installation in the pipeline is small.
3. Original pressure of the liquid can be recovered completely.
4. Accuracy is 1% of full range
5. Not much wear and tear.
6. Characteristics are well established and it is in use since years.
7. Can be used for large flow rates and large diameter pipes.

24.2.4 Disadvantages

1. Space requirements are more.
2. Expensive in installation.
3. Has to be designed as per requirement.

24.3 Pitot Tube

Pitot tube is used to measure flow velocity. The construction of Pitot tube is shown in Fig. 24.4. Outer body of pilot tube consist of ports at point A, for sensing the static pressure of fluid. At point B fluid vel. become zero and inner tube is for sensing the stagnation pressure. The outlet C & D is connected to U-tube manometer for measuring the pressure difference between the points A and B.

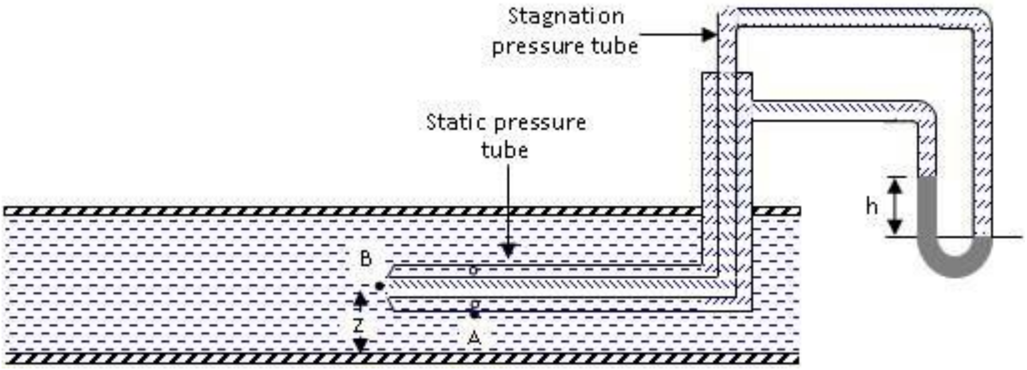


Fig. 24.4 Pitot tube

The flow velocity is given by the following equation:

Flow velocity $V = \sqrt{2gh}$

Lesson 25

ROTAMETER, WATER LEVEL POINT GAUGE, HOOK GAUGE

25.1 Rotameter

A Rotameter is a device that measures the flow rate of liquid or gas in a closed tube. It belongs to a class of meters called variable area meters, which measure flow rate by allowing the cross-sectional area the fluid travels through to vary, causing some measurable effect.

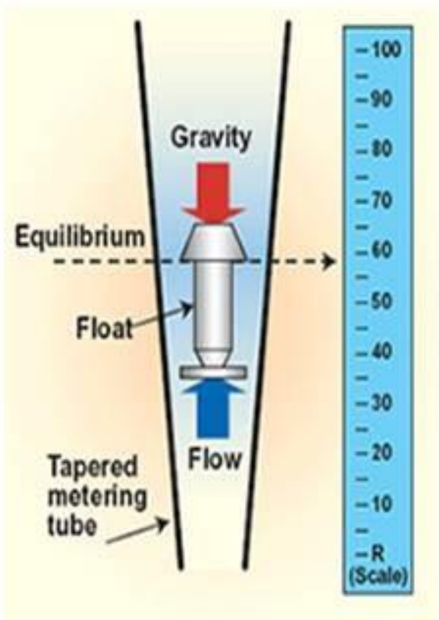


Fig. 25.1 Rotameter

25.1.1 Construction

It consist of tapered, metered metering glass tube inside of which is located a rotor or active element (float). The tube is provided a suitable inlet and outlet connection. The float has a specific gravity higher than that of fluid. As the fluid flows through the tube, the float rises. Equilibrium will be reached when pressure and the buoyancy of the float counterbalance gravity. The float's height in the tube is then used to reference a flow rate on a calibrated measurement reference.

25.1.2 Principle of working

The rotameter's operation is based on the variable area principle: fluid flow raises a float in a tapered tube, increasing the area for passage of the fluid. The greater the flow, the higher the float is raised. The height of the float is directly proportional to the flowrate. With liquid, the float is raised by a combination of the buoyancy of the liquid and the velocity head of the fluid. With gases, buoyancy is negligible, and the float responds to the velocity head alone.

The float moves up or down in the tube in proportion to the fluid flowrate and the annular area between the float and the tube wall. The float reaches a stable position in the tube when the upward force exerted by the flowing fluid equals the downward gravitational force exerted by the weight of the float. A change in flowrate upsets this balance of forces. The float then moves up or down, changing the annular area until it again reaches a position where the forces are in equilibrium. To satisfy the force equation, the rotameter float assumes a distinct position for every constant flowrate. However, it is important to note that because the float position is gravity dependent, rotameters must be vertically oriented and mounted.

Discharge (Q) is given as:

$$Q = C_d a_{ft}\left[2gV_f\frac{(\rho_f - \rho)}{A_f\rho}\right]^{1/2}$$

Where,

Q = Discharge

C_d = Discharge coefficient

ρ_f = Density of fluid

ρ = Density of float

A_f= Maximum cross sectional area of float.

a_{ft} = annular area between float & tube.

25.1.3 Advantages

1. Simpler in operation.
2. No external power or energy required for its operation.
3. Ease of handling & installation.
4. Relatively low cost.
5. Can handle corrosive fluids.

25.1.4 Types of rotameters

On basis of construction rotameters can be classified as follows:

a. Glass tube Rotameters
b. Metal Tube Flowmeters
c. Plastic Tube Rotameters

a. Glass tube rotameters

The basic rotameter is the glass tube indicating-type. The tube is precision formed of borosilicate glass, and the float is precisely machined from metal, glass or plastic. The metal float is usually made of stainless steel to provide corrosion resistance. The float has a sharp metering edge where the reading is observed by means of a scale mounted alongside the tube.

(Source: http://www.omega.com/prodinfo/rotameters.html)

Fig. 25.2 Glass Tube Rotameter

End fittings and connections of various materials and styles are available. The important elements are the tube and float, often called the tube-and-float combination, because it is this portion of the rotameter which provides the measurement. In fact, similar glass tube and stainless steel float combinations are generally available. The scale of the rotameter can be calibrated for direct reading of water, or it may have a scale to read a percent of range or an arbitrary scale to be used with conversion equations or charts. Safety-shielded glass tube rotameters are in general use throughout industry for measuring both liquids and gases. They are manufactured with end fittings of metal or plastic to meet the chemical characteristics of the fluid being metered. The only fluids for which these meters are not suited are those which attack glass metering tubes, such as water over 90°C (194°F), with its high pH which softens glass; wet steam, which has the same effect; caustic soda, which dissolves glass; and hydrofluoric acid, which etches glass.

b. Metal tube rotameter

(Source: http://www.omega.com/prodinfo/rotameters.html)

Fig. 25.3 Metal tube rotameter

For higher pressures and temperatures beyond the practical range of glass tubes, metal tubes are used. These are usually manufactured in aluminium, brass or stainless steel. The position of the piston is determined by magnetic or mechanical followers that can be read from the outside of the metal metering tube. Similar to glass tube rotameters, the spring-and-piston combination determines the flowrate, and the fittings and materials of construction must be chosen so as to satisfy the demands of the applications. These meters are used for services where high operating pressure or temperature, water hammer, or other forces would damage glass metering tubes. Spring and piston flowmeters can be used for most fluids, including corrosive liquids and gases. They are particularly well suited for steam applications, where glass tubes are unacceptable.

c. Plastic tube rotameters

(Source: http://www.omega.com/prodinfo/rotameters.html)

Fig. 25.4 Plastic tube rotameter

Plastic tubes are also used in some rotameter designs due to their lower cost and high impact strength. They are typically constructed of polycarbonate, with either metal or plastic end fittings. With plastic end fittings, care must be taken in installation, not to distort the threads. Rotameters with all plastic construction are available for applications where metal wetted parts cannot be tolerated, such as with deionized water or corrosives.

25.2 Water Level Gauge

a. Gauge glass

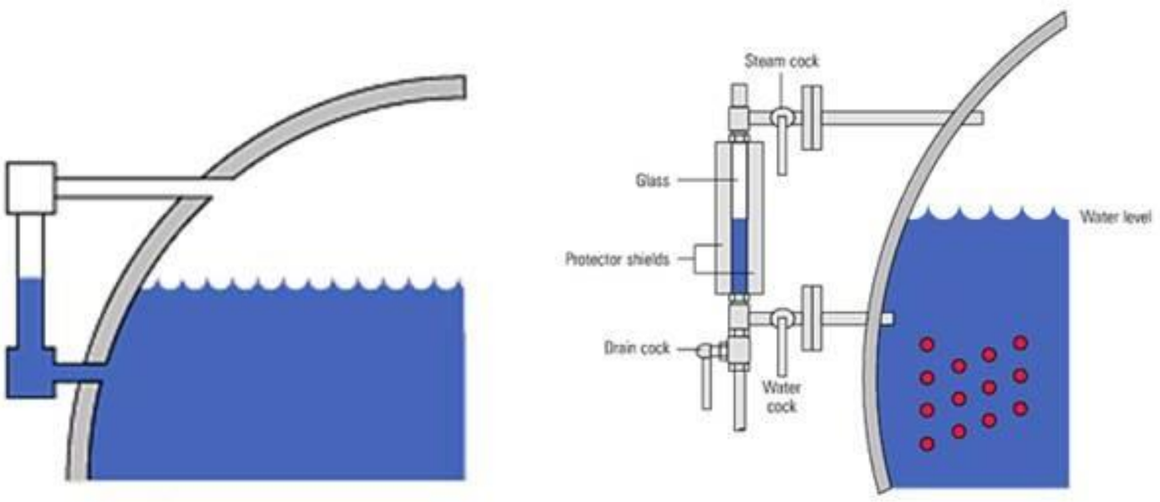


Fig 25.5 Gauge glass indicating liquid level in horizontal tank

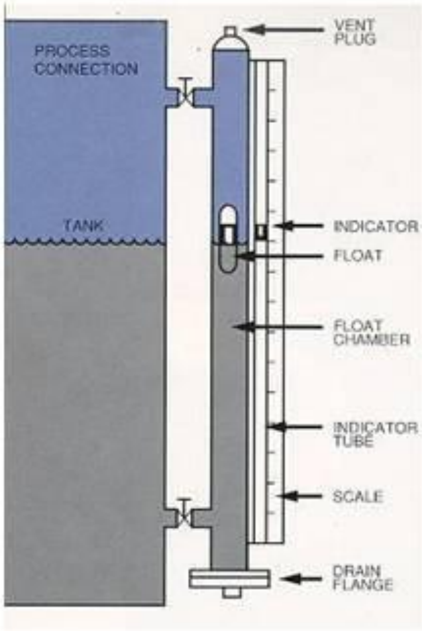


Fig 25.6 Gauge glass indicating liquid level in vertical tank

b. Float type gauge

i. Ball Float

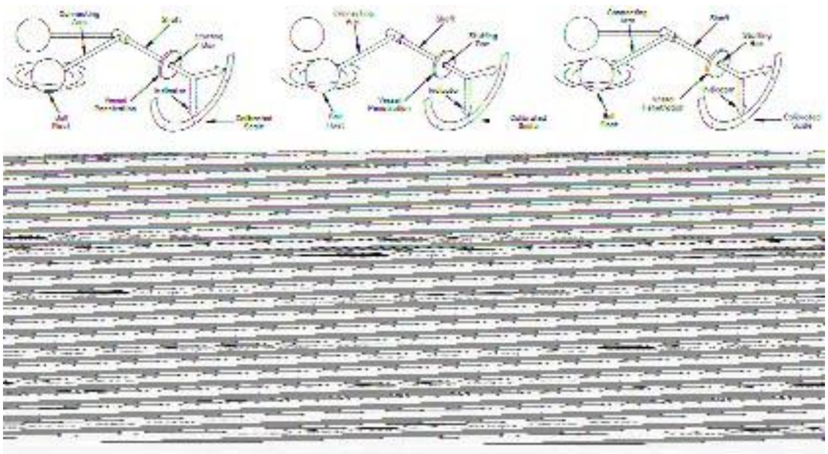


Fig 25.7 Ball Float

The ball float method is a direct reading liquid level mechanism. The most practical design for the float is a hollow metal ball or sphere. However, there are no restrictions to the size, shape, or material used. The design consists of a ball float attached to a rod, which in turn is connected to a rotating shaft which indicates level on a calibrated scale (Fig. 25.7). The operation of the ball float is simple. The ball floats on top of the liquid in the tank. If the liquid level changes, the float will follow and change the position of the pointer attached to the rotating shaft.

ii. Chain Float

This type of float gauge has a float ranging in size up to 12 inches in diameter and is used where small level limitations imposed by ball floats must be exceeded. The range of level measured will be limited only by the size of the vessel. The operation of the chain float is similiar to the ball float except in the method of positioning the pointer and in its connection to the position indication. The float is connected to a rotating element by a chain with a weight attached to the other end to provide a means of keeping the chain taut during changes in level (Fig. 25.8).

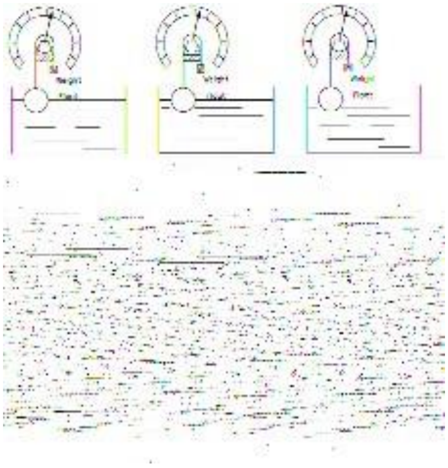


Fig. 25.8 Chain Float

25.3 Hook Gauge

Pointer gauge has a circular rod having one end attached to scale and the other being a pointed one. The pointed end touches the water and the other is attached to scale. The device consists of a sharp hook suspended from a micrometer cylinder, with the body of the device having arms which rest on the rim of a still well. The change in water level is determined by the difference in readings.

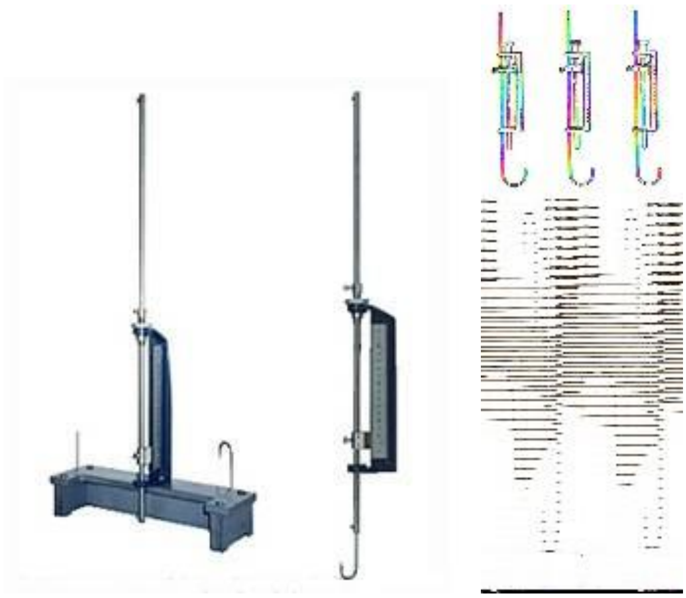


Fig 25.9 Hook Gauge

Fig 25.10 Hook Gauge with digital display

Lesson 26

BUCKINGHAM'S THEOREM APPLICATION TO FLUID FLOW PHENOMENA

26.1 Dimensions

In the SI system there are seven fundamental units: kilogram, meter, candela, second, ampere, kelvin, and mole. These are also known as fundamental quantities. In fluid mechanics mostly kg, m, s and K is used. Quantities like force, pressure, energy, acceleration etc are expressed in combination of fundamental quantities and are therefore called derived quantities. Few of them are mentioned in Table 26.1.

Table 26.1 Quantities, dimensions, and units

Quantity	**Dimensions** (M LT)	**Preferred units** (SI)
Length (L)	L	m
Time (T)	T	s
Mass (M)	M	kg
Area (A)	L^2	m^2
Volume (Vol)	L^3	m^3
Velocity (V)	LT^{-1}	m/s
Acceleration (a)	LT^{-2}	m/s^2
Discharge (Q)	$L^3\,T^{-1}$	m^3/s
Force (F)	MLT^{-2}	N
Pressure (p)	$ML^{-1}T^{-2}$	Pa
Shear stress (τ)	$ML^{-1}T^{-2}$	N/m^2
Density (ρ)	ML^{-3}	kg/m^3
Specific weight (ω)	$ML^{-2}T^{-2}$	N/m^3
Energy/Work/Heat (E)	ML^2T^{-2}	J
Power (P)	ML^2T^{-3}	W
Dynamic viscosity (μ)	$ML^{-1}T^1$	N s/m^2 or Pa.s
Kinematic viscosity (υ)	$L^2\,T^{-1}$	m^2/s

26.2 Dimensional Analysis

The principal use of dimensional analysis is to deduce from a study of the dimensions of the variables in any physical system certain limitations on the form of any possible relationship between those variables. The method is of great generality and mathematical simplicity. Dimensional Analysis is a process of arranging various factors is arranged in a manner that it forms dimensionless number.

26.3 Dimensional Homogeneity

An equation will be dimensionally homogeneous if the dimensions of various terms on the two sides of the equation are identical. A dimensionally homogeneous equation is independent of the fundamental units of measurement and is applicable to all systems of units For e.g. taking the following equation:

$T = 2\pi\sqrt{\frac{l}{g}}$

Dimension of L.H.S [T]

$Dimension\ of\ R.H.S\ \sqrt{\frac{L}{LT^{-2}}}\quad [T]$

The dimension of L.H.S is same as that of R.H.S. = [T]. Thus the equation may be said to be dimensionally homogeneous.

Other examples of dimensional homogeneity:

- $\rho = \delta gh$
- Q=VA
- $V = \sqrt{2gh}$

26.4 Application of Dimensional Analysis

- The first step to dimensional analysis is to determine the dimensional homogeneity of any equation.
- Analysis helps to determine the dimensions and thus the units of any quantity.
- Units can be easily transformed from one system to another.
- To establish relationship between number of variables.
- To reduce equations by arranging variables in dimensionless forms.

26.5 Steps for Dimensional Analysis

The application of dimensional analysis to mathematically represent engineering problems consists of several steps. These are:

(1) formulate a differential equations and/or algebraic expressions which adequately describe the problem, together with the required boundary conditions;

(2) select the appropriate dimensionless variables tor all independent and dependent variables involved, using arbitrary terms or boundary values for the denominator in each case;

(3) substitute the new dimensionless variables into each differential equation and boundary condition to normalize them;

(4) group into functional form all dimensionless variables and those parameters generated;

(5) reduce the functionality to the minimum possible number of independent groups; and

(6) drop any groups in which the arbitrary terms can not be divided out using other groups of the functionality.

26.6 Rayleigh's Method for Dimensional Analysis

Rayleigh's method for dimensional analysis involves following steps:

- Independent variables are identified which will express the functional relationship most closely. A maximum of three to four variables must be selected.
- If the dependent variable X is some function of the independent variables x_1, x_2, x_3….,the

equation can be given as:

$$X = f(x_1, x_2, x_3 \dots)$$

Above equation may be written as:

$$X = C(x_1^a, x_2^b, x_3^c \dots)$$

Here C is a dimensionless coefficient to be determined through experiments

Values of a, b, c etc are determined by comparing the powers on both sides.

26.7 Buckingham's π- method

It states that if there are 'n' variables in a dimensionally homogeneous equation and if these variables contain 'm' fundamental (primary) dimensions then the variables can be grouped into (n-m) dimensionless terms. These dimensionless terms are called π terms.

We use M L T fundamental quantities in fluid mechanics.

Mathematically if x_1 depends on other variables x_2, x_3, $x_4 \dots x_n$ the equation can be given as

$x_1 = f(x_2, x_3, x_4 \dots\dots\dots\dots, x_n)$

$f(x_1, x_2, x_3, x_4 \dots\dots\dots\dots, x_n) = 0$-------------------------- (i)

Where x are dimensional physical quantities such as velocity, density, pressure, area, diameter etc. Then the phenomenon given in equation (i) can be described by (n-m) dimensionless 'π' terms.

$f(\pi_1, \pi_2, \dots\dots\dots\dots \pi_{n-m})$

where 'm' represents the fundamental dimensions such as mass, length and time.

26.8 Numerical

Q 1. The resistance *R* experienced by a partially submerged body depends upon the velocity *V*, length of the body *l*, viscosity of the fluid μ, density of the fluid ρ and gravitational acceleration *g*. Obtain a dimensionless express for *R*.

Solution:

Mathematically following relation can be given:

$R = f(V, l, \mu, \rho, g)$

$f(R, V, l, \mu, \rho, g) = 0$

Number of variable n = 6

Number of fundamental variable m = 3

Therefore, number of π-terms = n-m = 6-3 = 3 π-terms

Choosing length l, velocity V, and density ρ as the 3 repeating variables.

Analysis of π-terms.

a. $\boldsymbol{\pi_1 = l^{a_1} V^{b_1} \rho^{c_1} R}$

$M^0 L^0 T^0 = [L]^{a_1} [LT^{-1}]^{b_1} [ML^{-3}]^{c_1} MLT^{-2}$

Equating powers of *M*, *L* and *T* on both sides

$0 = c_1 + 1; 0 = a_1 + b_1 - 3c_1 + 1; 0 = -b_1 - 2$

$c_1 = -1;\ b_1 = -2;\ a_1 = -2$

$\pi_1 = l^{-2} V^{-2} \rho^{-1} R$

$\pi_1 = \frac{R}{l^2 V^2 \rho}$

b. $\pi_2 = l^{a_2} V^{b_2} \rho^{c_2} \mu$

$$M^0 L^0 T^0 = [L]^{a_2} [LT^{-1}]^{b_2} [ML^{-3}]^{c_2} ML^{-1}T^{-1}$$

Equating powers of *M*, *L* and *T* on both sides

$$0 = c_2 + 1;\ 0 = a_2 + b_2 - 3c_2 - 1;\ 0 = -b_2 - 1$$

$$c_2 = -1;\ b_1 = -1;\ a_2 = -1$$

$$\pi_2 = l^{-1} V^{-1} \rho^{-1} \mu$$
$$\pi_2 = \frac{\mu}{lV\rho}$$

c. $\pi_3 = l^{a_3} V^{b_3} \rho^{c_3} g$

$$M^0 L^0 T^0 = [L]^{a_3} [LT^{-1}]^{b_3} [ML^{-3}]^{c_3} LT^{-2}$$

Equating powers of *M*, *L* and *T* on both sides

$$0 = c_3 + 1;\ 0 = a_3 + b_3 - 3c_3 + 1;\ 0 = -b_3 - 2$$
$$c_3 = -1;\ b_3 = -2 \text{ and } a_3 = 1$$
$$\pi_3 = l^1 V^{-2} \rho^0 g$$

$$\pi_3 = \frac{lg}{V^2}$$

The functional relationship can be given as:

$$\phi[\pi_1,\ \pi_2,\ \pi_3] = 0$$

$$\phi\left[\frac{R}{l^2V^2\rho}, \frac{\mu}{\rho V l}, \frac{lg}{V^2}\right] = 0$$

$$\frac{R}{l^2V^2\rho} = \Phi\left(\frac{\mu}{\rho V l}, \frac{lg}{V^2}\right)$$

$$\frac{R}{l^2V^2\rho} = \phi\left(\frac{\rho V l}{\mu}, \frac{V}{\sqrt{lg}}\right)$$

Since reciprocal of π-term and its square root is non-dimensional.

$$R = l^2V^2\rho\, \Phi\left(\frac{\rho V l}{\mu}, \frac{V}{\sqrt{lg}}\right)$$

It is evident that resistance *R* is a function of Reynolds number $(\frac{\rho V l}{\mu})$ and Froude's number $(\frac{V}{\int lg})$.

Lesson 27

FROUDE NUMBER, REYNOLDS NUMBER, WEBER NUMBER

27.1 Dimensionless Numbers

In dimensional analysis, a dimensionless quantity is a quantity without an associated physical dimension. Dimensionless quantities are often defined as products or ratios of quantities that are not dimensionless, but whose dimensions cancel out when their powers are multiplied. This is the case, for instance, with the engineering strain, a measure of deformation. It is defined as change in length over initial length but, since these quantities both have dimensions L (length), the result is a dimensionless quantity.

27.2 Importance of Dimensionless Numbers

- The use of dimensionless numbers in engineering and physics allows the important task of data reduction of similar problems. This means that a lot of experimental runs are avoided if data is correlated using appropriate dimensionless parameters.
- Dimensionless numbers often correlate with some performance parameter and greatly aid engineering analysis and design.
- The value of the dimensionless numbers often reflects certain properties. For example, a flow problem with a low Reynolds Number will be laminar, while a larger value will imply turbulent behaviour.
- The number of dimensionless numbers determines the dimensionality of the space of solutions. For example, if a problem has two dimensionless numbers, then by varying both numbers, all the different behaviours in the problem can be accounted for.

- A dimensionless number can be used in the analysis of prototype models, to predict behaviour in similar full-scale systems. Dimensionless numbers help to compare two systems that are vastly different by combining the parameters of interest. For example, the Reynolds number, Re = velocity * length / kinematic viscosity. If an airfoil has to be tested with a particular Re, and simulation is conducted on a scaled-down model (length is smaller), one could increase fluid velocity or lower kinematic viscosity (change fluids) or both to establish the same Re and ensure working under comparable circumstances.

Table 27.1 List of few dimensionless numbers

Dimensionless No.	Significance	Group of Variables	Application
1. Reynolds Numbers	$\frac{Inertia\ force}{Viscous\ force}$	$\frac{\rho V d}{\mu}$	Laminar viscous flow in confined passage (pipes)
2. Froude's Number	$\sqrt{\frac{Inertia\ force}{Gravity\ force}}$	$\frac{V}{\sqrt{Lg}}$	Free surface flow where effect of gravity is important.
3. Weber's Number	$\sqrt{\frac{Inertia\ force}{Surface\ force}}$	$\frac{V}{\sqrt{\sigma/\rho L}}$	Capillary & Sheet flow where surface tension is important.

27.3 Reynold Number (R_e)

In fluid mechanics, the Reynolds number Re is a dimensionless number that gives a measure of the ratio of inertial forces to viscous forces and consequently quantifies the relative importance of these two types of forces for given flow conditions.

Assume l represent a characteristic length in the flow system:

Inertia force F_i = mass × acceleration

= density x volume x $\frac{Velocity}{time}$

= density x Area x velocity x velocity

= $\rho A V^2$

[Since volume/time = Q = area * velocity]

Viscous force (F_v) = shear stress x area = $\mu \frac{velocity}{Length} X\ Area$

$$= \mu VA/l$$

Thus

$$R_e = \frac{inertia\ force}{viscous\ force} = \frac{\rho AV^2}{\mu VA/l} = \frac{\rho Vl}{\mu}$$

In the flow situations where the viscous forces plays an important, Reynolds number is taken as the criterion of dynamic similarity. Examples are as follows:

i. Incompressible flow through small diameter pipes,
ii. Objects moving completely under water,
iii. Air movement under low velocity around airplanes and automobiles
iv. Open channel flow.

27.4 Froude's Number (F_r)

Froude's number (Fr) is the ratio of the square root of the inertia force to the square root of the force due to gravity.

$Inertia\ force\ F_r = \rho l^2 V^2$

Gravity force = mass × gravitational acceleration

$$= \rho l^3 g$$

Froude number

$$F_r = \left(\frac{inertia\ force}{gravity\ force}\right)^{1/2}$$

$$F_r = \left(\frac{\rho l^2 V^2}{\rho l^3 g}\right)^{1/2} = \frac{V}{\sqrt{lg}}$$

In flow situations where gravitational force is more important, Froude number governs the dynamic similarity. Other forces are comparatively small and negligible. Examples are:

i. Flow through open channels
ii. Flow of liquid jets from orifices
iii. Flow over notches and weirs
iv. Flow over the spillway of a dam

27.5 Weber Number (W)

Weber number (W) is square root r*atio of the inertia force to the force of surface tension.*

$Inertia\ force = \rho l^2 V^2$

Surface tension force = σ l

Where, σ is the surface tension per unit length.

$$W = \sqrt{\frac{inertia\ force}{surface\ tension\ force}}$$

$$= \sqrt{\frac{\rho l^2 V^2}{\sigma l}} = \frac{V}{\sqrt{\sigma/\rho L}}$$

Application of Weber number:

i. Flow in Capillary tubes
ii. Thin sheet flow
iii. Liquid atomization

Lesson 28

HYDRAULIC SIMILITUDE

28.1 Prototype and Model

Prototype

Prototype is the physical structure for which engineering design is required. The predictions are made for the prototype to work under actual field conditions.

Model

Model is the scale down representation of any physical structure. A model is smaller than the prototype so as to conduct laboratory studies and it is less expensive to construct and operate. Sometimes size of model can be larger than the prototype if amplified and more focused studies have to be carried out.

28.2 Importance of Model Studies

- Many of the equipments and hydraulic structures like dams, canals etc require lot of investment and time for construction. A scale down model is tested under simulated condition to determine its performance. It saves time and resources. If the model fails it does not put financial pressure on the investors. Model can be easily improved and re-designing can be done if required.
- In many fields, there is great uncertainty as to whether a new design will actually do what is desired. New designs often have unexpected problems. A model is often used as part of the product design process to allow engineers and designers the ability to explore design alternatives, test theories and confirm performance prior to starting production of a new product
- Sometimes mathematical relationships and equations are not sufficient to aid in engineering design. A model provides valuable data on geometrical appearance, force and pressure distribution, performance, capacity etc.
- Experiments are conducted on models and the problems can be rectified before actual design and commissioning is done.
- By using model studies, alternate plans and modifications can be tested within a relatively short time with all flow conditions that can be expected. Also, the design and operating engineers can observe conditions resulting with a particular arrangement and satisfy themselves as to the adequacy of the plan.
- The cost of model studies varies with area of study, characteristics of the streams, nature of the problem, and number of plans and alternate plans to be tested before an acceptable solution is developed. The cost of model studies has usually been less than 0.10 percent of the cost of the project, a small price to pay for the assurance that the most practical and economical design has been developed.

28.3 Examples of Model Studies

- Models for builds are made to determine geometric appearance, stability under different wind velocities and patterns.
- Equipment and its components are first tested by making models for its performance.
- Hydraulic structures like dams, canals, reservoirs, spill ways etc require model testing before its design is finalized.
- Automobiles, planes, rockets require model testing in wind tunnels.

28.4 Similitude

"Similitude" in a general sense is the indication of a known relationship between a model and prototype i.e. model tests must yield data that can be scaled to obtain the similar parameters for the prototype.

The results obtained model experiments can be applied to the prototype only if a complete similarity exists between the model and prototype and for that the two systems must be (*i*) geometrically (*ii*) kinematically and (*iii*) dynamically similar.

28.4.1 Geometric similarity

A model and prototype are geometric similar if and only if all body dimensions in all three coordinates have the same linear-scale ratio. It requires that the model and the prototype be of the same shape and that all the linear dimensions of the model be related to corresponding dimensions of the prototype by a constant scale factor. Usually, one or more of these pi terms will involve ratios of important lengths, which are purely geometrical in nature.

Thus for geometric similarity,

$$\frac{l_m}{l_p} = \frac{b_m}{b_p} = \frac{h_m}{h_p} = \frac{d_m}{d_p} = L_r \text{ (scale ratio)}$$

Where,

l_m, b_m, h_m, and d_m = respective dimensions of the model

l_p, b_p, h_p and d_p = corresponding linear dimensions of the prototype.

L_r = constant known as scale ratio or the scale factor.

Area scale ratio $A_r = \frac{A_m}{A_p} = \frac{l_m \times b_m}{l_p \times b_p} = L_r^2$

Volume scale ratio

$$V_r = \frac{V_m}{V_p} = \frac{l_m \times b_m \times h_m}{l_p \times b_p \times h_p} = L_r^3$$

28.4.2 Kinematic similarity

The motions of two systems are kinematically similar if homogeneous particles lie at homogeneous points at homogeneous times. In a specific sense, the velocities at corresponding points are in the same direction and are related in magnitude by a constant scale factor. This also requires that streamline patterns must be related by a constant scale factor. The flows that are kinematically similar must be geometric similar because boundaries form the bounding streamlines. The factors like compressibility or cavitations must be taken care of to maintain the kinematic similarity.

Let points 1 and 2 represent the homologous (corresponding) points in the model and prototype. The conditions of kinematic similarity are as follows:

$$\frac{(V_1)_m}{(V_1)_p} = \frac{(V_2)_m}{(V_2)_p} = V_r \text{, velocity ratio}$$

Geometric similarity is a must for kinematic similarity. Therefore,

$$\frac{V_m}{V_p} = \frac{l_m/T_m}{l_p/T_p} = \frac{l_m}{l_p} \times \frac{T_p}{T_m} = \frac{L_r}{T_r}$$

T_r is the time scale ratio, $T_r = \frac{T_m}{T_p}$

$$\frac{(a_1)_m}{(a_1)_p} = \frac{(a_2)_m}{(a_2)_p} = a_r \text{ acceleration scale ratio}$$

$$\frac{Q_m}{Q_p} = \frac{l_m^3/T_m}{l_p^3/T_p} = \frac{L_r^3}{T_r} = Q_r \text{ the discharge scale ratio}$$

28.4.3 Dynamic similarity

When two flows have force distributions such that identical types of forces are parallel and are related in magnitude by a constant scale factor at all corresponding points, then the flows are dynamic similar. For a model and prototype, the dynamic similarity exists, when both of them have same length-scale ratio, time-scale ratio and force-scale (or mass-scale ratio).

- For compressible flows, the model and prototype Reynolds number, Mach number and specific heat ratio are correspondingly equal.
- For incompressible flows,

With no free surface: model and prototype Reynolds number are equal.

With free surface: Reynolds number, Froude number, Weber number and Cavitation numbers for model and prototype must match.

Table 28.1. Flow characteristics and similitude scale ratios (ratio of prototype quantity to model quantity).

Characteristic	Dimension	Scale ratios for laws of Reynolds	Froude	Mach
Geometric				
Length	L	L_r	L_r	L_r

Area	L^2	L_r^2	L_r^2	L_r^2
Volume	L^3	L_r^3	L_r^3	L_r^3
Kinematic				
Time	T	$\left(\frac{L^2\rho}{\mu}\right)_r$	$\left(L^{1/2}g^{-1/2}\right)_r$	$\left(\frac{L\rho^{1/2}}{E_v^{1/2}}\right)_r$
Velocity	LT^{-1}	$\left(\frac{\mu}{L\rho}\right)_r$	$\left(L^{1/2}g^{1/2}\right)_r$	$\left(\frac{E_v^{1/2}}{\rho^{1/2}}\right)_r$
Acceleration	LT^{-2}	$\left(\frac{\mu^2}{\rho^2 L^3}\right)_r$	g_r	$\left(\frac{E_v}{L\rho}\right)_r$
Discharge	L^3T^{-1}	$\left(\frac{L\mu}{\rho}\right)_r$	$\left(L^{3/2}g^{1/2}\right)_r$	$\left(\frac{L^2E_v^{1/2}}{\rho^{1/2}}\right)_r$
Dynamic				
Mass	M	$(L^3\rho)_r$	$(L^3\rho)_r$	$(L^3\rho)_r$
Force	MLT^{-2}	$\left(\frac{\mu^2}{\rho}\right)_r$	$(L^3\rho g)_r$	$(L^2E_v)_r$
Pressure	$ML^{-1}T^{-2}$	$\left(\frac{\mu^2}{L^2\rho}\right)_r$	$(L\rho g)_r$	$(E_v)_r$
Impulse and Momentum	MLT^{-1}	$(L^2\mu)_r$	$\left(L^{7/2}\rho g^{1/2}\right)_r$	$\left(L^3\rho^{1/2}E_v^{1/2}\right)_r$
Energy and Work	ML^2T^{-2}	$\left(\frac{L\mu^2}{\rho}\right)_r$	$(L^4\rho g)_r$	$(L^3E_v)_r$
Power	ML^2T^{-3}	$\left(\frac{\mu^3}{L\rho^2}\right)_r$	$\left(L^{7/2}\rho g^{3/2}\right)_r$	$\left(\frac{L^2E_v^{3/2}}{\rho^{1/2}}\right)_r$

Lesson 29

CLASSIFICATION OF PUMPS

29.1 Introduction

Pumps are used to transfer and distribute liquids in various industries. Pumps convert mechanical energy into hydraulic energy. Electrical energy is generally used to operate the various types of pumps.

Pumps have two main purposes.

- Transfer of liquid from one place to another place (e.g. water from an underground into a water storage tank).
- Circulate liquid around a system (e.g. cooling water or lubricants through machines and equipment).

29.2 Components of a Pumping System

- Pump casing and impellers
- Prime movers: electric motors, diesel engines or air system
- Piping used to carry the fluid
- Valves, used to control the flow in the system
- Other fittings, controls and instrumentation
- End-use equipment, which have different requirements (e.g. pressure, flow) and therefore determine the pumping system components and configuration. Examples include heat exchangers, tanks and hydraulic machines.

29.3 Classification

There exist a wide variety of pumps that are designed for various specific applications. However, most of them can be broadly classified into two categories as mentioned below.

i. positive displacement
ii. dynamic pressure pumps

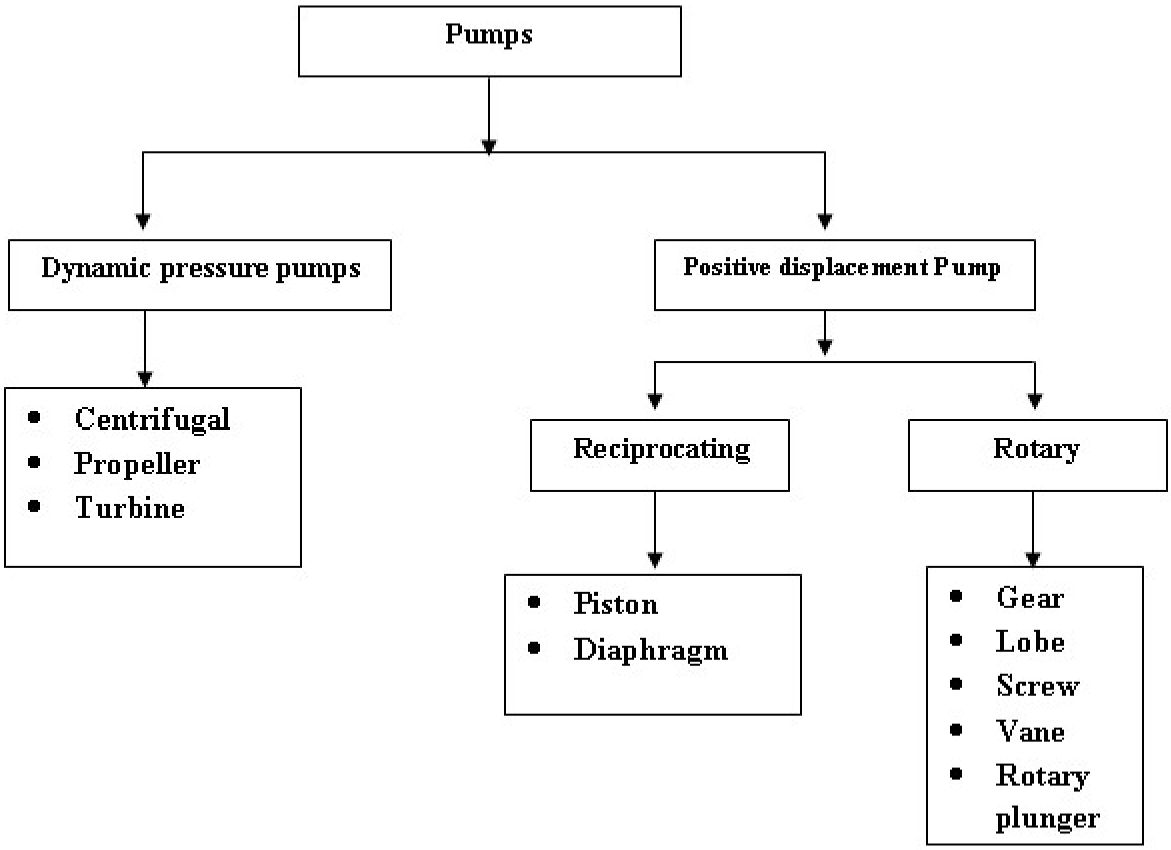


Fig. 29.1 Classification of pumps

29.4 Positive Displacement Pumps

The term positive displacement pump is quite descriptive, because such pumps are designed to displace a more or less fixed volume of fluid during each cycle of operation. The volumetric flow rate is determined by the displacement per cycle of the moving member (either rotating or reciprocating) times the cycle rate (e.g. rpm). The flow capacity is thus fixed by the design, size, and operating speed of the pump. The pressure (or head) that the pump develops depends upon the flow resistance of the system in which the pump is installed and is limited only by the size of the driving motor and the strength of the parts. Consequently, the discharge line from the pump should never be closed off without allowing for recycle around the pump or damage to the pump could result. They can be further classified as:

29.5 Types of Positive Displacement Pumps

29.5.1 Reciprocating pumps

Pumping takes place by to and fro motion of the piston or diaphragm in the cylinder. It is often used where relatively small quantity of liquid is to be handled and where delivery pressure is quite large.

Piston pump: A piston pump is a type of positive displacement pump where the high-pressure seal reciprocates with the piston. The pump has a piston cylinder arrangement. As the piston, goes away after the delivery stoke, low pressure is created in the cylinder which opens the suction valve. On forward stoke, the fluid filled inside the cylinder is compressed which intern opens the delivery valve for the delivery of liquid.

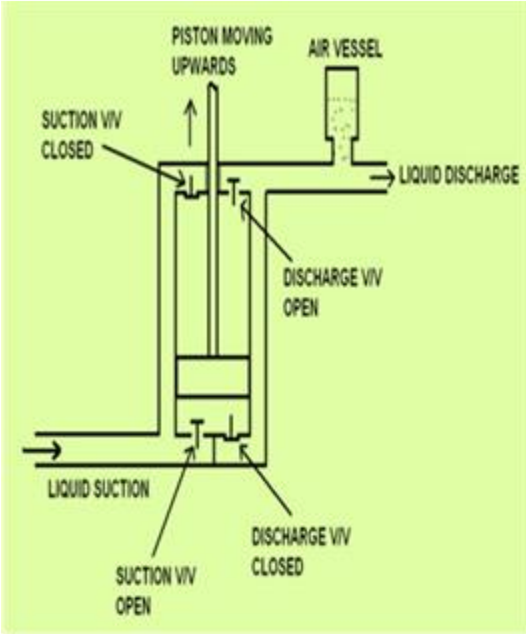


Fig. 29.2 Piston pump

Diaphragm pump: uses a combination of the reciprocating action of a rubber, thermoplastic or Teflon diaphragm and suitable non-return check valves to pump a fluid. Sometimes this type of pump is also called a membrane pump.

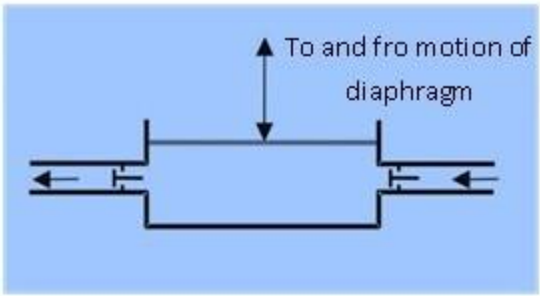


Fig. 29.3 Diaphragm pump

29.5.2 Rotary pumps

In rotary pumps, relative movement between rotating elements and the stationary element of the pump cause the pumping action. The operation is different from reciprocating pumps, where valves and a piston are integral to the pump. They also differ from centrifugal pumps, where high velocity is turned into pressure. Rotary pumps are designed so that a continuous seal is maintained between inlet and outlet ports by the action and position of the pumping elements and close running clearances of the pump. Therefore, rotary pumps do not require valve arrangements similar to reciprocating pumps.

Gear pumps: uses the meshing of gears to pump fluid by displacement. They are one of the most common types of pumps for hydraulic fluid power applications. The rigid design of the gears and houses allow for very high pressures and the ability to pump highly viscous fluids.

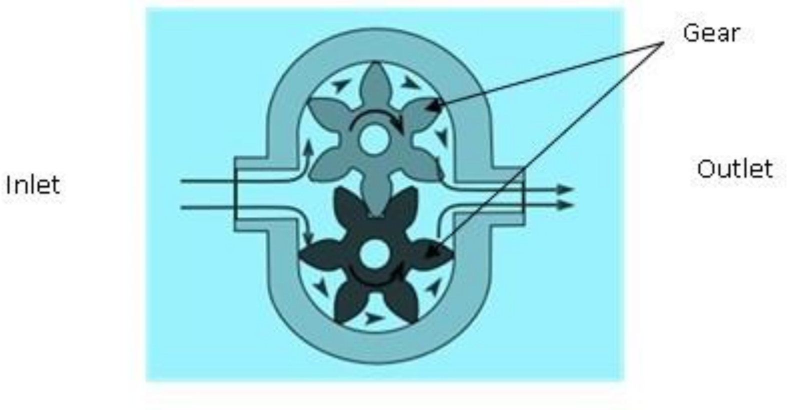


Fig. 29.4 Gear pump

Lobe pump: Lobe pumps are similar to external gear pumps in operation in that fluid flows around the interior of the casing. As the lobes come out of mesh, they create expanding volume on the inlet side of the pump. Liquid flows into the cavity and is trapped by the lobes as they rotate. Liquid travels around the interior of the casing in the pockets between the lobes and the casing. Finally, the meshing of the lobes forces liquid through the outlet port under pressure.

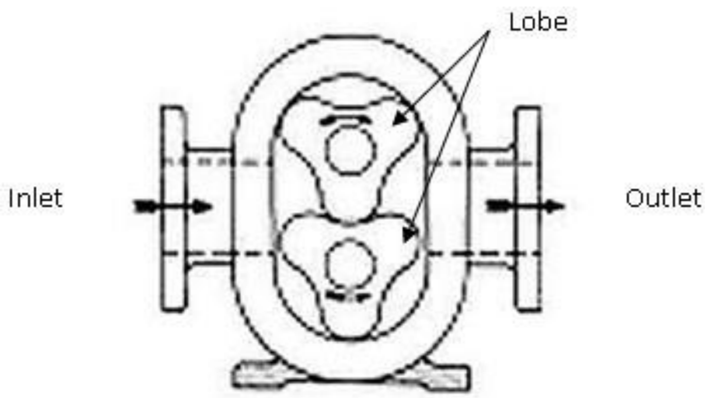


Fig. 29.5 Lobe pump

Screw Pump: These pumps are rotary, positive displacement pumps that can have one or more screws to transfer high or low viscosity fluids along an axis. Although progressive cavity pumps can be referred to as a single screw pumps, typically screw pumps have two or more intermeshing screws rotating axially clockwise or counterclockwise. Each screw thread is matched to carry a specific volume of fluid. Screw pumps provide a specific volume with each cycle and can be dependable in metering applications.

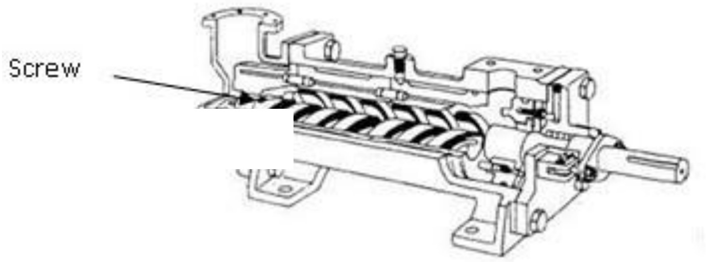


Fig. 29.6 Screw pump

Vane pump: A rotary vane pump is a positive-displacement pump that consists of vanes mounted to a rotor that rotates inside of a cavity. In some cases, these vanes can be variable length and/or tensioned to maintain contact with the walls as the pump rotates.

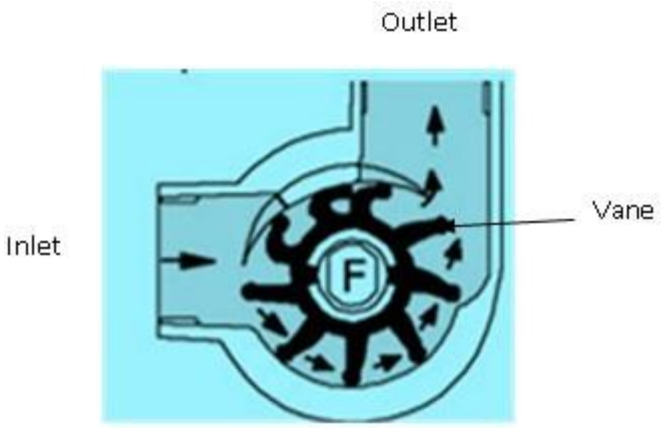


Fig. 29.7 Vane pump

Rotary plunger pump: The pumping action takes place by rotating rotor and reciprocating plunger. In a rotary plunger rotary pump, the axes of the plungers are perpendicular to the rotational axis of the rotor or at an angle of not less than 45° to the axis; the rotor is located eccentrically with respect to the axis of the case.

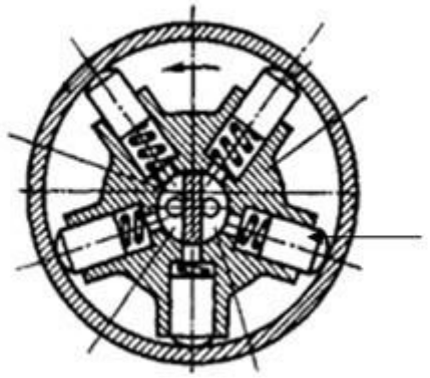


Fig. 29.8 Rotary plunger pump

Suction and forced delivery of the liquid occur with the reciprocating motion of the plungers as a result of centrifugal forces and spring action. Rotary pumps of this type may have as many as 72 plungers arranged in multiple rows, provide a delivery $Q \leq 400$ liters/min, and build up a pumping pressure $\rho \leq 100$ MN/m^2 .

29.6 Dynamic Pressure Pumps

In dynamic pressure pump, during pumping action, tangential force is imparted which accelerates the fluid normally by rotation of impeller. Some systems which contain dynamic pump may require positive displacement pump for priming. They are normally used for moderate to high discharge rate. The pressure differential range for this type of pumps is in a range of low to moderate. They are popularly used in a system where low viscosity fluids are used.

29.6.1 Centrifugal pumps

They use a rotating impeller to increase the pressure of a fluid. Centrifugal pumps are commonly used to move liquids through a piping system. The fluid enters the pump impeller along or near to the rotating axis and is accelerated by the impeller, flowing radially outward into a diffuser or volute chamber (casing), from where it exits into the downstream piping system. Centrifugal pumps are used for large discharge through smaller heads. These types of pumps are used for supply of water and handling of milk in dairy plants.

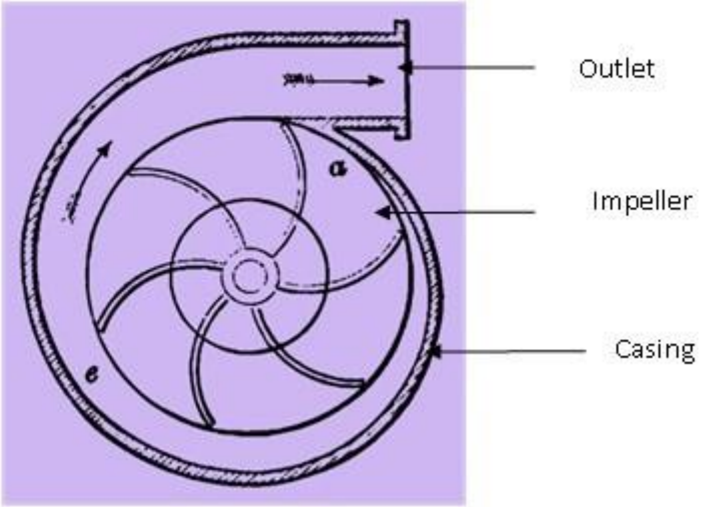


Fig. 29.9 Centrifugal pump

29.6.2 Propeller pump

A propeller pump is a high flow, low lift impeller type device featuring a linear flow path. The propeller pump may be installed in a vertical, horizontal, or angled orientation and typically has its motor situated above the water level with the impeller below water. These pumps function by drawing water up an outer casing and out of a discharge outlet via a propeller bladed impeller head.

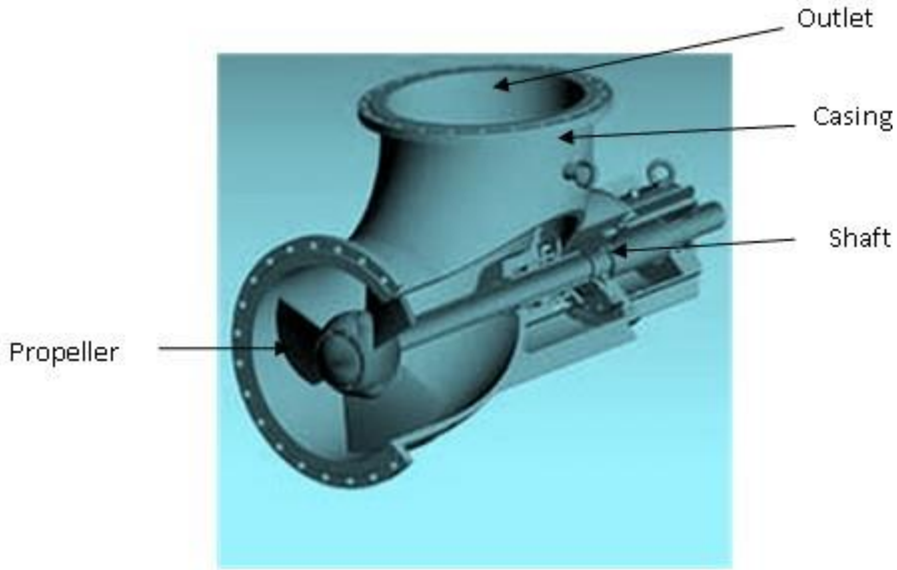


Fig. 29.10 Propeller pump

29.6.3 Turbine pump

Turbine pumps are centrifugal pumps that use pressure and flow in combination with a rotary mechanism to transfer fluid. They typically employ blade geometry, which causes fluid circulation around the vanes to add pressure from inlet to outlet. Turbine pumps operate using kinetic energy to move fluid utilizing an impeller. The centrifugal force drives the liquid to the housing wall in close proximity to the vanes of the impeller or propeller. The cyclical movement of the impeller produces pressure in the pumping bowl. The shape of turbine pumps also contributes to suction and discharge rates.

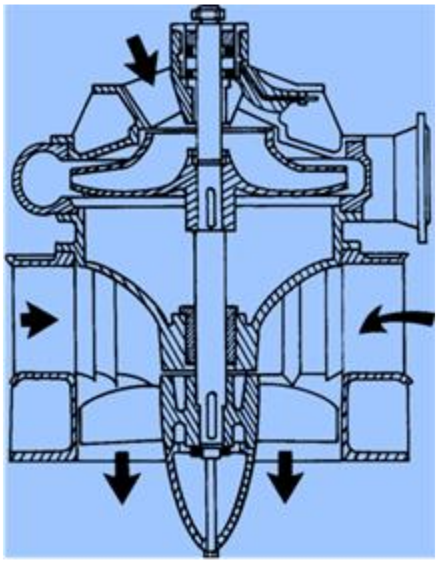


Fig. 29.11 Turbine pump

Lesson 30

RECIPROCATING PUMPS

30.1 Introduction

Reciprocating pumps create and displace a volume of liquid by the action of a reciprocating element. The pump consists of cylinder, piston or plunger and drive arrangement for to and fro motion of the piston. These pumps are called positive displacement pump as they delivers volume of the fluid filled in the cylinder irrespective of the delivery head and develops higher pressure as compared to centrifugal pumps. However, liquid discharge pressure is limited only by strength of structural parts. A pressure relief valve and a discharge check valve are normally required for reciprocating pumps. Reciprocating pump is used in milk homogenizer in dairy industry in order to develop high pressure. Reciprocating pumps can be further classified into two types, as given below.

i) Piston Pumps

ii) Diaphragm Pumps

30.2 Piston Pump

Hand pump is a simplest form of piston pump used in villages for lifting water from the tube well. Though, these pumps are replaced by sub-mersible electrically operated pumps even in villages. The piston pump is one of the most common reciprocating pumps for a broad range of applications prior to the development of high speed centrifugal pumps. Reciprocating pumps are used in low flow rate applications with very high pressure.

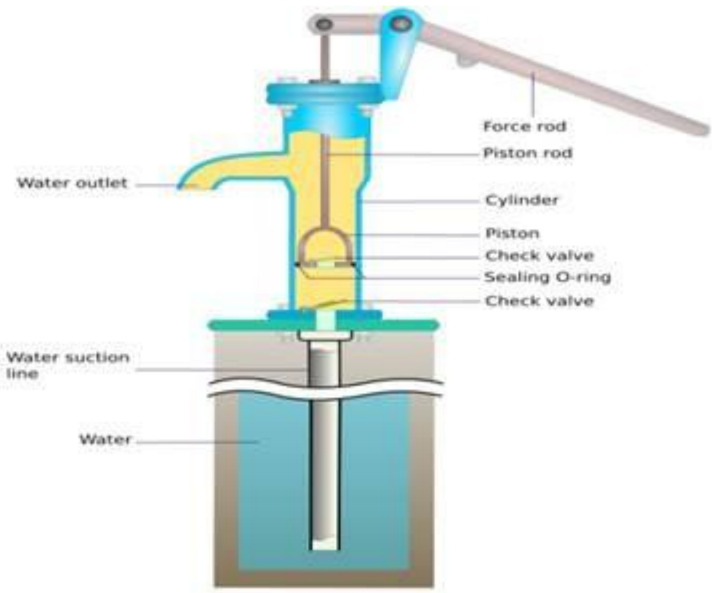


Fig. 30.1 Hand pump

30.2.1 Working of reciprocating pump

Fig. 30.2 shows a single acting reciprocating pump having piston which moves forwards and backwards in a close fitting cylinder. The movement of piston in the cylinder is obtained by connecting the piston rod to crank by means of connecting rod. The crank is rotated by means of an electric motor. The suction and delivery valves are suitably placed which are one way valves.

As the crank rotates from A to C ($\theta = 0°$ to $\theta = 180°$), the piston moves towards right in the cylinder. This creates partial vacuum in the cylinder. As the pressure on the water surface is higher, the liquid enters the cylinder through suction valve. The movement of piston from C to A ($\theta = 180°$ to $\theta = 360°$), closes the suction valve and opens delivery valve and the liquid is forced in the delivery pipe.

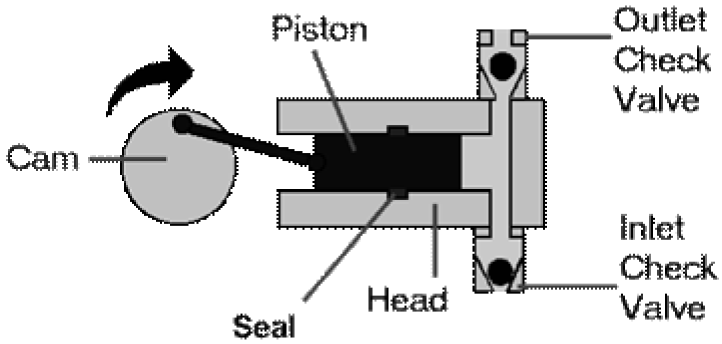


Fig. 30.2 Working and main parts of reciprocating pump (single acting)

The reciprocating pumps are further classified as single acting and double acting. In case of single acting pump as shown in Fig. 30.2, the pump delivers one effective discharge stoke per one revolution of the crank (i.e. one suction stoke and one delivery stroke). Double acting reciprocating pump has suction valve and delivery valve on both the sides of the piston as shown in Fig. 30.3.

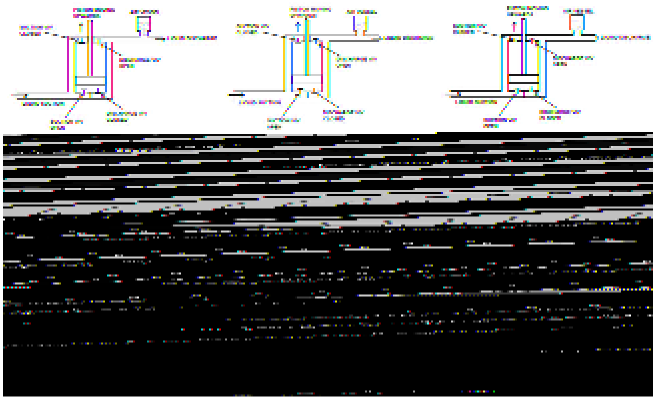


Fig. 30.3 Working of double acting piston pump

The single acting pump discharges water only on its forward stroke while the double acting pump discharges on its return stroke as well.

30.2.2 Capacity of reciprocating pump

The capacity (Q) of a single acting piston or plunger pump is proportional to its displacement per unit time. The displacement is the calculated capacity of the pump, assuming 100% hydraulic efficiency, and is proportional to the cross sectional area of the piston (A), the length of its stroke (L), the number of cylinders (n), and the speed (N) of the pump (r.p.m. of the crank).

$$\therefore\ Discharge\ per\ second, Q = A \times L \times \frac{N}{60} \times n \times number\ of\ acting$$

It is obvious that the rate of discharge will be double neglecting the volume of piston rod in case of double acting pump.

The actual discharge under field conditions will be less as compared to theoretical discharge. The difference between actual discharge and theoretical discharge of the pump is called slip of a pump. It is expressed as % slip which is given by:

$$\%\ slip = \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100$$

$$\%\ slip = \left(1 - \frac{Q_{act}}{Q_{th}}\right) \times 100$$

The major component of slip is the leakage of fluid back through the discharge or suction valve as it is closing (or seated). It can reduce calculated displacement from 2 to 10% depending upon valve design and condition. Another important factor that affects a capacity of reciprocating pump is called volumetric efficiency (VE). VE is expressed as a percentage and is proportional to the ratio of the total discharge volume to the piston or plunger displacement.

The ratio (r) is shown to be (c+d)/d where d is the volume displaced by the piston or plunger and c is the additional volume between the discharge and suction valves. The smaller is this ratio, the better the volumetric efficiency. Expressed mathematically as:

$$VE = 1 - (P * b * r) – S$$

Where, P is pressure, b is the liquid's compressibility factor, r is the volume ratio, and S is slip. The compressibility factor for water is quite small but at pressures greater than 10,000 PSI it does become a factor. Although there is no cylinder wall around the plunger at the bottom of its stroke, it still displaces fluid equal to its own volume. The actual capacity of the pump is given by:

$$Q_{act} = Q_{th} * VE$$

30.2.3 Power requirement for reciprocating pump

The work done by reciprocating pump is given as under:

work done/second = (weight of water lifted/second)*(total height through which water is lifted)

$$= \frac{\omega ALN}{60} \times (h_s + h_d)$$

30.2.4 Simplex and duplex pumps

A simplex pump, sometimes referred to as a single pump, is a pump having a single liquid (pump) cylinder. A duplex pump is the equivalent of two simplex pumps placed side by side on the same foundation. The driving of the pistons of a duplex pump is arranged in such a manner that when one piston is on its upstroke the other piston is on its downstroke, and vice versa. This arrangement doubles the capacity of the duplex pump compared to a simplex pump of comparable design.

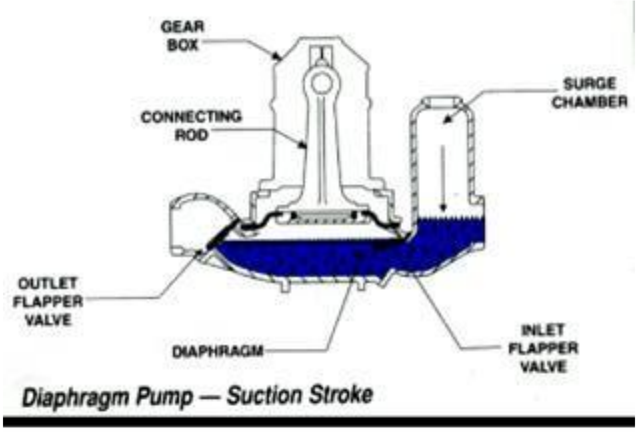
30.2.5 Triplex pump

A positive-displacement reciprocating pump that is configured with three plungers. Generally milk homogenizer are of triplex type.

Fig. 30.4 Triplex pump

30.3 Diaphragm Pumps

Diaphragm pumps are reciprocating positive displacement pumps that employ a flexible membrane instead of a piston or plunger to displace the pumped fluid. They are truly self priming (can prime dry) and can run dry without damage. They operate via the same volumetric displacement principle described earlier.



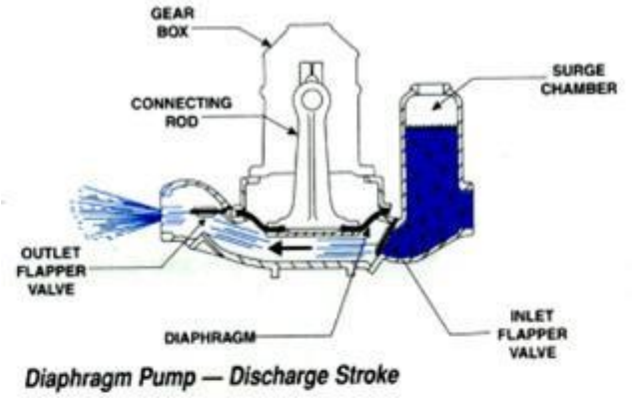


Fig. 30.5 Diaphragm pump

Were its operation any simpler, it would compete with gravity. The upper portion of the figure shows the suction stroke. The handle lifts the diaphragm creating a partial vacuum which closes the discharge valve while allowing liquid to enter the pump chamber via the suction valve. During the discharge stroke the diaphragm is pushed downward and the process is reversed.

You will note that, unlike pistons and plungers, diaphragms do not require a sealing system and therefore operate leak free. This feature does, however, preclude the possibility of a double acting design. If nearly continuous flow is required, a double-diaphragm or duplex pump is usually employed. The figure below is a cross section of an air operated, double diaphragm pump.

The double diaphragm pump utilizes a common suction and discharge manifold teamed with two diaphragms rigidly connected by a shaft. The pumped liquid resides in the outside chamber of each while compressed air is routed to and from their inner chambers. In the figure, the right hand chamber has just completed its suction stroke and, simultaneously, the left chamber completed its discharge stroke. As would be expected, the suction check is open so that liquid can flow into the right chamber and the discharge check of the left chamber is open so that liquid can flow out. Except for the double chamber configuration, its operation is just like the double acting piston pump seen earlier. The difference, of course, resides within the inner chambers and the method in which the reciprocating motion is maintained. This is accomplished by an air distribution valve that introduces compressed air to one diaphragm chamber while exhausting it from the other. Upon completion of the stroke the valve rotates 90 degrees and reciprocation occurs.

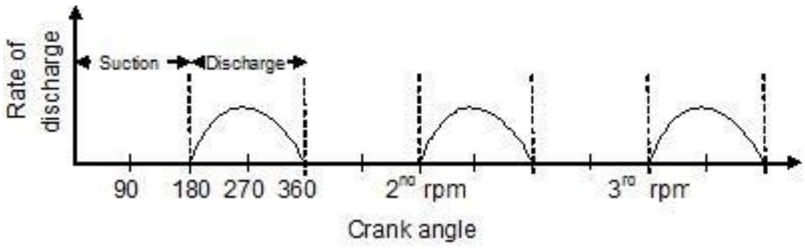


Fig. 30.6 Single cylinder single acting pump

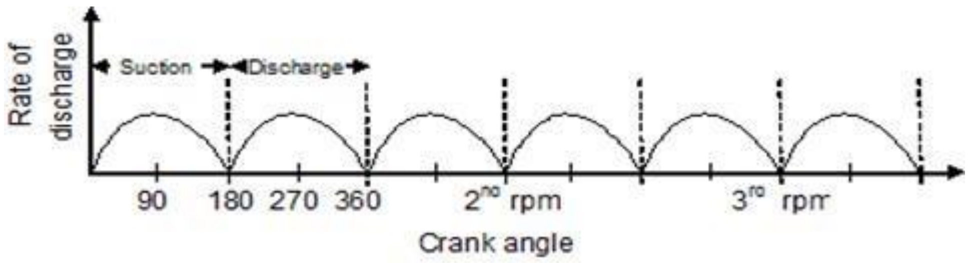


Fig. 30.7 (a) Single cylinder double acting pump or double cylinder single acting pump

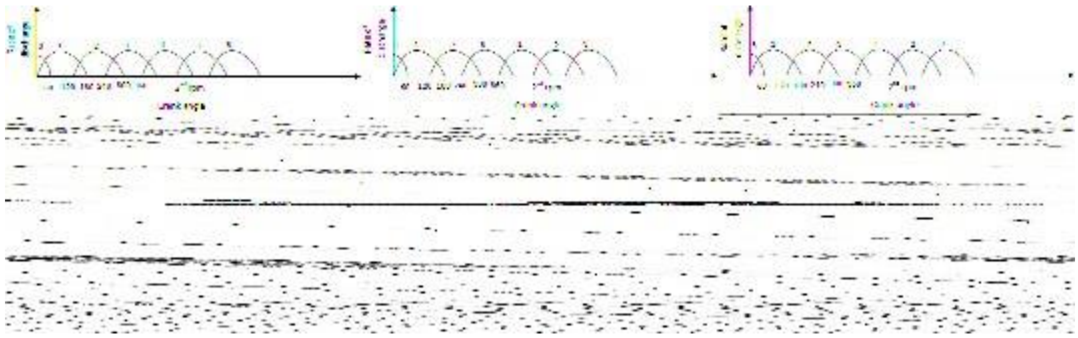


Fig. 30.7 (b) Triplex single acting reciprocating pump

Lesson 31

CENTRIFUGAL PUMP, PRESSURE VARIATION, WORK DONE, EFFICIENCY

31.1 Centrifugal Pump

Centrifugal pump uses a rotating impeller to increase the pressure of a fluid. Centrifugal pumps are very common in food and dairy processing industry. These pumps deliver large flow rate at low or medium head and are used for low viscous liquid like water, milk, juices, cream etc.

Fig. 31.1 Centrifugal pump

31.2 Construction of a Centrifugal Pump

Different parts of centrifugal pumps are as follows (Fig. 31.2):

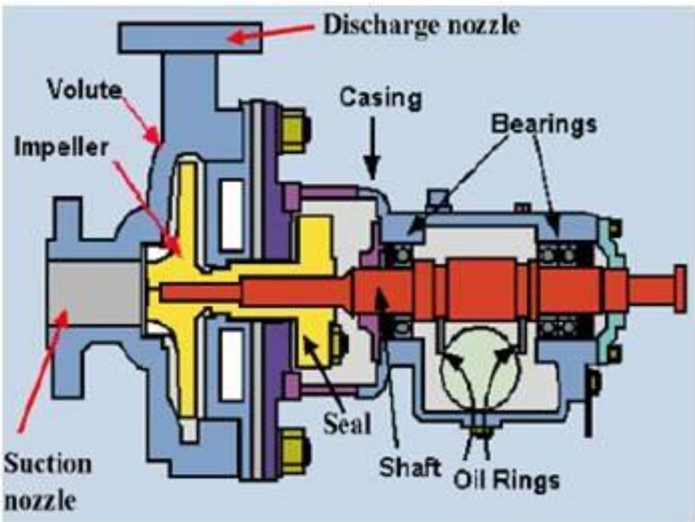


Fig. 31.2 Parts of a centrifugal pump

31.2.1 Rotating component

It consists of shaft and impeller. Shaft is coupled with a motor to give drive to the pump. An impeller is a rotating component of a centrifugal pump, usually made of iron, steel, bronze, brass or aluminium which transfers energy from the motor that drives the pump to the fluid being pumped by accelerating the fluid outwards from the center of rotation. Different types of impeller are:

***Open type impeller*:** It consists of vane which accelerates the fluid. The number of vanes can vary from 2 to 8 or more.

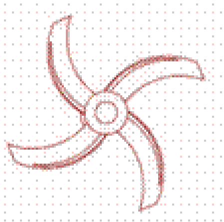


Fig. 31.3 Open type impeller

***Semi-closed impeller*:** It consists of vane attached to plate on one side. Industrial application of such types of impeller is for pumping liquids containing suspensions.

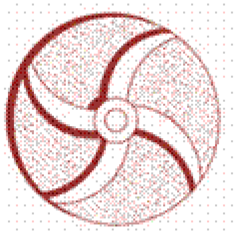


Fig. 31.4 Semi-closed impeller

***Closed impeller*:** The vane is enclosed by two disc or plates. Shrouds or sidewall encloses the vanes. The liquid moves in between the cavity between the vanes and plates.

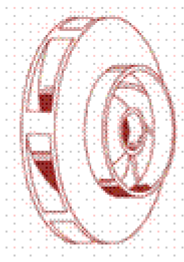


Fig. 31.5 Closed impeller

It can also be classified as based on major direction of flow in reference to the axis of rotation

1. Radial flow
2. Axial flow
3. Mixed flow

31.2.2 Casing

It is the cover housing the impeller. The rotating impeller increases the kinetic energy of the liquid. The casing helps to covert this kinetic energy into pressure energy. The main function of casing is to enclose the impeller at suction and delivery ends and thereby form a pressure vessel. The pressure at suction end may be as little as one-tenth of atmospheric pressure and at delivery end may be twenty times the atmospheric pressure in a single-stage pump.

The vanes of the rotating impeller impart a radial and rotary motion to the liquid, forcing it to the outer periphery of the pump casing where it is collected in the outer part of the pump casing called the volute. The volute is a region that expands in cross-sectional area as it wraps around the pump casing. Purpose of the volute is to collect the liquid discharged from the periphery of the impeller at high velocity and gradually cause a reduction in fluid velocity by increasing the flow area This converts the velocity head to static pressure. The fluid is then discharged from the centrifugal pump through the discharge connection.

32.2.2.1 Types of chamber/casing

Volute casing: This is named from the spiral shape of the casing which is so constructed to act as a collector for the liquid as it leaves the outer edge of the impeller vanes. In volute pumps area of flow gradually increases from throat towards the delivery pipe. The increase in area of flow decreases the exit velocity and hence pressure increases in the casing.

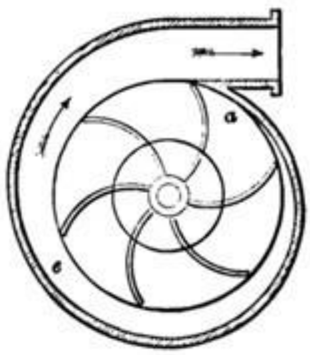


Fig. 31.6 Volute casing

***Vortex*:** Vortex casing is a casing in which circular chamber is provided between the casing and the impeller. Vortex casing will increase pump efficiency by reducing eddies formation to a considerable extent.

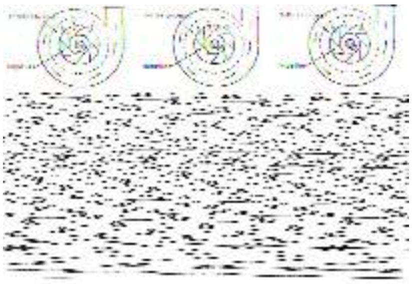


Fig. 31.7 Vortex

Diffuser: In diffuser casing, the fluid passes through a ring of fixed vanes or diffuser after the fluid has left the impeller, that diffuse the liquid, this provides a more controlled flow and a more efficient conversion of velocity head into pressure head. Providing fixed diffuser increases the efficiency of the pump up to 90 percent.

Fig. 31.8 Diffuser

31.2.3 Suction and delivery pipe

A pipe whose one end is connected to the inlet of the pump and other end dips into water in a pump is known as suction pipe. A foot value is a non- return value fitted at the lower end of the suction pipe which helps to fill water during priming. The suction pipe is connected to the opening (eye) of the impeller. Discharge pipe is provided on the pump casing to attach delivery side.

31.2.4 Bearing housing

The bearing housing encloses the bearings mounted on the shaft. The bearings keep the shaft or rotor in correct alignment with the stationary parts under the action of radial and transverse loads. The bearing house also includes an oil reservoir for lubrication, constant level of oil, jacket for cooling by circulating cooling water.

31.3 Working Principle of Centrifugal Pumps

The centrifugal pump operates by the transfer of energy (or angular momentum) from a rotating impeller to the fluid, which is normally inside a casing. The fluid enters at the axis or ‘eye’ of the impeller (which may be open or closed and usually contains radial curved vanes) and is discharged from the impeller periphery. On rotation of impeller, partial vacuum is created in the casing which causes the suction of liquid from the reservoir. The maximum possible suction lift is 10.3 m of water. The kinetic energy and momentum of the fluid are increased by the angular momentum imparted by the high-speed impeller. This kinetic energy is then converted to pressure energy (‘head’) in a diverging area (the ‘volute’) between the impeller discharge and the casing before the fluid exits the pump. The head that these pumps can develop depends upon the pump design and the size, shape, and speed of the impeller and the flow capacity is determined by the flow resistance of the system in which the pump is installed.

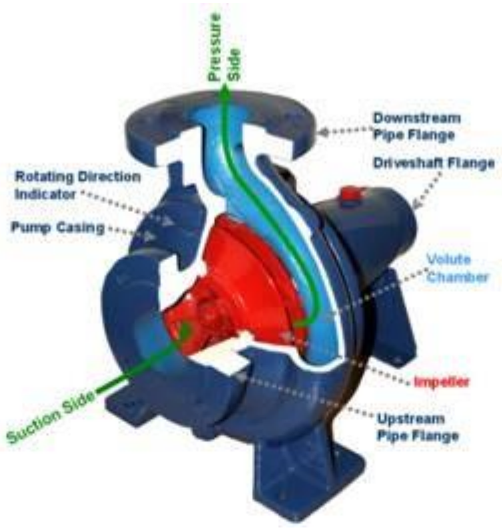


Fig 31.9 Cut view of centrifugal pump

Thus, these pumps operate at approximately constant head and variable flow rate within limits. Centrifugal pumps can be operated in a ‘closed off’ condition (i.e. closed discharge line), because the liquid can re-circulate within the pump without causing damage. However, such conditions should be avoided, because energy dissipation within the pump could result in excessive heating of the fluid and/or the pump or unstable operation, with adverse consequences. Centrifugal pumps are most appropriate for ‘ordinary’ (i.e. low to moderate viscosity) liquids under a wide variety of flow conditions and are thus the most common type of pump. The following discussion applies primarily to centrifugal pumps.

Priming of centrifugal pump

The operation in which the suction pipe, casing of pump and a portion of delivery pipe is filled by the outside source of liquid before starting the pump. Foot value is necessary at the end of suction pipe to fill water in the pipe for priming. This process removes air from impeller and casing necessary for pumping of liquid.

31.4 Few Important Terms

Capacity**:** It is the water handling capability of a pump commonly expressed as either cubic meter per minute (m^3/min).

***Cavitation*:** It is a phenomenon caused as a result of vapor bubbles imploding. This is the result of bubble formation at the suction point due to pressure difference.

***Discharge Port*:** It is the point where the discharge hose or pipe is connected to the pump.

***Datum*:** It is used as reference of the horizontal plane for which all the elevations and head are measured.

***Dynamic Discharge Head*:** It is the static discharge head plus the friction in the discharge line also referred to as Total Discharge Head.

***Dynamic Suction Head*:** It is the static suction lift plus the friction in the suction line also referred to as Total Suction Head.

***Friction Head*:** It is the head required to overcome the resistance to flow in the pipe and fittings. It is dependent upon the size, condition and type of pipe, number and type of pipe fittings, flow rate, and nature of the liquid.

***Friction Loss*:** It refers to the reductions in flow due to turbulence as water passes through hoses, pipes, fittings and elbows.

***Priming*:** Most centrifugal pumps are not self-priming. In other words, the pump casing must be filled with liquid before the pump is started, otherwise the pump will not function. If the pump casing becomes filled with vapors or gases, the pump impeller becomes gas-bound and incapable of pumping. To ensure that a centrifugal pump remains primed and does not become gas-bound, it is necessary to attach a foot valve at the end of the suction pipeline.

31.5 Terminologies Associated to Pumping Systems

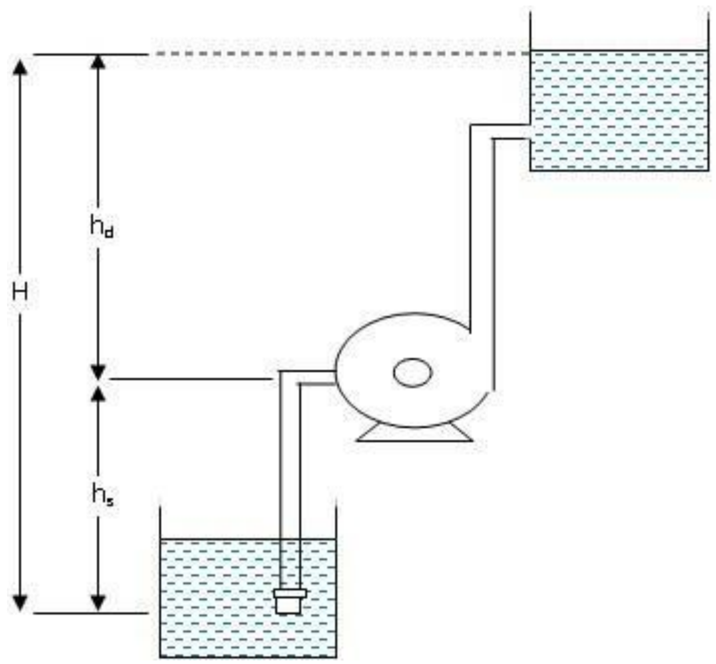


Fig. 31.10 Pressure variations in a pump

***Static Suction Head (hs)*: ** It is the vertical distance between the centre of the impeller to the free surface of the water from where water is to be pumped. If the liquid level is above pump centerline, *hs* is positive. If the liquid level is below pump centerline, *hs* is negative. Negative *hs* condition is commonly denoted as a 'suction lift'.

***Static Discharge Head (hd)*: ** It is the vertical distance between the pump centerline and the point of free discharge or the surface of the liquid in the discharge tank.

***Total Suction Head (hs)*: ** It is the suction reservoir pressure head (*hps*) plus the static suction head (*hs*) plus the velocity head at the pump suction flange (*hvs*) minus the friction head in the suction line (*hfs*).

HS = *hps* + *hs* + *hvs* + *hfs*

***Total Discharge Head (hd)*: ** It is the sum total of discharge reservoir pressure head (*hpd*), static discharge head (*hd*), the velocity head at the pump discharge flange (*hvd*) and the total friction head in the discharge line (*hfd*).

hd = *hpd* + *hd* + *hvd* + *hfd*

***Manometric head*: ** It is defined as the head against which a centrifugal pump has to work.

$$H_m = h_s + h_{fs} + h_{fd} + \frac{{V_d}^2}{2g}$$

Where,

h_s = Suction head

h_d = delivery head

h_{fs} = Frictional head loss in suction pipe

h_{fd} = Frictional head loss in delivery pipe

V_d = Velocity of pipe in delivery pipe

31.6 Pump Performance Curve

The main characteristic curves of a centrifugal pump consistes pf variation of head (h_m), power (P) and discharge (Q) with respect to speed (Fig. 31.11) The operating characteristic curves is shown in fig. 31.12.

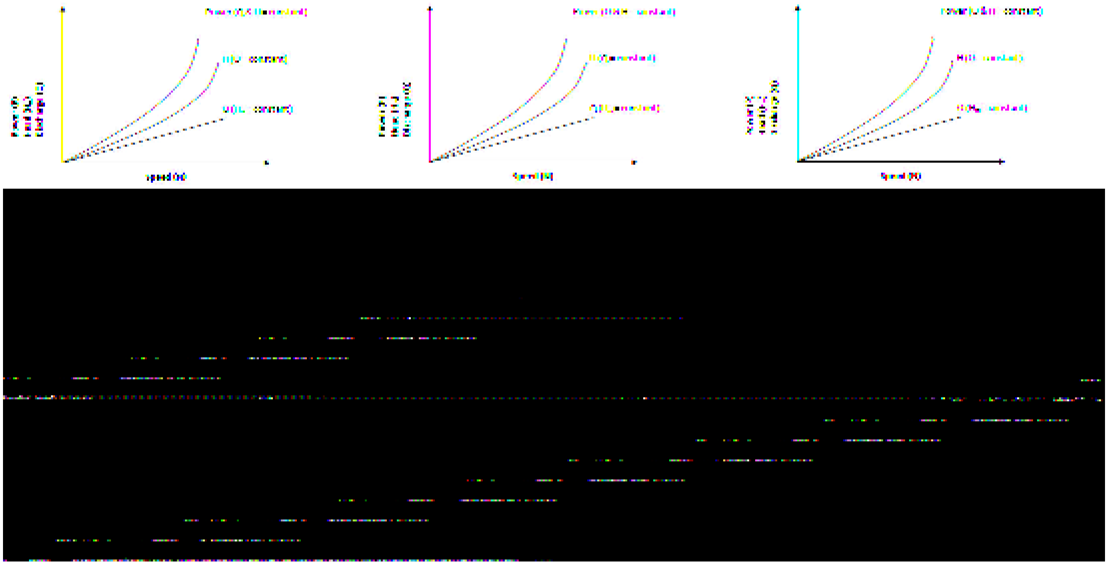


Fig. 31.11 Main characteristic curve

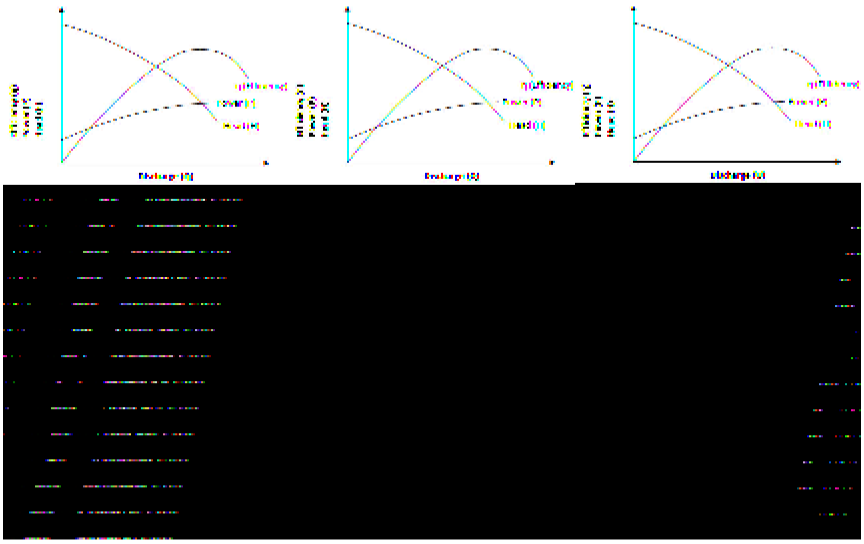


Fig. 31.12 Operating characteristic curves

31.7 Pump Efficiency

Pump efficiency, η (%) is a measure of the efficiency with which the pump uses the input power to convert the energy into useful output.

$\eta = P_{out}/P_{in}$

where

η = efficiency (%)

P_{in} = power input

P_{out} = power output

31.8 Water Horse Power (WHP) and Brake Horse Power (BHP)

The work performed by a pump is a function of the total head and the weight of the liquid pumped in a given time period. Pump input or brake horsepower (BHP) is the actual horsepower delivered to the pump shaft. Pump output or hydraulic or water horsepower (WHP) is the liquid horsepower delivered by the pump. These two terms are defined by the following formulas.

$$WHP = \rho g H Q$$

$$BHP = \frac{\rho g H Q}{\eta}$$

where:

BHP is the brake horse power required (W)

ρ is the fluid density (kg/m^3)

g is the standard acceleration of gravity (9.81 m/s^2)

H is the energy Head added to the flow (m)

Q is the flow rate (m^3/s)

η is the efficiency of the pump (decimal)

$$\eta = \frac{WHP}{BHP}$$

The head added by the pump (*H*) is a sum of the static lift, the head loss due to friction and any losses due to valves or pipe bends all expressed in metres of fluid. Power is more commonly expressed as kilowatts (10^3 W) or horsepower (multiply kilowatts by 0.746). The value for the pump efficiency η may be stated for the pump itself or as a combined efficiency of the pump and motor system. The energy usage is determined by multiplying the power requirement by the length of time the pump is operating.

31.8 Pump selection criteria

Pump is selected on the basis of the following criteria:

a) Nature of liquid

- Type- Water, Beverage, Juice, milk, cream etc
- Density
- Viscosity
- Clear liquid/suspended

b) Capacity (m^3/min or m^3/h)

c) Suction head (m)

d) Discharge head (m)

e) Total head (m) = suction head + discharge head

f) Pump installation (horizontal or vertical)

g) Power requirement (kW)

Lesson 32
NUMERICALS ON PUMPS

32.1 Numericals

1. **Calculate the discharge of reciprocating pump (single cutting) if area of cylinder is 0.25 m^2, length of stroke is 0.15 m, number of cylinder =1 and speed of pump is 50 rpm.**

Sol.

$$Discharge\ Q = A \times L \times \frac{N}{60} \times n \times number of\ cutting$$

A = 0.25 m^2
L = 0.15 m
N = 50 rpm
n = 1
number of acting =1

$$Q = 0.25 \times 0.15 \times \frac{50}{60} \times 1 \times 1$$

= 0.03125 m^3 /s

2. **Calculate the power required to drive the single cutting reciprocating pump for water of following specification**.

Sol.

Area of cylinder = 0.85 m^2
Length of stroke = 0.35 m
Number of cylinder = 2
Speed of pump = 15 rpm
Suction head = 0.5 m
Discharge head = 1 m

$$Power\ (hp) = \frac{WALNn\ (h_s + h_d)}{4500}$$

$$Where\ w = \rho_g$$

$$power = \frac{1000 \times 9.81 \times 0.85 \times 0.35 \times 15 \times 2 \times (0.5 + 1)}{4500} = 29.1\ h.p.$$

3. Calculate water horse power for centrifugal water pump if flow rate is 4500 liter/h. head added to the flow is 10 m.

Sol.

1 m^3 = 1000 L

$$Q = \frac{4500 \times 10^{-3}}{60 \times 60} \; m^3/s$$

$$water\ horse\ power\ WHP = \rho_g\ HQ$$

$$= \frac{1000 \times 9.81 \times 10 \times 4500 \times 10^{-3}}{60 \times 60} = 12.26\ hp$$

4. Calculate brake horse power for centrifugal water pump if flow rate is 1000 litres/h. Head added to the flow is 10 m pump efficiency is 86%.

Sol.

1 m^3 = 1000 L

$$Q = \frac{10000 \times 10^{-3}}{60 \times 60}$$

$$Brake\ horse\ power\ bhp = \frac{\rho_g\ HQ}{n}$$

$$bhp = \frac{1000 \times 9.81 \times 10 \times 10000 \times 10^{-3}}{0.86 \times 60 \times 60} = 316.86\ hp$$