REINFORCED CONCRETE STRUCTURES DESIGN AND DRAWING (ACE009)

B.Tech V Semester

Department of Civil Engineering

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COURSE GOAL

- To introduce students to various concepts of Limit state design.
- To impart knowledge regarding the design of structures of buildings like slabs , beams and columns etc

UNIT	TITLE	CONTENTS
1	INTRODUCTION	Concepts of RC Design – Limit state method – Material Stress–Strain curves – Safety factors – Characteristic values – Stress block parameters – IS-456:2000 – Working stress method. BEAMS: Limit state analysis and design of singly reinforced, doubly reinforced, T, and L beam sections

2	SHEAR TORSION AND BOND	Limit state analysis and design of section for shear and torsion – concept of bond, anchorage and development length, I.S. code provisions. Design examples in simply supported and continuous beams, detailing Limit state design for serviceability for deflection, cracking and codal provision.
3	DESIGN OF SLABS	Design of Two-way Slabs, one-way slabs, Continuous slabs using I.S. coefficients, Cantilever slab/ Canopy slab.

4	**SHORT AND LONG COLUMNS**	**Axial loads, uni-axial and bi-axial bending I.S. Code provisions.**
5	**DESIGN OF FOOTINGS**	**Isolated(square, rectangle) and Combined Footings. Design of Stair Case**

UNIT-I
DESIGN OF BEAMS

Plain Cement Concrete

- Plain Cement Concrete is a hardened mass obtained from a mixture of cement, sand, gravel and water in definite proportions.
- These ingredients are mixed together to form a plastic mass which is poured into desired shape moulds called as forms. This plastic mass hardens on setting and we get plain cement concrete.

Plain Cement Concrete

- The hardening of this mixture is caused by a chemical
- reaction between cement and water.
- Plain cement concrete has good compressive strength but very little tensile strength, thus limiting its use in construction.
- Plain concrete is used where good compressive strength and weight are the main requirements and tensile stresses are very low.

REINFORCED CEMENT CONCRETE

- Plain cement concrete has very low tensile strength.
- To improve the tensile strength of concrete, some sort of reinforcement is needed which can take up the tensile stresses developed in the structure.
- The most common type of reinforcement is in the form of steel bars which are quite strong in tension.
- The reinforcing steel is placed in the forms and fresh concrete is poured around it.
- This solidified composite mass is called as Reinforced cement concrete and is abbreviated as R.C.C.

REINFORCED CEMENT CONCRETE

- Thus, Reinforced cement concrete is a composite material which is made up of concrete and steel reinforcement.
- The steel reinforcement generally in the form of steel bars, are placed in the tensile zone of the structure and take up the tensile stresses R.C C. is a versatile construction material which is strong in compression as well as tension.
- The use of reinforcement in concrete not only increases its strength but also helps in preventing the temperature and shrinkage stresses.

REINFORCED CEMENT CONCRETE

The composite action of steel and concrete in a reinforced concrete section is dependent on the following important factors.

(i) The bond between steel and concrete.

(ii) Prevention of corrosion of steel bars embedded in the concrete.

(iii) Practically equal thermal expansion of both concrete and steel.

Uses of Reinforced Concrete

Reinforced cement concrete has innumerable uses in construction some of which are list below:

1. Buildings

2 Flyovers

3. Water Tanks

4 Road and Rail Bridges

5 Chimneys and Towers

6 Retaining Walls

7 Bunkers and Silos

Advantages

Reinforced cement concrete has following advantages over other construction materials.

1. Strength: R.C.C. has very good strength in tension as well as compression.
2. Durability: R.C.C. structures are durable if designed and laid properly. They can last up to 100 years.
3. Mouldability: R.C.C. sections can be given any shape easily by properly designing the formwork. Thus, it is more suitable for architectural requirements.
4. Ductility: The steel reinforcement imparts ductility to the R.C.C. structures.

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5. Economy: R.C.C. is cheaper as compared to steel and prestressed concrete. There is an overall economy by using R.C.C. because its maintenance cost is low.

6. Transportation: The raw materials which are required for R.C.C. i.e. cement, sand, aggregate, water and steel are easily available and can be transported easily. Nowadays Ready Mix Concrete (RMC is used for faster and better construction. (RMC is the concrete which is manufactured in the factory and transported to the site in green or plastic state)

7. Fire Resistance: RCC structures are more fire resistant than other commonly used construction materials like steel and wood

8 Permeability: RC.C is almost impermeable to moisture.

9. Seismic Resistance: Properly designed R.C C structures are extremely resistant to earthquakes

Disadvantages:

Despite the above mentioned advantages, R.C.C. has following disadvantages:

1. R.C.C. structures are heavier than structures of other materials like steel, wood and glass etc.
2. R.C.C. needs lot of formwork, centering and shuttering to be fixed, thus require lot of site space and skilled labour.
3. Concrete takes time to attain its full strength. Thus, R.C.C. structures can't be used immediately after construction unlike steel structures.

MATERIALS USED IN R.C.C.

R.C.C. consists of concrete and reinforcing material. The strength of an R.C.C. section depends on the kind of concrete and reinforcement used.

The properties of concrete depend upon the proportions and type of its ingredients. A properly designed concrete mix is very durable. A good concrete mix should satisfy the following requirements.

(i) The concrete should be mixed thoroughly to from a homogeneous mix.

(ii) Concrete should be compacted properly to prevent it from being porous.

(iii) Sufficient curing of concrete is required for developing full strength.

(iv) The water cement ratio should be appropriate, considering the strength and workability criteria.

(v) The concrete mix should be designed properly and should have all the ingredients in right proportions.

(vi) The water used for mixing should be free from all harmful organic substances.

(vii) The aggregate should be hard, durable and properly graded. For most R.C.C. works, 20 mm size of aggregate is suitable

(Viii) The cement used for R.C.C. work should be of good quality and measured by weight only, and not by volume.

Concrete grades are expressed by letter M followed by a number. The letter 'M' refers to the mix and the number represents the characteristic compressive strength of concrete in N/mm^2.

- The specified characteristic strength is determined for 150 mm size cube at 28 days.

The characteristic compressive strength of concrete is defined as that strength below which not more than 5 percent of the test results are expected to fall.

The various grades of concrete as per their use are listed below:

1. **For R.C.C. work - not lower than M20.**
2. **For post tensioning works - M35 and above.**
3. **For pretensioned prestressed concrete - M40 and above.**

Concrete of grades lower than M20 may be used for plain concrete works, lean concrete, simple foundations, masonry walls and other simple constructions works.

SUITABILITY OF STEEL AS REINFORCING MATERIAL

Many traditional materials such as bamboo and natural fibres have been tried as reinforcement in earlier times.

But steel is found to be the most appropriate form of reinforcement. It is the most suitable reinforcing material in R.C.C. because of following reasons:

1. Steel is very strong in tension, compression, shear and torsion.
2. Concrete develops very good bond with steel.

3. Steel is ductile in behavior. More ductility means more elongation of steel before failure. This results in sufficient warning time before failure.

4. The steel bars can be cut, bent, lifted and welded easily with commonly available tools.

and machines.

5. Steel has longer life.

6 Steel is easily available.

Steel reinforcement has various advantages as listed above, which make it a suitable reinforcing material. However, steel has a few disadvantages which are listed below.

1. The biggest disadvantage of steel reinforcement is rusting. If concrete is porous or if cover to the reinforcement is not sufficient, steel gets rusted and loses strength.
2. Steel loses its strength at high temperatures.

Mild Steel Reinforcement

Mild steel bars are also known as Fe 250 because the yield strength of this steel is 250 N/mm^2.

Although mild steel bars are very ductile are not preferred over high yield strength deformed bars because of their less strength and weak bond. The modulus of elasticity of mild steel is taken as equal to 2 x 10^5N/mm2. However, they are used as lateral ties in columns and also places where nominal reinforcement is required. Mild steel plain bars are represented by symbol

The stress-strain curve for mild steel is given in Fig. 1.1. It shows a clear, definite yield point.

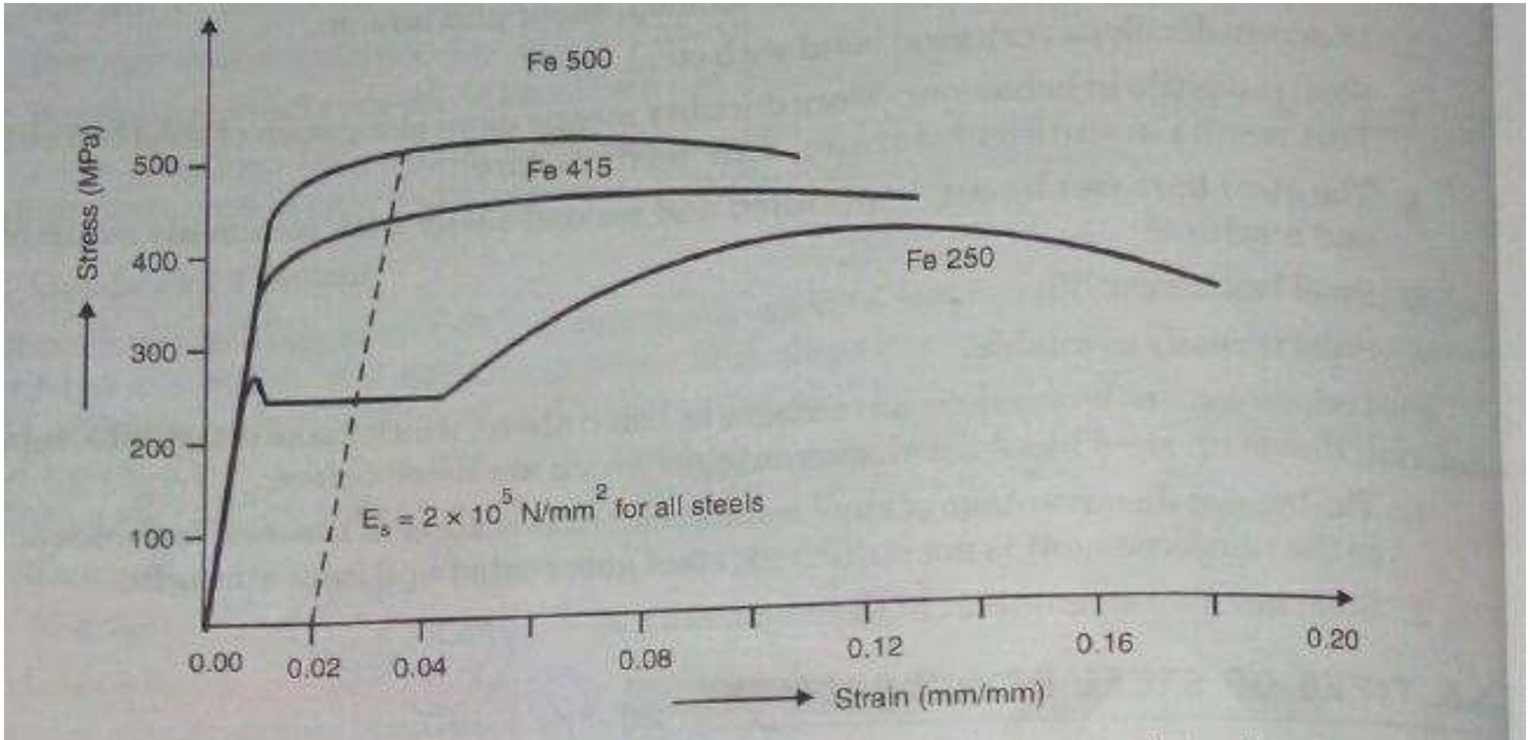


Fig. 1.1. Typical stress strain curves for various types of steel.

METHODS OF R.C.C. DESIGN:

Design of any R.C.C. member comprises of the following:

(i) To decide the size (dimensions) of the member and the amount of reinforcement required.

(ii) To check whether the adopted section will perform safely and satisfactorily during the life time of the structure.

Various methods used for the design of R.C.C. structures are as follows:

(i) Working stress method.

(ii) Load factor or ultimate load method.

(iii) Limit state method.

Working Stress Method:

This method of design was the oldest one. It is based on the elastic theory and assumes that both steel and concrete are elastic and obey Hook's law. It means that the stress is directly proportional to strain up to the point of collapse. Based on the elastic theory, and assuming that the bond between steel and concrete is perfect, permissible stresses of the materials are obtained.

- The basis of this method is that the permissible stresses are not exceeded any where in the structure when it is subjected to worst combination of working loads.
- In this method, the ultimate stresses of concrete and yield strength or 0.2% proof stress of steel are divided by factors of safety to obtain permissible stresses.
- These factor of safety take into account the uncertainties in manufacturing of these materials.
- Modular Ratio between steel and concrete remains constant.

The main drawbacks of the working stress method of design are as follows:

(i) It assumes that concrete is elastic which is not true.

(ii) It uses factors of safety for stresses only which does not give true margin of safety because we do not know the failure load.

(iii) It does not use any factor of safety with respect to loads. It means, there is no provision for uncertainties associated with the load.

(iv) It does not account for shrinkage and creep which are time dependent and plastic in nature.

(v) This method gives uneconomical sections.

(vi) It pays no attention to the conditions that arise at the time of collapse.

- The working stress method is very simple and reliable but as per IS 456 : 2000 the working stress method is to be used only if it is not possible to use limit state method of design. Working stress method is the basic method and its knowledge is essential for understanding the concepts of design.

Load Factor Method or Ultimate Load Method:

- In this method, ultimate load or collapse load is used as design load. Ultimate load is obtained by multiplying working load with the load factor. The load factor gives exact margin of safety in terms of load.

- This method uses the real stress-strain curve and takes into account the plastic behavior of materials. This method gives very thin sections which result in excessive deformation and cracking
- thus, making the structure almost unserviceable. This method is not at all used by designers.

Reasons

- If the sections are designed based on ultimate strength design, there is a danger that although the load factor is adequate. The cracking and the deflections at the service loads may be excessive.

- Cracking may be excessive if the steel stresses are high or if the bars are badly distributed.

Limit State Method

This is the most rational method which takes into account the ultimate strength of the structure and also the serviceability requirements. It is a judicious combination of working stress and ultimate load method of design. The acceptable limit of safety and serviceability requirements before failure occurs is called a limit state. This method is based on the concept of safety at ultimate loads (ultimate load method) and serviceability at working loads (working stress method).

The two important limit states to be considered in design are:

(i) Limit state of collapse.

(ii) Limit state of serviceability.

What is Limit State?

- *The acceptable limit of safety and serviceability requirements before failure occurs is called a limit state.*

DESIGN CONCEPTS

- Safety: implies that the likelihood of (partial or total) collapse of structure is acceptably low not only under (normal loads) service loads but also under overloads.
- Serviceability: satisfactory performance of structure under service loads without discomfort to user due to excessive deflections, cracking, vibration etc.
- Other considerations such as durability, impermeability, acoustic and thermal insulation etc.

Limit States

- Purpose: to achieve acceptable probability that a structure will not become unfit for its intended use i.e. that it will not reach a limit state.
- Thus, a structure ceases to be fit for use will constitute a limit state and the design aims to avoid any such condition being reached during the expected life of the structure.
- Two principle types of limit state are:

  I. Ultimate Limit State

  II. Serviceability Limit State

Limit State Design

Ultimate Limit State

- This requires that the structure must be able to withstand, with an adequate factor of safety against collapse, the loads for which it is designed.
- Limit state of Collapse: flexure, shear, compression, torsion, bearing, etc.
- Possibility of buckling or overturning must also be taken into account, as must the possibility of accidental damage as caused, for example, by an internal explosion.

Serviceability Limit States

Most important serviceability limit states are

- Deflection: appearance or efficiency of any part of the structure must not be adversely affected by deflections.
- Cracking: local damage due to cracking and spalling must not affect the appearance, efficiency or durability of structure.
- Durability: this must be considered in terms of the proposed life of the structure and its conditions of exposure.

Serviceability Limit States

Other limit states include

- Excessive vibration: which may cause discomfort or alarm as well as damage.
- Fatigue: must be considered if cyclic loading is likely.
- Fire resistance: this must be considered in terms of resistance to collapse, flame penetration and heat transfer.
- Special circumstances: any special requirements of the structure which are not covered by any of the more common limit states, such as earthquake resistance, must be taken into account.

Partial safety factor

What is Partial safety factor?

- In Limit State Design, the load actually used for each limit state is called the “Design Load” for that limit state
- “Design Load” is the product of the characteristic load and the relevant partial safety factor for loads
- Design load = γ_f x (characteristic load)

Partial safety factor

Why do we use partial safety factors?

- Partial safety factor is intended to cover those variations in loading in design or in construction which are likely to occur after the designer and the constructor have each exercised carefully their skill and knowledge.
- Also takes into account nature of limit state in question.

Partial safety factor

- Other possible variations such as constructional tolerances are allowed for by partial factors of safety applied to the strength of materials and to loadings.
- Lack of adequate data, however, makes this unrealistic and in practice the values adopted are based on experience and simplified calculations.

Partial safety factor

- Errors and inaccuracies may be due to a number of causes:
- Design assumptions and inaccuracy of calculation.
- Possible unusual load increases.
- Unforeseen stress redistributions.
- Constructional inaccuracies
- These are taken into account by applying a particular factor of safety (γ_f) on the loadings, so that
- Design load = characteristic load x partial factor of safety (γ_f)
- This factor should also take into account the importance of the limit state under consideration and reflect to some extent the accuracy with which different type of loading can be predicted, and the probability of particular load combinations occurring.

Partial Factors of Safety for Materials(γm)

- The strength of material in an actual member will differ from that measured in a carefully prepared test specimen and it is particularly true for concrete where placing, compaction and curing are so important to the strength. Steel, on the other hand, is a relatively consistent material requiring a small partial factor of safety.
- The severity of the limit state being considered. Thus, higher values are taken for the ultimate limit state than for the serviceability limit state.

Partial safety factor

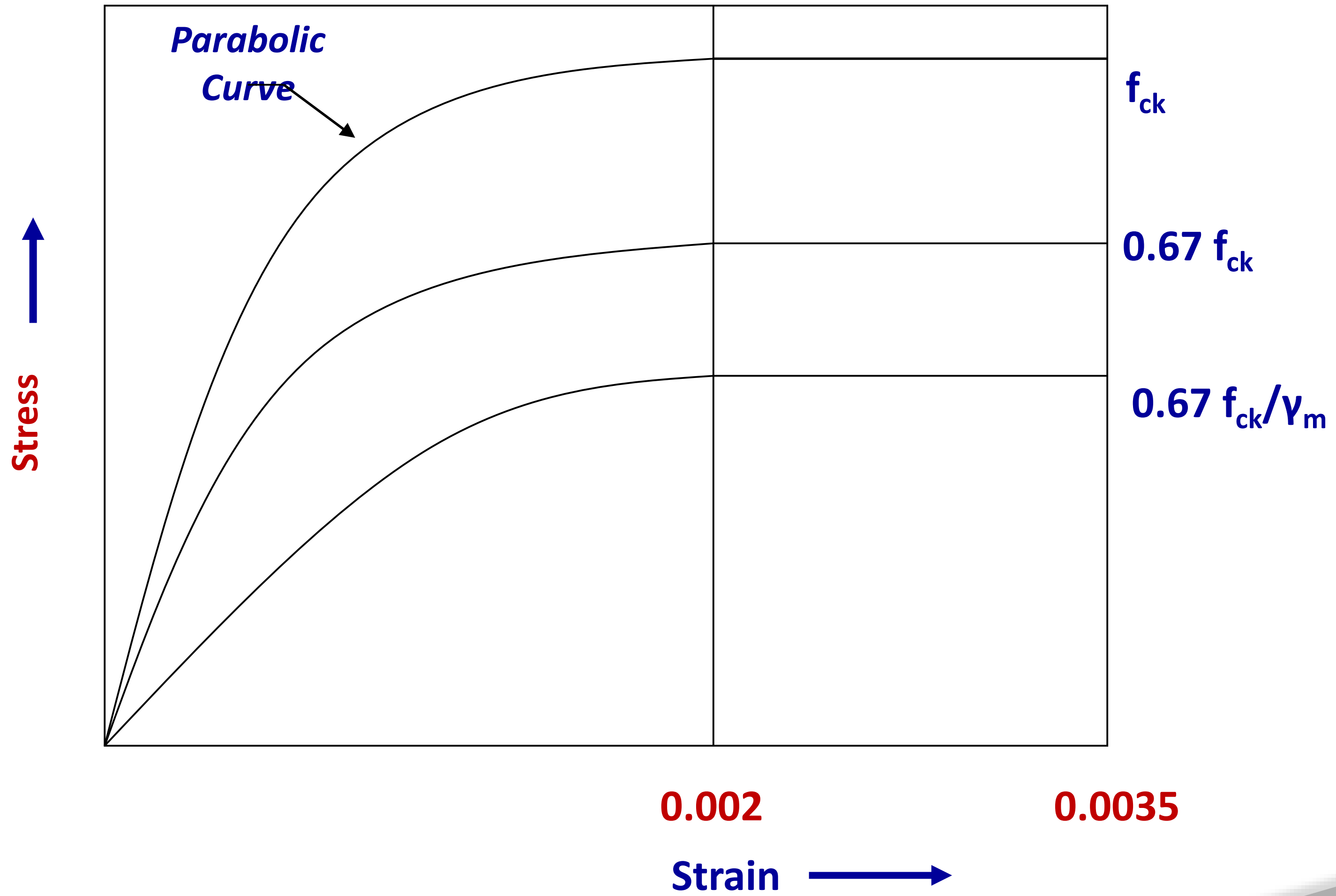
Partial Factor of Safety for loads

Load Combination	Limit State of Collapse			Limit State of Serviceability		
	DL	LL	WL/EL	DL	LL	WL/EL
DL + LL	1.5	1.5	1.0	1.0	1.0	--
DL + WL or EL	1.5 or 0.9*	-	1.5	1.0	---	1.0
DL + LL + WL/EL	1.2	1.2	1.2	1.0	0.8	0.8

What is Design Strength?

- In design calculations “Design Strength” for a given material and limit state is obtained by dividing the characteristic strength by the partial safety factor for strength, appropriate to that material and that limit state.

- When assessing the strength of a structure or structural member for the limit state of collapse, the partial safety factor should be taken as 1.5 for concrete and 1.15 for steel.



Stress-Strain Curve for Concrete in Flexural Compression

DESIGN CONCEPTS

Safety: implies that the likelihood of (partial or total) collapse of structure is acceptably low not only under (normal loads) service loads but also under overloads.

Serviceability: satisfactory performance of structure under service loads without discomfort to user due to excessive deflections, cracking, vibration etc.

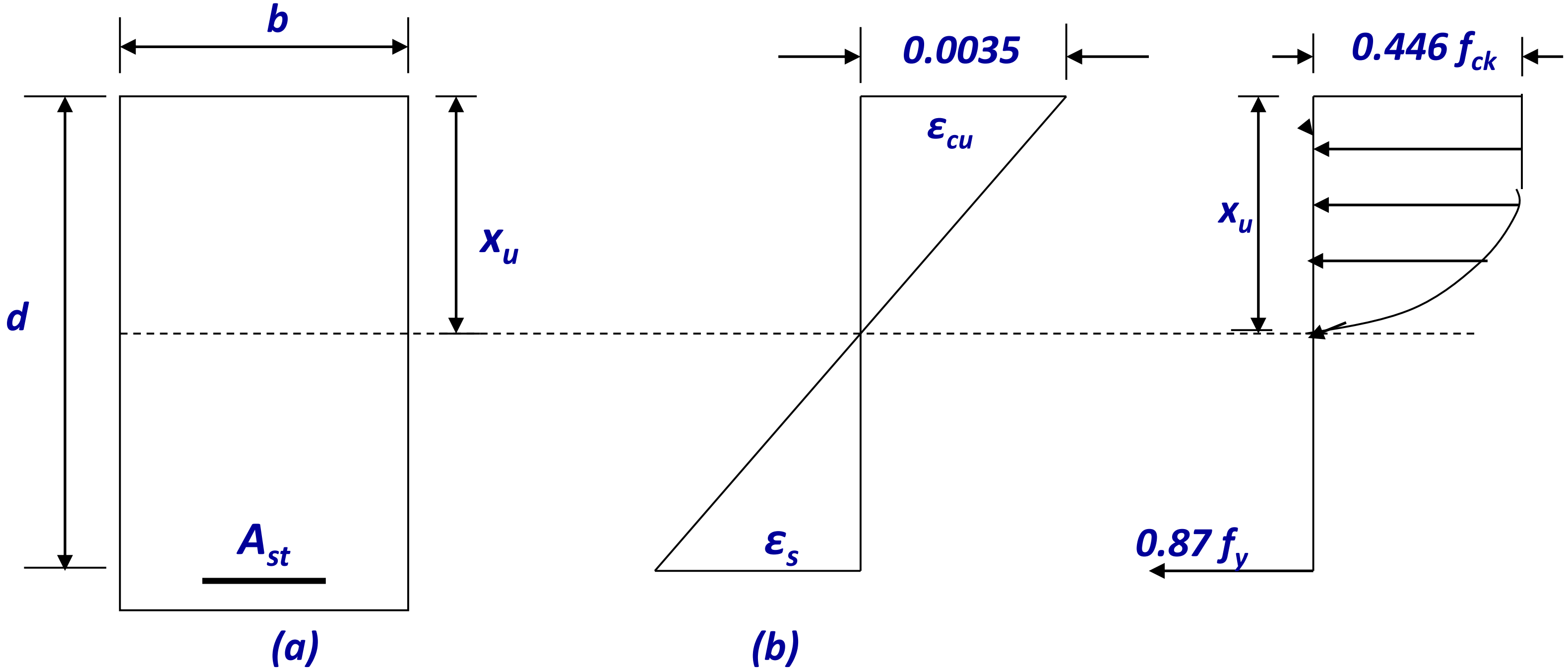
Other considerations such as durability, impermeability, acoustic and thermal insulation etc.

Assumptions for Design in Flexure

1. At any cross-section, sections which are plane prior to bending remain plane after bending. Or strain varies linearly with distance from neutral axis i.e. plane sections remain plane in bending.
2. The maximum strain in concrete at the outermost fiber is 0.0035.
3. Stress-strain relationship in concrete could be either rectangular, parabolic or combination of rectangular and parabolic curves which should be agreeable with the experimental results.

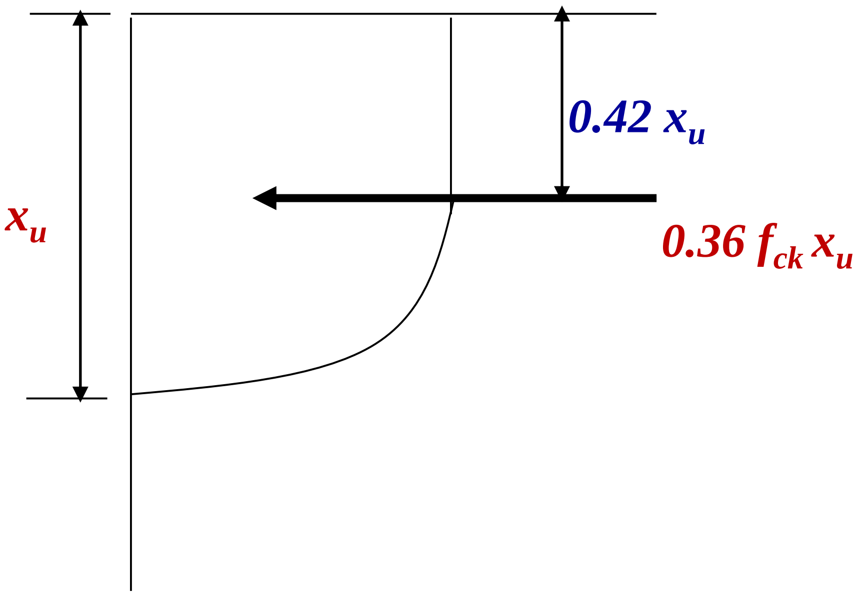
Assumptions for Design in Flexure

4. The stresses in steel bars used for reinforcement are derived from the representative stress-strain curve for the type of steel used.
5. Perfect bond between reinforced steel and adjoining concrete.
6. Tensile strength of concrete is neglected.
7. Minimum strain in steel reinforcement should not be less than $((0.87f_y/E_s) + 0.002)$.



Strain diagram and Stress blocks:

(a) Section; (b) Strain diagram; (c) Stress block

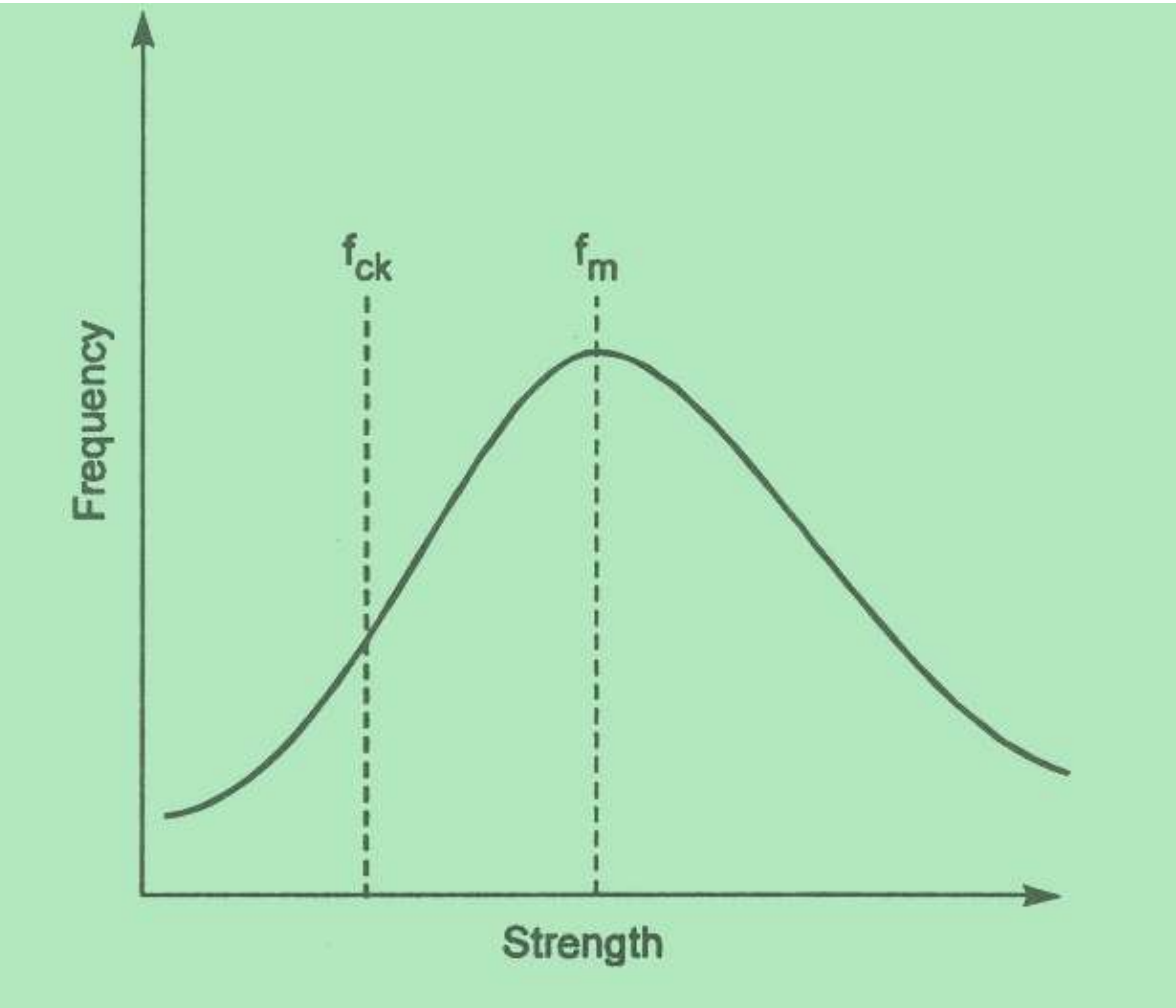


Stress Block Parameters

Characteristic strength

- "Characteristic strength is defined as the strength of material below which not more than 5 percent of the test results are expected to fall".
- Strength of concrete varies for the same concrete mix, which give different compressive strength in laboratory tests.
- Variability in strength evidently depends on degree of quality control.
- Variability in strength is measured in terms of either the "Standard Deviation" or the Coefficient of Variation (COV), which is the ratio of standard deviation to mean strength(f_{cm}).

Characteristic strength



Characteristic strength

- It is well established that the probability distribution of concrete strength (for a given mix) is approximately “Normal”.
- Coefficient of variation is generally in the range of 0.01 to 0.02.
- Due to significant variability in strength, it is necessary to ensure that the designer has a reasonable assurance of a certain minimum strength of concrete.
- Characteristic strength provides minimum guaranteed strength.

Idealized Normal Distribution

- Accordingly, the mean strength of concrete, f_{cm} (as obtained from 28 days tests) has to be significantly greater than the 5 percentile characteristic strength, that is sp f_{ck} fied by the designer.

$$\sigma = \sqrt{\frac{\sum (x-\bar{x})^2}{n}}$$

$$\sigma = \sqrt{\frac{\sum f_i (x_i-\bar{x})^2}{n}}$$

$$COV = \frac{\sigma}{\bar{x}} = S$$

Normal Probability Curve

$$The\ \ probability\ \ function,\ y = \frac{1}{\sigma\sqrt{(2\pi)}}\exp\left\{-\frac{\frac{1}{2}(x-\bar{x})^2}{\sigma^2}\right\}$$

$$where\ \ e = 2.71828$$

$$Let\ \ z = \frac{x-\bar{x}}{\sigma}$$

$$Then\ \ the\ \ probability\ \ function\ \ is\ \ y = \frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{1}{2}z^2\right\}$$

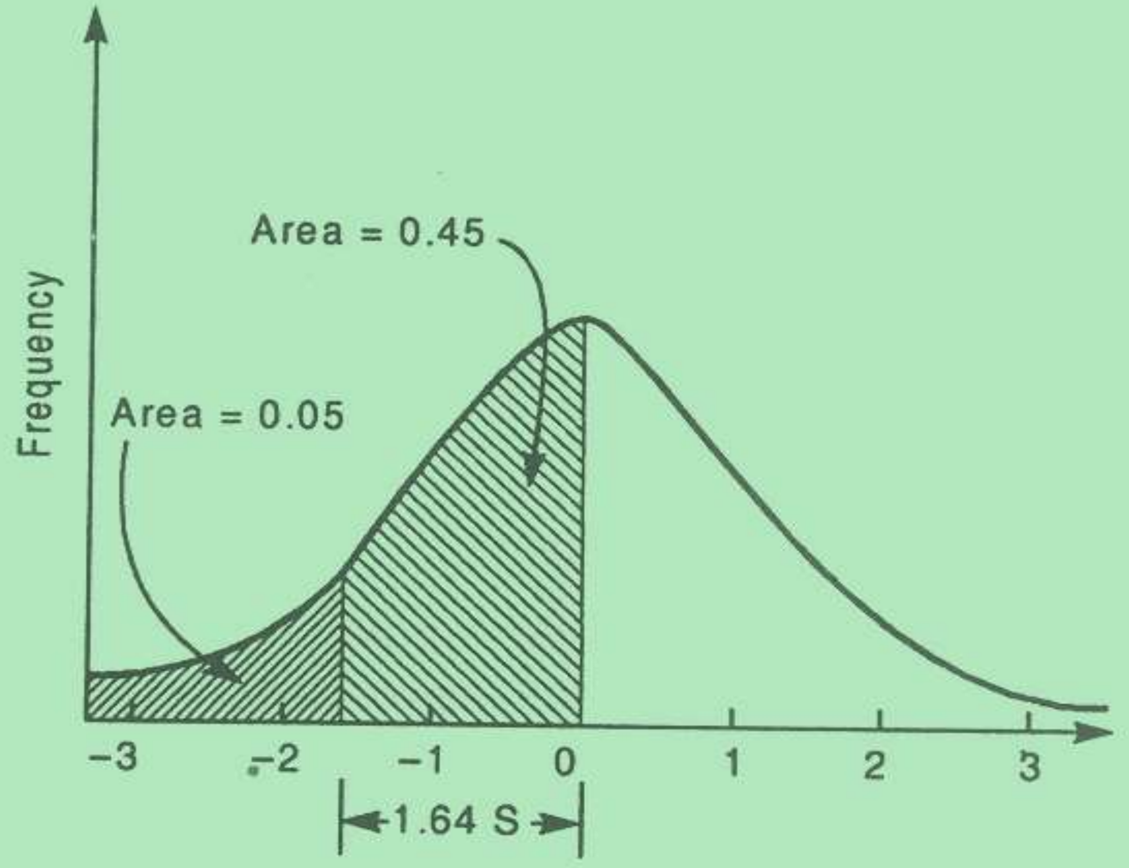


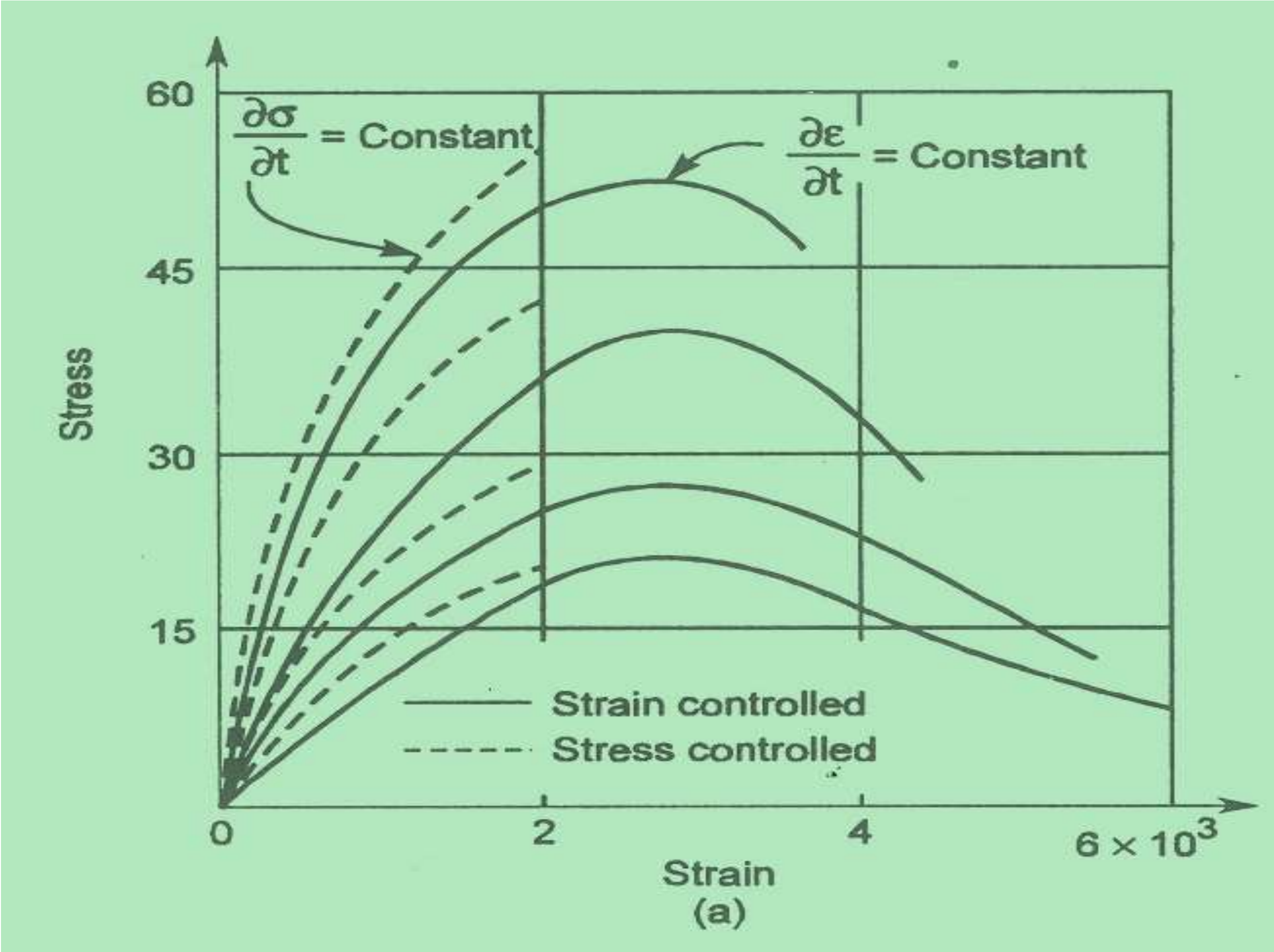
Fig. 2.1 Areas under the normal probability curve.

Normal Probability Curve

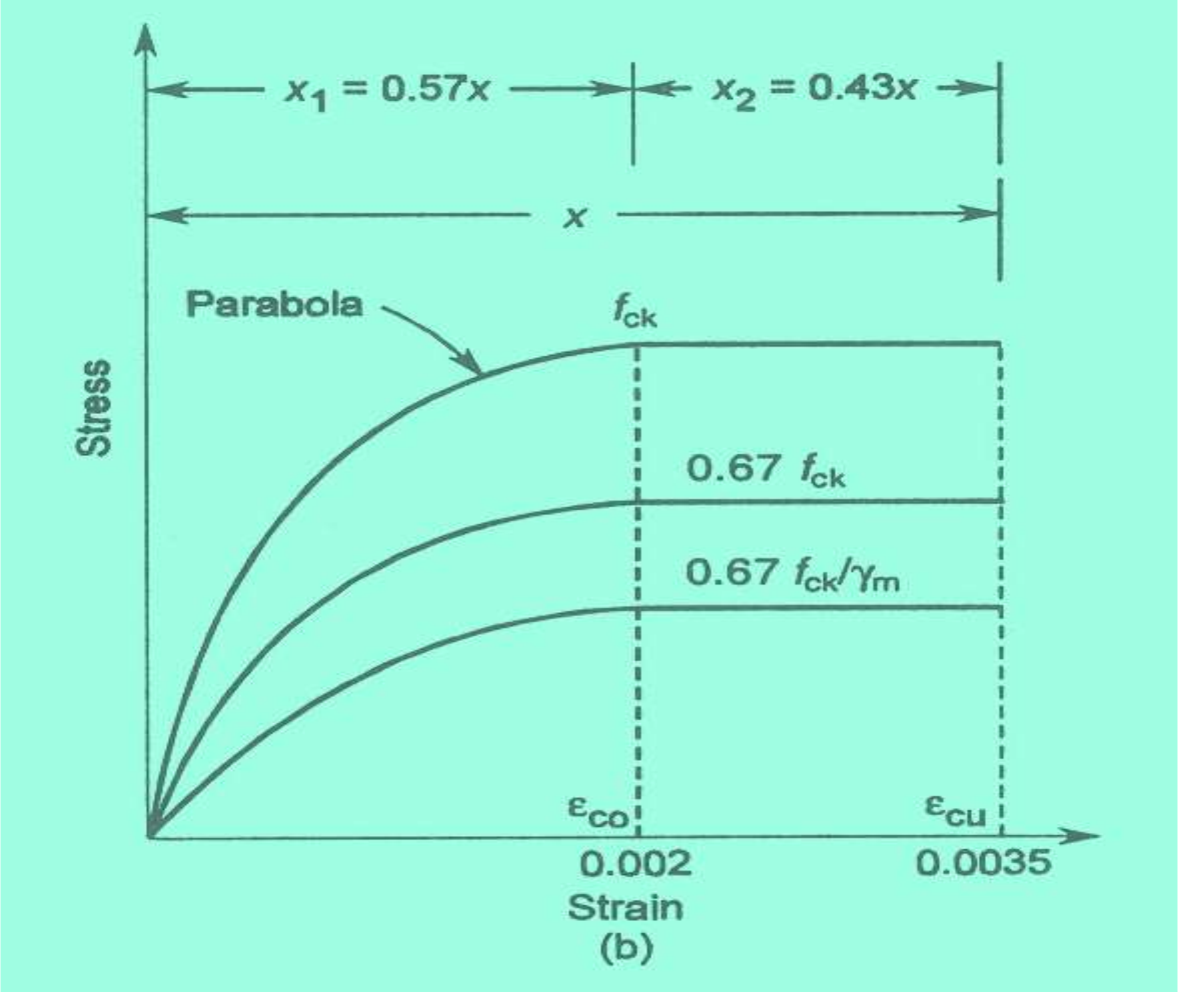
- Strength of materials upon which design is based on that strength is assumed to be normal.
- Characteristic value is defined as that value below which it is unlikely that more than 5% of the results will fall.
- $$f_{ck} = f_m - 1.64\sigma$$
-
- f_{ck} = Characteristic Strength
- f_m = Standard Deviation
- The relationship between f_{ck} anc f_m accounts for variations in results of test specimens and with the method, and control of manufacture, quality of construction and type of materials

Characteristic Loads

- Loads on structures can also be assessed stastically.
- Characteristic Load = Mean Load ± 1.64 (standard deviation).
- In most cases, it is the maximum loading on a structural member that is critical and the upper, positive value given by the above expression.
- But the lower, minimum value may apply when considering the stability of the behaviour of continuous members.



Design stress-strain curves for concrete in compression



Design stress-strain curves for concrete in compression

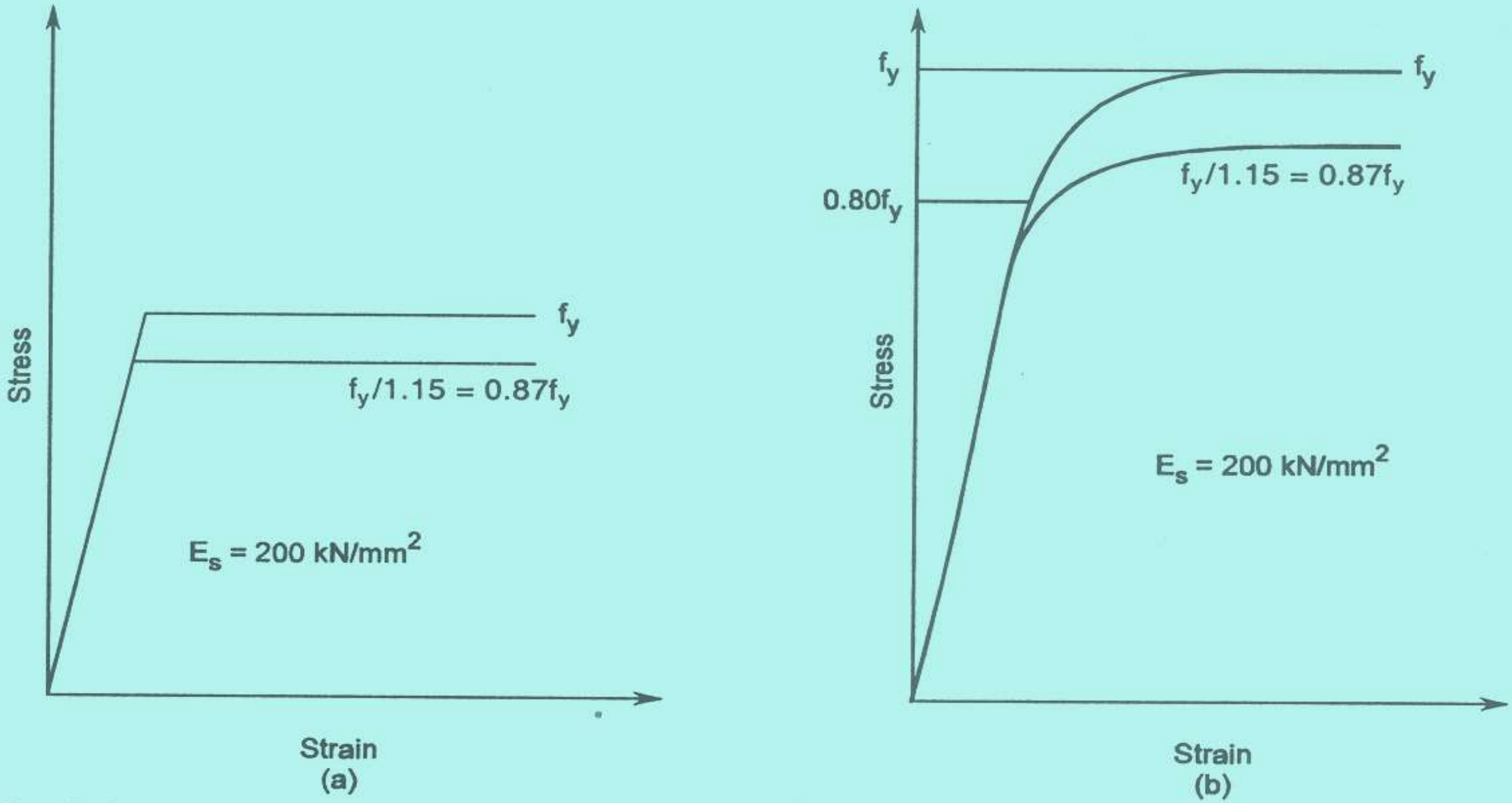


Fig. 2.4 **Stress-strain curves for steel reinforcements: (a) Mild steel; and (b) Cold worked bars. (Fig. 23 of IS 456–2000)**

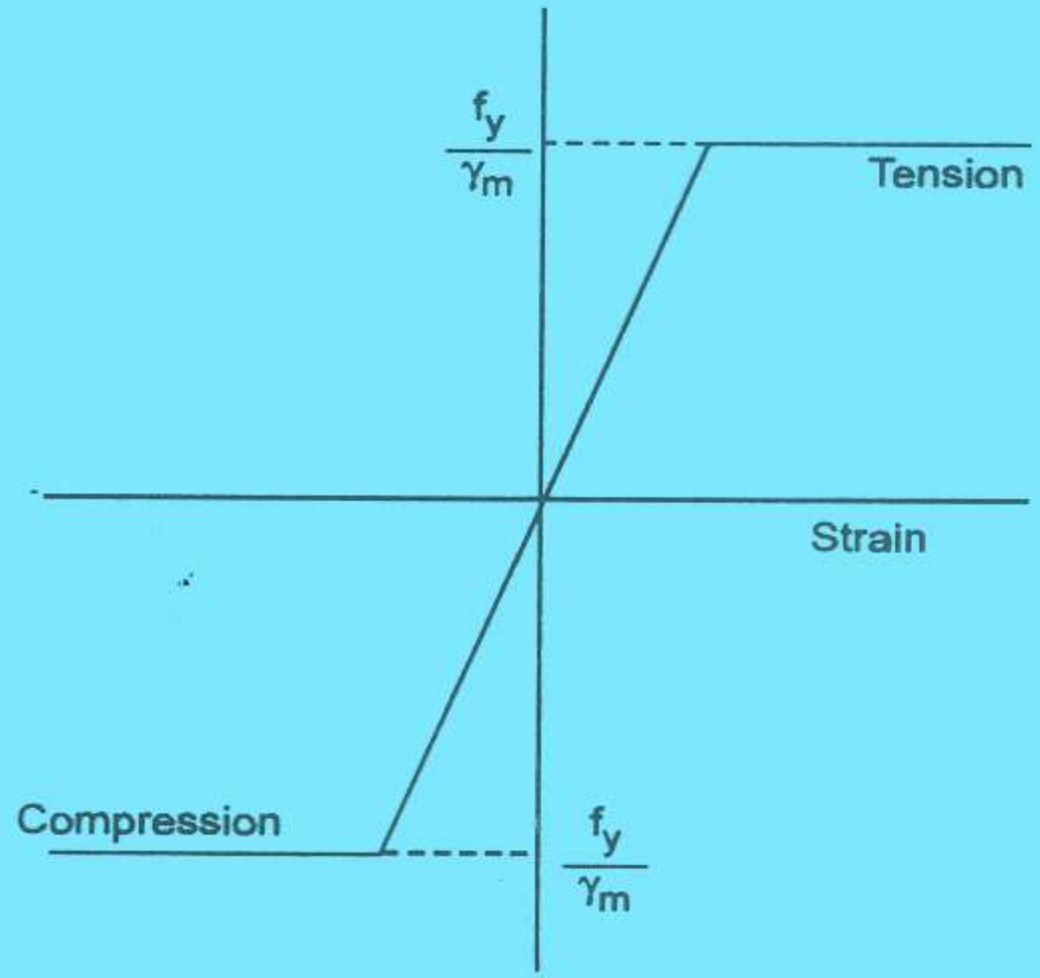


Fig. 2.5 Stress-strain curve for steel (BS 8110).

Doubly Reinforced Section

The R.C.C. beams in which the steel reinforcement is placed in the tension as well as compression zone are called as doubly reinforced beams. The moment of resistance of a balanced R.C.C. beam of dimension b x d is Rbd^2. Sometimes due to head room constraints or architectural considerations the size of the beam is restricted and the same beam (b x d) is required to resist a moment greater than Rbd^2. There are only two ways in which it can be done.

Doubly Reinforced Section

(i) By using an over reinforced section.

(ii) By using a doubly reinforced section.

The option (i) is not a good choice because over reinforced sections are uneconomical and the failure of these beams is sudden without warning.

Therefore, it is better to use doubly reinforced beam section in such circumstances.

The extra steel provided in the tension and compression zone constitutes, the additional moment of resistance (greater than Rbd^2) required.

NECESSITY OF DOUBLY REINFORCED SECTION

Doubly reinforced sections are used in the following conditions:

1. When the dimensions (b x d) of the beam are restricted due to any constraints like availability of head room, architectural or space considerations and the moment of resistance of singly reinforced section is less than the external moment.
2. When the external loads may occur on either face of the member i. e. , the loads are alternating or reversing and may cause tension on both faces of the member.
3. When the loads are eccentric.

Doubly Reinforced Section

4. When the beam is subjected to accidental or sudden lateral loads.

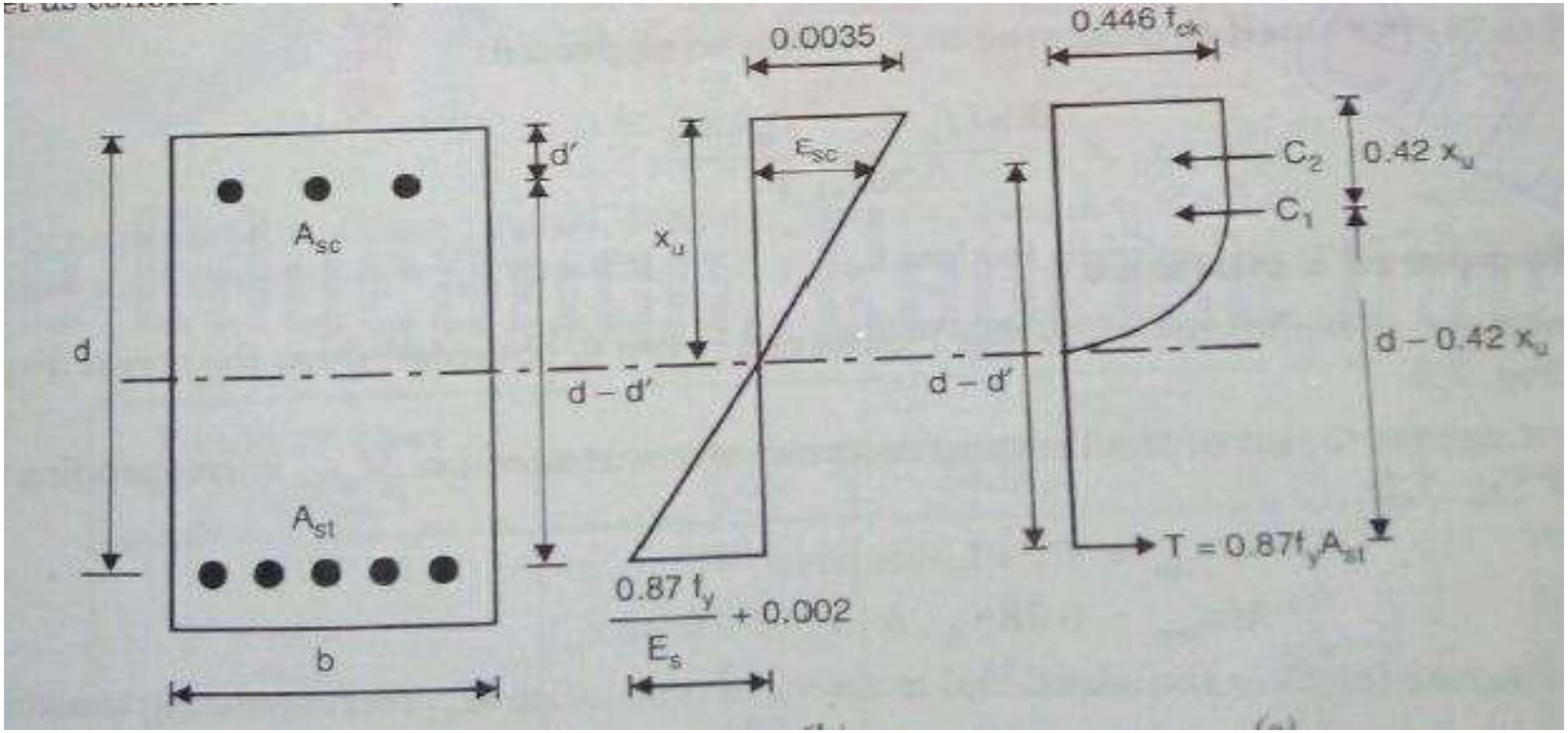
5. In the case of continuous beams or slab, the sections at supports are generally designed as doubly reinforced sections.

Section 1: Section 1 consists of a singly reinforced balanced section having area of steel A_{st1} and moment of resistance M_u lim.

Section 2: Section 2 consists of compression steel A_{sc} and additional tensile steel Ast2 corresponding to A_{sc}. The moment of resistance of this section is M_{u2} such that

$$M_u = M_{u\,lim} + M_{u2}$$

Let us consider a doubly reinforced beam shown in Fig.



where

b= Width of beam

x_u=Depth of neutral axis

d=Effective depth of beam

f_{sc}= Stress in compression steel

d'= Effective cover to compression steel

f_{cc}=Stress in concrete at the level of steel

A_{sc}=Area of compression steel

A_{st}=Area of tension steel

Depth of Neutral Axis (x_u)

The depth of neutral axis of a doubly reinforced beam section is obtained by equating the total compression and total tension.

Total compression $C_1 + C_2$

Where

C_1 is the force carried by the concrete area

C_2 is the compressive force carried by compression steel Asc.

$$C_1 = 0.36\, f_{ck}.\, b.\, x_u$$

$$C_2 = f_{sc}\,.\,A_{sc} - f_{cc}\,.\,A_{sc}$$

The term fcc. Asc accounts for the loss of concrete area occupied by compression steel.

The term $f_{cc} \cdot A_{sc}$ accounts for the ...

$\therefore$ Total compression $= 0.36 f_{ck}\, b\, x_u + f_{sc} A_{sc} - f_{cc} A_{sc}$

$= 0.36 f_{ck}\, b x_u + (f_{sc} - f_{cc}) A_{sc}$

Total tension $= T$

$$T = 0.87 f_y \cdot A_{st}$$

Equating total compression and total tension, we get

$$0.36 f_{ck}\, b\, x_u + (f_{sc} - f_{cc}) A_{sc} = 0.87 f_y \cdot A_{st}$$

$$x_u = \frac{0.87 f_y A_{st} - (f_{sc} - f_{cc}) A_{sc}}{0.36 f_{ck} \cdot b}$$

Since f_{cc} is very small as compared to f_{sc}, it can be neglected.

$$\therefore \quad x_u = \frac{0.87 f_y A_{st} - f_{sc} \cdot A_{sc}}{0.36 f_{ck} \cdot b}$$

DESIGN OF FLANGED BEAMS

Design of Flanged Beams

- In reinforced concrete construction, slab is supported over beams.
- Simple concrete slabs of moderate depth and weight are limited to spans of 3m to 5m
- If it is desired for long spans without excessive weight and material, slab is built monolithically with RC beams and beams are considered as flanged beams.
- At the interior portions of floor, slab with beam acts as a T-beam and at an end the portion acts as an L-beam.
- Shear reinforcement of beams and bent bars extend into slab and Complete construction is cast integrally. A part of slab acts with upper part in bending compressive stresses.

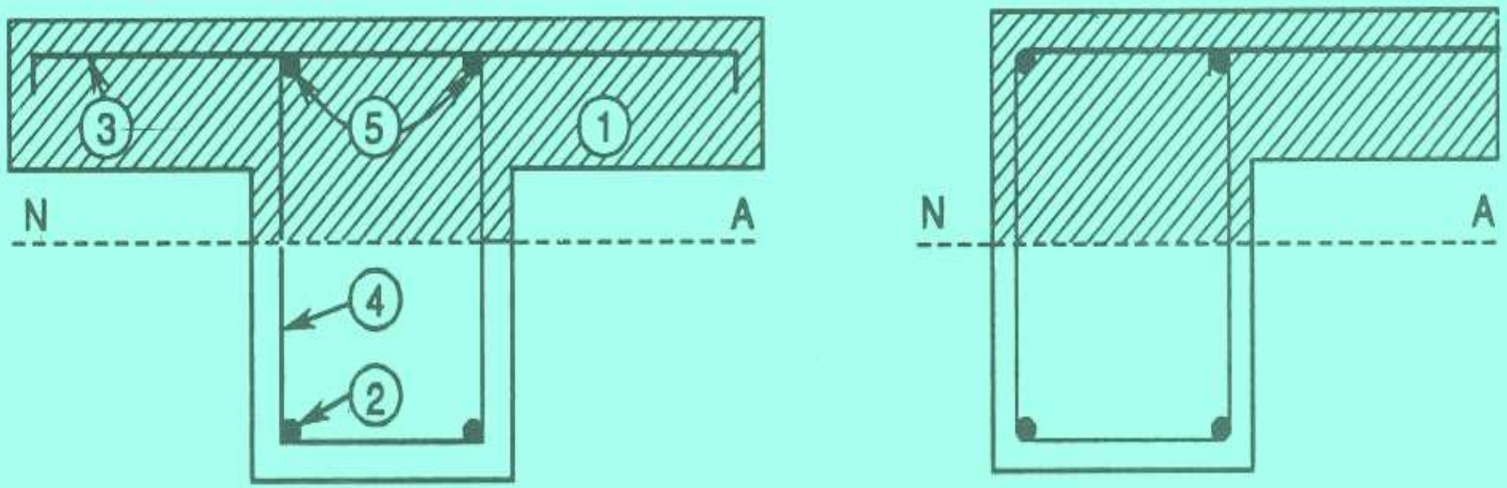


Fig. 8.1 Flanged beams: (a) T beam—1–Compression in concrete; 2–Tension steel; 3–Transverse steel; 4–Stirrups for shear; 5–Anchorage of stirrups; (b) L beam.

T-Beam

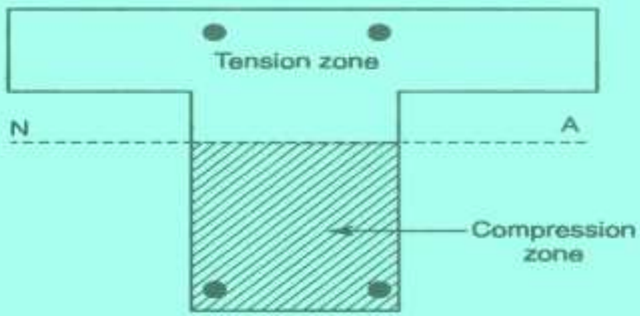


Fig. 8.2 Flanged beams over supports with negative moments.

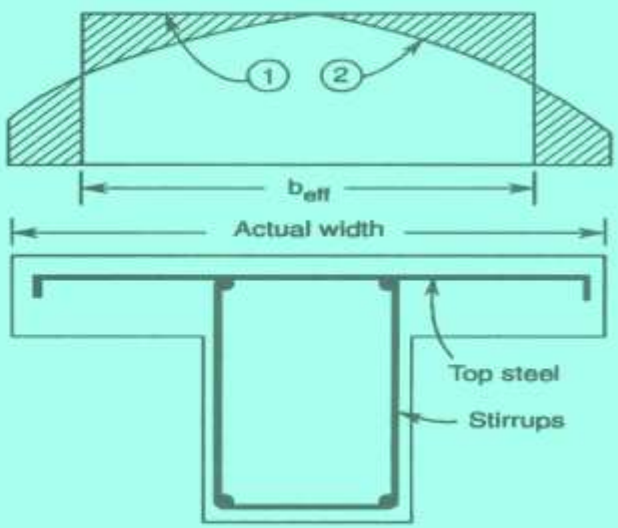


Fig. 8.3 Effective width of T beams: 1. Actual stress distribution in compression flange; 2. Assumed stress distribution in compression flange.

Effective Width of Flange

- Theoretically width of flange is supposed to act as top flange of beam.
- Elements of flange midway between webs of two adjacent beams are less highly stressed in longitudinal compression than those elements directly over webs of beams.
- An effective width of flange, b_f is used in the design of flanged beam and is treated to be uniformly stressed at the maximum value, which is smaller than actual width of flange.
- Effective width of flange primarily depends on span of the beam, breadth of web, b_w and thickness of flange, D_f.

Effective Width of Flange

- IS: 456-2000 recommends for effective width of flanges of T- and L-beams.
  - For symmetrical T-beams

$$b_f = [(l_0\backslash 6) + b_w + 6D_f]$$

  - For beams with slab on one side only

$$b_f = [(l_0\backslash 12) + b_w + 3D_f]$$

  - For isolated T-beams

$$b_f = [(l_0\backslash((l_0\backslash b)+4)) + b_w]$$

  - For Isolated L-beams

$$b_f = [(0.5l_0\backslash((l_0\backslash b)+4)) + b_w]$$

Effective Width of Flange

- **Calculated effective flange width, b_f shall be not greater than the breadth of web plus half the sum of clear distances to the adjacent beams on either side**

  - $b_f < 0.5\,[l_1 + l_2] + b_w$
  - $b_f < 0.5\,[l_2 + L_3] + b_w$

Location of Neutral Axis

- **Depending upon proportions of cross-section, area of steel reinforcement in tension, strength of materials**

  1. **Neutral axis of a T-beam in one case may lie in the flange i.e. depth of NA, x_u is less than or equal to thickness of flange or depth of slab, D_f (Neutral axis lies within flange ($x_u < D_f$))**

  2. **NA may lie in web i.e. depth of neutral axis, x_u is more than thickness of slab, D_f.**

- **Stress diagram consists of a rectangular portion of depth $0.43x_u$ and a parabolic portion of depth $0.57x_u$.**

Stress Block in T-Beam

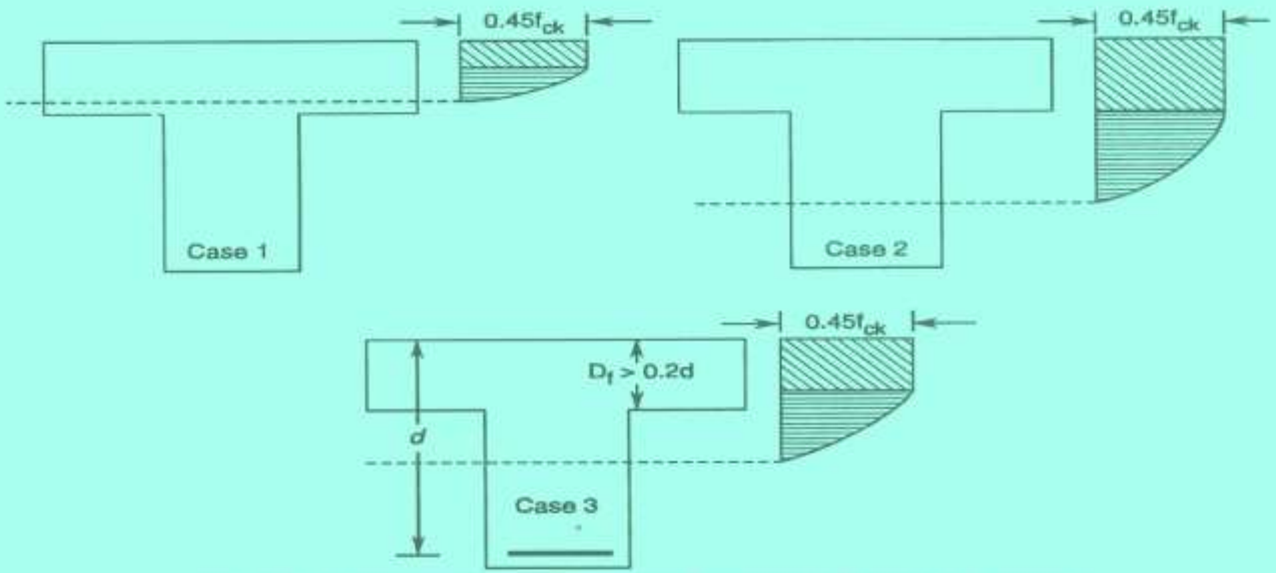


Fig. 8.4 Three possible positions of neutral axis in T beams.

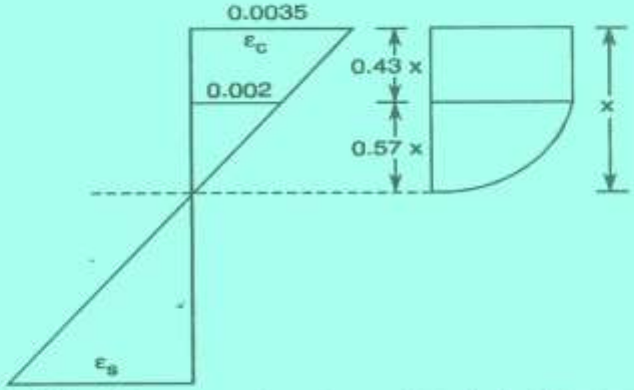


Fig. 8.5 . Compression stress block in T beams.

(2) NEUTRAL AXIS LIES OUT SIDE FLANGE [i.e. xu > Df]

- **When NA of T-section lies outside flange, it lies in web of T-beam. However, there are two possibilities depending upon whether depth of flange D_f is less than or equal to $0.43x_u$ or D_f is more than $0.43\ x_u$.**
- **Comparison of D_f with $0.43x_u$ (i.e. $3/7x_u$) is more rational as $0.43x_u$ is actual depth of rectangular portion of stress block.**
- **In IS:456-2000, if (D_f/d) is less than 0.2, the flange of T-beam is considered as small.**

i. **D_f is less than $0.43x_u$**

- **Total area in compression consists of sum of compressive force in concrete in web of width, b_w, $C_{w,\ cu}$ and compressive force in concrete in the flange excluding web, $C_{f,\ cu}$.**

(2) NEUTRAL AXIS LIES OUT SIDE FLANGE [i.e. xu > Df]

D_f > 0.43 x_u or (D_f > 0.2d)

Depth of flange D_f is more than 0.43x_u, some portion is subjected to uniform stress equal to 0.446f_{ck} (0.43x_u) and remaining portion is subjected to parabolic stress.

To obtain compressive force in portion of flange, concept of modified thickness of flange equal to

$$y_f = (0.15x_u + 0.65D_f)$$

is recommended by IS456-2000.

Average stress is assumed to be 0.446f_{ck}

Moment of Resistance

I. A singly reinforced slab 120mm thick is supported by T-beam spaced at 3.5m c/c has an effective depth, d= 550mm, width, b_w = 400mm. The beam is provided with steel reinforcement consisting of 5 bars of 20mm diameter in one layer, d' = 50mm. l_e = 3.7m. Use M20 grade concrete and Fe415 steel. Determine the depth of neutral axis and the moment of resistance of the beam, MR?

II. Calculate the moment of resistance of a T-beam for M20 and Fe415, D_f = 120 mm, b_f = 750mm, b_w = 250mm, d' = 50mm, D = 500mm

III. T-beam floor, D_f = 150mm, b_w = 250mm spacing = 3.5m c/c, l_e = 8.0m. LL = 6.5 kN/m. Design an intermediate beam using M20 and Fe415 steel.

IV. T-beam d = 750mm, b_f= 1400mm, D_f = 100mm, b_w = 300mm, A_{st} = ? M= 100kN-m. Use M20 and Fe 415 HYSD bars.

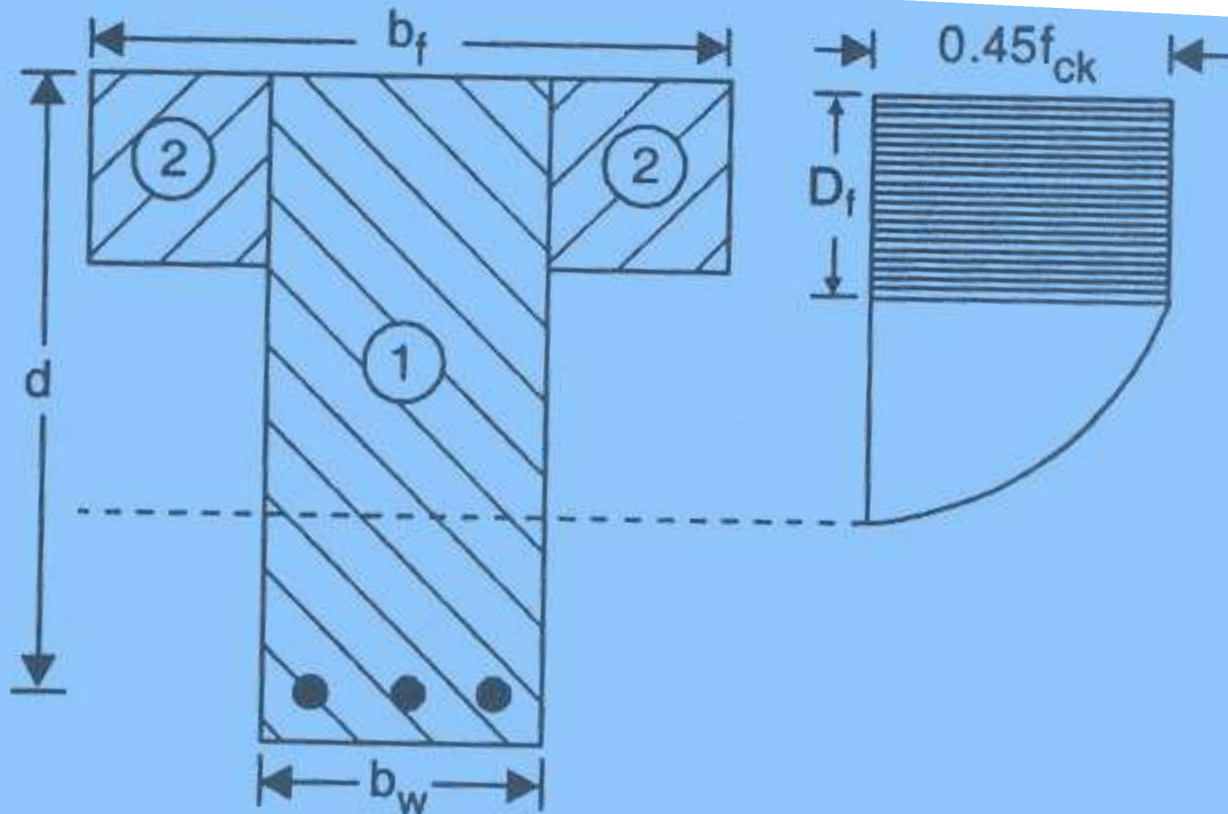


Fig. 8.6 Calculation of moment of resistance of T beams.

DESIGN OF
SINGLY REINFORCED BEAM

BEAM:-

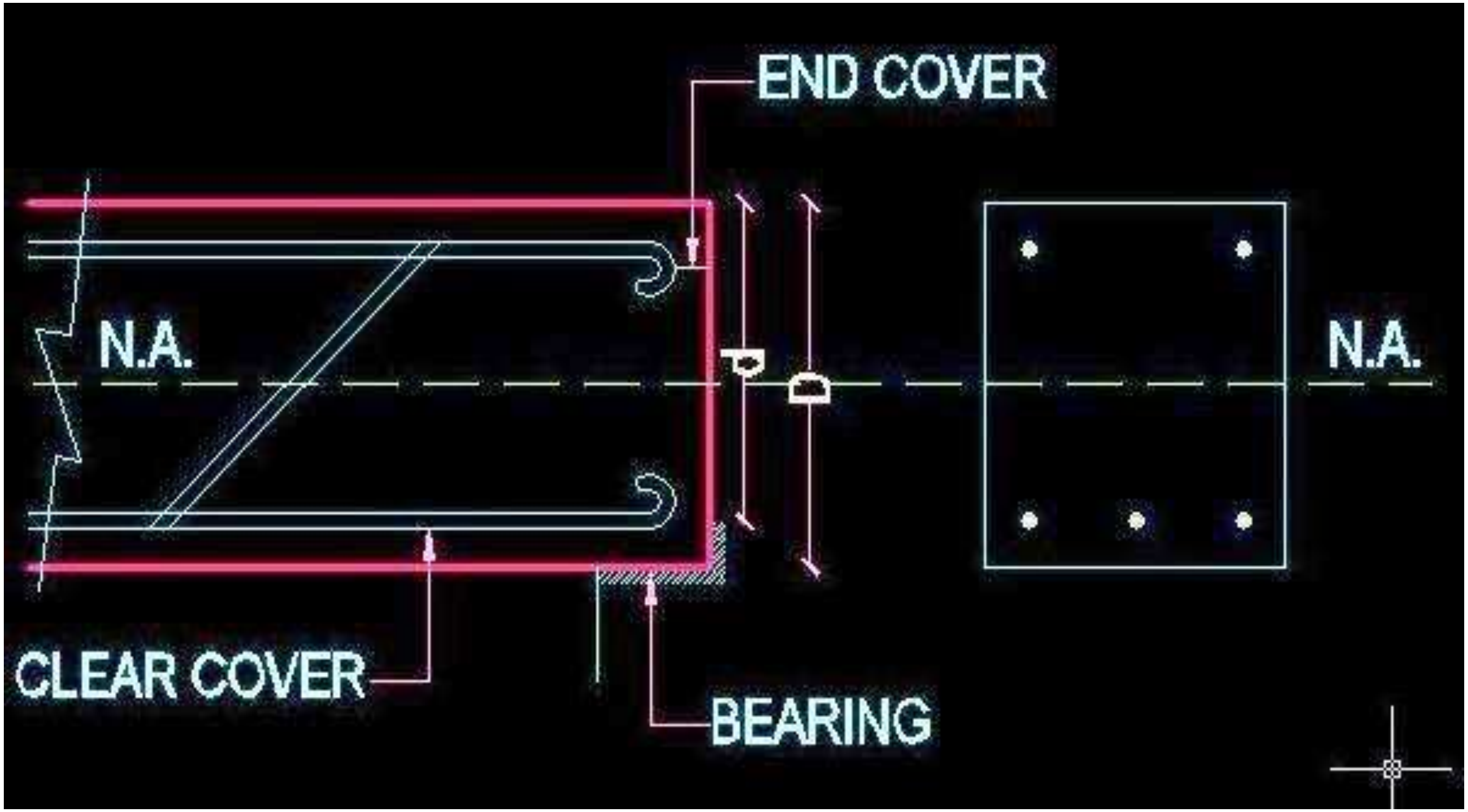
A Beam is any structural member which resists load mainly by bending. Therefore it is also called flexural member. Beam may be singly reinforced or doubly reinforced. When steel is provided only in tensile zone (i.e. below neutral axis) is called singly reinforced beam, but when steel is provided in tension zone as well as compression zone is called doubly reinforced beam.

The aim of design is:

- **To decide the size (dimensions) of the member and the amount of reinforcement required.**

- **To check whether the adopted section will perform safely and satisfactorily during the life time of the structure.**

FEW DEFINITIONS



OVER ALL DEPTH :-

THE NORMAL DISTANCE FROM THE TOP EDGE OF THE BEAM TO THE BOTTOM EDGE OF THE BEAM IS CALLED OVER ALL DEPTH. IT IS DENOTED BY ‘D’.

EFFECTIVE DEPTH:-

THE NORMAL DISTANCE FROM THE TOP EDGE OF BEAM TO THE CENTRE OF TENSILE REINFORCEMENT IS CALLED EFFECTIVE DEPTH. IT IS DENOTED BY ‘d’.

CLEAR COVER:-

THE DISTANCE BETWEEN THE BOTTOM OF THE BARS AND BOTTOM MOST THE EDGE OF THE BEAM IS CALLED CLEAR COVER.

CLEAR COVER = 25mm OR DIA OF MAIN BAR, (WHICH EVER IS GREATER).

EFFECTIVE COVER:-

THE DISTANCE BETWEEN CENTRE OF TENSILE REINFORCEMENT AND THE BOTTOM EDGE OF THE BEAM IS CALLED EFFECTIVE COVER. EFFECTIVE COVER = CLEAR COVER + ½ DIA OF BAR.

END COVER:-

END COVER = 2XDIA OF BAR OR 25mm (WHICH EVER IS GREATER)

NEUTRAL AXIS:- THE LAYER / LAMINA WHERE NO STRESS EXIST IS KNOWN AS NEUTRAL AXIS. IT DIVIDES THE BEAM SECTION INTO TWO ZONES, COMPRESION ZONE ABOVE THE NETURAL AXIS & TENSION ZONE BELOW THE NEUTRAL AXIS.

DEPTH OF NETURAL AXIS:- THE NORMAL DISTANCE BETWEEN THE TOP EDGE OF THE BEAM & NEUTRAL AXIS IS CALLED DEPTH OF NETURAL AXIS. IT IS DENOTED BY ‘n’.

LEVER ARM:- THE DISTANCE BETWEEN THE RESULTANT COMPRESSIVE FORCE (C) AND TENSILE FORCE (T) IS KNOWN AS LEVER ARM. IT IS DENOTED BY ‘z’. THE TOTAL COMPRESSIVE FORCE (C) IN CONCRETE ACT AT THE C.G. OF COMPRESSIVE STRESS DIAGRAM i.e. n/3 FROM THE COMPRESSION EDGE. THE TOTAL TENSILE FORCE (T) ACTS AT C.G. OF THE REINFORCEMENT.

LEVER ARM = d-n/3

TENSILE REINFORCEMENT:-

THE REINFORCEMENT PROVIDED TENSILE ZONE IS CALLED TENSILE REINFORCEMENT. IT IS DENOTED BY A_{st}.

COMPRESSION REINFORCEMENT :-

THE REINFORCEMENT PROVIDED COMPRESSION ZONEIS CALLED COMPRESSION REINFORCEMENT. IT IS DENOTED BY A_{sc}

TYPES OF BEAM SECTION:- THE BEAM SECTION CAN BE OF THE FOLLOWING TYPES:

1.BALANCED SECTION

2.UNBALNCED SECTION

(a) UNDER- REINFORCED SECTION

(b) OVER-REINFORCED SECTION

1.BALANCED SECTION:- A SECTION IS KNOWN AS BALANCED SECTION IN WHICH THE COMPRESSIVE STREE IN CONCRETE (IN COMPRESSIVE ZONES) AND TENSILE STRESS IN STEEL WILL BOTH REACH THE MAXIMUM PERMISSIBLE VALUES SIMULTANEOUSLY.

THE NEUTRAL AXIS OF BALANCED (OR CRITICAL) SECTION IS KNOWN AS CRITICAL NEUTRAL AXIS (n_c). THE AREA OF STEEL PROVIDED AS ECONOMICAL AREA OF STEEL. REINFORCED CONCRETE SECTIONS ARE DESIGNED AS BALANCED SECTIONS.

2. UNBALNCED SECTION:-THIS IS A SECTION IN WHICH THE QUANTITY OF STEEL PROVIDED IS DIFFERENT FROM WHAT IS REQUIRED FOR THE BALANCED SECTION.

UNBALANCED SECTIONS MAY BE OF THE FOLLOWING TWO TYPES:

(a) UNDER-REINFORCED SECTION

(b) OVER-REINFORCED SECTION

(a)UNDER-REINFORCED SECTION:- IF THE AREA OF STEEL PROVIDED IS LESS THAN THAT REQUIRED FOR BALANCED SECTION, IT IS KNOWN AS UNDER-REINFORCED SECTION. DUE TO LESS REINFORCEMENT THE POSITION OF ACTUAL NEUTRAL AXIS (n) WILL SHIFT ABOVE THE CRITICAL NEUTRAL AXIS (n_c)i.e. $n< n_c$. IN UNDER-REINFORCED SECTION STEEL IS FULLY STRESSED AND CONCRETE IS UNDER STRESSED (i.e. SOME CONCRETE REMAINS UN-UTILISED). STEEL BEING DUCTILE, TAKES SOME TIME TO BREAK. THIS GIVES SUFFICIENT WARNING BEFORE THE FINAL COLLAPSE OF THE STRUCTURE. FOR THIS REASON AND FROM ECONOMY POINT OF VIEW THE UNDER-REINFORCED SECTIONS ARE DESIGNED.

(b) OVER-REINFORCED SECTION:- IF THE AREA OF STEEL PROVIDED IS MORE THAN THAT REQUIRED FOR A BALANCED SECTION, IT IS KNOWN AS OVER-REINFORCED SECTION. AS THE AREA OF STEEL PROVIDED IS MORE, THE POSITION OF N.A. WILL SHIFT TOWARDS STEEL, THEREFORE ACTUAL AXIS (n) IS BELOW THE CRITICAL NEUTRAL AXIS (n_c)i.e. $n > n_c$. IN THIS SECTION CONCRETE IS FULLY STRESSED AND STEEL IS UNDER STRESSED. UNDER SUCH CONDITIONS, THE BEAM WILL FAIL INITIALLY DUE TO OVER STRESS IN THE CONCRETE. CONCRETE BEING BRITTLE, THIS HAPPENS SUDDENLY AND EXPLOSIVELY WITHOUT ANY WARNING.

Basic rules for design of beam

1. Effective span:- In the case of simply supported beam the effective length,

l = i. Distance between the centre of support

ii. Clear span + eff. Depth

eff. Span = least of i. & ii.

2. Effective depth:- The normal distance from the top edge of beam to the centre of tensile reinforcement is called effective depth. It is denoted by ‘d’.

d= D- effect. Cover

where D= over all depth

3. Bearing :- Bearings of beams on brick walls may be taken as follow:

Up to 3.5 m span, bearing = 200mm

Up to 5.5 m span, bearing =300mm

Up to 7.0 m span, bearing =400mm

4. Deflection control:- The vertical deflection limits assumed to be satisfied if (a) For span up to 10m

Span / eff. Depth = 20

(For simply supported beam)

Span / eff. Depth = 7

(For cantilever beam)

(b) For span above 10m, the value in (a) should be multiplied by 10/span (m), except for cantilever for which the deflection calculations should be made.

(c) Depending upon the area and type of steel the value of (a&b) modified as per modification factor.

5. Reinforcement :-

(a) Minimum reinforcement:- The minimum area of tensile reinforcement shall not be less than that given by the following:

$$A_{st} = 0.85\ bd / f_y$$

(b)Maximum reinforcement:- The maximum area of tensile reinforcement shall not be more than 0.4bD

(c)Spacing of reinforcement bars:-

i. The horizontal distance between to parallel main bars shall not be less than the greatest of the following:

- Diameter of the bar if the bars are of same diameter.
- Diameter of the larger bar if the diameter are unequal.
- 5mm more than the nominal maximum size of coarse aggregate.

ii. When the bars are in vertical lines and the minimum vertical distance between the bars shall be greater of the following:

- 15mm.
- 2/3rd of nominal maximum size of aggregate.
- Maximum diameter of the bar.

6. Nominal cover to reinforcement :- The Nominal cover is provided in R.C.C. design:

- To protect the reinforcement against corrosion.
- To provide cover against fire.
- To develop the sufficient bond strength along the surface area of the steel bar.

Exposure conditions	***Nominal cover(mm) Not less than***
Mild Moderate Severe Very severe Extreme	*20 30 45 50 75*

Procedure for Design of Singly Reinforced Beam by Working Stress Method

Given :

(i) Span of the beam (**l**)

(ii) Loads on the beam

(iii)Materials-Grade of Concrete and type of steel.

1. Calculate design constants for the given materials (k, j and R)

$$k = m\ \sigma_{cbc} / \ m\ \sigma_{cbc} + \sigma_{st}$$

where k is coefficient of depth of Neutral Axis

$$j = 1 - k/3$$

where j is coefficient of lever arm.

$$R = 1/2\ \sigma_{cbc}\ kj$$

where R is the resisting moment factor.

2. *Assume dimension of beam:*

d = Span/10 to Span/8

Effective cover = 40mm to 50mm

b = D/2 to 2/3D

3. Calculate the effective span (l) of the beam.

4. Calculate the self weight (dead load) of the beam.

Self weight = D x b x 25000 N/m

5. Calculate the total Load & maximum bending moment for the beam.

Total load (w) = live load + dead load

Maximum bending moment, $M = wl^2 / 8$ at the centre of beam for simply supported beam.

$M = wl^2 / 2$ at the support of beam for cantilever beam.

6. Find the minimum effective depth

$$M = M_r$$

$$= Rbd^2$$

$$d_{reqd.} = \sqrt{M / R.b}$$

7. Compare $d_{reqd.}$ With assumed depth value.

(i) If it is less than the assumed d, then assumption is correct.

(ii) If $d_{reqd.}$ is more than assumed d, then revise the depth value and repeat steps 4, 5 & 6.

8. Calculate the area of steel required (A_{st}).

$$A_{st} = M / \sigma_{st} jd$$

Selecting the suitable diameter of bar calculate the number of bars required

Area of one bar = $\pi/4 \times \phi^2 = A\phi$

No. of bars required = $A_{st} / A\phi$

9. *Calculate minimum area of steel (A_S) required by the relation:*

$$A_S = 0.85\, bd / f_y$$

Calculate maximum area of steel by the area relation:

Maximum area of steel = 0.04bD

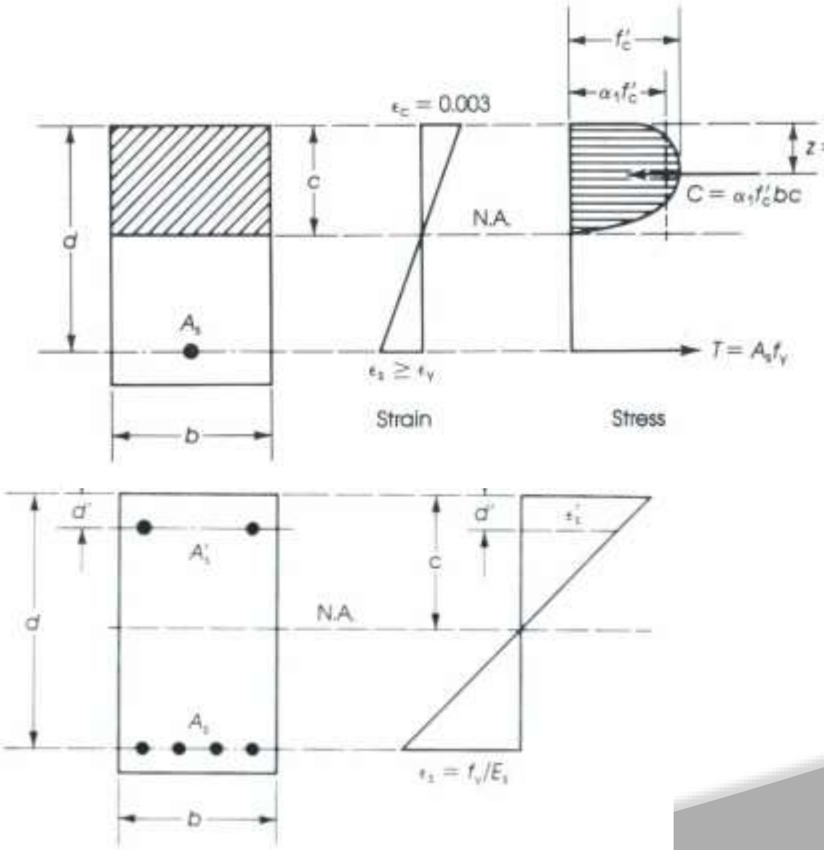
Check that the actual A_{St} provided is more than minimum and less than maximum requirements.

10. Check for shear and design shear reinforcement.
11. Check for development length.
12. Check for depth of beam from deflection.
13. Write summary of design and draw a neat sketch.

Lecture Goals

- Doubly Reinforced beams
- T Beams and L Beams

Analysis of Doubly Reinforced Sections



$$T = A_s f_y$$

$$C = T$$

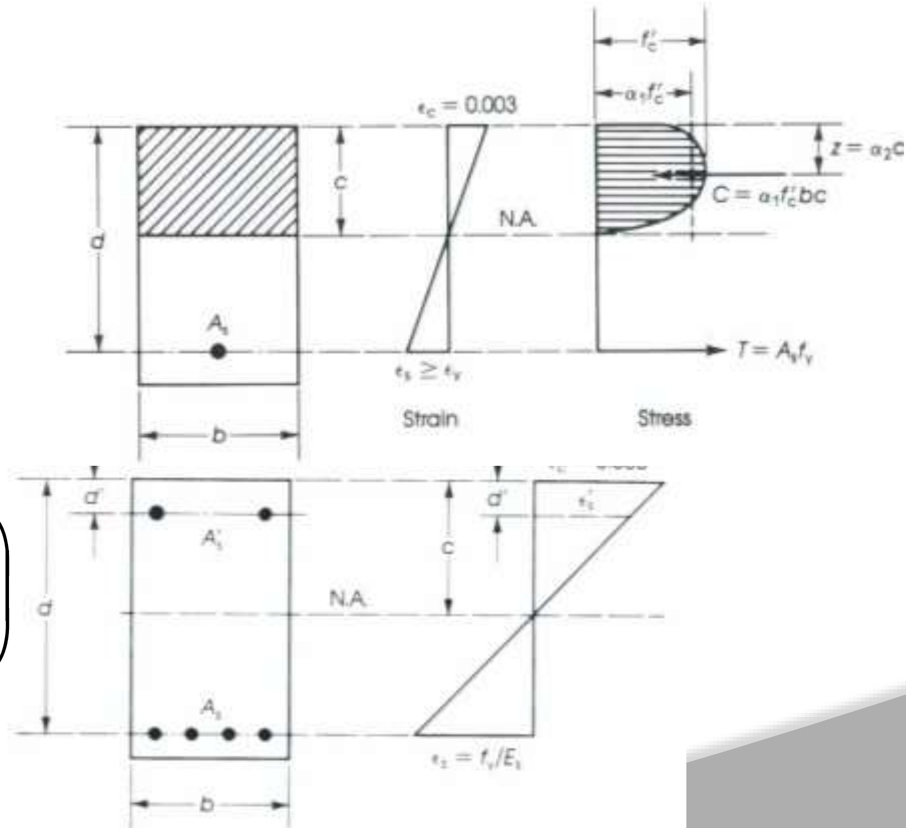
Singly Reinforced $\Rightarrow$

$$C = C_c \;;\; M_n = A_s f_y \left(d - \frac{a_1}{2} \right)$$

Doubly Reinforced $\Rightarrow$

$$C = C_c + C'_s \;;\; M_n = A_s f_y \left(d - \frac{a_2}{2} \right)$$

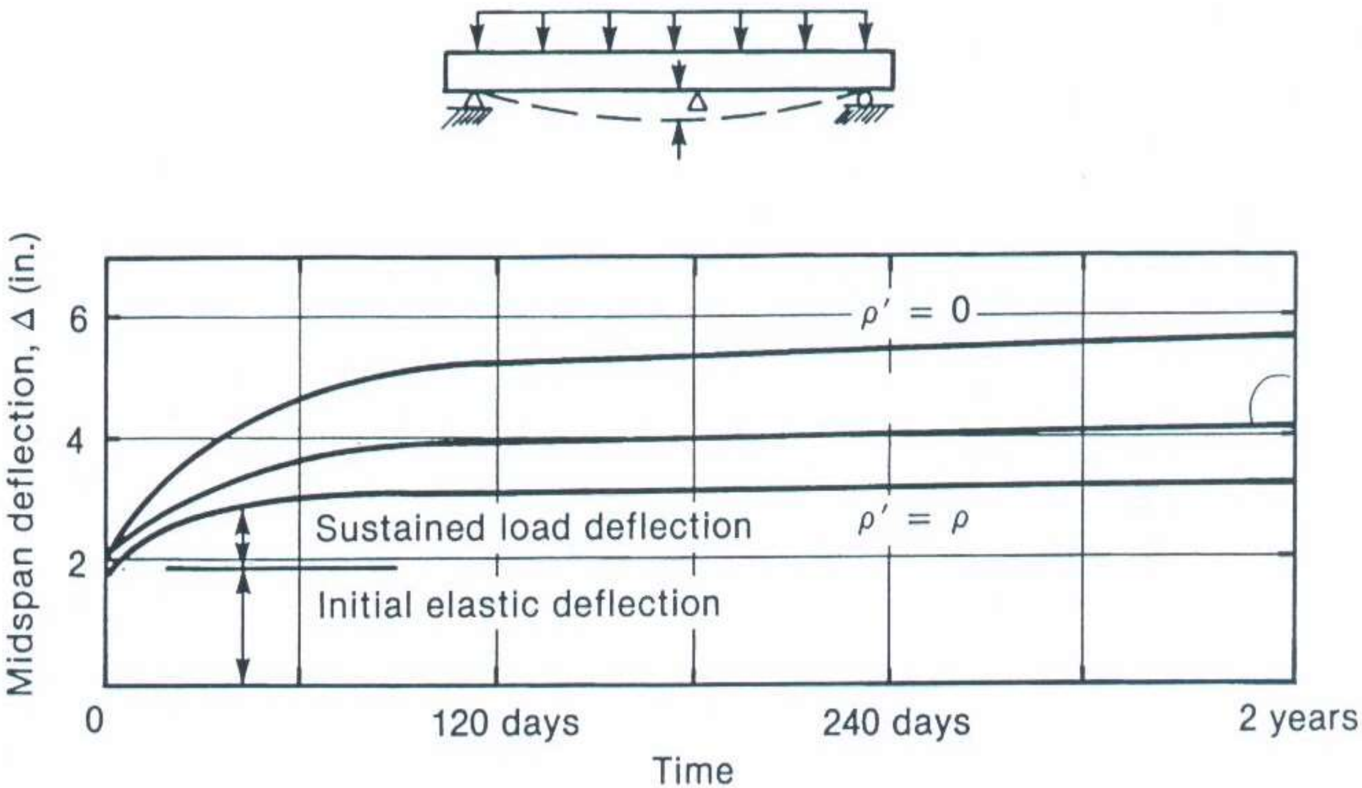
and $(a_2 < a_1)$



Reasons for Providing Compression Reinforcement

- **Reduced sustained load deflections.**
  - Creep of concrete in compression zone
  - transfer load to compression steel
  - reduced stress in concrete
  - less creep
  - less sustained load deflection

Reasons for Providing Compression Reinforcement



Reasons for Providing Compression Reinforcement

- **Increased Ductility**

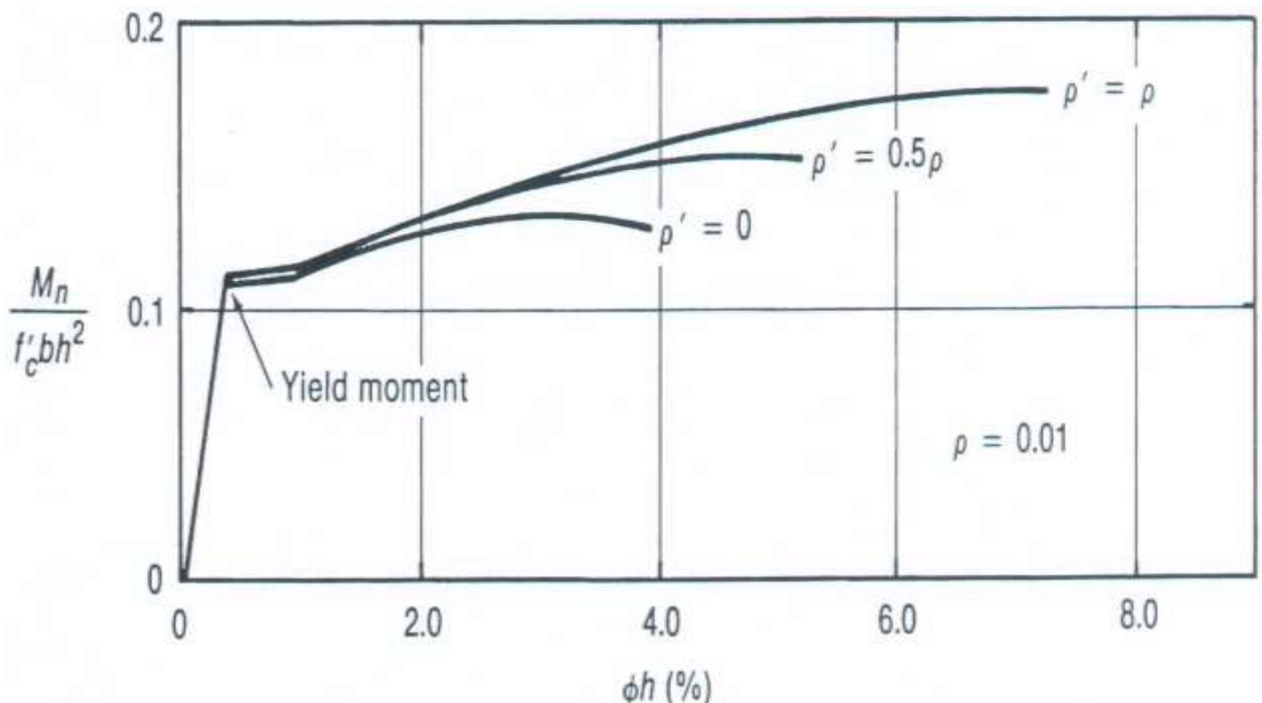
reduced stress block depth

increase in steel strain larger curvature are obtained.

Reasons for Providing Compression Reinforcement

Effect of compression reinforcement on strength and ductility of under reinforced beams.

$r < r_b$

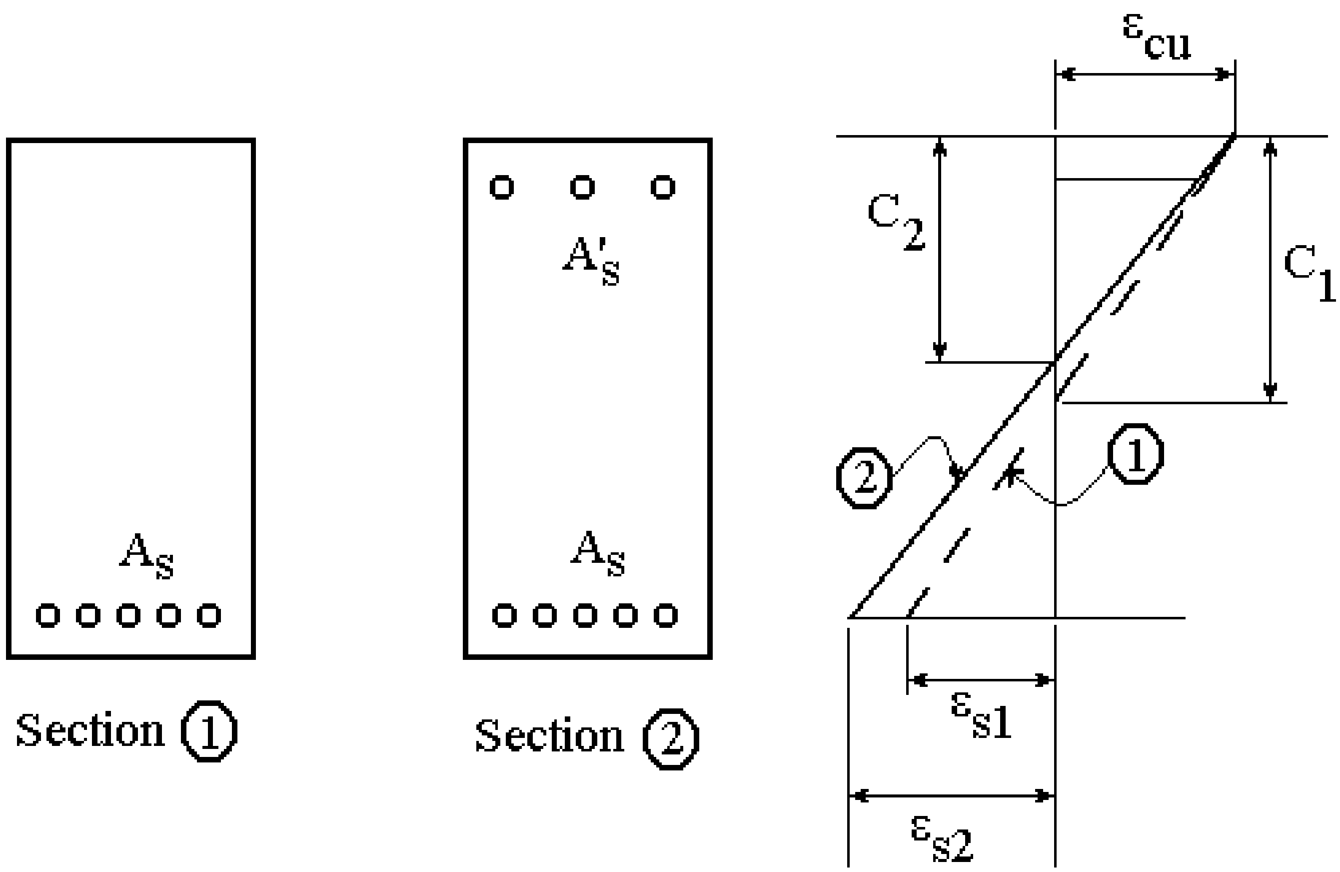


Reasons for Providing Compression Reinforcement

- Change failure mode from compression to tension. When $r > r_{bal}$ addition of A_s strengthens.
- Eases in Fabrication - Use corner bars to hold & anchor stirrups.

Effect of Compression Reinforcement

Compare the strain distribution in two beams with the same A_s



Effect of Compression Reinforcement

$$T = A_s f_s$$

$$T = C_{c1} = 0.85 f_c' ba = 0.85 f_c' b\beta_1 c_1$$

$$c_1 = \frac{A_s f_s}{0.85 f_c' b\beta_1}$$

$$T = A_s f_s$$

$$T = C_s' + C_{c1}$$

$$= A_s' f_s' + 0.85 f_c' ba_2$$

$$= A_s' f_s' + 0.85 f_c' b\beta_1 c_2$$

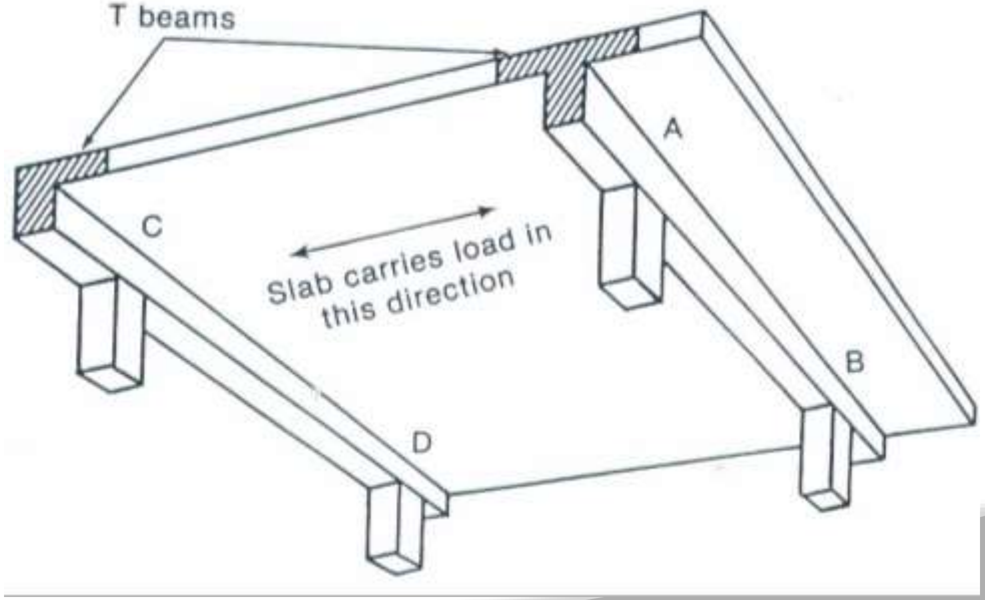
$$c_2 = \frac{A_s f_s - A_s' f_s'}{0.85 f_c' b\beta_1}$$

Effect of Compression Reinforcement

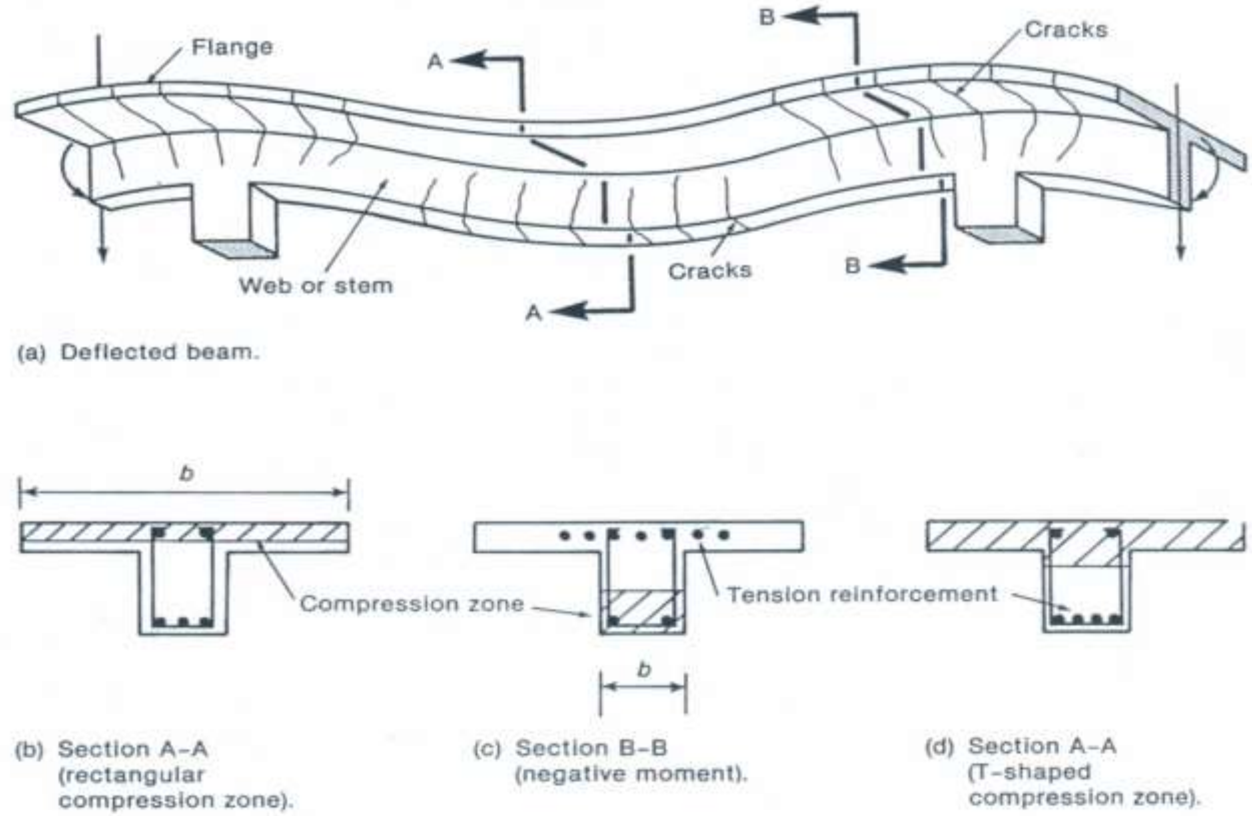
- Under reinforced Failure
  - (Case 1) Compression and tension steel yields
  - (Case 2) Only tension steel yields
- Over reinforced Failure
  - (Case 3) Only compression steel yields
  - (Case 4) No yielding Concrete crushes

Analysis of Flanged Section

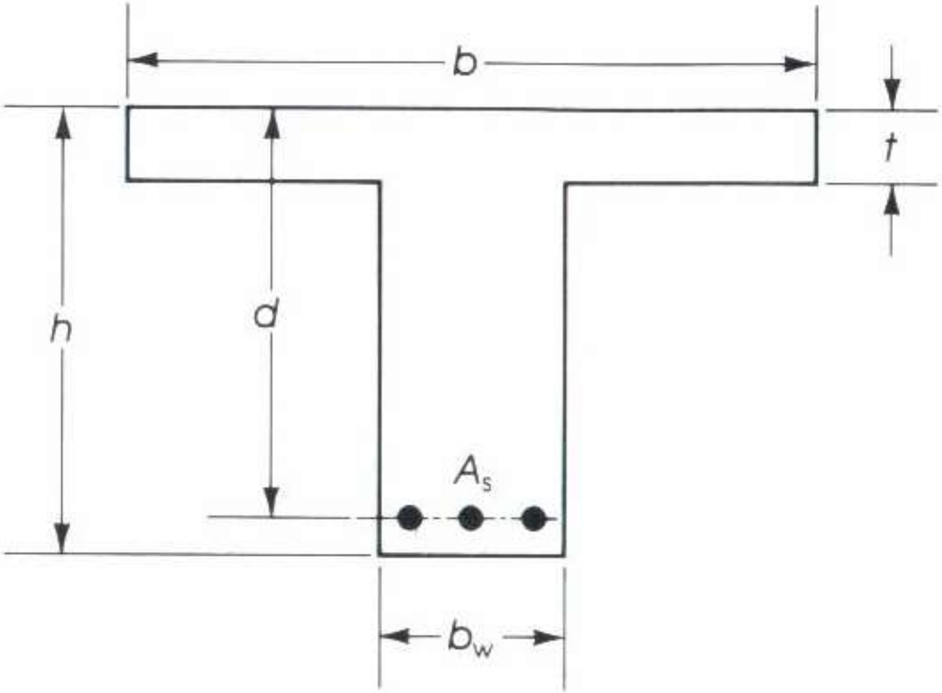
- Floor systems with slabs and beams are placed in monolithic pour.
- Slab acts as a top flange to the beam; *T-beams*, and *Inverted L(Spandrel) Beams*.



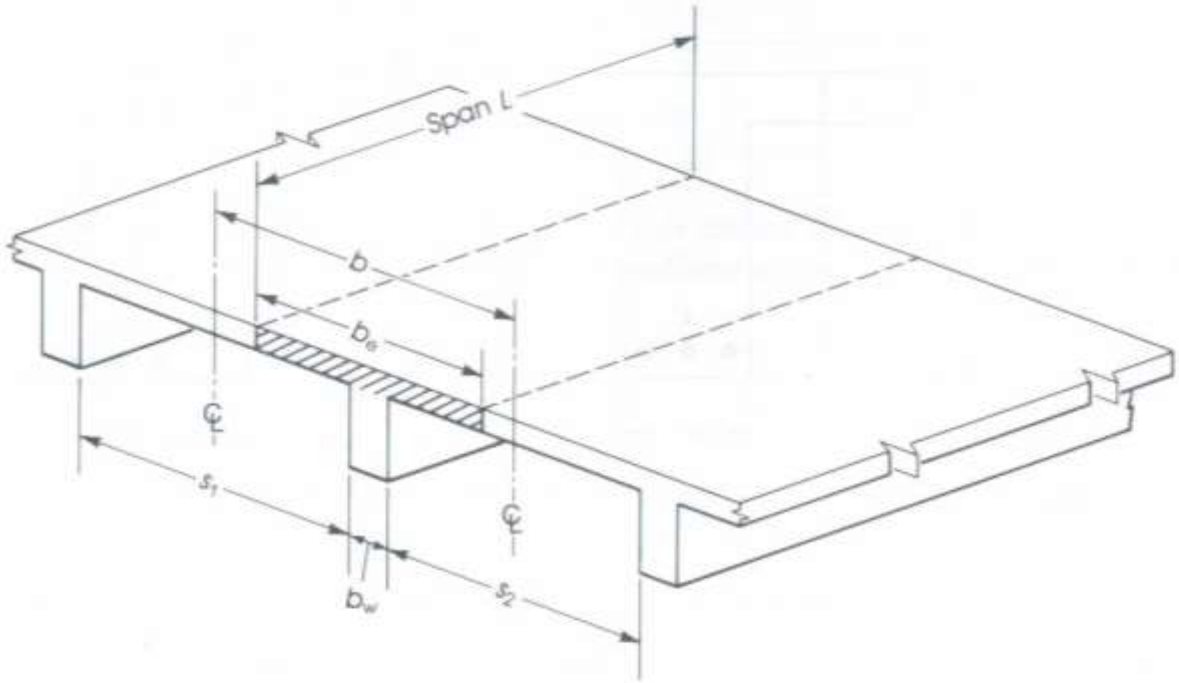
Positive and Negative Moment Regions in a T-beam



If the neutral axis falls within the slab depth analyze the beam as a rectangular beam, otherwise as a T-beam.



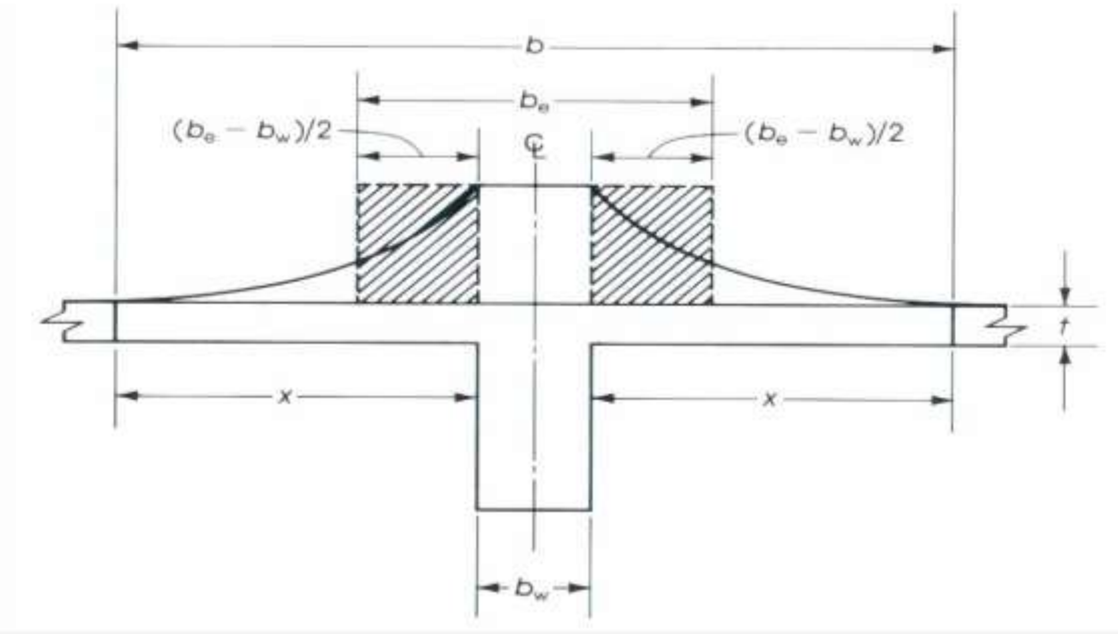
Analysis of Flanged Section



Analysis of Flanged Section

Effective width (b_{eff})

b_{eff} is width that is stressed uniformly to give the same compression force actually developed in compression zone of width $b_{(actual)}$



ACI Code Provisions for Estimating beff

$$b_{\text{eff}} \leq \frac{L}{4}$$

$$\leq 16h_{\text{f}} + b_{\text{w}}$$

$$\leq b_{\text{actual}}$$

ACI Code Provisions for Estimating beff

From ACI 318, Section 8.10.3

Inverted L Shape Flange

$$
\begin{aligned}
b_{\text{eff}} &\le \frac{L}{12} + b_{\text{w}} \\
&\le 6h_{\text{f}} + b_{\text{w}} \\
&\le b_{\text{actual}} = b_{\text{w}} + 0.5*(\text{clear distance to next web})
\end{aligned}
$$

ACI Code Provisions for Estimating beff

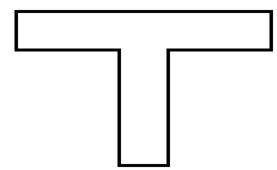
From ACI 318, Section 8.10

Isolated T-Beams

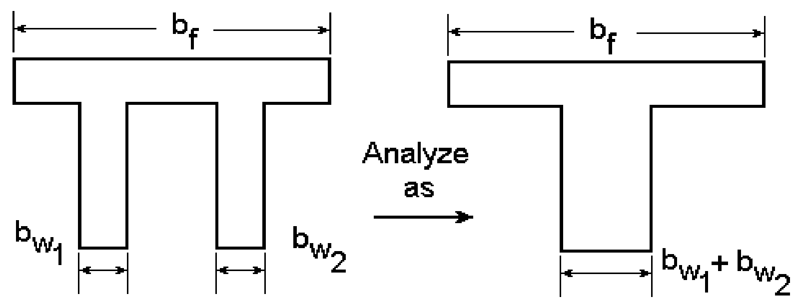
$$h_f \geq \frac{b_w}{2}$$

$$b_{eff} \leq 4b_w$$

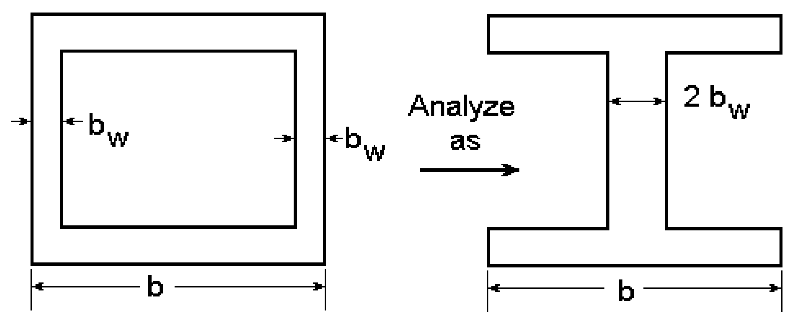
Single Tee



Twin Tee



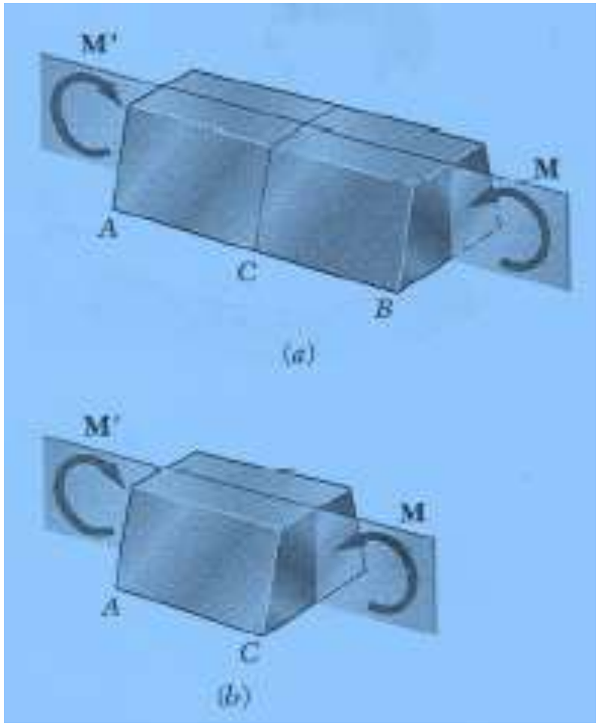
Box

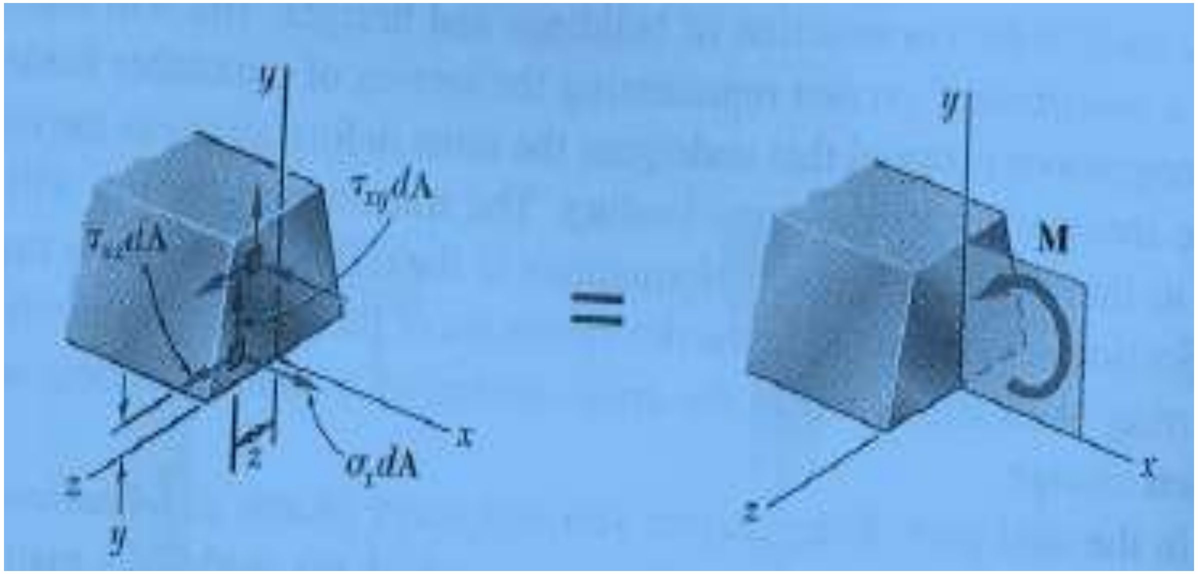


UNIT-II

SHEAR, TORSION & BOND

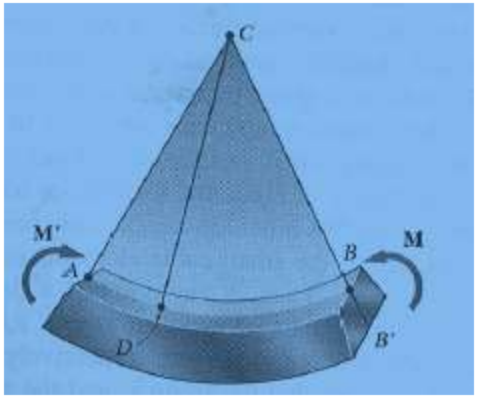
Beam subjected to pure bending moment

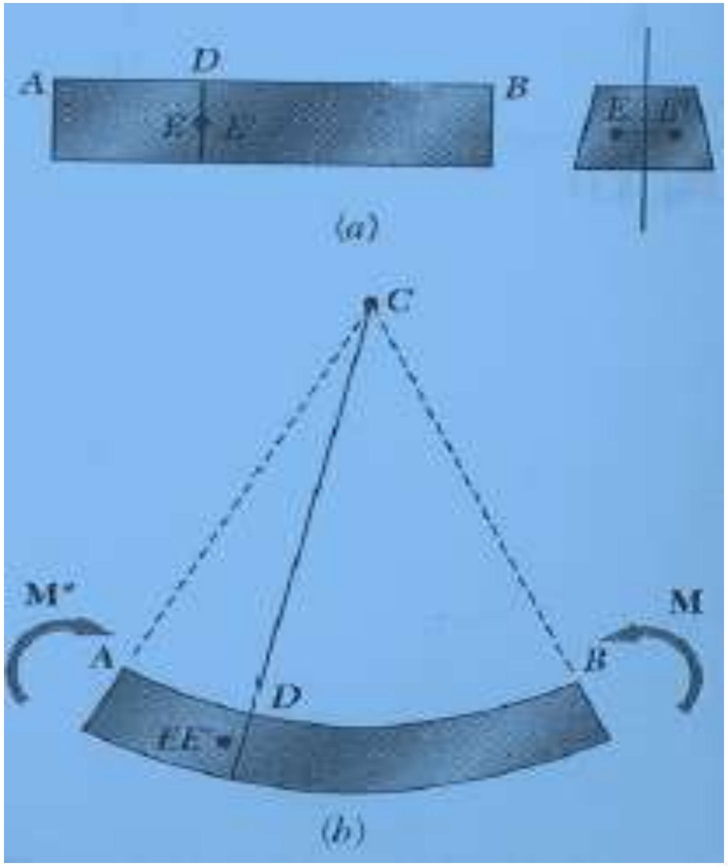




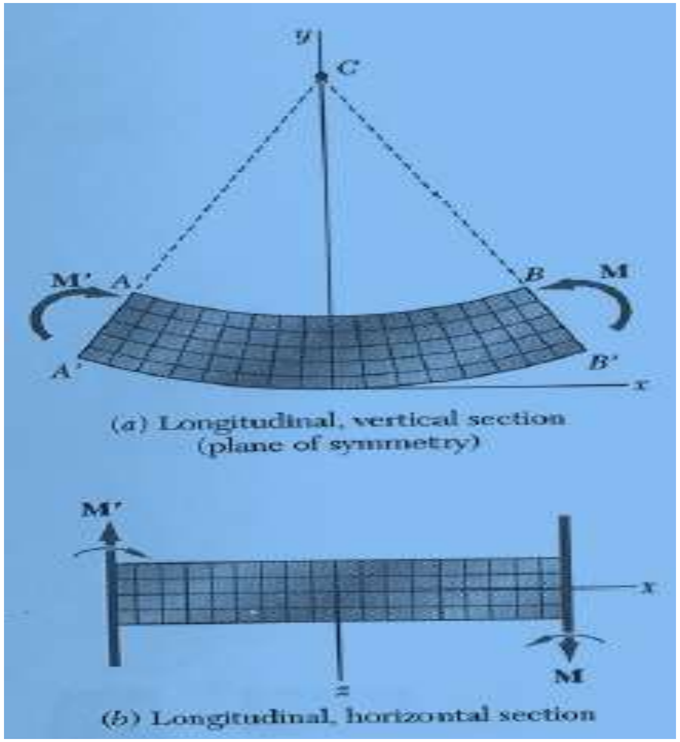
Stresses developed in beam under pure bending moment

Deflection of Beam under Pure BM

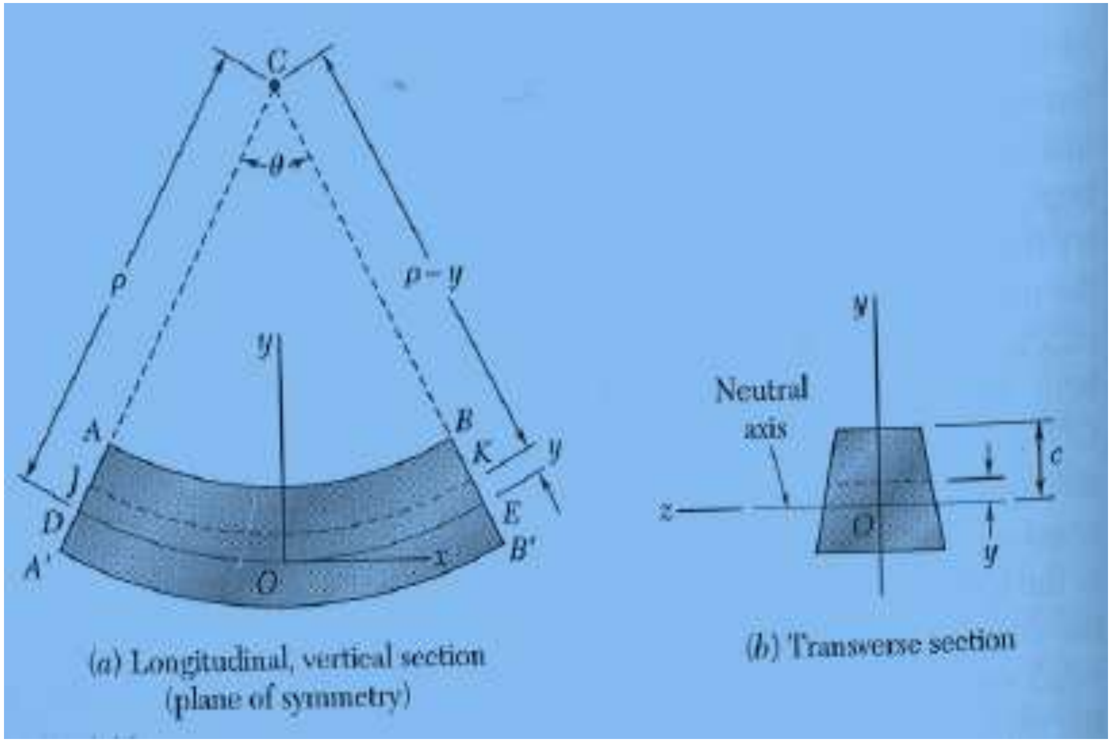




Deflected Shape of Beam

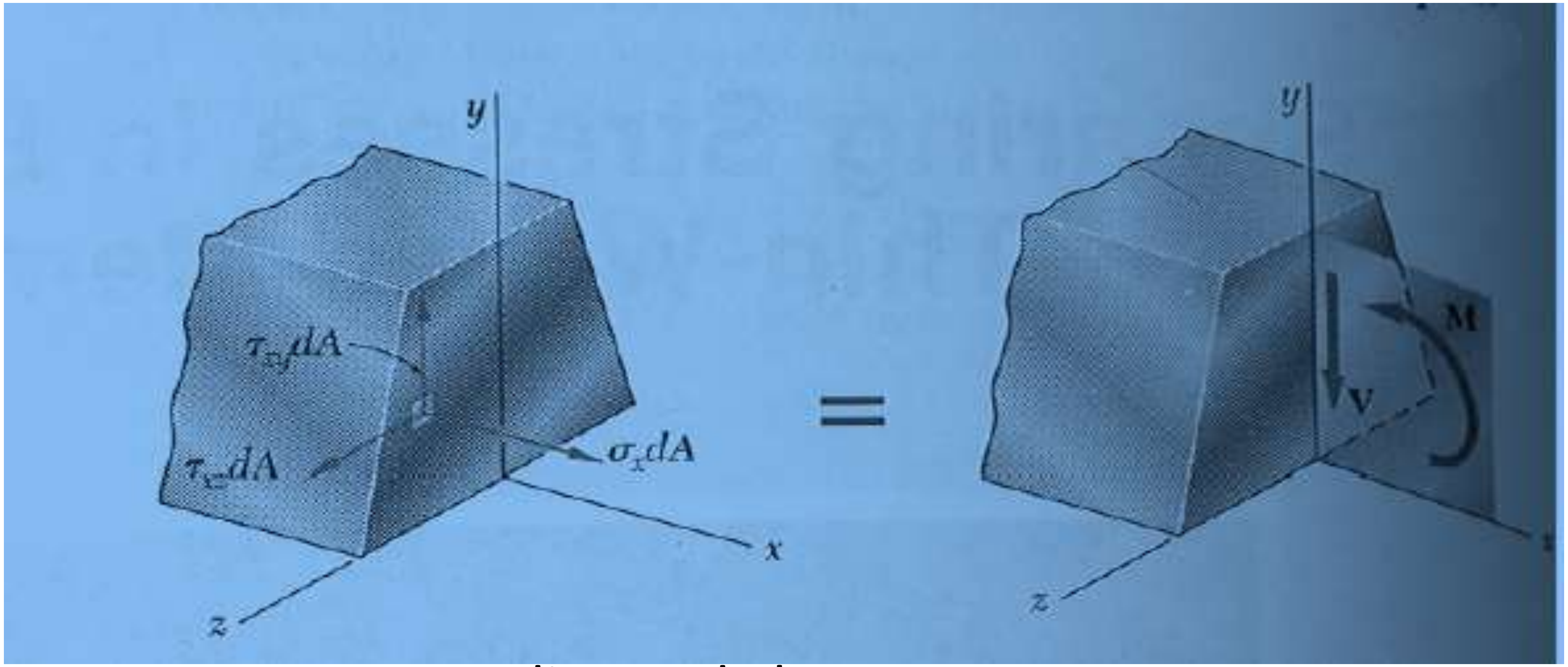


Sectional View of Beam



Neutral Surface/Axis

Shear Stress in Beams-General Loading

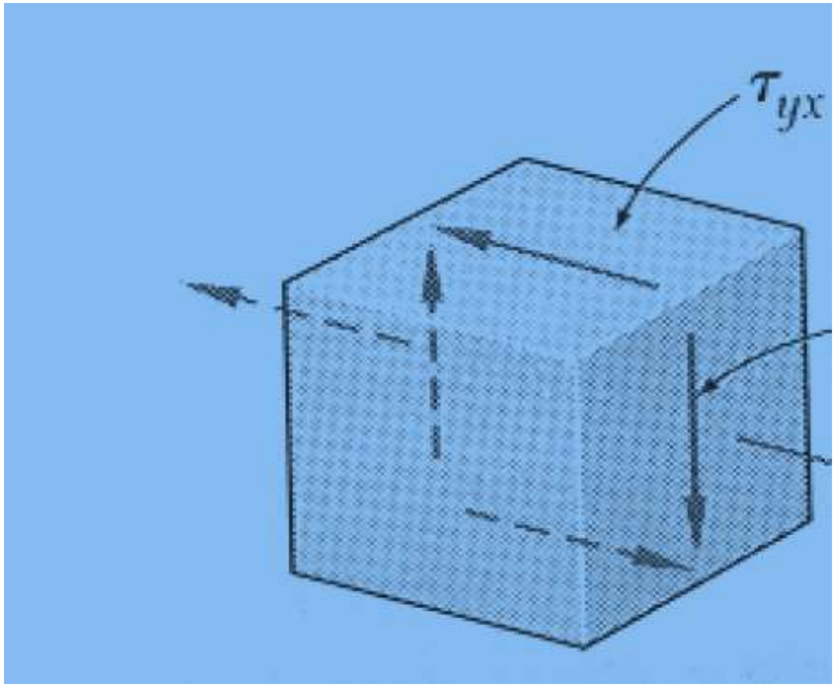


Bending and Shear Stresses

✓Equilibrium of Forces

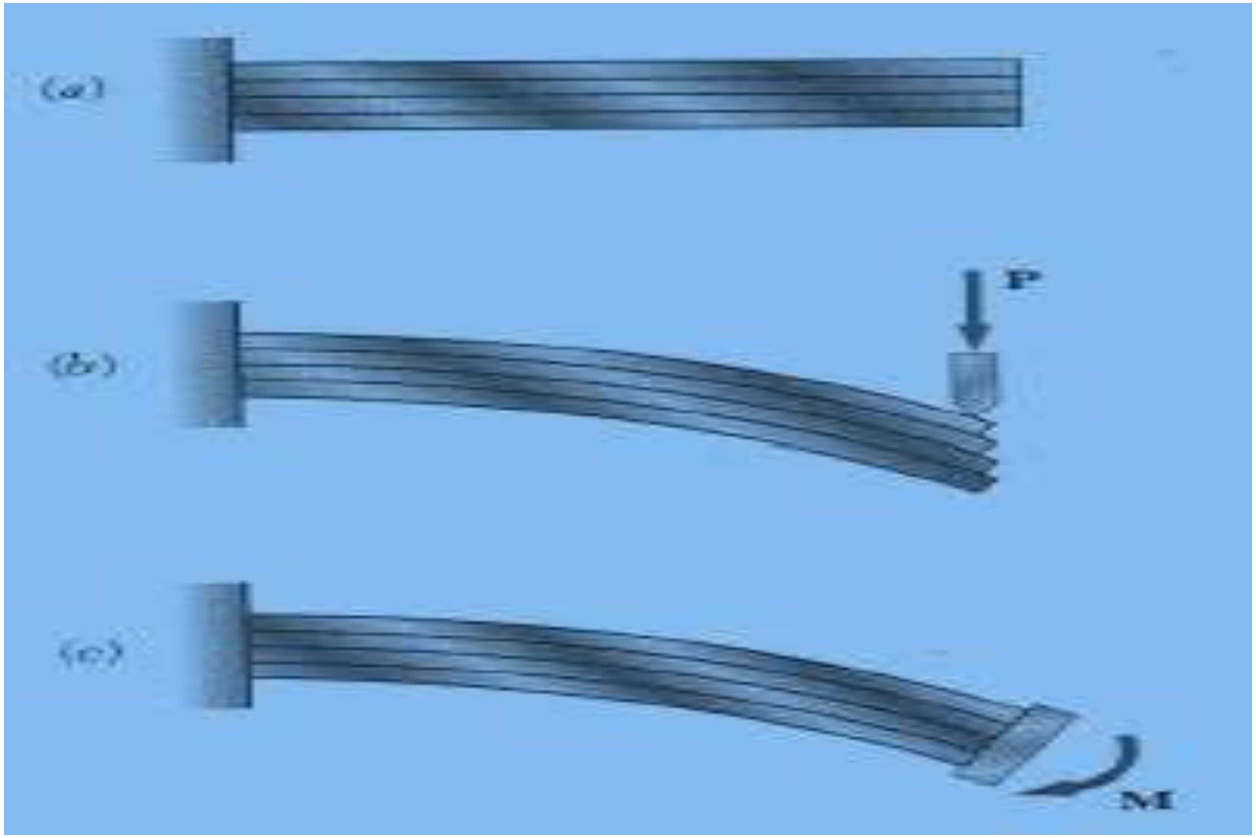
- Vertical Shear Force
- Bending Moment

Horizontal Shear Stress



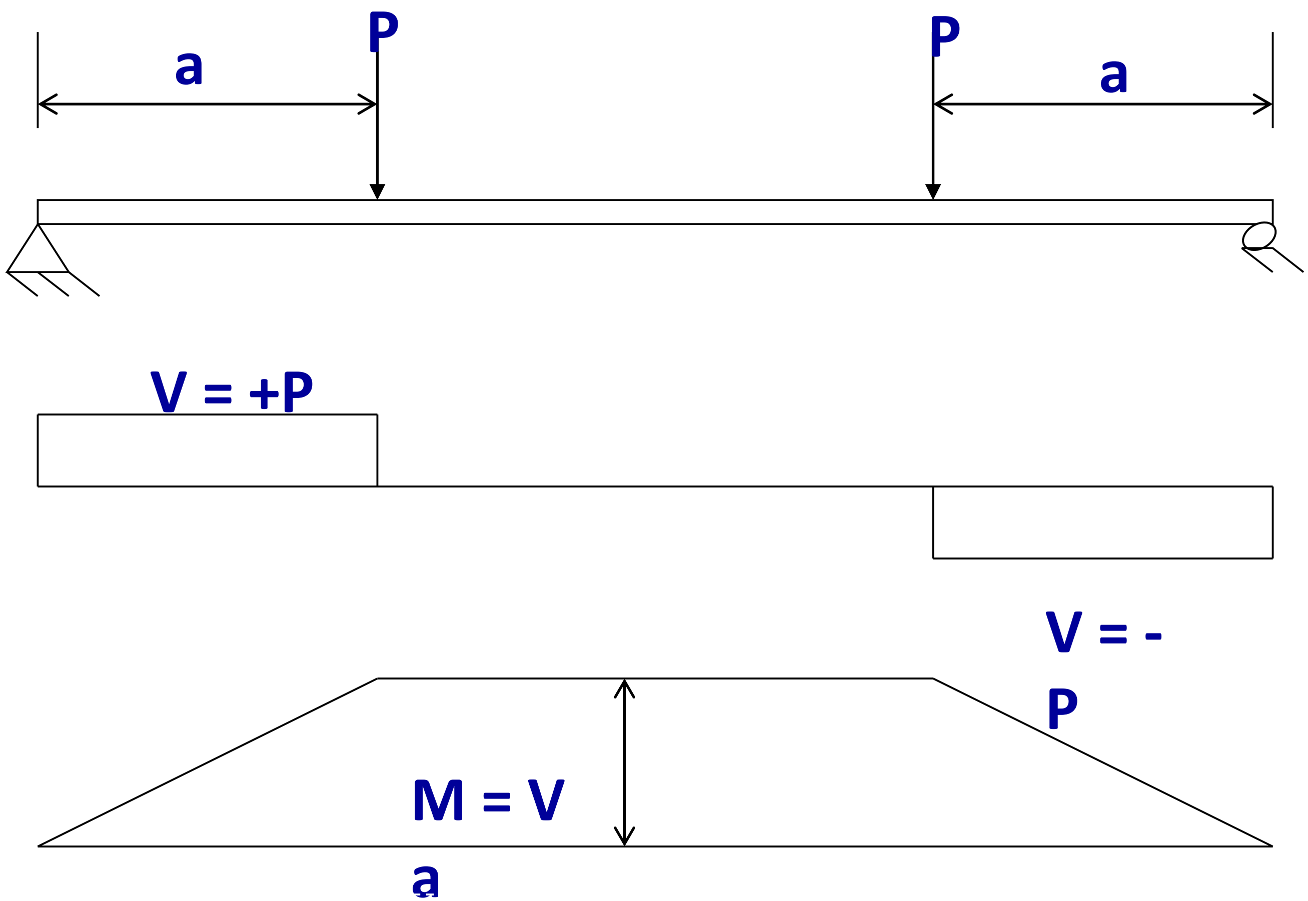
Shear stresses develop on horizontal planes

Deflection of Beam with Planks



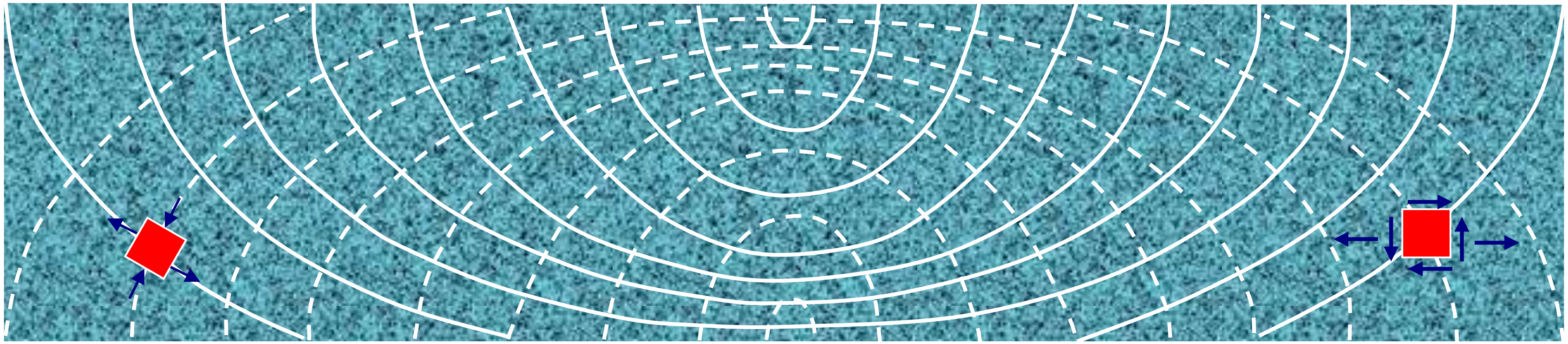
Beams with and without Planks

Definition of Shear Span

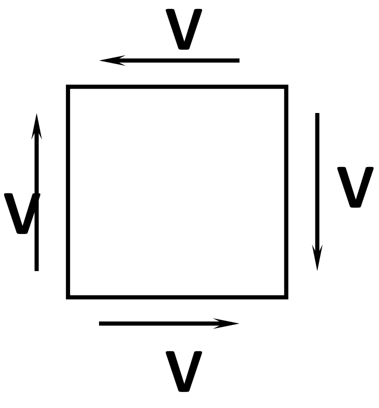


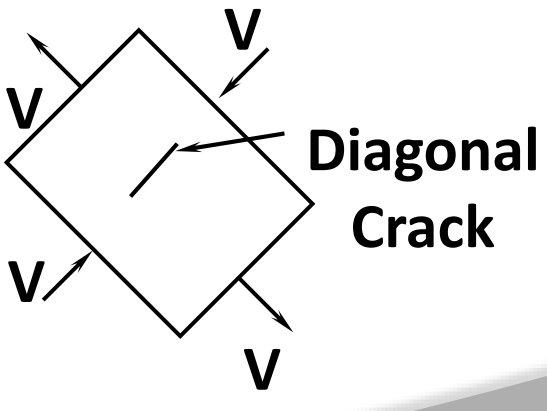
Basic Definition of Shear span, a

Principal Stress Trajectories

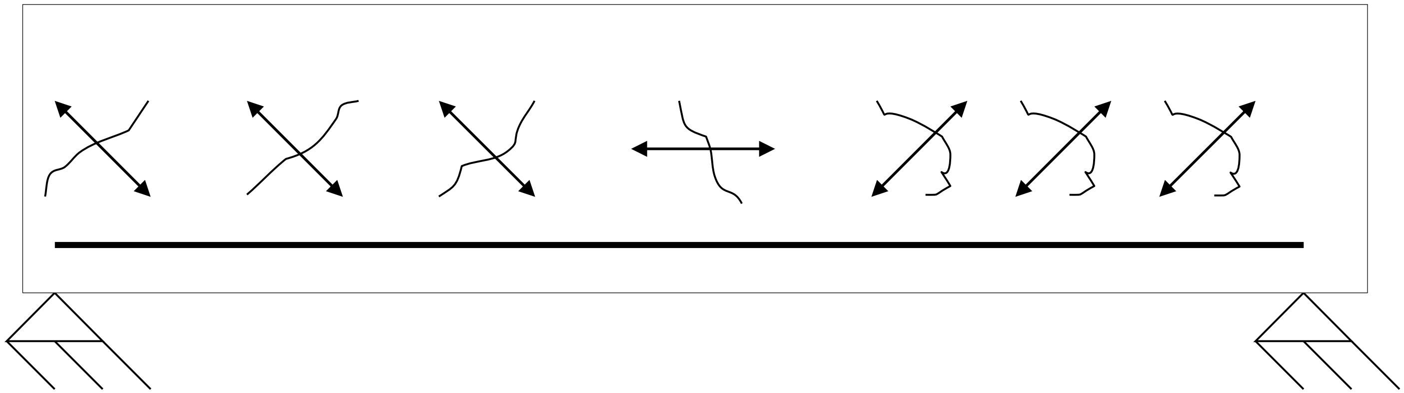


Diagonal Tension





Cracking in RC Beams



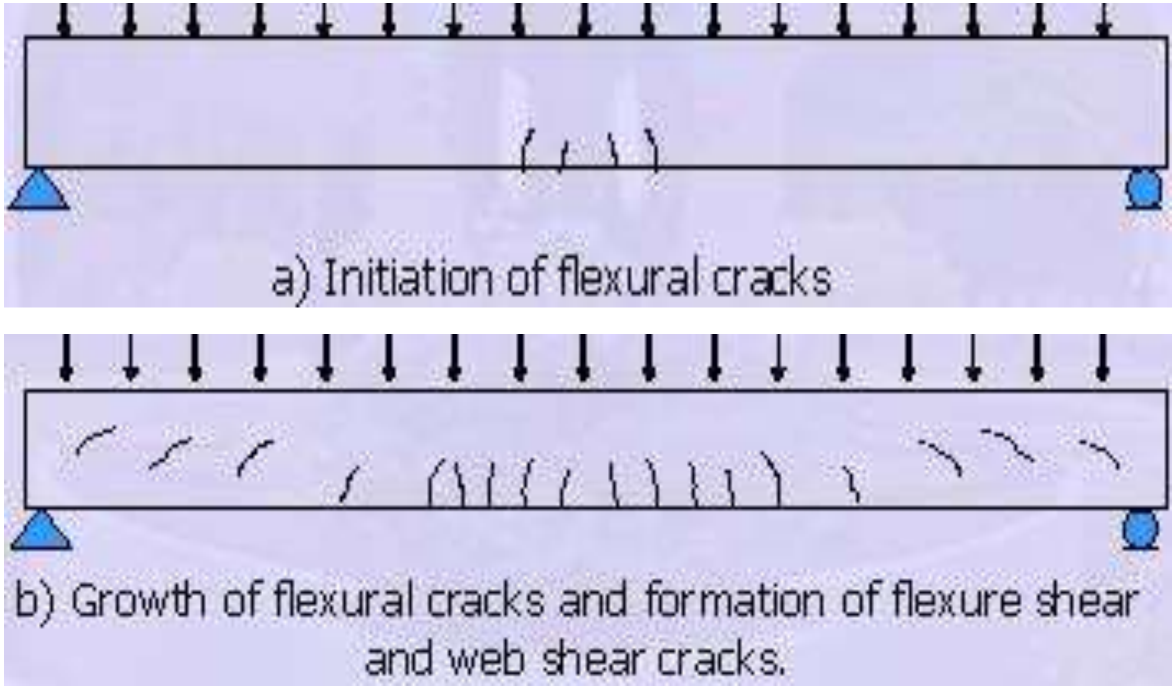
Direction of potential cracks in a simple beam.

Types of Cracks

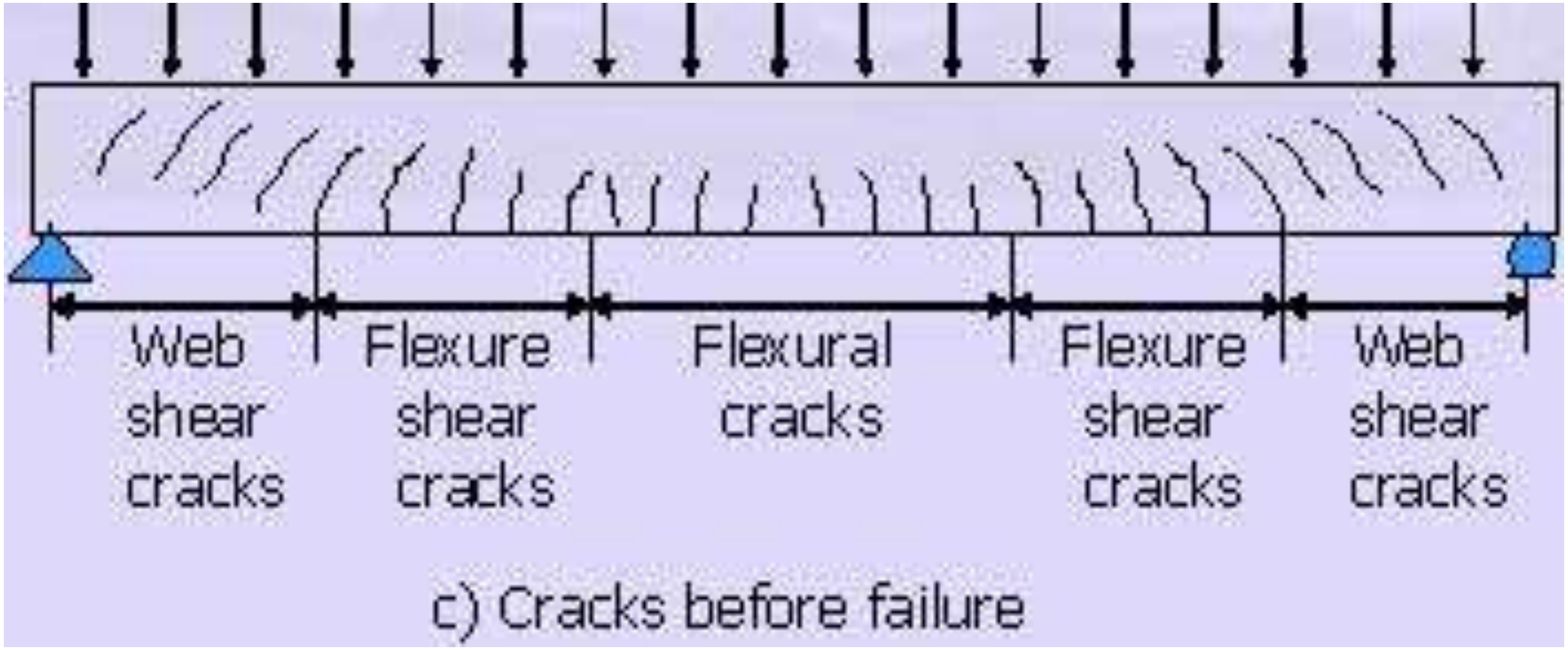
- Type and formation of cracks depend on span-to-depth ratio of beam and loading.
- For simply supported beam under uniformly distributed load, three types of cracks are identified.

1. Flexural cracks: form at the bottom near mid span and propagate upwards.
2. Web shear cracks: form near neutral axis close to support and propagate inclined to the beam axis.
3. Flexure shear cracks: These cracks form at bottom due to flexure and propagate due to both flexure and shear.

• Formation of cracks for a beam with large span-to-depth ratio and uniformly distributed loading is shown.



b) Growth of flexural cracks and formation of flexure shear and web shear cracks.



c) Cracks before failure

Fig. 7 Formation of cracks in a reinforced concrete beam

Components of Shear Resistance

1. Components of shear resistance at a flexure shear crack are shown in the following figure.

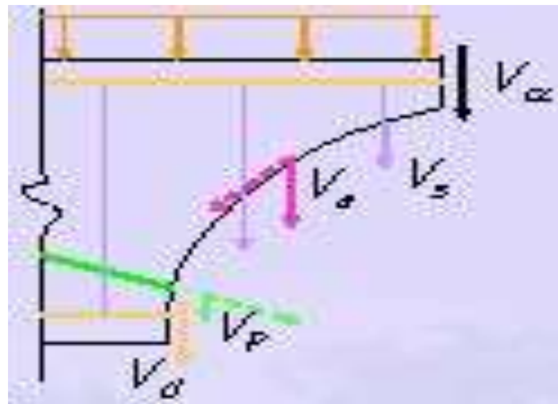


Fig. 8 Internal forces at a flexure shear crack

The notations in the previous figure are as follows.

V_{cz} = Shear carried by uncracked concrete

V_a = Shear resistance due to aggregate interlock

V_d = Shear resistance due to dowel action

V_s = Shear carried by stirrups

- Magnitude and relative value of each component change with increasing load.

Modes of Failure

- *For beams with low span-to-depth ratio or inadequate shear reinforcement, the failure may be due to shear.*
- *Failure due to shear is sudden as compared to failure due to flexure.*
- Five modes of failure due to shear are identified.

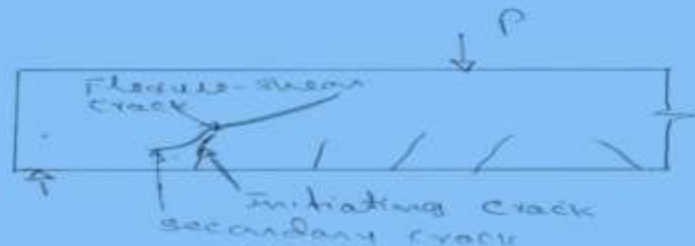
    1. *Diagonal tension failure*
    2. *Shear compression failure*
    3. *Shear tension failure*
    4. *Web crushing failure*
    5. *Arch rib failure*

- Mode of failure depends on span-to-depth ratio, loading, cross-section of beam, amount and anchorage of reinforcement.

Shear in Concrete

* Direction of potential cracks in a simple beam.

(a) Web-shear crack



(b) Flexure shear crack

Types of Inclined cracks.

Diagonal tension failure: inclined crack propagates rapidly due to inadequate shear reinforcement.

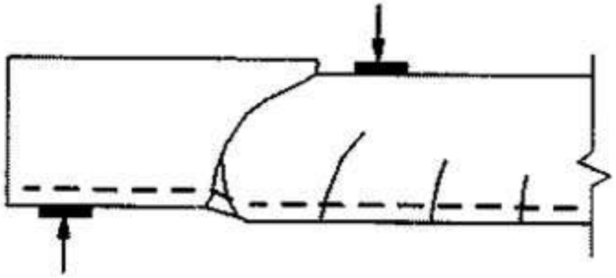


Fig. 9 Diagonal tension failure.

Shear compression failure: crushing of concrete near the compression flange above the tip of the inclined crack.

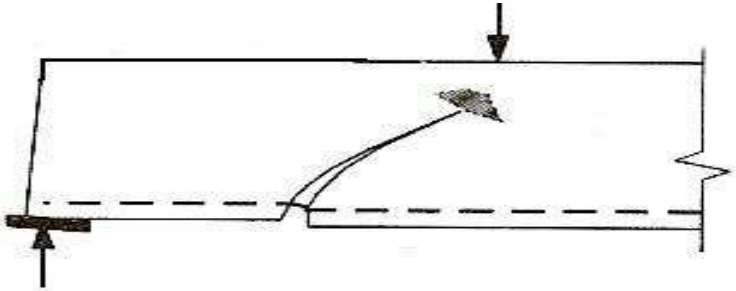


Fig. 10 Shear compression failure

Shear tension failure: inadequate anchorage of longitudinal bars, diagonal cracks propagate horizontally along the bars.

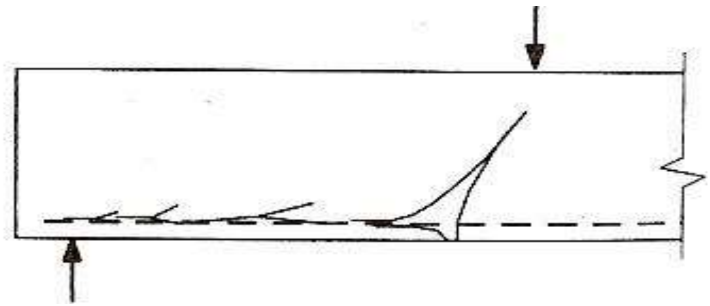


Fig. 11 Shear tension failure

Web crushing failure: concrete in the web crushes due to inadequate web thickness.

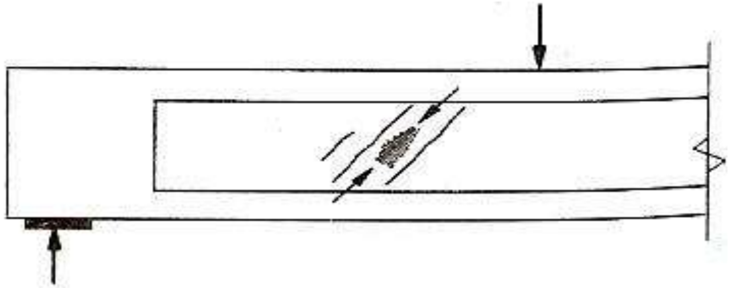
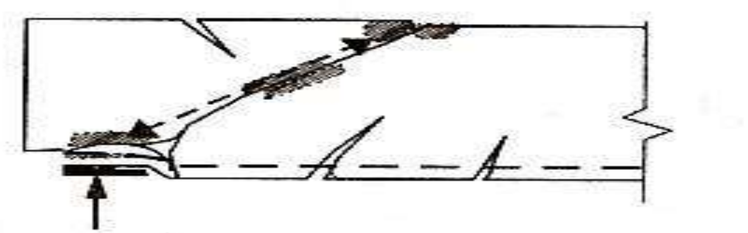


Fig. 12 Web crushing failure

Arch rib failure: in deep beams, web may buckle and subsequently crush. There can be anchorage failure or failure of the bearing.



- Design for shear is to avoid shear failure; beam should fail in flexure at its ultimate flexural strength.
- Design involves not only design of the stirrups, but also limiting average shear stress in concrete, providing adequate thickness of web and adequate development length of longitudinal bars.

Shear in Reinforced Concrete Members

Behavior of RC members under Shear (including combined loads with other loads) is very complex

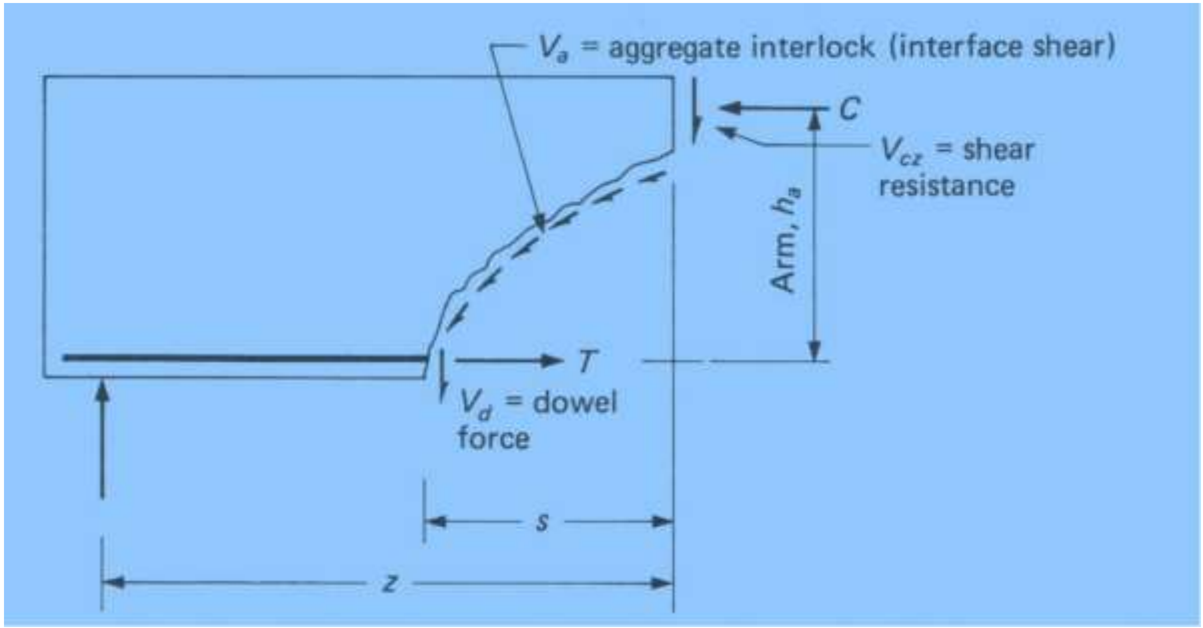
1. Non-homogeneity of materials
2. Presence of Cracks and Reinforcement
3. Nonlinearity in Material Response

Current design (Code) procedures,

I. Based on results of extensive tests on small size members with simplifying assumptions

II. No unified and universally accepted method for prediction of shear strength of beams without web reinforcement

Shear Transfer Mechanisms



v_{cz} = Shear in compression zone (20-40%)

v_a = Aggregate Interlock forces (35-50%)

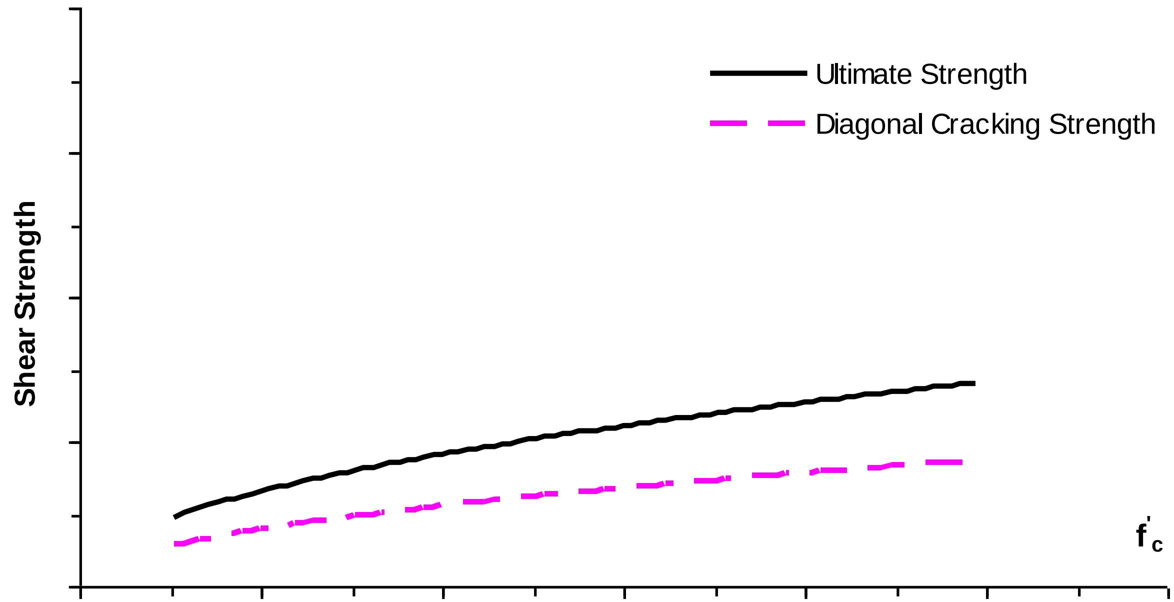
v_d = Dowel action from longitudinal bars (15-25%)

Total Resistance = (v_{cz} + v_{ay} +v_d) (For Beams without stirrups)

Factors Influencing Shear Strength

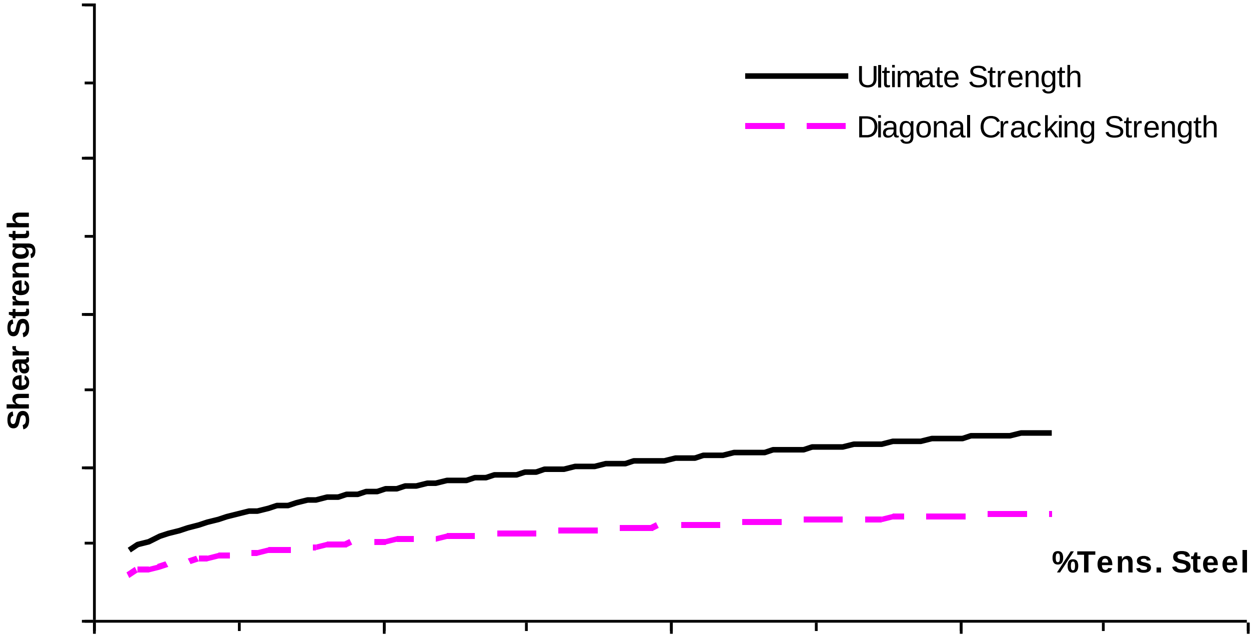
1. Strength of Concrete (f'_c)
2. Percentage of Flexural (Tensile) Reinforcement (ρ_t)
3. Shear Span-to-Depth Ratio (a/d)
4. Depth of Member (d)
5. Size of Aggregate (d_a)

Shear Strength with Compressive Strength of Concrete



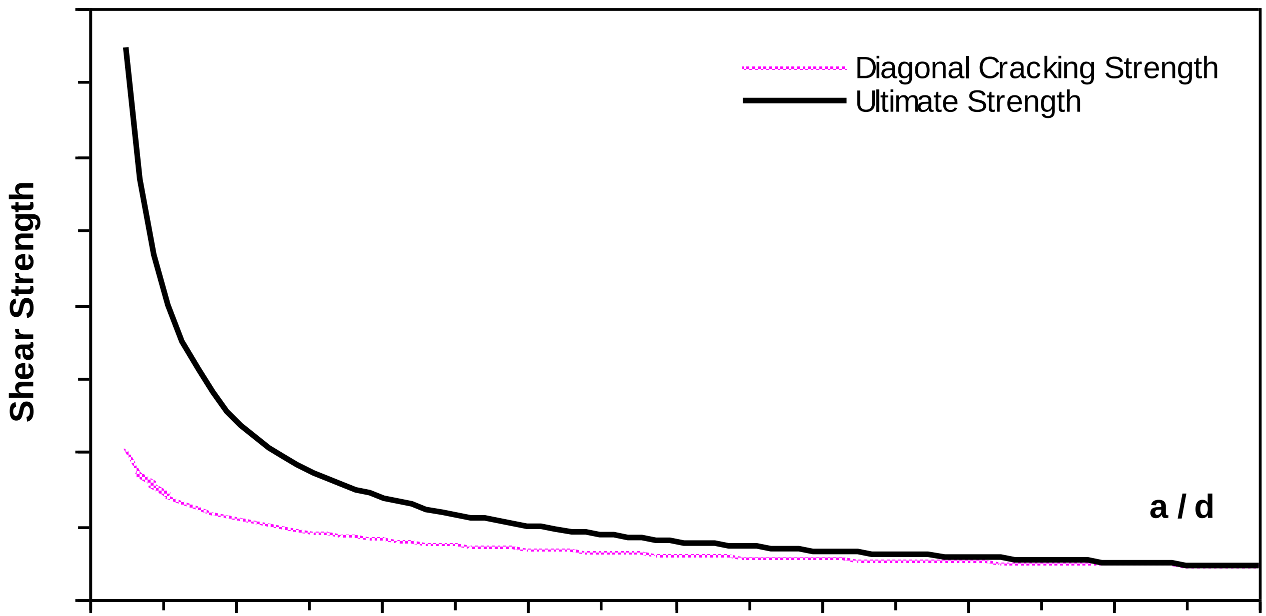
Qualitative variation of ultimate and Diagonal cracking strength with Compressive Strength of Concreet

Shear Strength with Tension Reinforcement



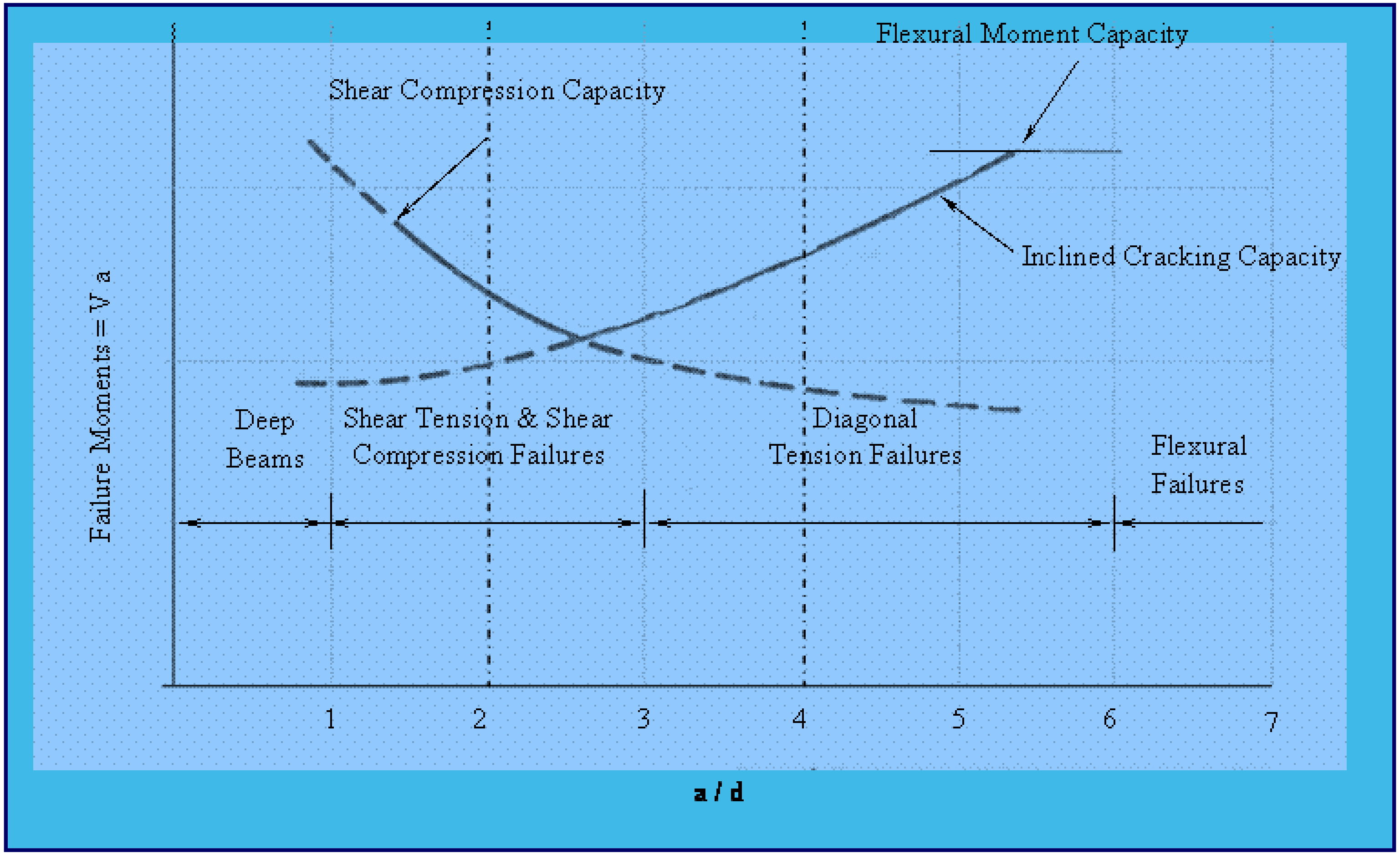
Qualitative variation of ultimate and Diagonal cracking strength with % Tension Steel

Shear Strength vs. Shear Span-to-Depth Ratio cal



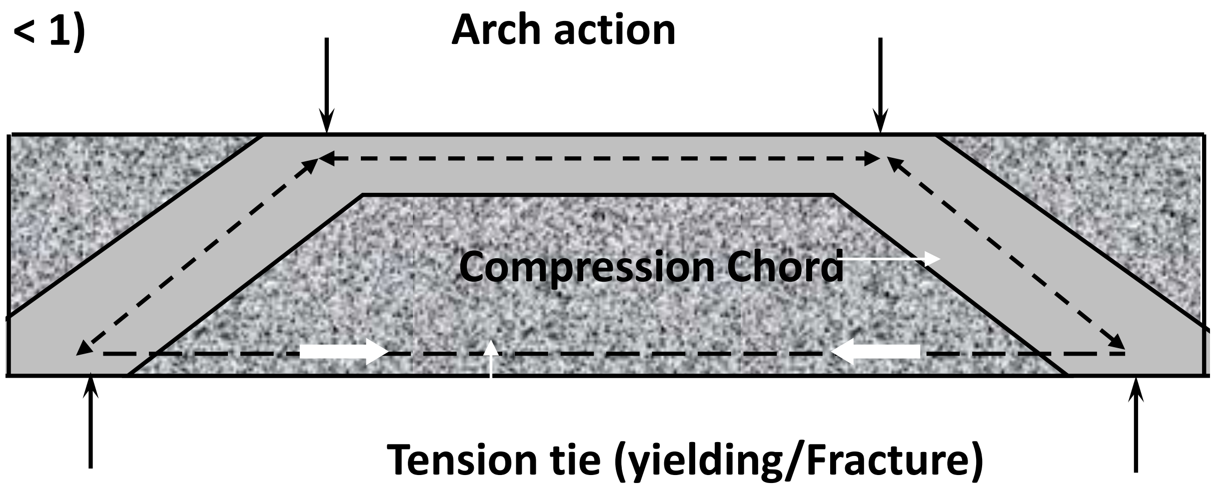
Qualitative variation of ultimate and Diagonal cracking strength with a/d ratio

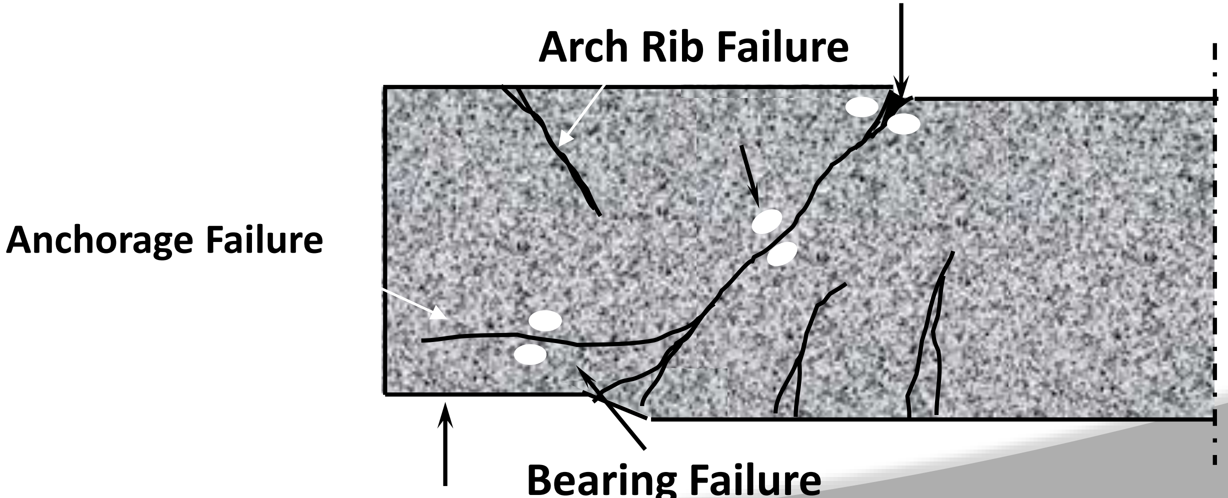
Failure Mechanism of RC Elements at different a/d Ra



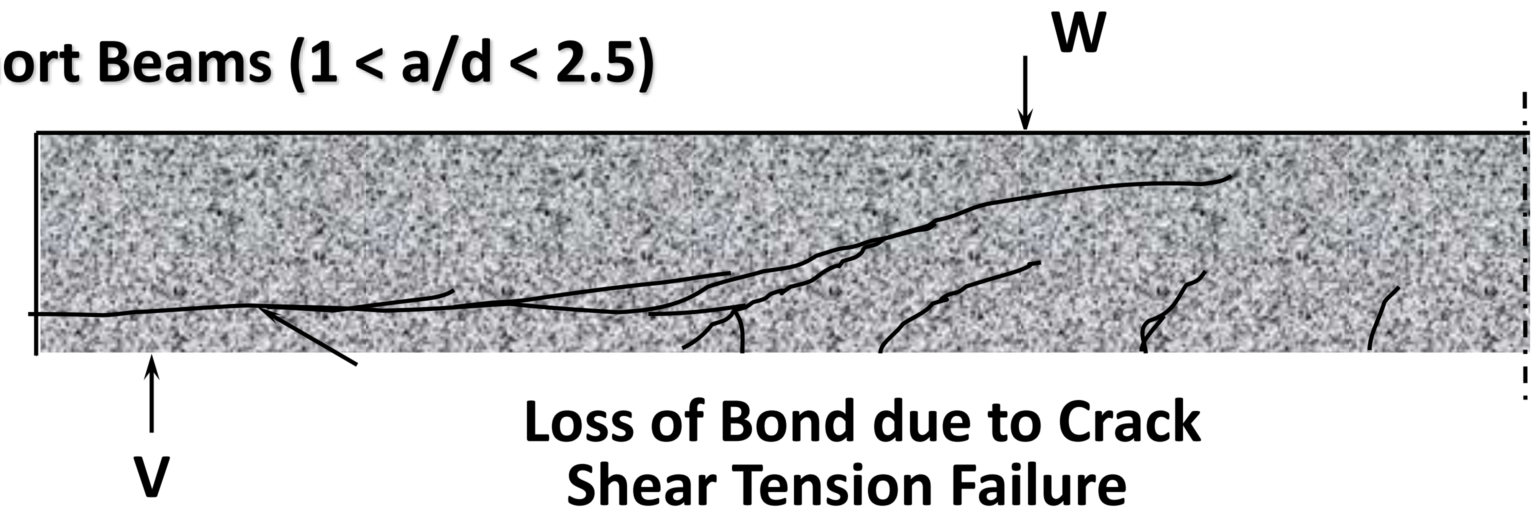
Failure Mechanisms

Deep Beams (a/d < 1)

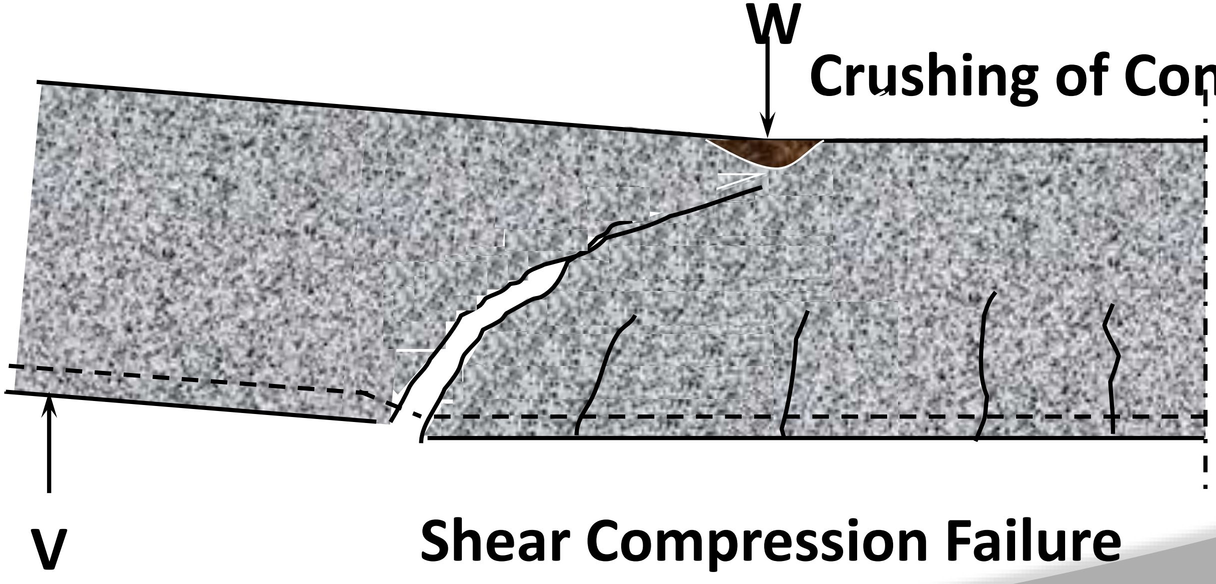




Short Beams (1 < a/d < 2.5)

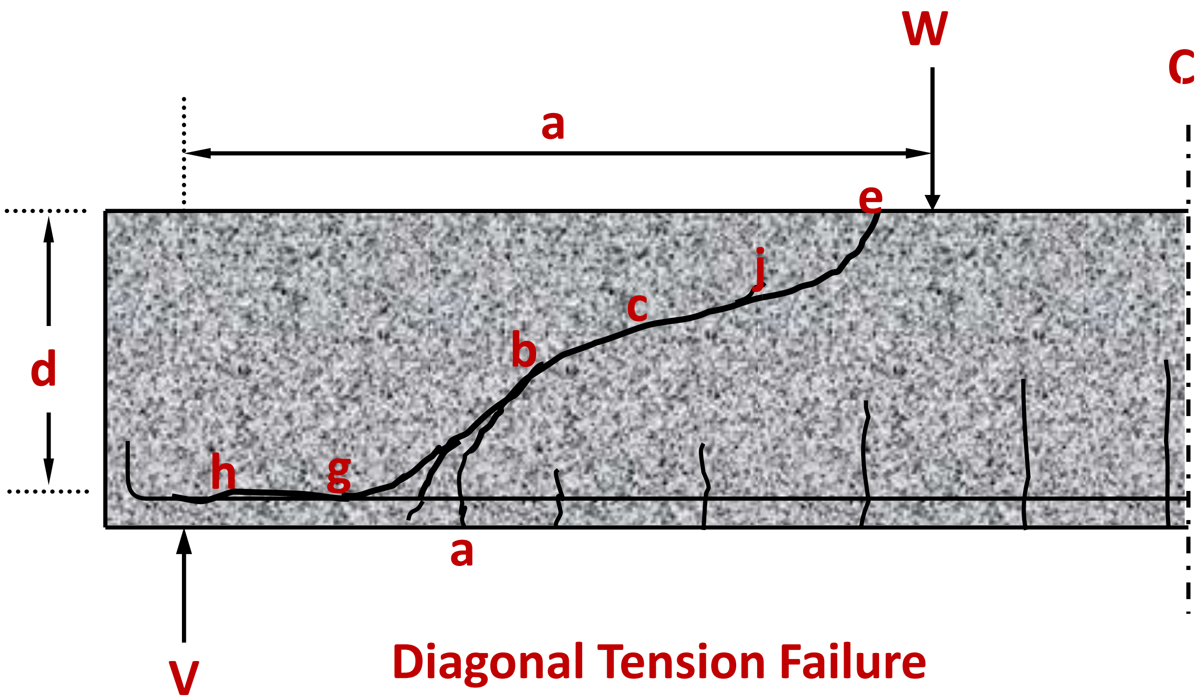


Loss of Bond due to Crack
Shear Tension Failure



Shear Compression Failure

Slender Beams (2.5 < a/d < 6)



Diagonal Tension Failure

Does beam depth effect the strength?

- A phenomenon related to change in strength with member size, decrease in strength with increase in member size
- Size effect in real structures:

  i) material heterogeneity and

  ii) discontinuities (flaws) to flow of stress either in the form of micro cracking before application of any load load induced micro cracking or macro cracking

- Size effect in shear is more serious due to
  - failure mode highly brittle - small deflections and lack of ductility
  - shear strength related to tensile strength of concrete
- Evidence
  - Sudden failure of Wilkins Air force depot warehouse in Shelby, Ohio (1955)
  - Catastrophic failure of structures leading to loss of human lives and property due to Hyogo-Ken Nambu Earth Quake in 1995

Wilkins Air Force Depot in Shelby, Ohio (1955)

Collapse of Superstructure - Hyogo-Ken Nambu EQ (1995)

Diagonal Cracking and Ultimate Strength

Ultimate Strength: “load corresponding to the total and complete failure due to shear and diagonal tension”

Diagonal Cracking Strength: “load corresponding to formation of first fully developed inclined crack”

a. An inclined crack is considered to be fully developed when it has progressed sufficiently towards both the mid span and the support while intersecting the tensile reinforcement

Diagonal Cracking and Ultimate Strength

How is it measured in the laboratory?

1. Normally, visual methods are deployed to measure the diagonal cracking loads in the laboratory
2. load at the onset of formation of first diagonal crack
3. when the diagonal crack crosses the mid height of the beam

The design provisions in most of the codes

1) are based on diagonal tension cracking and
2) by using a suitable multiplication factor, strength of short or deep beams is obtained
3) Need to be re-examined

Owing to complex nature of stress distribution, Code Provisions for prediction of shear strength are Empirical in Nature for Beams without Web Reinforcement

Prediction of Shear Strength

ACI - 2002

$$v_c = \left(0.16\sqrt{f_c'} + 17.2\,\rho\frac{V_u d}{M_u}\right) \le 0.3\sqrt{f'_c} \quad MPa \quad \rightarrow (6)$$

$$v_c = 0.17\sqrt{f'_c} \quad MPa \quad \rightarrow 6\,(a)$$

Simplified

$$v_c = \left(3.5 - 2.5\frac{M_u}{V_u d}\right)\left(0.16\sqrt{f'_c} + 17.2\rho\frac{V_u d}{M_u}\right) \quad MPa \quad \rightarrow (7)$$

$$\le 0.5\sqrt{f'_c} \qquad (ACI - 1999)$$

$$where \left(3.5 - 2.5\frac{M_u}{V_u d}\right) \le 2.5$$

$$and \quad \frac{V_u d}{M_u} \le 1.0$$

Prediction of Shear Strength

BS 8110 - 1997

$$v_c = \frac{0.79}{\gamma_m}\left(\frac{100A_s}{b_v d}\right)^{1/3}\left(\frac{400}{d}\right)^{1/4}\left(\frac{f_{cu}}{25}\right)^{1/3} \quad MPa \rightarrow (8)$$

Where $\frac{100\,A_s}{b_v d} \leq 3.0, \quad \frac{400}{d} \geq 1.0, \quad \gamma_m = 1.25 \quad \text{and} \quad f_{cu} \leq 40.0 \quad MPa$

For Short Beams

$$v_c = (Eqn.8)\left(\frac{2}{a/d}\right) \quad \text{for} \quad a/d < 2.0$$

Prediction of Shear Strength

IS 456-2000

$$v_{cr} = \frac{0.85\sqrt{0.8f_{ck}}\,(\sqrt{(1+5\beta}-1)}{6\beta} \quad \text{MPa} \rightarrow (9)$$

$$\leq 0.62\sqrt{f_{ck}}$$

Where 0.8 f_{ck} = Cylinder strength in terms of cube strength and

0.85 reduction factor = 1/γm ~ 1/1.2

$$and \quad \beta = \frac{0.8f_{ck}}{6.89\rho} \geq 1.0$$

For Short Beams $$v_{cr} = (Eqn.9)\left[\frac{2}{a/d}\right]$$

Design of Beams for Shear

- Nominal Shear stress $$\tau_v = \frac{V_u}{bd}$$

Where

- v_u = shear force due to design loads
- b = breadth of the member which for flanged sections shall be taken as the breadth of web b_w and
- d = effective depth

Shear Strength of RC beams

- ***Calculate the nominal shear stress and compare with the shear strength of RC beams from Table 19 of IS 456-2000.***
- ***If the nominal shear stress*** τ_v ***exceeds the shear strength*** τ_c ***of RC beams without shear reinforcement, then the beam needs to be designed for shear reinforcement.***
- ***When*** τ_v ***is less than*** τ_c ***obtained from Table 19, minimum shear reinforcement is provided which is given by***

$$\frac{A_{sv}}{bs_v} > \frac{0.4}{f_y}$$

$$s_v \leq \frac{A_{sv} f_y}{0.4b}$$

Table 19 Design Shear Strength of Concrete, τ_c, N/mm²

(*Clauses* 40.2.1, 40.2.2, 40.3, 40.4, 40.5.3, 41.3.2, 41.3.3 *and* 41.4.3)

$100\frac{A_s}{bd}$	Concrete Grade					
	M 15	M 20	M 25	M 30	M 35	M 40 and above
(1)	(2)	(3)	(4)	(5)	(6)	(7)
≤ 0.15	0.28	0.28	0.29	0.29	0.29	0.30
0.25	0.35	0.36	0.36	0.37	0.37	0.38
0.50	0.46	0.48	0.49	0.50	0.50	0.51
0.75	0.54	0.56	0.57	0.59	0.59	0.60
1.00	0.60	0.62	0.64	0.66	0.67	0.68
1.25	0.64	0.67	0.70	0.71	0.73	0.74
1.50	0.68	0.72	0.74	0.76	0.78	0.79
1.75	0.71	0.75	0.78	0.80	0.82	0.84
2.00	0.71	0.79	0.82	0.84	0.86	0.88
2.25	0.71	0.81	0.85	0.88	0.90	0.92
2.50	0.71	0.82	0.88	0.91	0.93	0.95
2.75	0.71	0.82	0.90	0.94	0.96	0.98
3.00 and above	0.71	0.82	0.92	0.96	0.99	1.01

NOTE — The term A_s is the area of longitudinal tension reinforcement which continues at least one effective depth beyond the section being considered except at support where the full area of tension reinforcement may be used provided the detailing conforms to **26.2.2** and **26.2.3**

Table 20 Maximum Shear Stress, $\tau_{c\,max}$, N/mm²

(*Clauses* 40.2.3, 40.2.3.1, 40.5.1 *and* 41.3.1)

Concrete Grade	M 15	M 20	M 25	M 30	M 35	M 40 and above
$\tau_{c\,max}$, N/mm²	2.5	2.8	3.1	3.5	3.7	4.0

Prediction of Shear Strength

IS 456-2000

$$\tau_c = \frac{0.85\sqrt{0.8f_{ck}}\left(\sqrt{(1+5\beta} - 1)\right)}{6\beta} \quad \text{MPa} \leq 0.62\sqrt{f_{ck}}$$

Where 0.8 f_{ck} = Cylinder strength in terms of cube strength and

0.85 reduction factor = 1/γm ~ 1/1.2

$$and \quad \beta = \frac{0.8 f_{ck}}{6.89\rho} \geq 1.0$$

For Short Beams

$$v_{cr} = (Eqn.9)\left[\frac{2}{a/d}\right]$$

Design of Shear Reinforcement

I. **When the shear stress is greater than shear strength given in Table 19 (IS 456), shear reinforcement shall be provided in any of the following forms**

   a. **Vertical stirrups**
   b. **Bent-up bars along with stirrups and**
   c. **Inclined stirrups**

- **Shear reinforcement shall be provided to carry a shear force equal to**

$$\left[V - \tau_c bd\right]$$

Forms of Shear Reinforcement

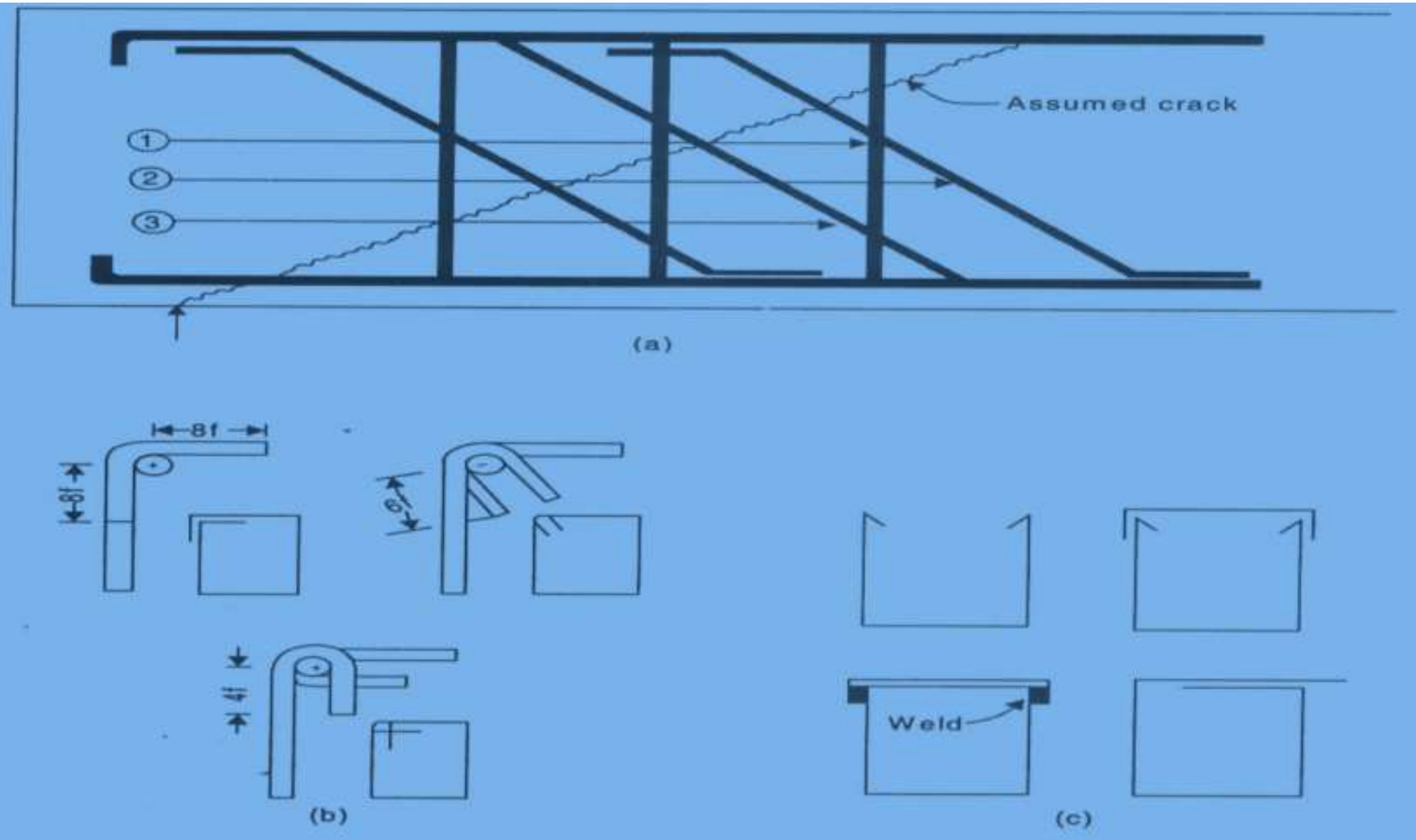


Fig. 7.4 Types of shear reinforcements in beams: (a) Shear reinforcements—(1) system of vertical stirrups; (2) system of inclined stirrups; (3) bent-up tension bar; (b) Detailing of ends of conventional types of stirrups; (c) Some special shapes of stirrups.

Forms of Shear Reinforcement

- ***For vertical stirrups***

$$v_s = \frac{A_{sv}.f_{sv}}{s_v} d$$

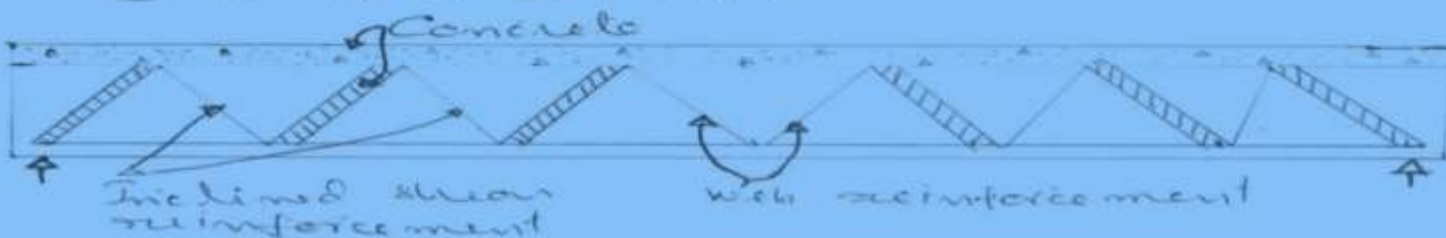
- ***For single bar or single group of parallel bars all bent up at the same cross-section***

$$V_s = \sigma_{sv}.A_{sv}.\sin\alpha$$

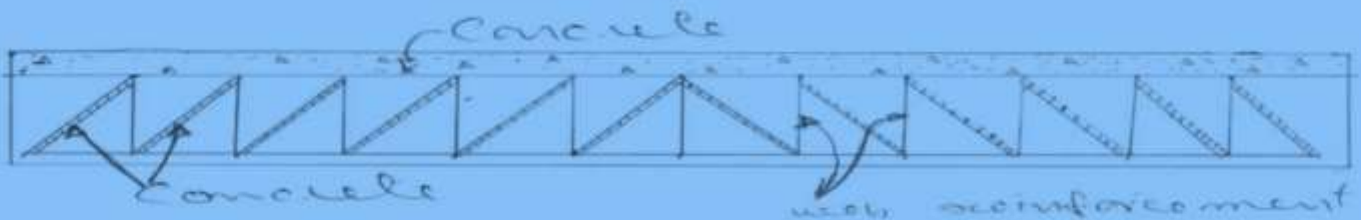
- α ***= angle between the inclined stirrup or bent up bar and the axis of the member not less than*** 45^0

Truss Analogy

(a) A steel truss



(b) Truss Action in a reinforced Concrete Beam.



(c) Truss Action in a Reinforced Concrete Beam

Truss Analogy

1. Action of vertical and inclined (stirrups) shear reinforcement may be described by the analogous truss action.
2. In a simple truss, the upper and lower chords are in compression and tension respectively; the diagonal members, called web members, are alternately in compression and tension.
3. Shear strength of RC beam may be increased by use of shear reinforcement similar in action to tensile web members in a truss.
4. Shear reinforcement must be anchored in compression zone of concrete and is usually hooped around longitudinal tension reinforcement.

Design of Stirrups

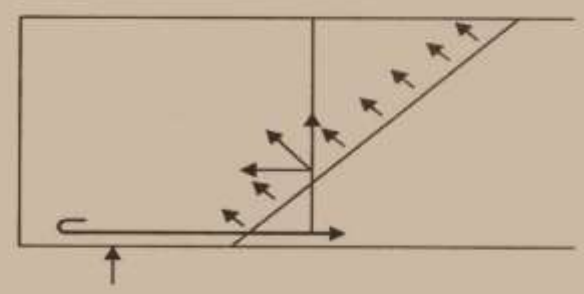


Fig. 7.6 Equilibrium of diagonal tension forces in concrete.

- A_{sv} = total area of legs of shear links
- s_v = spacing of links
- Number of links crossing 45^0 diagonal crack $N = \dfrac{d}{s_v}$
- Total strength of vertical stirrups $V_s = 0.87 f_y A_{SV} \dfrac{d}{s_v}$
- Spacing of stirrups required=

$$0.87 f_y A_{sv} \frac{d}{s_v} = (\tau_v - \tau_c) bd \Rightarrow s_v = \frac{0.87 f_y A_{sv}}{b(\tau_v - \tau_c)}$$

Design of Bent-up Bars

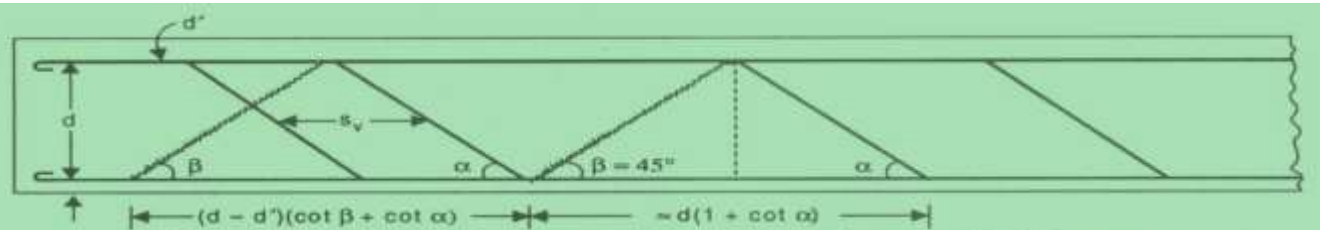


Fig. 7.7 Design of bent-up bar system for shear in RCC beams.

- ***Horizontal length over which the bar is effective can be taken as equal to d(cot β + cot α), where β = direction of shear compression, α = angle at which the bars bent***
- ***Let s_v = spacing of bent bars. Then the number of effective bars in this region are***

$$N = \frac{(\cot\beta + \cot\alpha)(d - d')}{s_v}$$

- ***The maximum shear carried by bent up bars =***

$$V_s = A_{sv}(0.87 f_y)\sin\alpha\,\frac{(\cot\beta + \cot\alpha)(d - d')}{s_v}$$

$$= A_{sv}(0.87 f_y)\frac{(\cos\alpha + \sin\alpha)(d)}{s_v};\therefore\left(\beta = 45^0; (d - d') \approx d\right)$$

Minimum Shear Reinforcement

1. Restrains the growth of inclined cracking.
2. Ductility is increased and gives warning before failure.
3. In an unreinforced web, such reinforcement is of great value if a member is subjected to an unexpected tensile force or an overload.
4. A minimum area of shear reinforcement is required whenever the total factored shear force V_u is greater than one-half the shear strength provided by concrete kV_c.
5. Need to increase minimum shear reinforcement as concrete strength increases to prevent sudden shear failure by inclined cracking.

UNIT-III
ONE-WAY SLAB

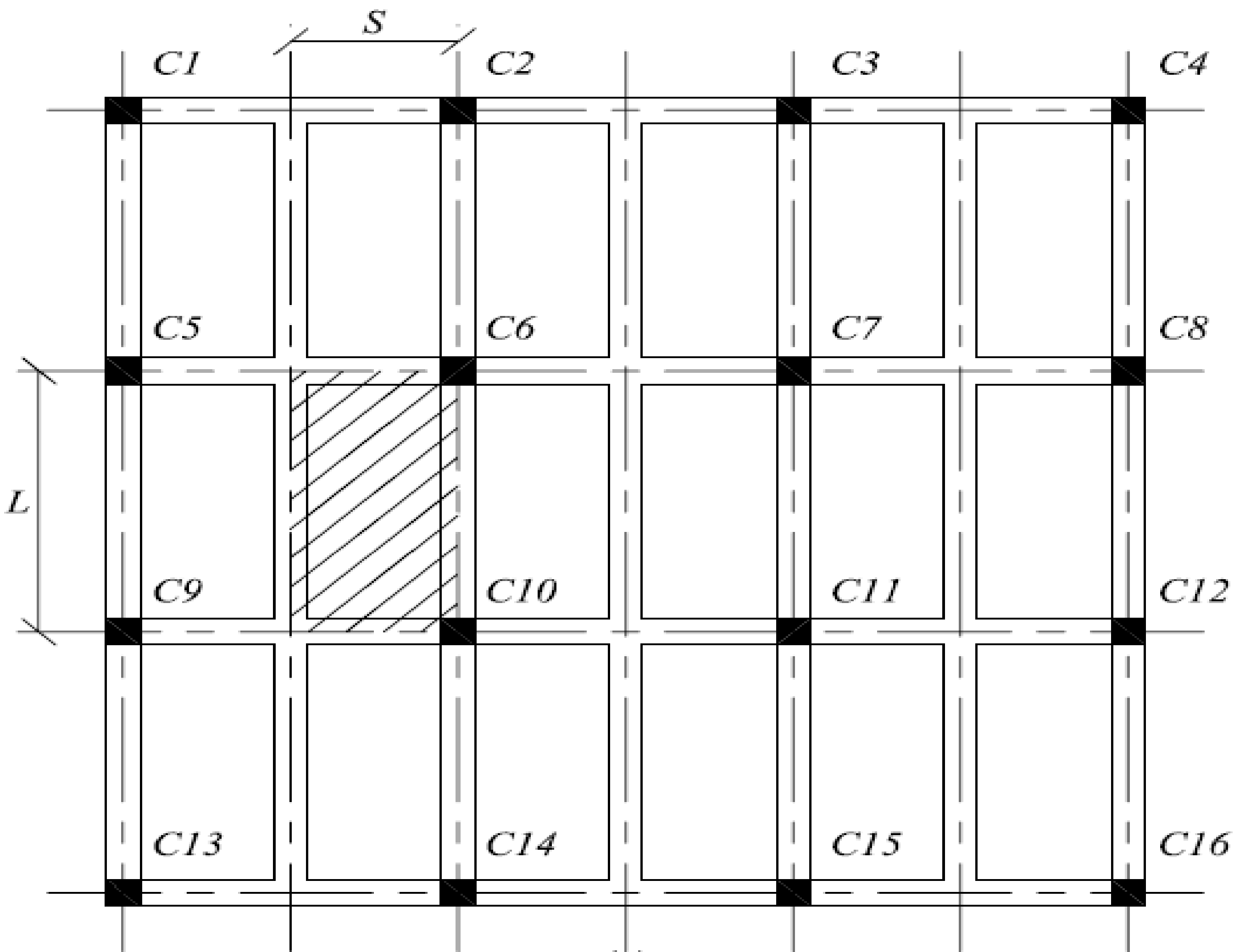
Introduction

A slab is structural element whose thickness is small compared to its own length and width. Slabs are usually used in floor and roof coverings.

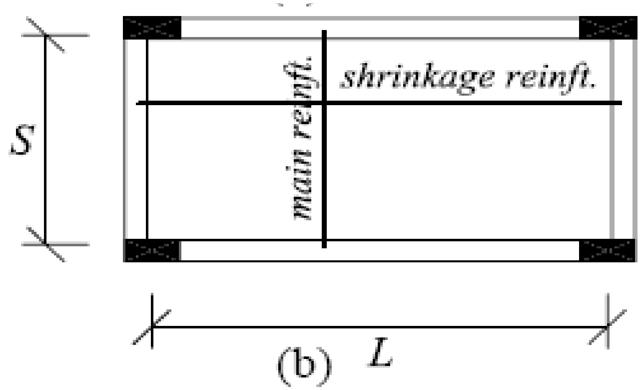
Length of the longer span(l

One-way slabs:

When the ratio of the longer to the shorter side (*L/ S*) of the slab is at least equal to 2.0, it is called one-way slab. Under the action of loads, it is deflected in the short direction only, in a cylindrical form. Therefore, main reinforcement is placed in the shorter direction, while the longer direction is provided with shrinkage reinforcement to limit cracking. When the slab is supported on two sides only, the load will be transferred to these sides regardless of its longer span to shorter span ratio, and it will be classified as one-way slab.



(a)



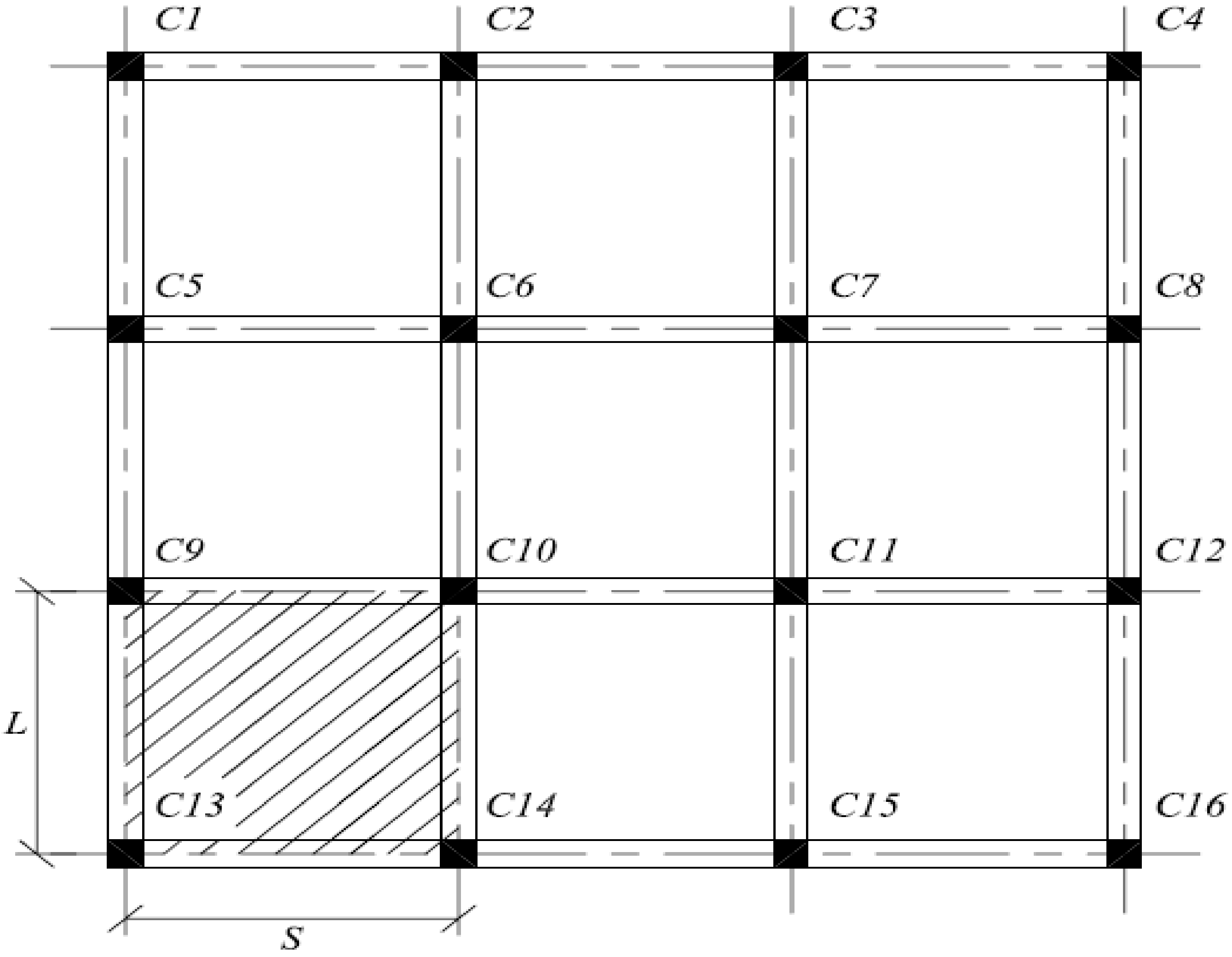
One way slab; (a) classification; (b) reinforcement

Two-way Slabs:

When the ratio (*L/ S*) is less than 2.0, it is called two-way slab. Bending will take place in the two directions in a dish-like form.

Accordingly, main reinforcement is required in the two directions.

Two way slabs



One-way Slabs

In this section, two types will be discussed, one-way solid slabs and one-way ribbed slabs.

One-way Solid Slabs

Minimum Thickness

To control deflection, *ACI Code 9.5.2.1* specifies minimum thickness values for one-way solid slabs, shown in Table.

Minimum thickness of one-way solid slabs

Element	Simply supported	One end continuous	Both ends continuous	Cantilever
One-way solid slabs	$l/20$	$l/24$	$l/28$	$l/10$

where l is the span length in the direction of bending.

Minimum Concrete Cover

According to *ACI Code 7.7.1*, the following minimum concrete cover is to be provided:

a. Concrete not exposed to weather or in contact with ground:

- ϕ 36 mm and larger bar -- 4 *cm*
- ϕ 36 mm and smaller bars ------------------------------------- 2 *cm*

b. Concrete exposed to weather or in contact with ground:

- ϕ 16 mm and larger bars --------------------------------------- 5 *cm*
- ϕ 16 mm and smaller bars ------------------------------------- 4 *cm*

c. Concrete cast against and permanently exposed to earth ----------- 7.5 *cm*

Design Concept

One-way solid slabs are designed as a number of independent 1 *m* wide strips which span in the short direction and supported on crossing beams.

Maximum Reinforcement Ratio

One-way solid slabs are designed as rectangular sections subjected to shear and moment. Thus, the maximum reinforcement ratio ρ_{max} is not to exceed

$$0.75\,\rho_b \text{ and } A_{s\,\max} = 0.75 A_{sb}$$

Shrinkage Reinforcement Ratio

According to *ACI Code 7.12.2.1* and for steels yielding at $f_y = 4200 kg/cm^2$, the shrinkage reinforcement is taken not less than 0.0018 of the gross concrete area, or $A_{s\,shrinkage} = 0.0018\, b\, h$

where, *b* = width of strip, and *h* = slab thickness.

Minimum Reinforcement Ratio

According to *ACI Code 10.5.4*, the minimum flexural reinforcement is not to be less than the shrinkage reinforcement, or

$$A_{s\,min} = 0.0018\, b\, h$$

Spacing Of Flexural Reinforcement Bars

Flexural reinforcement is to be spaced not farther than three times the slab thickness, nor farther apart than 45 *cm*, center-to-center.

Spacing Of Shrinkage Reinforcement Bars

Shrinkage reinforcement is to be spaced not farther than five times the slab thickness, nor farther apart than 45 *cm*, center-to-center.

Loads Assigned to Slabs

(1) Own weight of slab:

(2) Weight of slab covering materials:

- Sand fill with a thickness of about 5 *cm*,

$$0.05 \times 1.80 \ t/m^2$$

-Cement mortar, 2.5 *cm* thick.

$$0.025 \times 2.10 \ t/m^2$$

- Tiling

$$0.025 \times 2.30 \ t/m^2$$

-A layer of plaster about 2 *cm* in thickness.

$$0.02 \times 2.10 \ t/m^2$$

(3) Live Load:

Table shows typical values used by the *Uniform Building Code* (*UBC*).

Minimum live Load values on slabs

Type of Use	Uniform Live Load kg/m^2
Residential	200
Residential balconies	300
Computer use	500
Offices	250
Warehouses	
▪ Light storage	600
▪ Heavy Storage	1200
Schools	
▪ Classrooms	200
Libraries	
▪ Reading rooms	300
▪ Stack rooms	600
Hospitals	200
Assembly Halls	
▪ Fixed seating	250
▪ Movable seating	500
Garages (cars)	250
Stores	
▪ Retail	400
▪ wholesale	500
Exit facilities	500
Manufacturing	
▪ Light	400
▪ Heavy	600

(4) Equivalent Partition Weight:

This load is usually taken as the weight of all walls carried by the slab divided by the floor area and treated as a dead load rather than a live load.

Loads Assigned to Beams

The beams are usually designed to carry the following loads:
- Their own weights.
- Weights of partitions applied directly on them.
- Floor loads.

The floor loads on beams supporting the slab in the shorter direction may be assumed uniformly distributed throughout their spans.

ACI Code 8.3.3 permits the use of the following approximate moments and shears for design of continuous beams and one-way slabs, provided:

1. Positive Moment:

a. End Spans:

When discontinuous end unrestrained, $M_u = w_u l_n^2/11$

When discontinuous end is integral with support,

where l_n is the corresponding clear span length $M_u = w_u l_n^2/14$

b. Interior Spans: $M_u = w_u l_n^2/16$

2. Negative Moment:

a. Negative moment at exterior face of first interior support:

- Two spans, $M_u = w_u l_n^2 / 9$

More than two spans,

where l_n is the average of adjacent clear span lengths.

b. Negative moment at other faces of interior supports:

$$M_u = w_u l_n^2/10$$

c. Negative moment at interior face of exterior support:

Support is edge beam, $M_u = w_u l_n^2/11$

Support is a column, $M_u = w_u l_n^2/16$ $M_u = w_u l_n^2/24$

3. Shear:

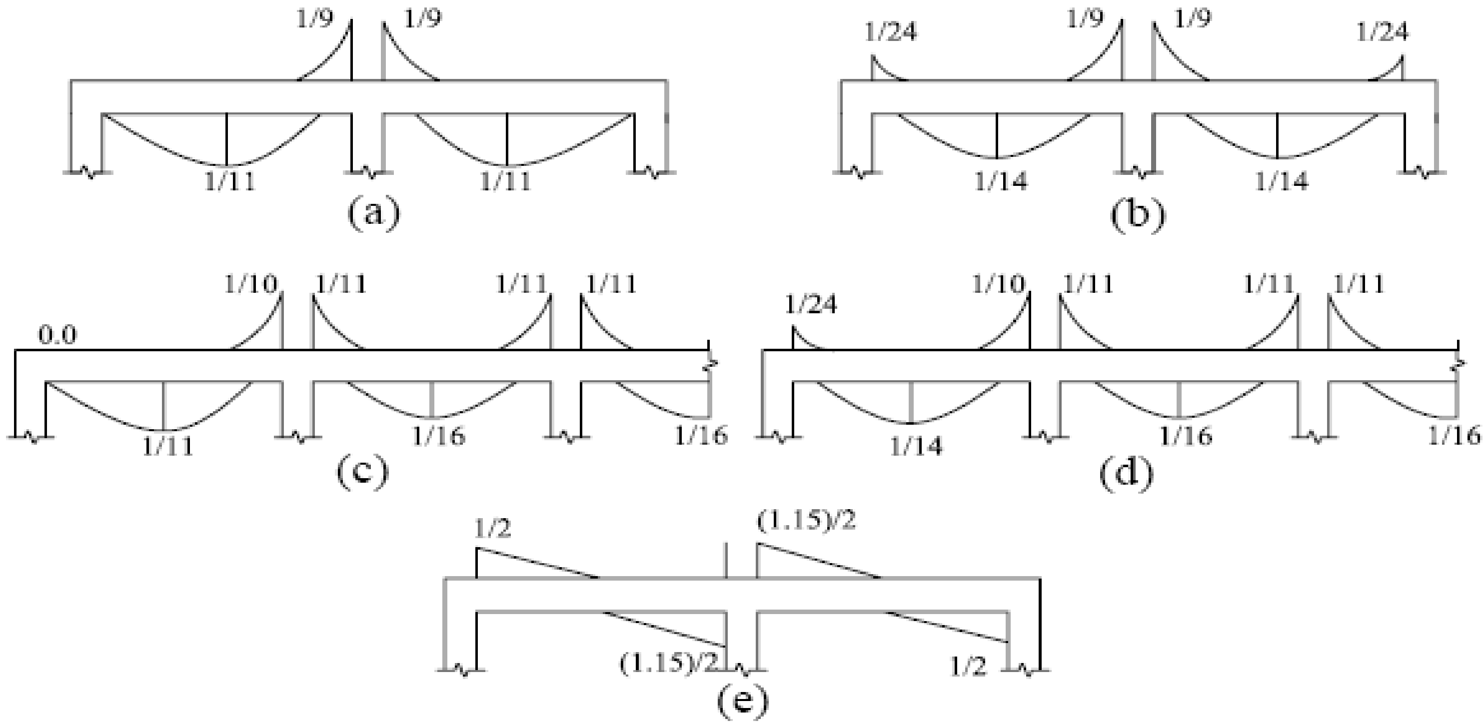
a. Shear in end members at face of first interior support:

$$V_u = 1,15 w_u l_n / 2$$

b. Shear at face of all other supports:

$$V_u = w_u l_n / 2$$

where l_n is the corresponding clear span length.


1/9
1/9
1/11
1/11
(a)
1/24
1/9
1/9
1/24
1/14
1/14
(b)
1/10
1/11
1/11
1/11
0.0
1/11
1/16
1/16
(c)
1/24
1/10
1/11
1/11
1/11
1/14
1/16
1/16
(d)
1/2
(1.15)/2
(1.15)/2
1/2
(e)

Summary of One-way Solid Slab Design Procedure

Once design compressive strength of concrete and yield stress of
reinforcement are specified, the next steps are followed:

1. Select representative 1 *m* wide design strip/strips to span in the short direction.
2. Choose a slab thickness to satisfy deflection control requirements.

When several numbers of slab panels exist, select the largest calculated thickness.

$$w_u = 1.40 w_d + 1.70 w_l$$

3. Calculate the factored load W_u by magnifying service dead and live loads according to this equation

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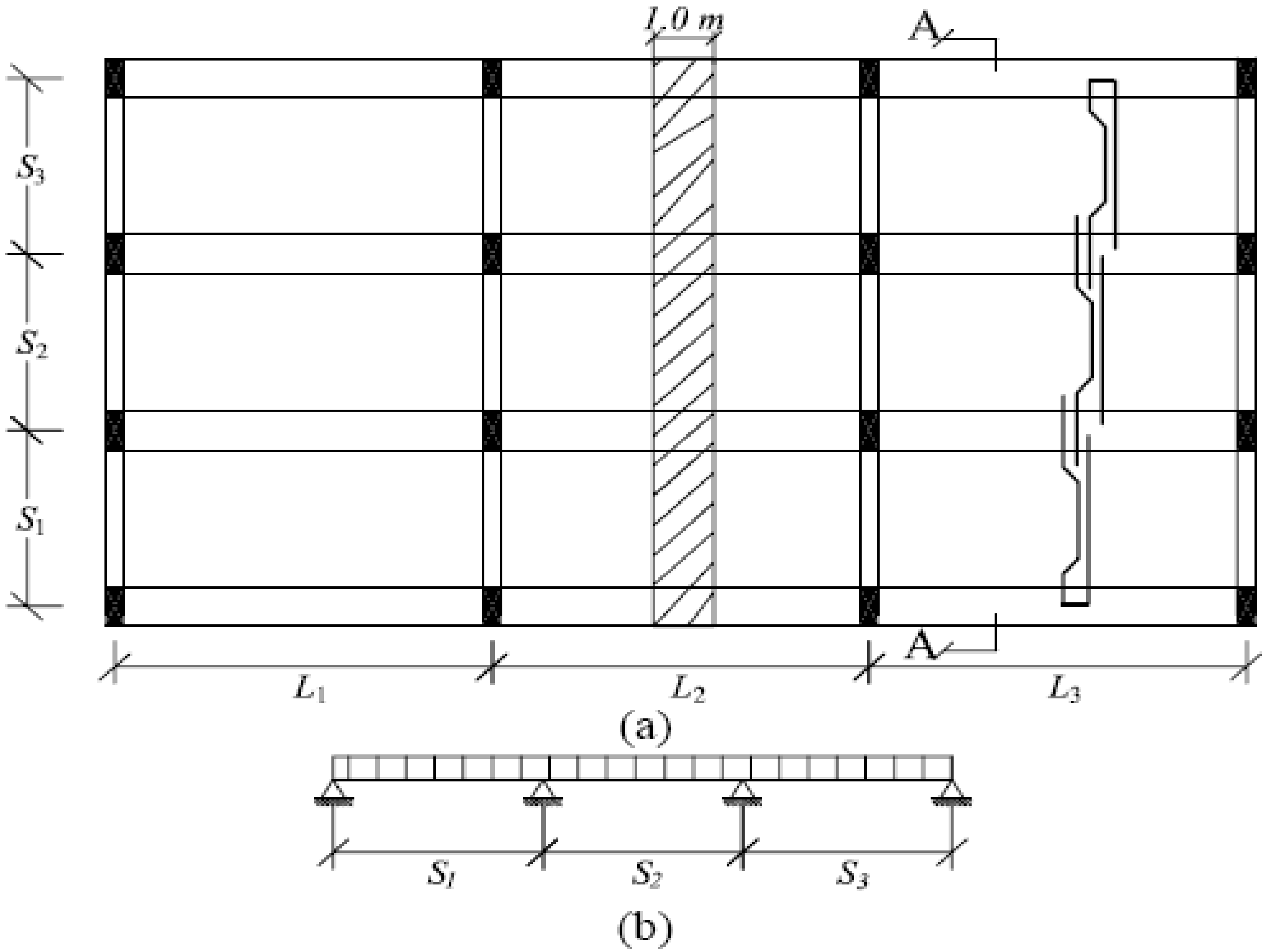


…strip and loads

4. Draw the shear force and bending moment diagrams for each of the strips.
5. Check adequacy of slab thickness in terms of resisting shear by satisfying the following equation:

$$V_u \leq 0.53\Phi\sqrt{f_c^{'}}bd$$

where

V_u = factored shear force

V_c = shear force resisted by concrete alone

Φ = strength reduction factor for shear is equal to 0.85.

b = width of strip = 100 cm

d = effective depth of slab

If the previous equation is not satisfied, go ahead and enlarge the thickness to do so.

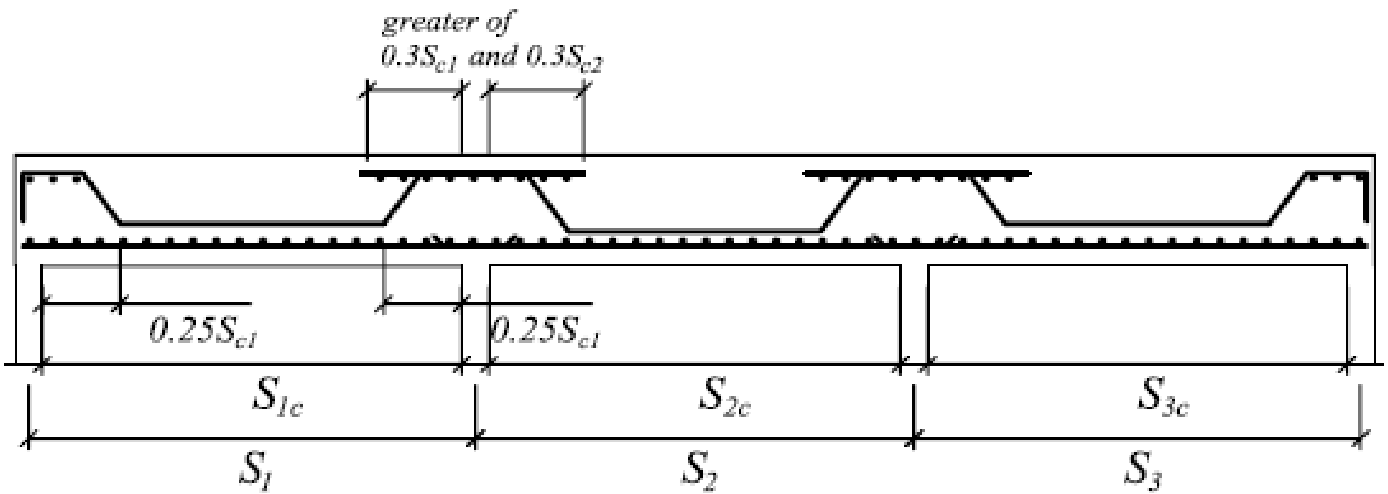
6. Design flexural and shrinkage reinforcement:
Flexural reinforcement ratio is calculated from the following equation:

$$\rho = \frac{0.85\, f_c'}{f_y}\left[1 - \sqrt{1 - \frac{2.61 \times 10^5\, M_u}{b\, d^2\, f_c'}}\right]$$

Make sure that the reinforcement ratio is not larger than ¾ ρ_b
Compute the area of shrinkage reinforcement, where
Select appropriate bar numbers and diameters for both, main and secondary reinforcement.

$$A_{s\,min} = 0.0018\, b\, h$$

Check reinforcement spacing, modify your bar selection if needed.
7. Draw a plan of the slab and representative cross sections showing the dimensions and the selected reinforcement.



greater of
0.3Sc1 and 0.3Sc2
0.25Sc1
0.25Sc1
S1c
S2c
S3c
S1
S2
S3

Design of Two Way Slabs

Moments in Two way Restrained Slabs with Corners Held Down

Analysis by Coefficients

- Restrained slabs are defined in codes as those slabs which are cast integral with RC frames and which are not free to lift up at the corners.
- These slabs may be continuous or discontinuous at the edges. Those which are discontinuous at edges are also referred to be simply supported.
- Coefficients specified in IS:456-2000, Table 26 in Annexure D can be used for analysis of such slabs.

Moments in Two way Restrained Slabs with Corners Held Down

Analysis by Coefficients

- Conditions to be satisfied for use of these coefficients are:
  1. Loading on adjacent spans should be the same
  2. Span in each direction should be approximately equal.

The span moment per unit width and the negative moments at continuous edges for the slabs are calculated from the equations in terms of "l_x",

$$M_x = \alpha_x w l_x^2$$ for short span

$$M_y = \alpha_y w l_x^2$$ for longer span

Design of Two-way Slabs

- **Restrained two way slab is divided into middle strip and edge strips. Middle strip is forming three-fourth of slab width in width directions.**
- **Torsion steel must be provided at discontinuous edges as specified in code.**
- **Coefficients are given in Table 26 of IS: 456-2000. These coefficients are derived from the yield line theory of slabs.**

Design of Two-way Slabs

- Table given for coefficients is applicable for slabs carrying uniformly distributed loads not for concentrated loads, for which analysis should be done, separately.
- Span moments/edge moments per unit width are calculated by determining ratio of "l_y" and "l_x" and for different edge conditions.c

Coefficients for Moments

- Slabs restrained against corners lifted up

$$\propto_y^+ = \frac{\left[24 + 2n_d + 1.5n_d^{\,2}\right]}{1000}$$

- n_d = Number of discontinuous edges = 0,1,2,3 and 4.

$$\propto_x^+ = \frac{2}{9}\left[\frac{3 - \sqrt{18 \propto_y^+ (C_{s1} + C_{s2})}}{r} \frac{1}{(C_{l1} + C_{l2})^2}\right]$$

- C = 1.0 for a discontinuous edge

  = $\sqrt{\frac{7}{3}}$ for continuous edge
- Subscripts “s” and “l” denote “short edge” and “long edge”
- Subscripts “1” and “2” represent two edges in short and longer direction

Important Design Issues from Table 26

1. Edge moments of continuous supports are 1.33 times the span moments.
2. Long span moment coefficient"α_y" is a constant for given end conditions of slab, irrespective of the span ratios.
3. Short span coefficient varies sharply with variation of the ratio of spans

Important Design Issues from Table 26

- While using Table 26 for a series of slabs, moments calculated at an interior support will sometimes be different on two sides of that support because of the differences in continuity condition of slabs on opposite sides of support.

Arrangement of Reinforcements

- While using design of two-way slabs with the help of coefficients, restrained slabs are considered to be divided into middle and edge strips.
- Moments given in Table 26 apply only to middle strips, and no further redistribution is allowed for these moments.
- Edge strips have to be reinforced only with nominal minimum steel for crack control.

Arrangement of Reinforcements

- Middle strip should have steel (+ve and –ve) calculated for various sections. In edge strips , steel is placed as positive steel at the bottom of slab.
- Negative moments may be experienced at discontinuous edges since, in practice, they are not supported on rollers but partially restrained at their ends.
- The magnitude of this moment depends on the degree of fixity at the edge of the slab and is intermediate.

Arrangement of Reinforcements

- Usual practice is to provide at these edges top reinforcement for negative moment equal to

$$\frac{wl^2}{24}$$

- As per IS:456.2000, 50 percent of the steel provided at mid span should be extended along these edges, and the negative steel has to extend into the span 0.1 times the span length from the face of the beam.

Design of Two-way Slabs

- Slab thickness should be calculated based on the greater value of the negative B.M on the short span.

$$M_u = Kf_{ck}bd^2$$

- Hence
- Total thickness = d (short) + 0.5ø + cover
- Total thickness = d (long) + 0.5ø + ø + cover

- The slab should satisfy span/effective depth ratio to control deflection.
  - Simply supported = 28
  - Continuous = 32
- Depth of slab selected from deflection criterion will be generally greater than the minimum required from strength criterion.
- Short span steel will be placed in the lower layer.

- Restrained moments are obtained for the middle strips only. The reinforcement is distributed uniformly in the middle strips.
- Each direction is to be provided only with the minimum reinforcement placed at the bottom of the slabs.

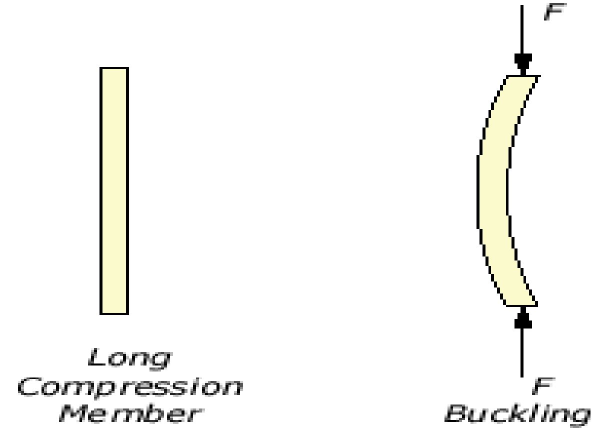
- In addition, corner steel reinforcement should be provided at the discontinuous edges.
- Corner reinforcement consists of two mats, are placed on the top and the other at the bottom of the slab, with each mat having steel in both x and y directions.
- Where the slab is discontinuous on both sides of a corner, fall torsion steel has to be provided.

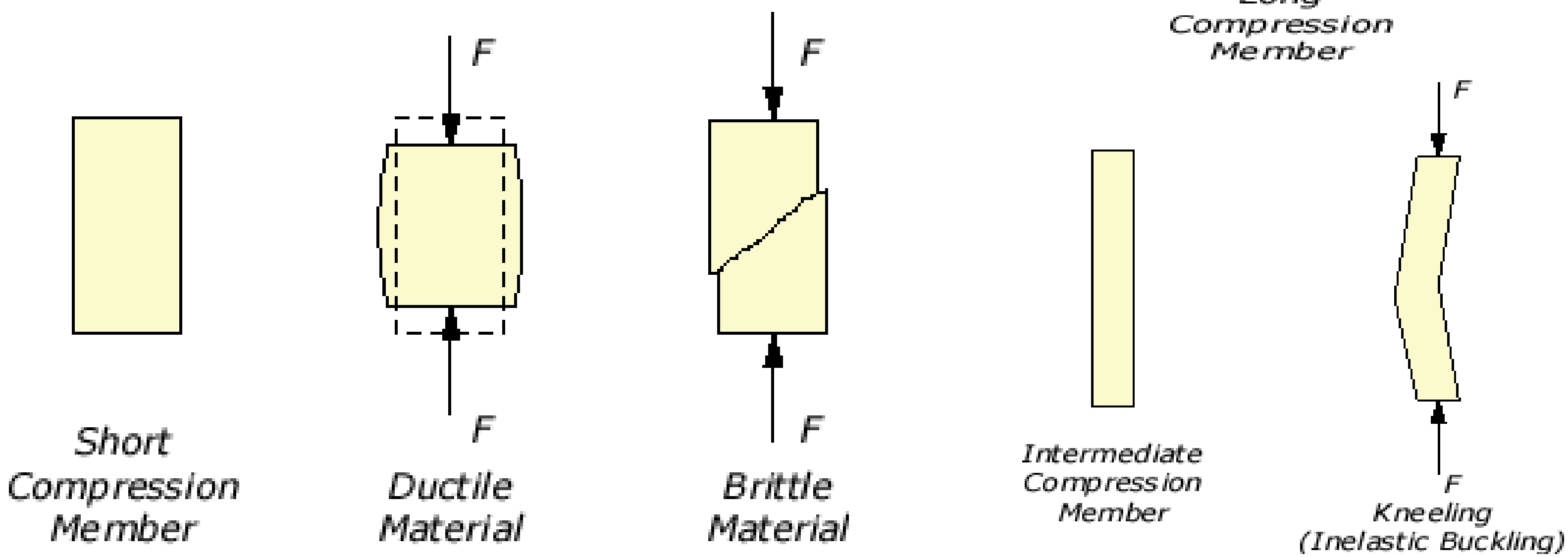
- The area of the full torsion reinforcement per unit width in each of the four layer should be as follows.
- (Area of full corner steel) = [3/4][Area required for the maximum span per unit width in each of four layers]
- These steels are to be provided for a distance of one-fifth the short span .

UNIT-IV
COLUMNS

Compression Members

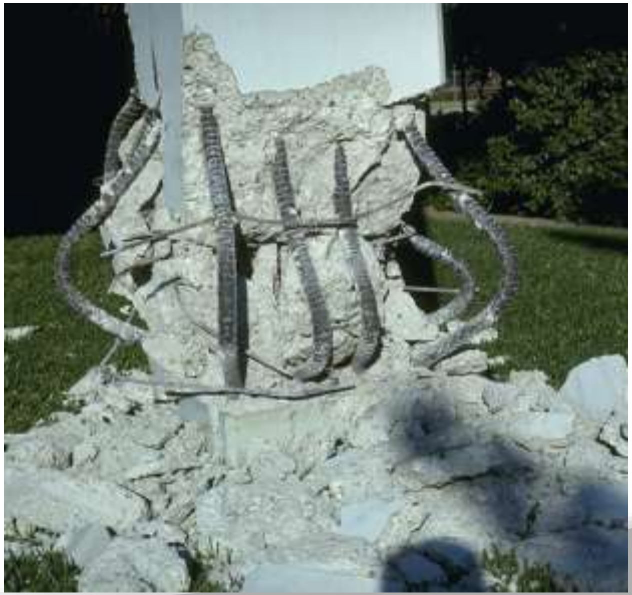
The failure of members in compression are due either to the load exceeding the ultimate strength in compression (crushing) or due to buckling under the load, because the applied load is larger than the critical buckling load.





Long members are referred to as columns. Columns, normally fail in buckling.

Compression Members

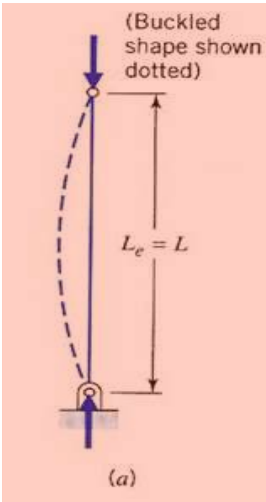


Crushing failure – 1985 Mexico earthquake.

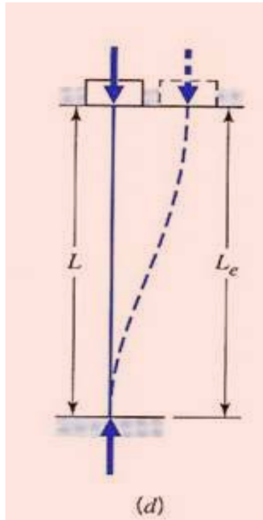
Buckling failure

Euler Column – End Conditions and Effective Length

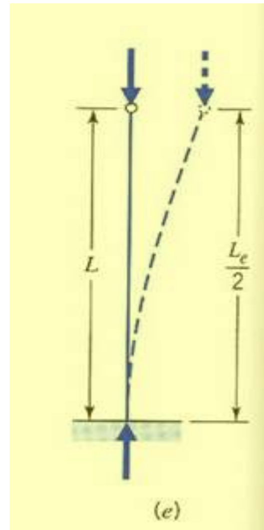
Euler's end condition



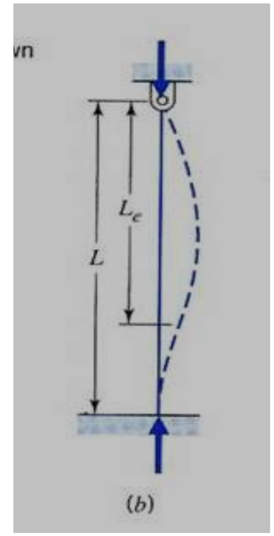
Pinned-Pinned, or rounded-rounded



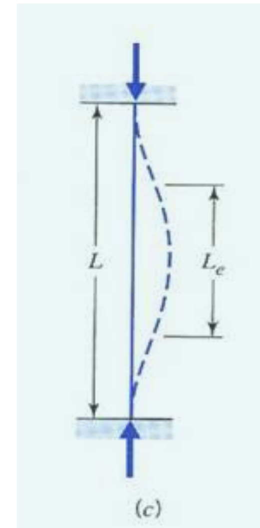
Fixed-sliding



Fixed-Free



Fixed-pinned



Fixed-Fixed

American **I**nstitute of **S**teel **C**onstruction

End Conditions	Theoretical Value	AISC* Recommended
Rounded-Rounded	$l_{eff} = l$	$l_{eff} = l$
Pinned-Pinned	$l_{eff} = l$	$l_{eff} = l$
Fixed-Free	$l_{eff} = 2l$	$l_{eff} = 2.1l$
Fixed-Pinned	$l_{eff} = 0.707l$	$l_{eff} = 0.80l$
Fixed-Fixed	$l_{eff} = 0.5l$	$l_{eff} = 0.65l$

$$P_{cr} = \frac{\pi^2 E I}{(l_{eff})^2}$$

Biaxial Bending

Biaxial bending in columns

As the position of live load on a floor varies, building columns may be subject to loading patterns that produce biaxial bending, i.e., bending about both principal axes of the cross section. Nevertheless, to simplify design, code provisions specify loading patterns that produce uniaxial bending in most building columns. Corner columns, routinely designed for biaxial bending, are the exception.

If the moments in the weak direction (y axis here) are rather small compared to bending in the strong direction (x axis), it is rather common to neglect the smaller moment. This practice is probably reasonable as long as e_y is less than about 20% of e_x,

Design Procedure for biaxial bending:

A- Determine reinforcement based on the biaxial bending capacity:

1- Determine the dimensions based on a reasonable stress in the column.
2- Determine *g* in the week axis direction (any direction for square sections.
3- Calculate the biaxial bending moment in the week axis direction.
4- Use an interaction diagram to design the reinforcement for the section.

B- Use the Bresler equation to check the axial capacity of the section

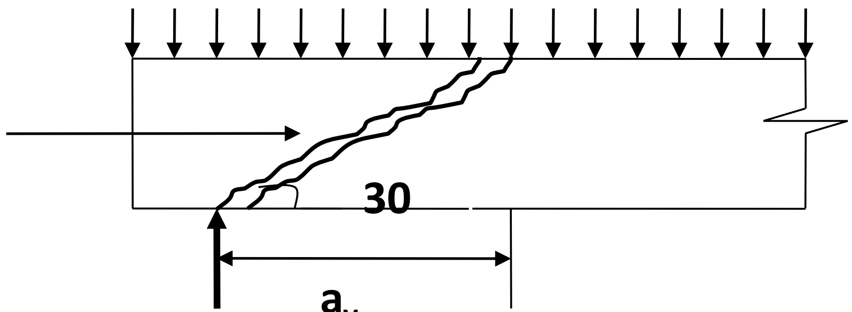
1- Calculate P_{nx} from the interaction diagram assuming only M_{nx} is applied.
2- Calculate P_{ny} from the interaction diagram assuming only M_{ny} is applied.
3- Calculate P_0.
4- Calculate P_n and check

$$\phi P_n \geq P_u$$

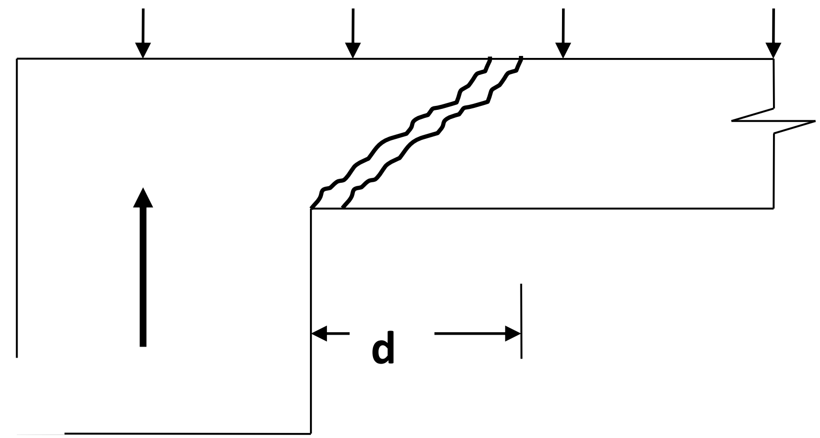
Enhanced Shear Near Supports

- Section near the supports can be assumed to have enhanced shear strength.
- Shear failure at sections of beams and cantilevers without shear reinforcement normally takes place on a plane making an angle 30^0 with the horizontal.
- Enhance shear capacity at sections near supports as would be done in design of brackets, ribs, corbels etc.

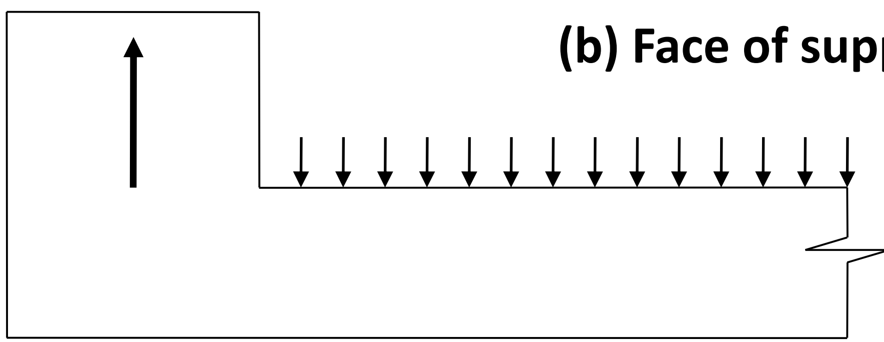
Critical Sections for shear in beams which are supported on members in compression, and tension



(a) Beams with compression at end region



(b) Face of support in compression

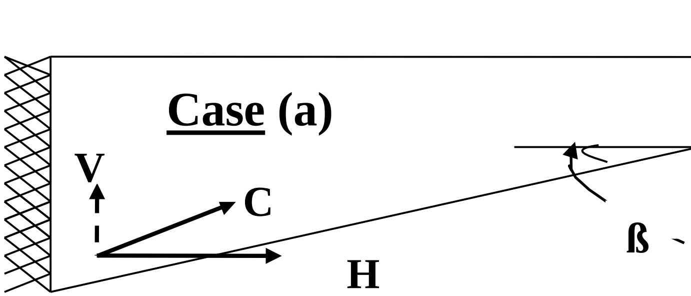


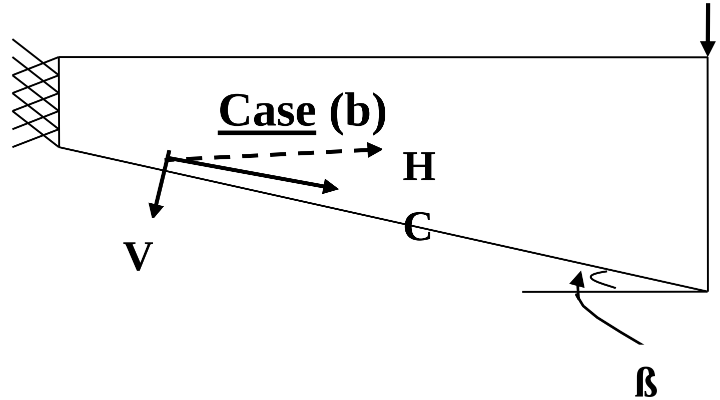
(c) Face of support in tension

Beams of Varying Depth

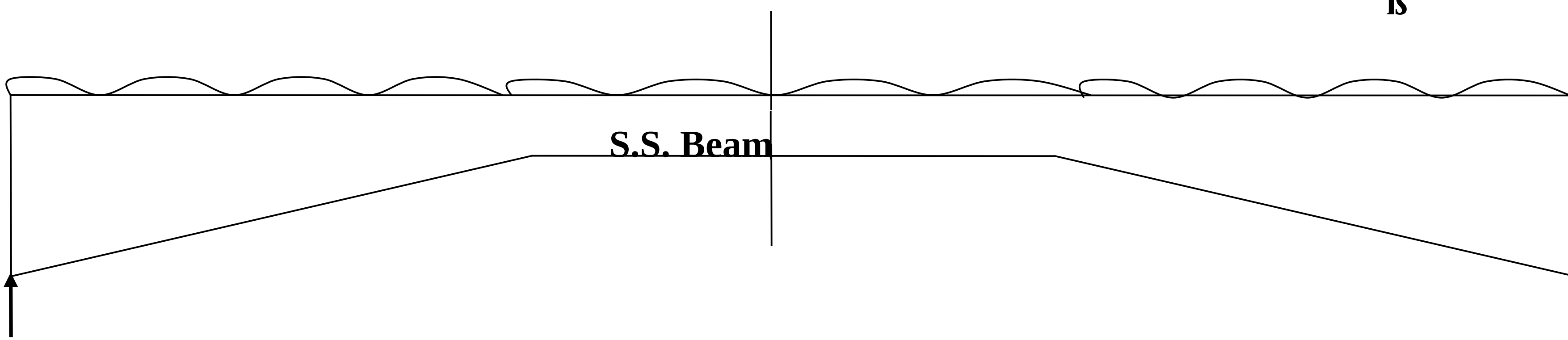
- Beams with varying depth are encountered in RC.
- Beam depth is varied according to the variation of bending moment and shear force.

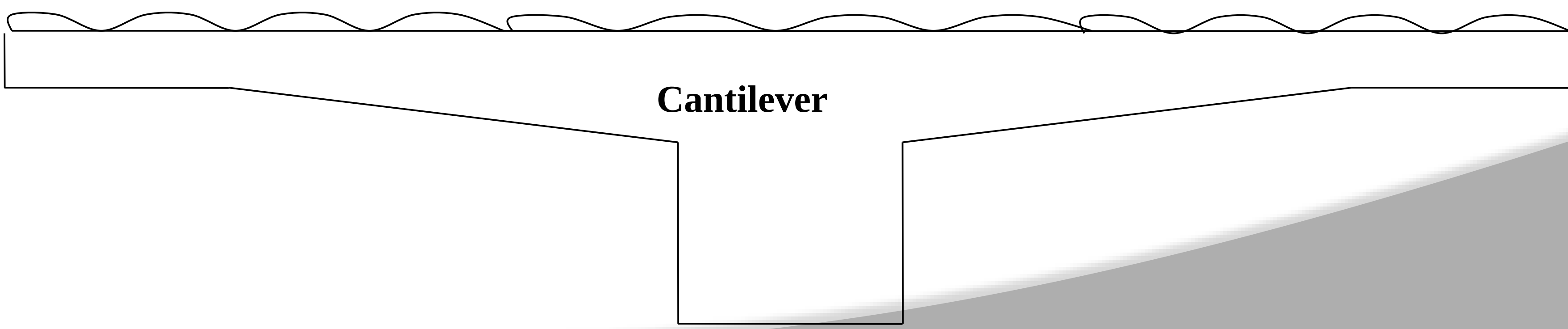
1. Case (a): Bending moment increases numerically in the direction in which effective depth increases.
2. Case (b). Bending moment decreases numerically in the direction in which effective depth increases.





M = H Jd





Effective shear force for determining the shear stress

$$V = V_w - \frac{M}{d} \tan\beta \text{ for case(a)}$$

$$V = V_w + \frac{M}{d} \tan\beta \text{ for case(b)}$$

Design of Stirrups at Steel Cut-off Points

- When flexural reinforcement in beams is terminated in tension region, at that section it should satisfy
  a) shear at cut-off point does not exceed two-thirds of combined strength of concrete and steel. $\tau_s > [1.5\tau - \tau_c]$
  b) Additional stirrups should be provided along each terminated direction over a distance from the cut-off point equal to three-fourth effective depth, equal to

$$A'_{su} = \frac{0.4\, b\, s}{f_y}$$

- Spacing of stirrups $< \dfrac{d}{8\,\beta}$ $\quad \beta = \dfrac{Area\ of\ cut-off\ bars}{Total\ area\ of\ bars}$

Deep Beams

- ***Depth much greater than normal, in relation to their span, while thickness is much smaller than either span or depth.***
- ***Main loads and reactions act in plane of member to achieve a state of plane stress in concrete***
- ***Members with span-to-depth ratio of about 5 or less, or with shear span, a, less than about twice depth are called deep beams.***

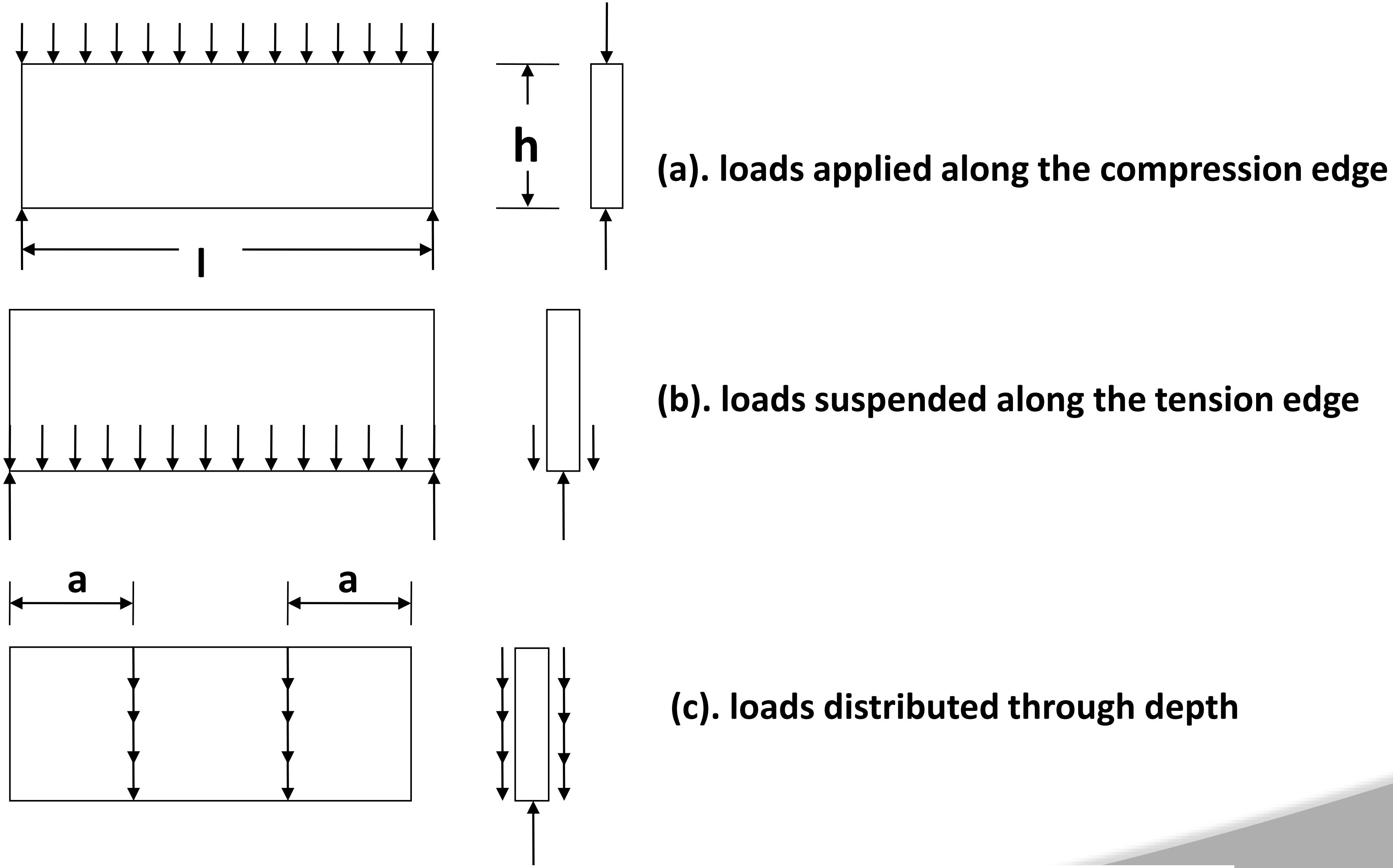
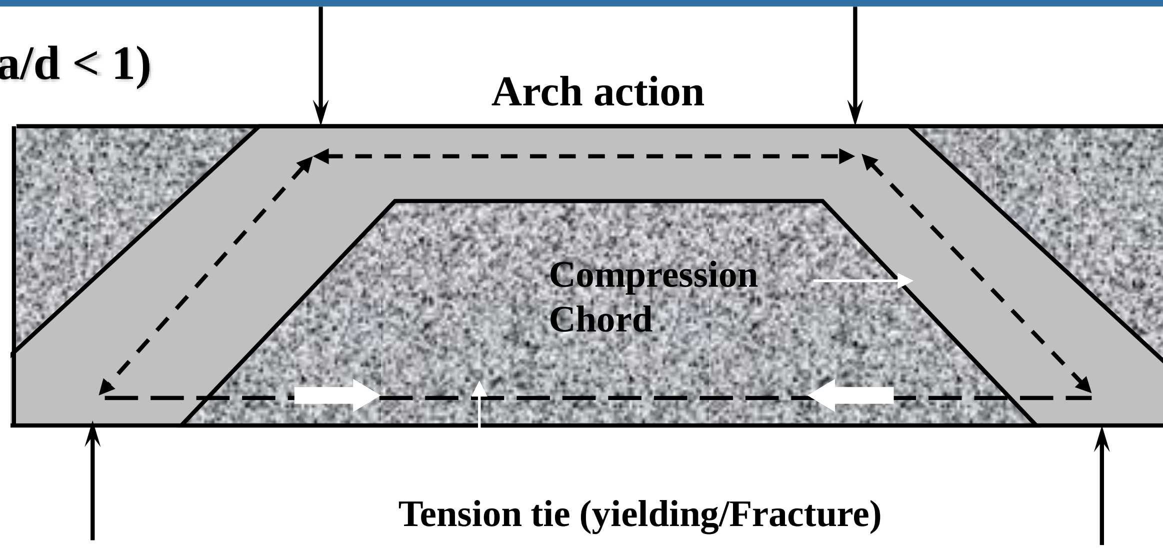


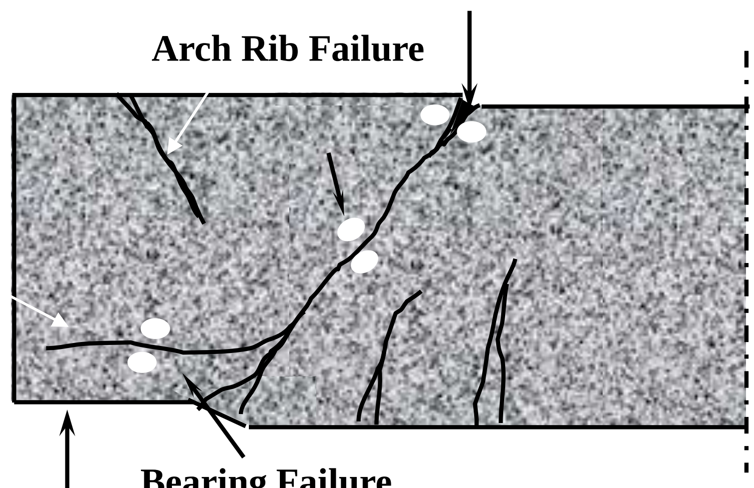
Fig. Placements of loads on deep beams.

- Examples of Deep Beams found in:
  - Column offsets,
  - Walls of rectangular tanks and bins,
  - Floor diaphragms
  - Shear walls,
  - in folded plate roof structures

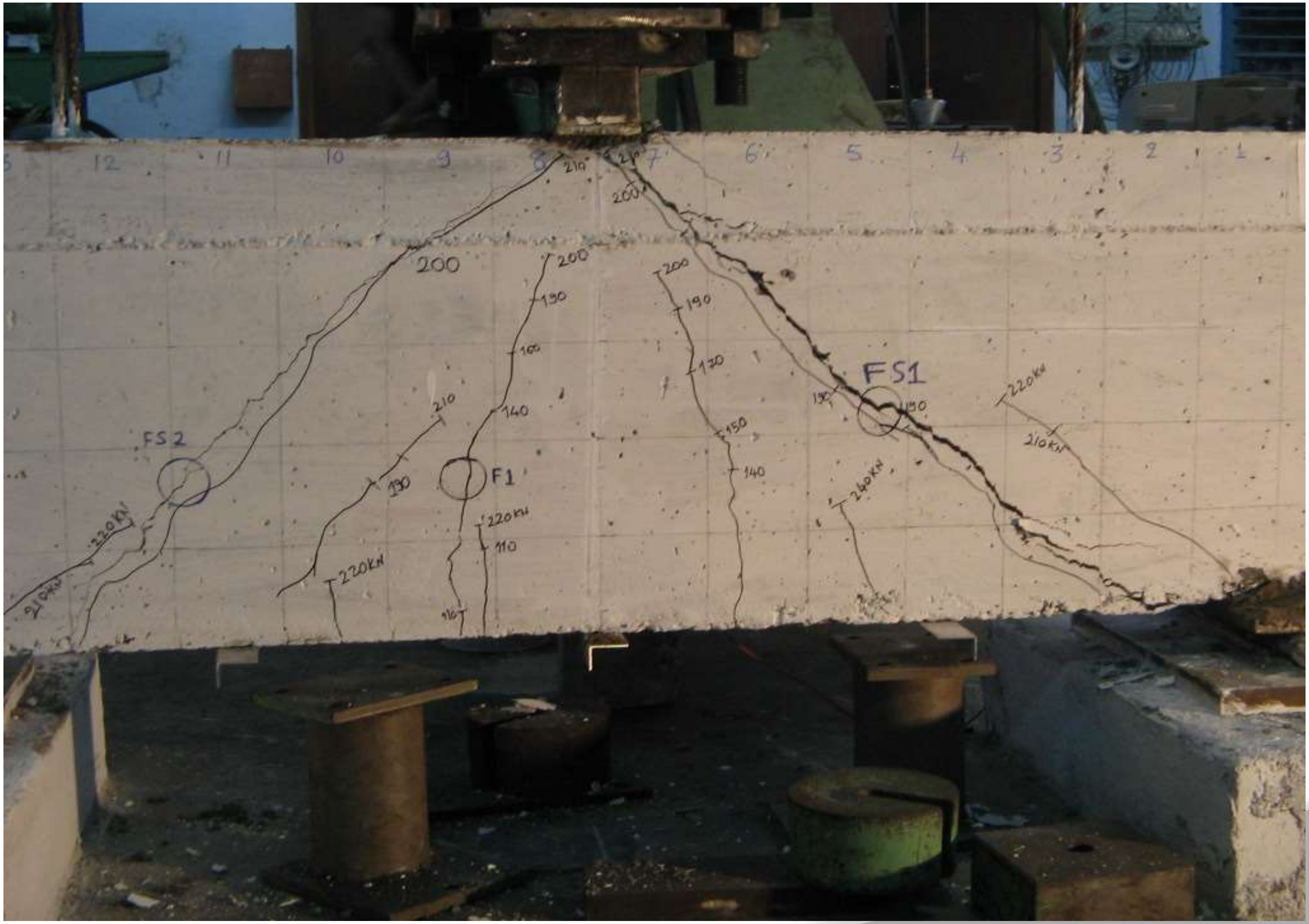
- Behavior of deep beams is significantly different from that of the normal beams, requires special consideration in analysis, design, and detailing of reinforcement.

Deep Beams (a/d < 1)





Failure in Deep Beams




129t
85t
70t
95t
121t
95t
67t
73t
131t
127
77t

UNIT-V

FOOTINGS

Lecture Goals

- Footing Classification
- Footing Design

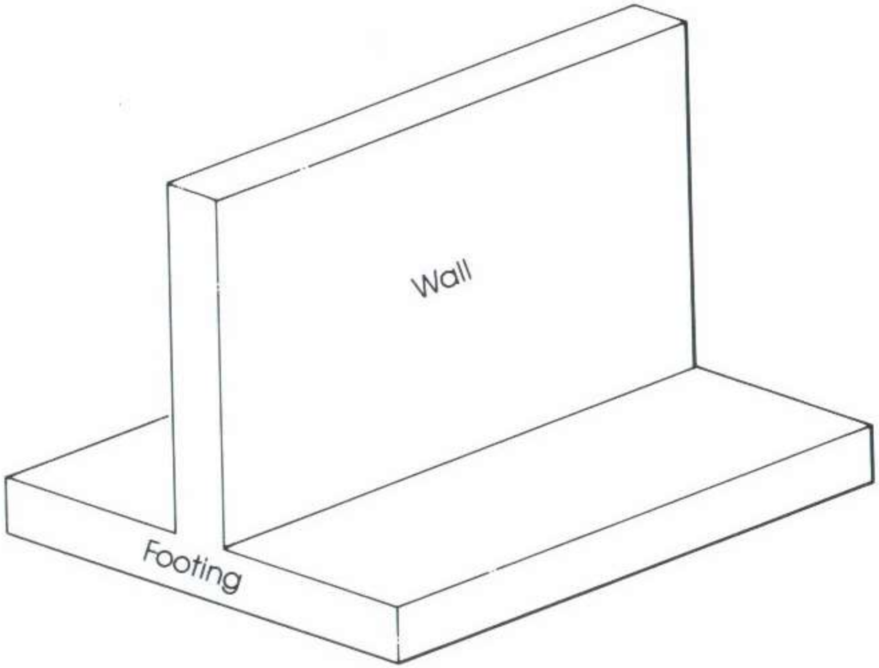
Footing

Definition

Footings are structural members used to support columns and walls and to transmit and distribute their loads to the soil in such a way that the load bearing capacity of the soil is not exceeded, excessive settlement, differential settlement,or rotation are prevented and adequate safety against overturning or sliding is maintained.

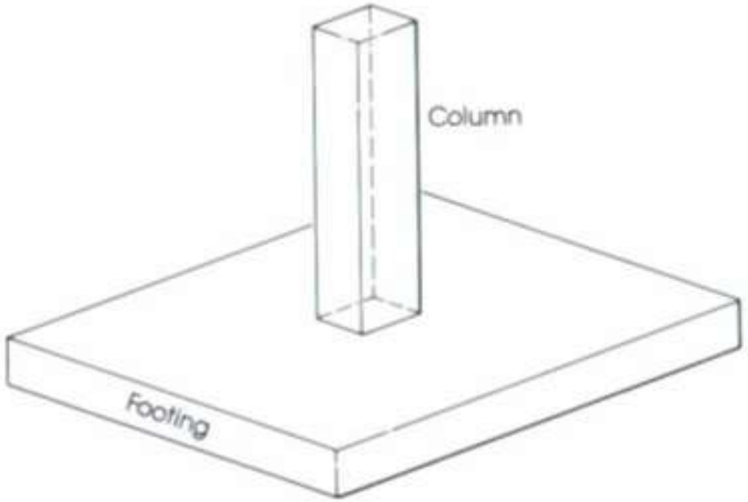
Types of Footing

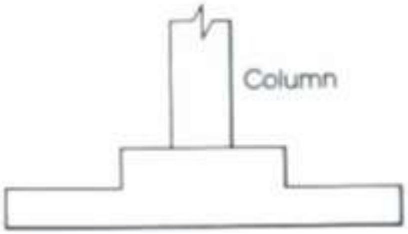
Wall footings are used to support structural walls that carry loads for other floors or to support nonstructural walls.

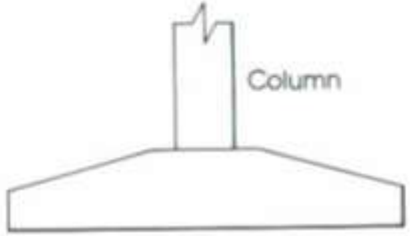


Wall footing.

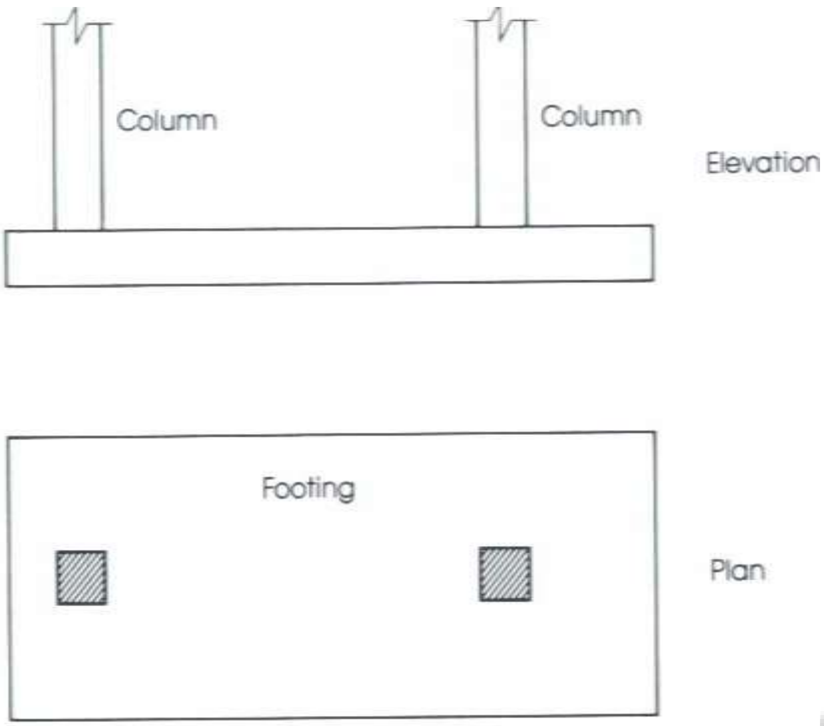
Isolated or single footings are used to support single columns. This is one of the most economical types of footings and is used when columns are spaced at relatively long distances.



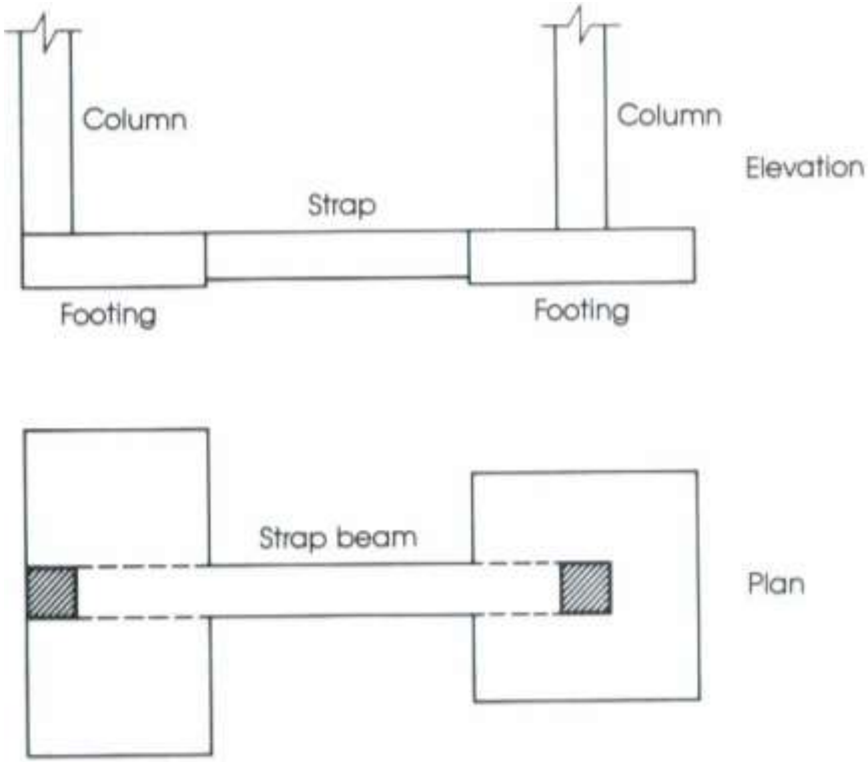




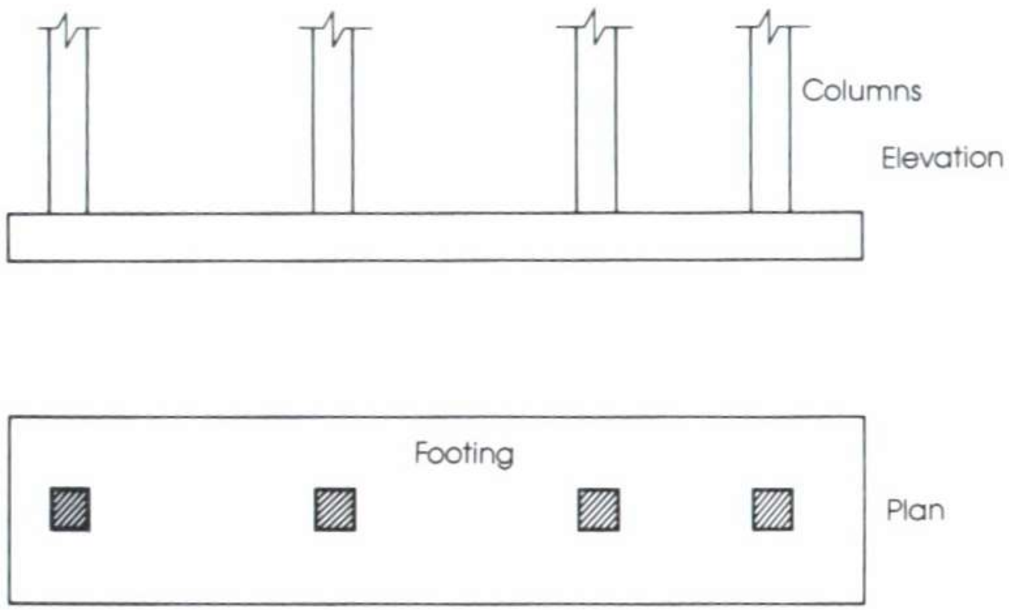
Combined footings usually support two columns, or three columns not in a row. Combined footings are used when two columns are so close that single footings cannot be used or when one column is located at or near a property line.



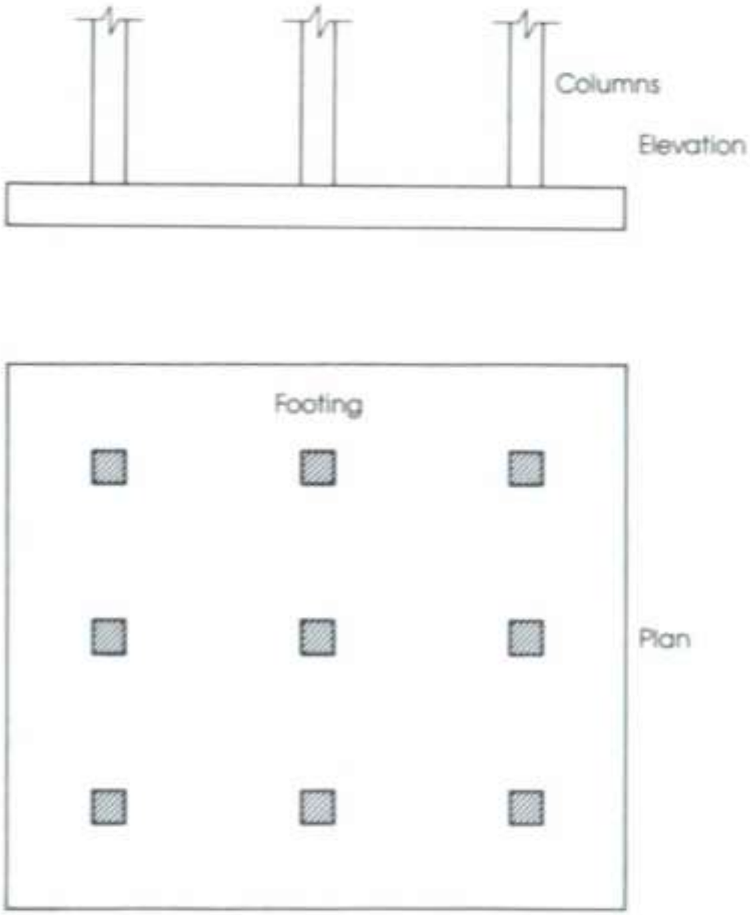
Cantilever or strap footings consist of two single footings connected with a beam or a strap and support two single columns. This type replaces a combined footing and is more economical.



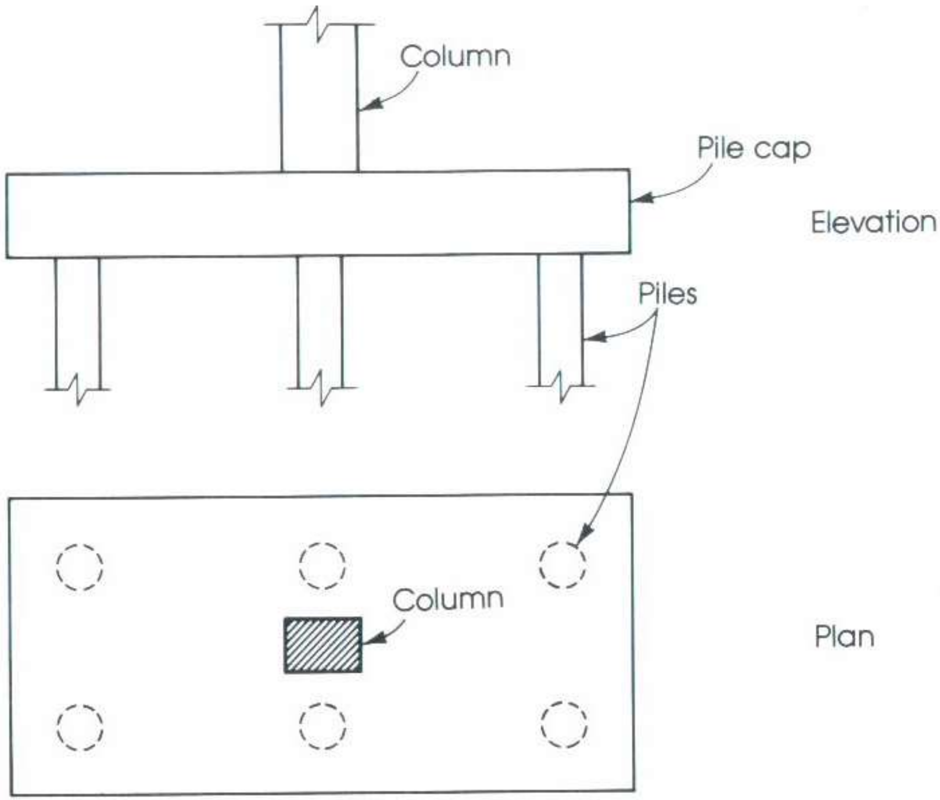
Continuous footings support a row of three or more columns. They have limited width and continue under all columns.



Rafted or mat foundation consists of one footing usually placed under the entire building area. They are used, when soil bearing capacity is low, column loads are heavy single footings cannot be used, piles are not used and differential settlement must be reduced.

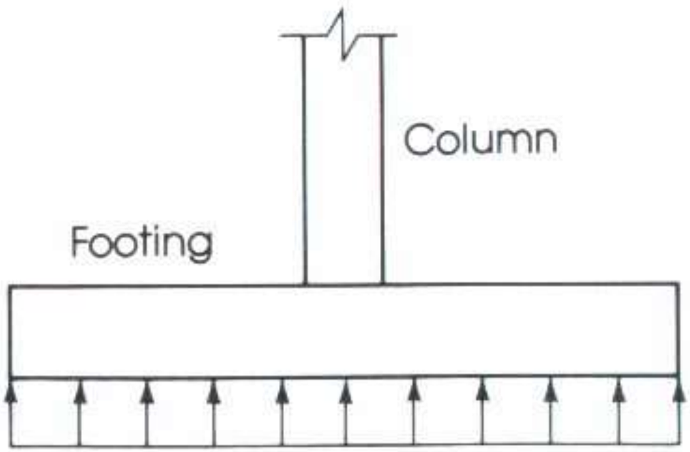


Pile caps are thick slabs used to tie a group of piles together to support and transmit column loads to the piles.



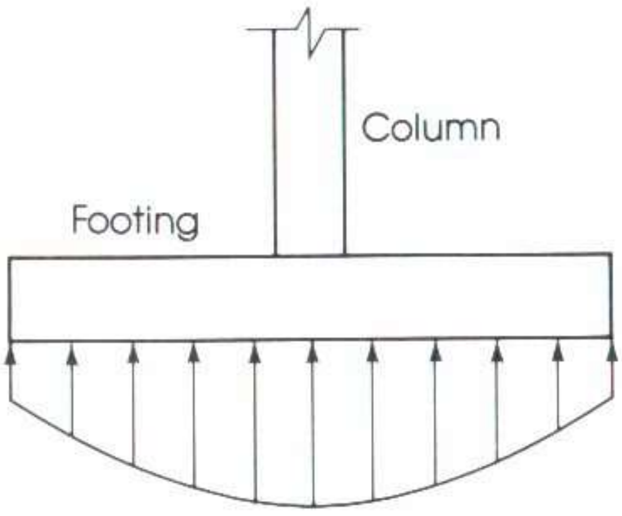
Distribution of Soil Pressure

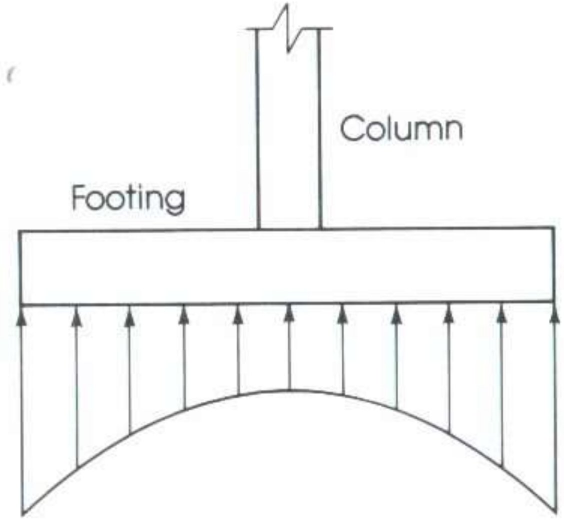
When the column load P is applied on the centroid of the footing, a uniform pressure is assumed to develop on the soil surface below the footing area.



However the actual distribution of the soil is not uniform, but depends on may factors especially the composition of the soil and degree of flexibility of the footing.

Distribution of Soil Pressure





Design Considerations

Footings must be designed to carry the column loads and transmit them to the soil safely while satisfying code limitations.

1. The area of the footing based on the allowable bearing soil capacity
2. Two-way shear or punch out shear.
3. One-way bearing

   Bending moment and steel reinforcement required

Design Considerations

Footings must be designed to carry the column loads and transmit them to the soil safely while satisfying code limitations.

1. Bearing capacity of columns at their base
2. Dowel requirements
3. Development length of bars
4. Differential settlement

Size of Footing

The area of footing can be determined from the actual external loads such that the allowable soil pressure is not exceeded.

$$\text{Area of footing} = \frac{\text{Total load (including self - weight)}}{\text{allowable soil pressure}}$$

$$q_u = \frac{P_u}{\text{area of footing}}$$

Two-Way Shear (Punching Shear)

$$V_c = \left(2 + \frac{4}{\beta_c}\right)\sqrt{f_c}\, b_0 d$$

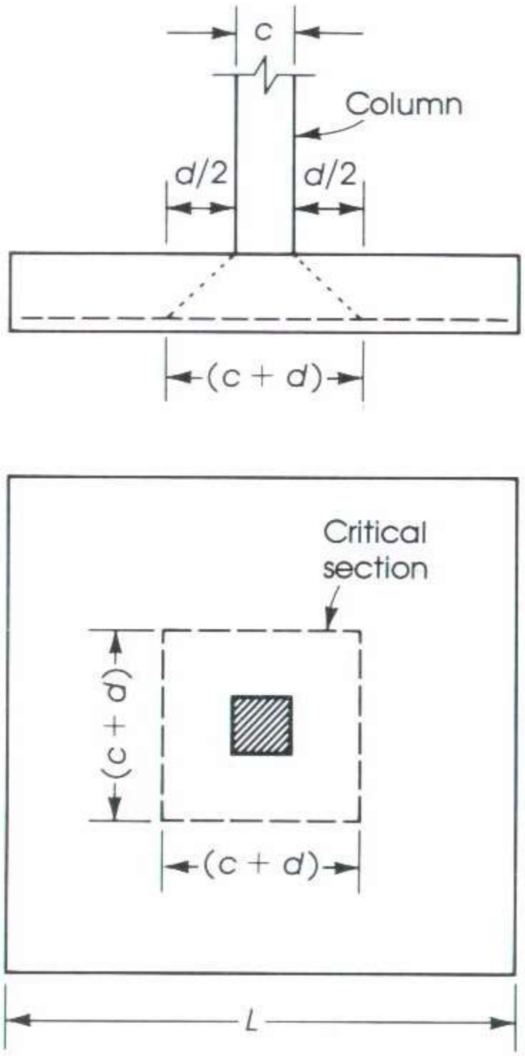
Design of two-way shear

1. Assume d.
2. Determine b_0.

$b_0 = 4(c+d)$ **for square columns where one side = c**

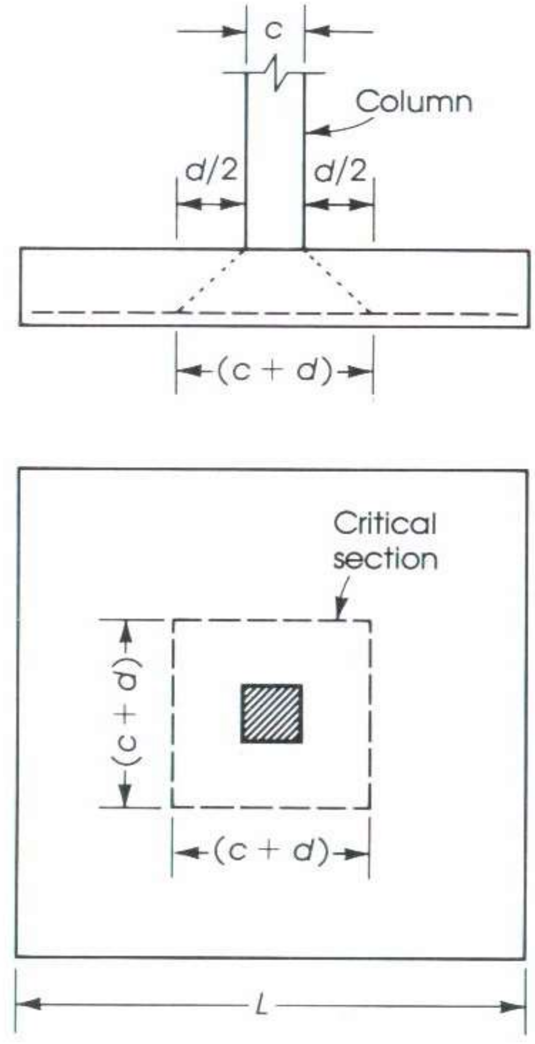
$b_0 = 2(c_1+d) + 2(c_2+d)$

for rectangular columns of sides c_1 and c_2.



Design of two-way shear

3. The shear force V_u acts at a section that has a length $b_0 = 4(c+d)$ or $2(c_1+d) + 2(c_2+d)$ and a depth d; the section is subjected to a vertical downward load P_u and vertical upward pressure q_u.

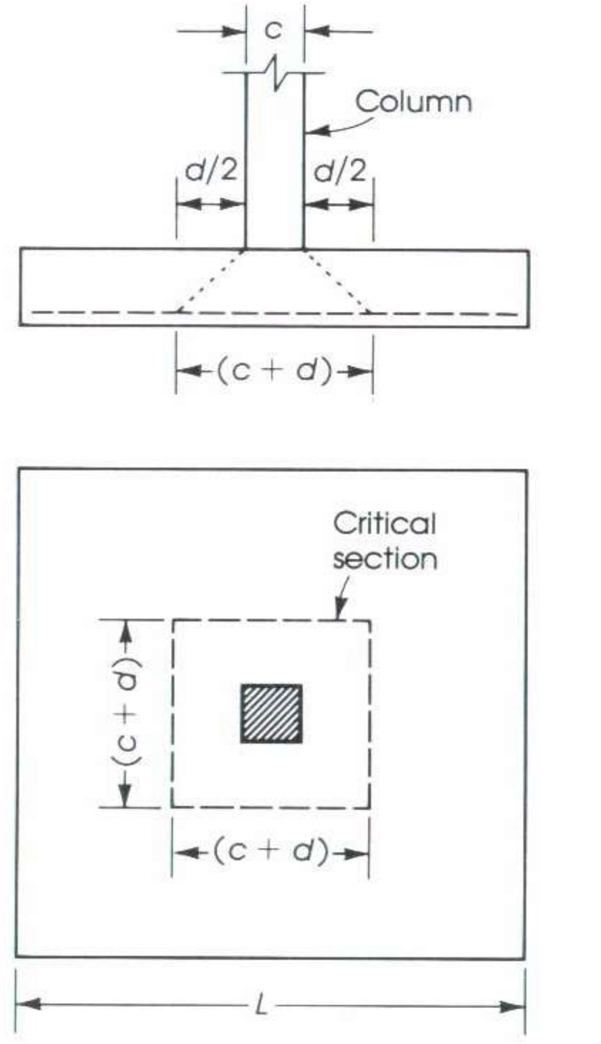


Design of two-way shear

$$\phi V_c = 4\phi\sqrt{f_c}\, b_0 d$$

$$d = \frac{V_u}{4\phi\sqrt{f_c}\, b_0}$$

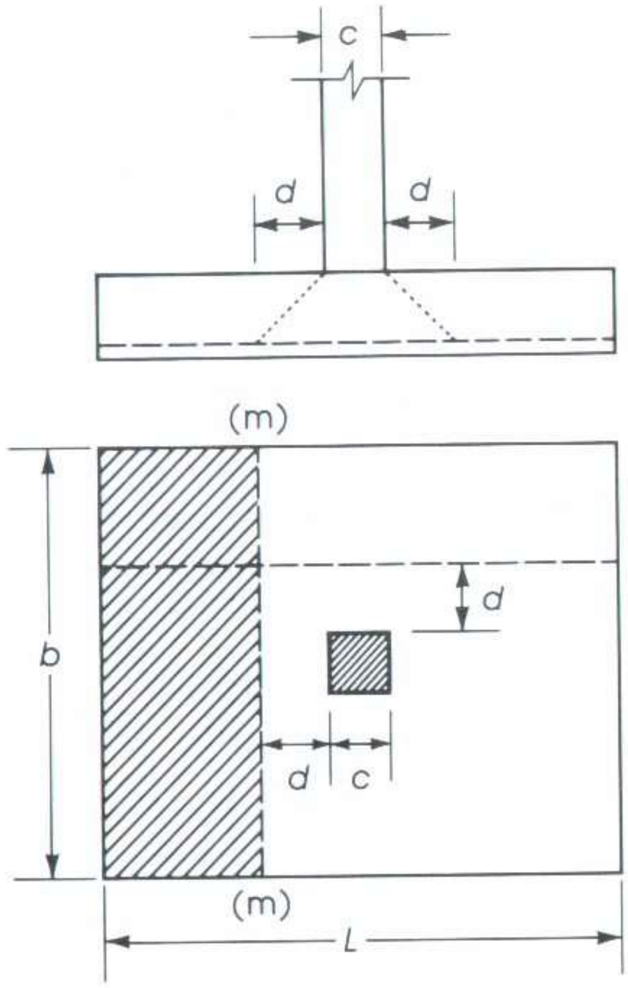
If d is not close to the assumed d, revise your assumptions



Design of one-way shear

For footings with bending action in one direction the critical section is located a distance d from face of column

$$\phi V_c = 2\phi\sqrt{f_c}\, b_0 d$$

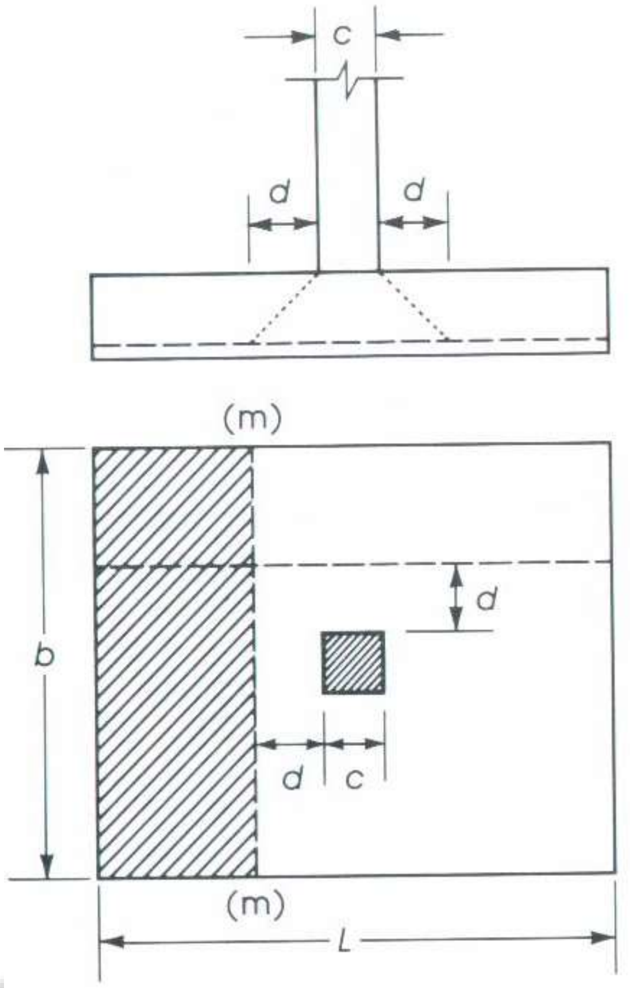


Design of one-way shear

The ultimate shearing force at section m-m can be calculated

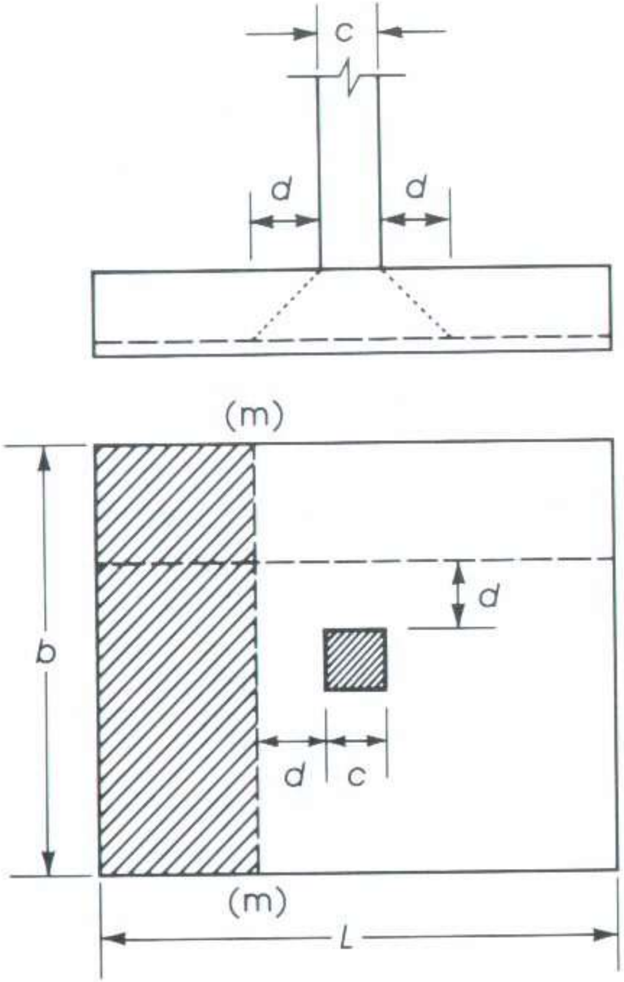
$$V_u = q_u\, b\left(\frac{L}{2} - \frac{c}{2} - d\right)$$

If no shear reinforcement is to be used, then d can be checked



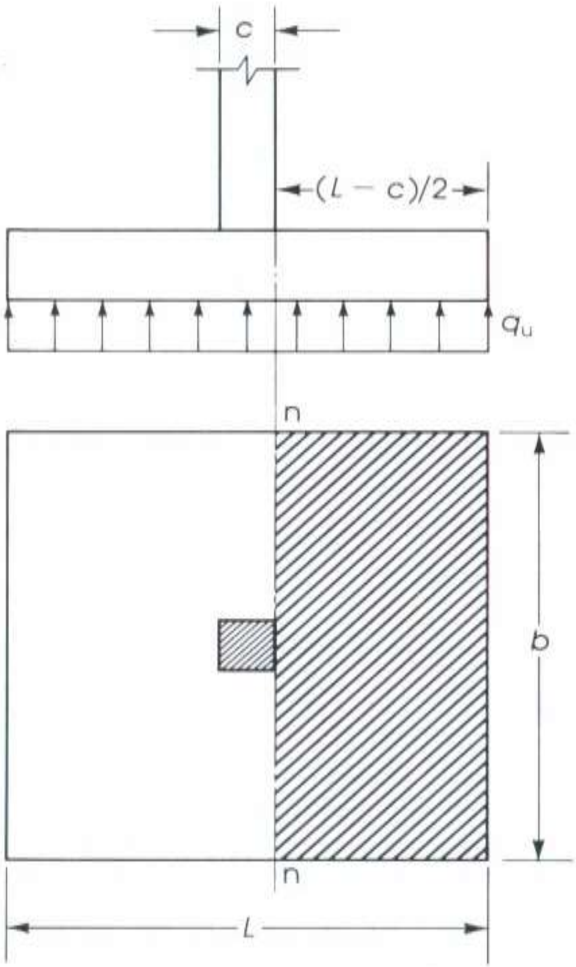
If no shear reinforcement is to be used, then d can be checked, assuming $V_u = fV_c$

$$d = \frac{V_u}{2\phi\sqrt{f_c}\, b}$$



The bending moment in each direction of the footing must be checked and the appropriate reinforcement must be provided.

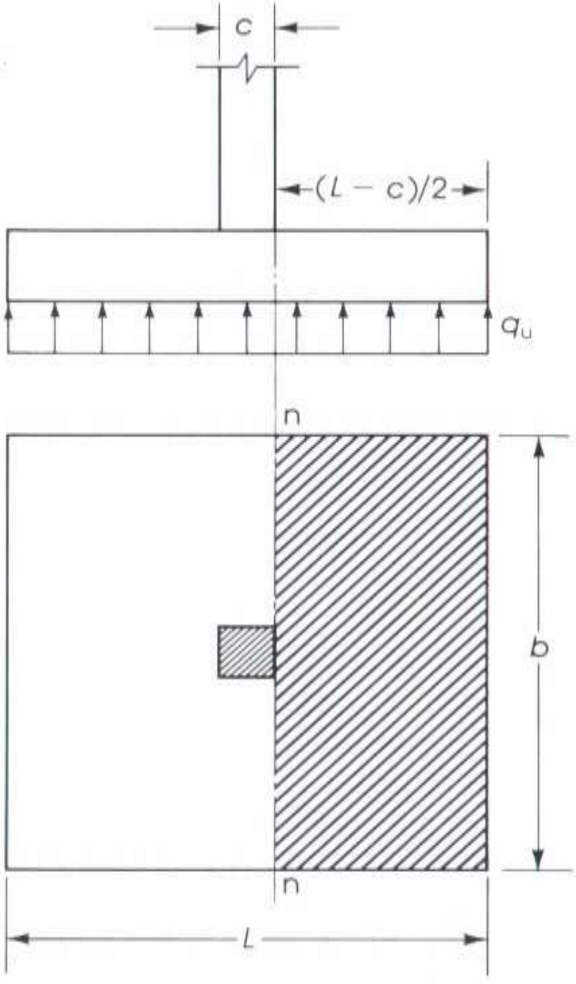
$$A_s = \frac{M_u}{\phi f_y \left(d - \frac{a}{2} \right)}$$



Another approach is to calculated

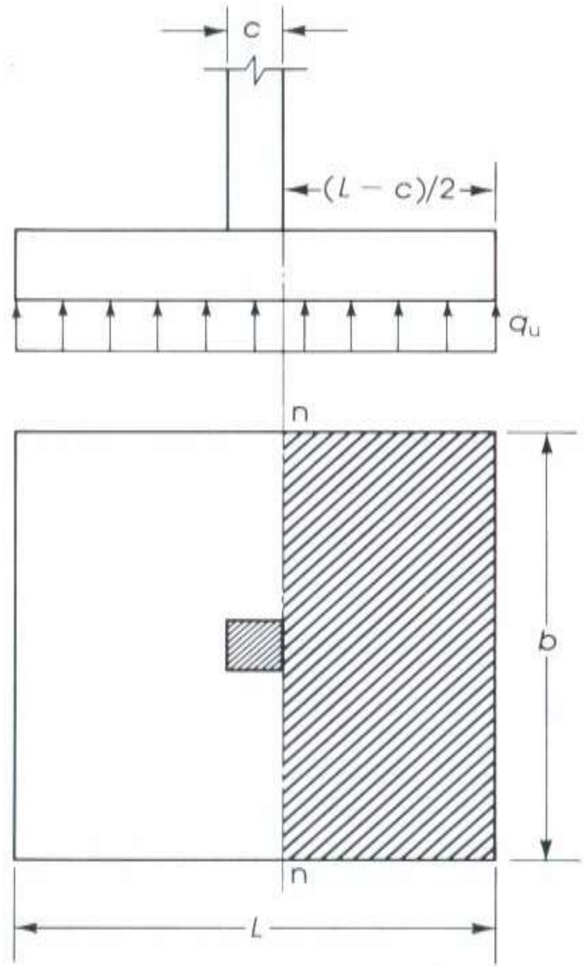
$R_u = M_u / bd^2$ and determine the steel percentage required r . Determine A_s then check if assumed a is close to calculated a

$$a = \frac{f_y A_s}{0.85 f_c b}$$

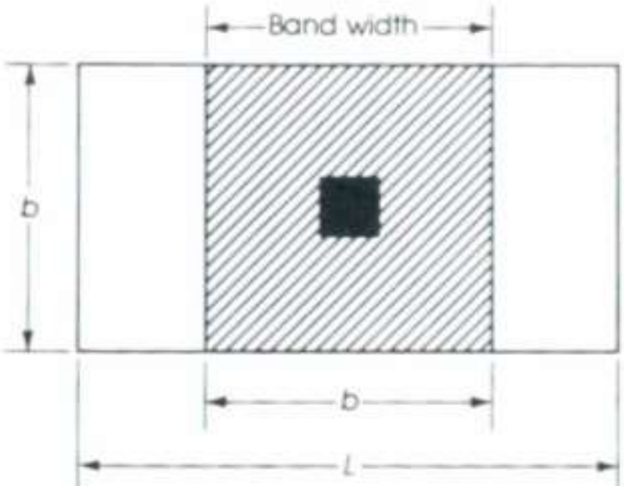


Flexural Strength and Footing reinforcement

The minimum steel percentage required in flexural members is 200/f_y with minimum area and maximum spacing of steel bars in the direction of bending shall be as required for shrinkage temperature reinforcement.



The reinforcement in one-way footings and two-way footings must be distributed across the entire width of the footing.



$$\frac{\text{Reinforcement in band width}}{\text{Total reinforcement in short direction}} = \frac{2}{\beta + 1}$$

where

$$\beta = \frac{\text{long side of footing}}{\text{short side of footing}}$$

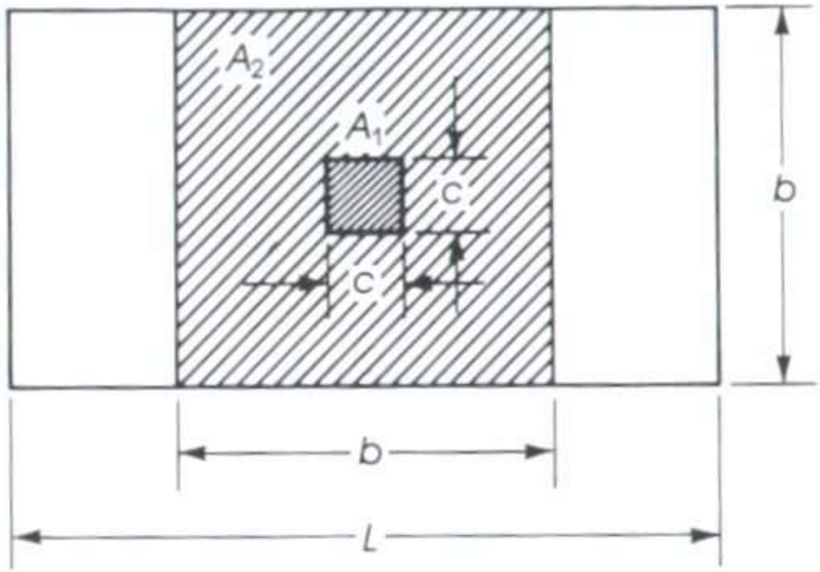
Bearing Capacity of Column at Base

The loads from the column act on the footing at the base of the column, on an area equal to area of the column cross-section. Compressive forces are transferred to the footing directly by bearing on the concrete. Tensile forces must be resisted by reinforcement, neglecting any contribution by concrete.

Bearing Capacity of Column at Base

Force acting on the concrete at the base of the column must not exceed the bearing strength of the concrete

$$N_1 = \phi\left(0.85 f_c A_1\right)$$



where f = 0.7 and

A_1 =bearing area of column

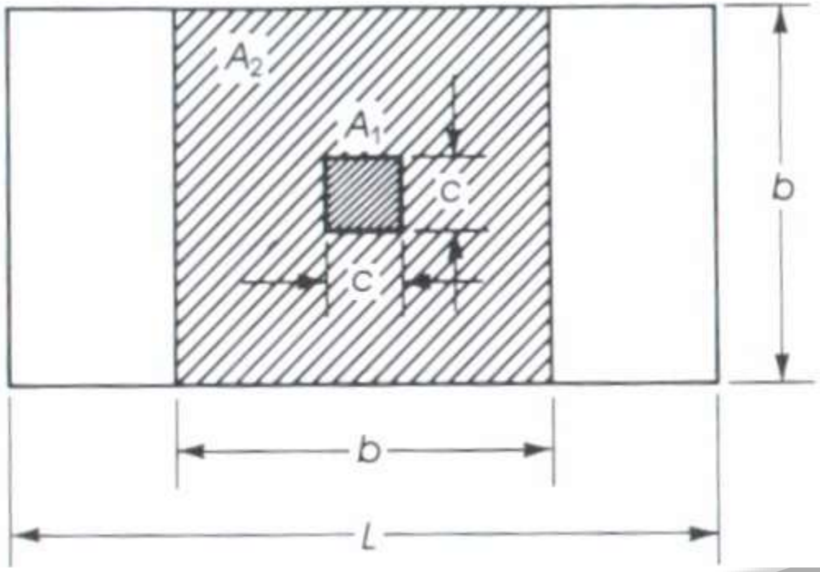
The value of the bearing strength may be multiplied by a factor for bearing on footing when the supporting surface is wider on all sides than the loaded area.

$$\sqrt{A_2 / A_1} \leq 2.0$$

The modified bearing strength

$$N_2 \leq \phi(0.85 f_c A_1)\sqrt{A_2 / A_1}$$

$$N_2 \leq 2\phi(0.85 f_c A_1)$$



Dowels in Footings

A minimum steel ratio r = 0.005 of the column section as compared to r = 0.01 as minimum reinforcement for the column itself. The number of dowel bars needed is four these may be placed at the four corners of the column. The dowel bars are usually extended into the footing, bent at the ends, and tied to the main footing reinforcement. The dowel diameter shall not exceed the diameter of the longitudinal bars in the column by more than 0.15 in.

Development length of the Reinforcing Bars

The development length for compression bars was given

$$l_{\mathrm{d}} = 0.02\, f_{\mathrm{y}} d_{\mathrm{b}} / \sqrt{f_{\mathrm{c}}}$$

but not less than Dowel bars must be checked for proper development length.

$$0.003\, f_{\mathrm{y}} d_{\mathrm{b}} \geq 8\,\mathrm{in.}$$

Differential Settlement

Footing usually support the following loads:

1. Dead loads from the substructure and superstructure
2. Live load resulting from material or occupancy
3. Weight of material used in back filling
4. Wind loads

General Requirements for Footing Design

1. A site investigation is required to determine the chemical and physical properties of the soil.
2. Determine the magnitude and distribution of loads form the superstructure.
3. Establish the criteria and the tolerance for the total and differential settlements of the structure.

General Requirements for Footing Design

4. Determine the most suitable and economic type of foundation.

5. Determine the depth of the footings below the ground level and the method of excavation.

   Establish the allowable bearing pressure to be used in design.

General Requirements for Footing Design

7. Determine the pressure distribution beneath the footing based on its width
8. Perform a settlement analysis.