# LECTURE NOTES

ON

# AC MACHINES

**2018 – 2019**

**II B. Tech II Semester (IARE-R16)**

Mr. K **Devender Reddy, Assistant Professor**

**Department of Electrical and Electronics Engineering**

**INSTITUTE OF AERONAUTICAL ENGINEERING**

**(Autonomous)**

Dundigal, Hyderabad - 500 043

# UNIT-I

## Three Phase Induction Motor

### Introduction

The induction machine was invented by NIKOLA TESLA in 1888. Right from its inception its ease of manufacture and its robustness have made it a very strong candidate for electromechanical energy conversion. It is available from fractional horsepower ratings to megawatt levels. It finds very wide usage in all various application areas. The induction machine is an AC electromechanical energy conversion device. The machine interfaces with the external world through two connections (ports) one mechanical and one electrical. The mechanical port is in the form of a rotating shaft and the electrical port is in the form of terminals where AC supply is connected. There are machines available to operate from three phase or single phase electrical input. In this module we will be discussing the three phase induction machine. Single phase machines are restricted to small power levels.

### Construction

In actual practice, the three coils form three windings distributed over several slots. These windings may be connected in star or delta and three terminations are brought out. These are conventional three phase windings which are discussed in greater detail in the chapters on alternators. Such windings are present n the stator as well as rotor

**Figure 1: stator of an induction machine**

A photograph of the stator of an induction machine is shown in fig. 1. A close up of the windings is shown in fig. 2.the several turns that makeup a coil are seen in this picture. The three terminations are connected to rings on which three brushes make a sliding contact. As the rotor rotates the brushes slip over the rings and provide means of connecting stationary external circuit elements to the rotating windings. A schematic of these arrangements is shown in fig. 8. A photograph of a wound rotor for an induction machine is shown in fig. 3 Fig. 4 shows a close up of the slip ring portion. Brushes are not shown in this picture.

Induction machines, which have these kinds of windings and terminals that are brought out, are called slip ring machines. The reader may note that in order that torque is produced current must flow in the rotor. To achieve that, the stationary brush terminals must either be shorted, or connected to a circuit allowing current flow.

Sometimes a star connected resistor bank is connected so that the developed starting torque is higher. There are also other forms of power electronic circuitry that may be connected to the rotor terminals to achieve various functions.

The popularity of the induction machine however, stems from another variety of rotor

**Figure2: Coils in the stator**

**Figure 3: A wound rotor with slip rings**

**Figure 4: slip rings**

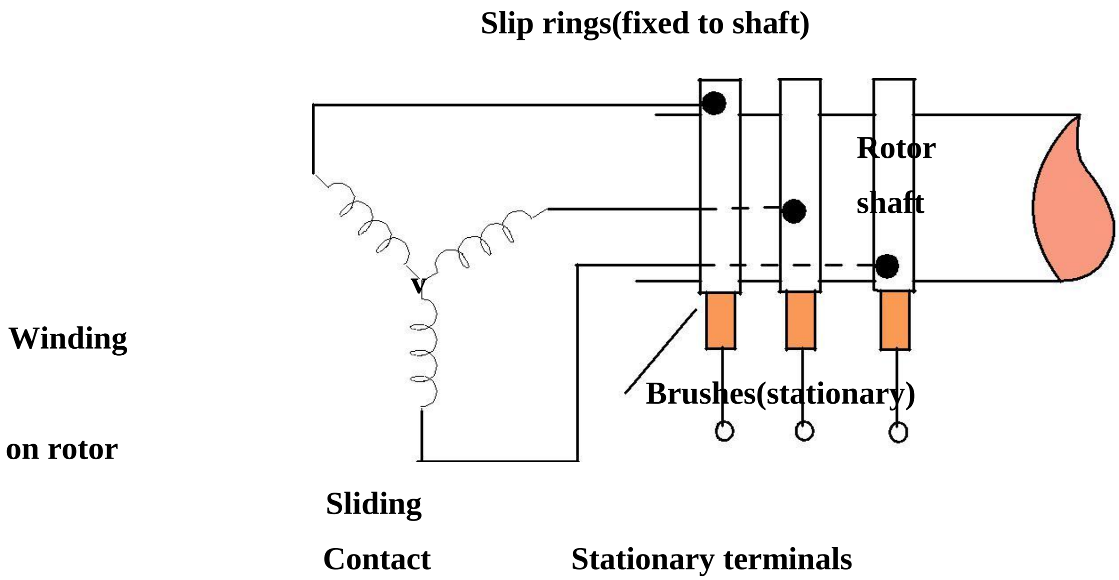



**Figure 5: Slip rings and brushes in induction machines — a schematic**

That is used. This rotor has slots into which copper or aluminum bars are inserted. These bars are then shorted by rings that are brazed on to each of the rotor ends. Figure 9 shows a simple schematic.

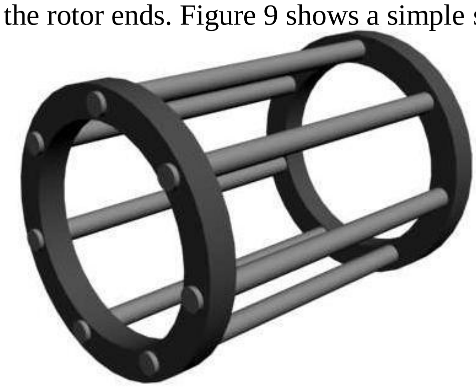


**Figure 6: Squirrel cage rotor — a schematic**

Such a rotor is called squirrel cage rotor. This rotor behaves like a short-circuited winding and hence the machine is able to perform electromechanical energy conversion. This type of rotor is easy to manufacture, has no sliding contacts and is very robust. It is this feature that makes induction machine suitable for use even in hazardous environments and reliable operation is achieved. The disadvantage of this type of rotor is that the motor behavior cannot be altered by connecting anything to the rotor — there are no rotor terminals.

Fig. 7 shows a photograph of a squirrel cage rotor. The rotor also has a fan attached to it. This is for cooling purposes. The bars (white lines on the surface) are embedded in the rotor iron which forms the magnetic circuit.

The white lines correspond to the visible portion of the rotor bar.

Sometimes two rotor bars are used per slot to achieve some degree of variability in the starting and running performances. It is to make use of the fact that while high rotor

**Figure 7: squirrel cage rotor**

Resistance is desirable from the point of view of starting torque; low rotor resistance is desirable from efficiency considerations while the machine is running. Such rotors are called double cage rotors or deep-bar rotors.

To summarize the salient features discussed so far,

1. The stator of the 3 - phase induction machine consists of normal distributed AC windings.
2. Balanced three phase voltages impressed on the stator, cause balanced three phase currents to flow in the stator.
3. These stator currents cause a rotating flux pattern (the pattern is a flux distribution which is sinusoidal with respect to the space angle) in the air gap.
4. The rotating flux pattern causes three phase induced EMFs in rotor windings (again normal ac windings). These windings, if shorted, carry three phase-balanced currents. Torque is produced as a result of interaction of the currents and the air gap flux.
5. The rotor may also take the form of a squirrel cage arrangement, which behaves in a manner similar to the short-circuited three phase windings.

## The Rotating Magnetic Field

The principle of operation of the induction machine is based on the generation of a rotating magnetic field. Let us understand this idea better.

Click on the following steps in sequence to get a graphical picture. It is suggested that the reader read the text before clicking the link.

- Consider a cosine wave from 0 to $360^\circ$. This sine wave is plotted with unit amplitude.

- Now allow the amplitude of the sine wave to vary with respect to time in a sinusoidal fashion with a frequency of 50Hz.Let the maximum value of the amplitude is, say, 10 units. This waveform is a pulsating sine wave.

$$i_{apk} = I_m \cos 2\pi.50.t \qquad (1)$$

- Now consider a second sine wave, which is displaced by $120^\circ$ from the first

- And allow its amplitude to vary in a similar manner, but with a $120^\circ$time lag.

$$i_{bpk} = I_m \cos(2\pi.50.t - 120^\circ) \qquad (2)$$

- Similarly consider a third sine wave, which is at $240^\circ$ lag. . .

- And allow its amplitude to change as well with a $240^\circ$ time lag. Now we have three pulsating sine waves.

$$i_{cpk} = I_m \cos(2\pi.50.t - 240^\circ \qquad (3)$$

Let us see what happens if we sum up the values of these three sine waves at every angle. The result really speaks about Tesla's genius. What we get is a constant amplitude travelling sine wave!

In a three phase induction machine, there are three sets of windings — phase A winding, phase B and phase C windings. These are excited by a balanced three-phase voltage supply. This would result in a balanced three phase current. Equations 1 — 3 represent the currents that flow in the three phase windings. Note that they have a $120^\circ$ time lag between them.

Further, in an induction machine, the windings are not all located in the same place. They are distributed in the machine $120^{\circ}$ away from each other (more about this in the section on alternators). The correct terminology would be to say that the windings have

their axes separated in space by $120^{\circ}$. This is the reason for using the phase A, B and C since waves separated in space as well by $120^{\circ}$.

When currents flow through the coils, they generate MMF. Since MMF is proportional to current, these waveforms also represent the MMF generated by the coils and the total MMF. Further, due to magnetic material in the machine (iron), these MMFs generate magnetic flux, which is proportional to the MMF(we may assume that iron is infinitely permeable and non-linear effects such as hysterisis are neglected). Thus the waveforms seen above would also represent the flux generated within the machine. The net result as we have seen is a travelling flux wave. The x-axis would represent the space angle in the machine as one travels around the air gap. The first pulsating waveform seen earlier would then represent the a-phase flux, the second represents the b-phase flux and the third represents the c-phase.

This may be better visualized in a polar plot. The angles of the polar plot represent the space angle in the machine, i.e., angle as one travels around the stator bore of the machine. Click on the links below to see the development on polar axes.

- This plot shows the pulsating wave at the zero degree axes. The amplitude is maximum at zero degree axes and is zero at $90^{\circ}$ axis. Positive parts of the waveform are shown in red while negative in blue. Note that the waveform is pulsating at the $0 - 180^{\circ}$ axis and red and blue alternate in any given side. This corresponds to the sine wave current changing polarity. Note that the maximum amplitude of the sine wave is reached only along the $0 - 180^{\circ}$ axis. At all other angles, the amplitude does not reach a maximum of this value. It however reaches a maximum value which is less than that of the peak occurring at the $0 - 180^{\circ}$ axis. More exactly, the maximum reached at any space angle $\theta$ would be equal to $\cos\theta$ times the peak at the $0 - 180^{\circ}$ axis. Further, at any space angle $\theta$, the time variation is sinusoidal with the frequency and phase lag being that of the excitation, and amplitude being that corresponding to the space angle.

- This plot shows the pulsating waveforms of all three cosines. Note that the first is

be represented as

$$f(t) = f_1 \cos\omega t \cos\theta + f_1 \cos(\omega t - \frac{2\pi}{3}) \cos(\theta - \frac{2\pi}{3}) + f_1 \cos(\omega t - \frac{4\pi}{3}) \cos(\theta - \frac{4\pi}{3})$$

$$= \frac{3}{2} f_1 \cos(\omega t - \theta) \qquad (4)$$

pulsating about the $0 - 180^\circ$ axis, the second about the $120^\circ - 300^\circ$ axis and the third at $240^\circ - 360^\circ$ axis.

- This plot shows the travelling wave in a circular trajectory. Note that while individual pulsating waves have maximum amplitude of 10, the resultant has amplitude of 15.

If $f_1$ is the amplitude of the flux waveform in each phase, the travelling wave can then

## Principles of Torque Production

In the earlier section, we saw how a rotating flux is produced. Now let us consider a rotor, which is placed in this field. Let the rotor have a coil such that the coil sides are placed diametrically opposite each other. This is shown in the fig. 1. Since the flux generated by the stator rotates flux linked by this rotor coil also changes.

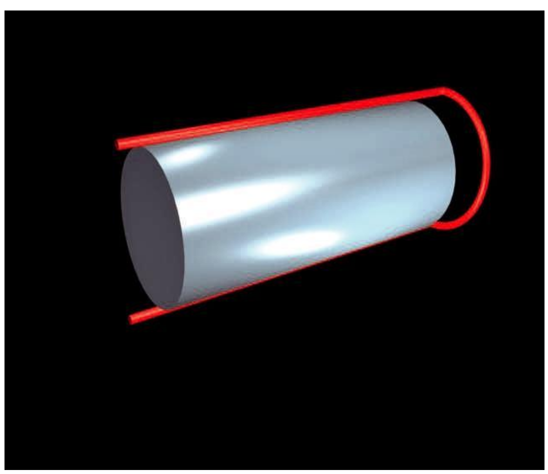


**Figure 8: A Coil on the rotor**

Since the flux pattern is varying sinusoidally in space, as the flux waveform rotates, the flux linkage varies sinusoidally. The rate of variation of this flux linkage will then be equal to the speed of rotation of the air gap flux produced. This sinusoidal variation of the flux linkage produces a sinusoidal induced EMF in the rotor coil. If the coil is short circuited, this induced EMF will cause a current flow in the coil as per Lenz's law.

Now imagine a second coil on the rotor whose axis is 120° away from the first. This is shown in fig. 9. The flux linkage in this coil will also vary sinusoidally with respect to time and therefore cause an induced voltage varying sinusoidally with time. However the flux linkages in these two coils will have a phase difference of 120° (the rotating flux wave will have to travel 120° in order to cause a similar flux linkage variation as in the first coil), and hence the time varying voltages induced in the coils will also have a 120° phase difference.

A third coil placed a further 120° away is shown in fig. 3. This will have a time varying induced emf lagging 240° in time with respect to the first.

When these three coils are shorted upon themselves currents flow in them as per Lenz's law. The mechanism by which torque is produced may now be understood as follows. Positive current is said to

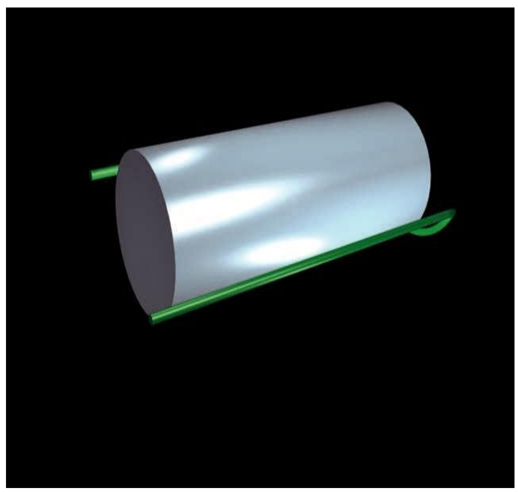


**Figure 9: A coil displaced 120° from the first**

Flow in these coils when current flows out of the page in a, b, c conductors and into $a'$, $b'$ and $c'$ respectively.

If we look at the voltage induced in these coils as phasor, the diagram looks as shown in fig. 12. The main flux is taken as the reference phasor. Considering that the induced emf is $-d\psi/dt$ where $\psi$ is the flux linkage, the diagram is drawn as shown.

As usual, the horizontal component of these phasor gives the instantaneous values of the induced emf in these coils.

Let these coils be purely resistive. Then these EMF phasor also represent the currents flowing in these coils. If we consider the instant t = 0, it can be seen that

1. The field flux is along $0^{\circ}$ axis.
2. The current in a phase coil is zero
3. The current in c phase coil is

These currents act to produce MMF and flux along the axes of the respective coils. Let us consider the space around $b'$ and c coil sides. The situation is shown in fig. 6.
The resulting flux pattern causes a tendency to move in the anticlockwise direction. This is easy to see through the so called whiplash rule. Alternatively, since the force on a current

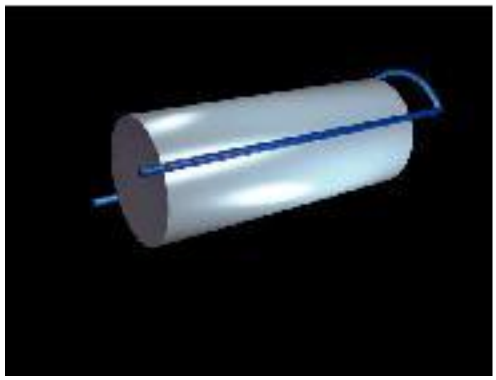


**Figure 10: A coil displaced $240^{\circ}$ from the first**

Carrying conductor is F = q(v X B), it can be seen that the torque produced tends to rotate the rotor counter-clockwise. The magnitude of the torque would increase with the current magnitude in the coils. This current is in turn dependent on the magnitude of the main field flux and its speed of rotation. Therefore one may say that motion of the main field tends to drag the rotor along with it.

When the rotor is free to move and begins moving, the motion reduces the relative speed between the main field and the rotor coils. Less EMF would therefore be induced and the torque would come down. Depending on the torque requirement for the load, the difference in speed between the rotor and the main field settles down at some particular value.
From the foregoing, the following may be noted.

1. The torque produced depends on a non-zero relative speed between the field and the rotor.
2. It is therefore not possible for the rotor to run continuously at the same speed of the field. This is so because in such a condition, no EMF would be induced in the rotor and hence no rotor current, no torque.
3. The frequency of currents induced in the rotor coils and their magnitude depends on this difference in speed.

These are important conclusions. The speed of the main field is known as the synchronous speed, $n_s$. If the actual speed of the rotor is $n_r$ then the ratio is known as slip and is frequently expressed as a percentage. Typically induction machines are designed to operate at about less than 4 percent slip at full load.

It is instructive to see the situation if the rotor resistance is neglected and is considered to be purely inductive. The phasor diagram of voltages and the currents would then look as shown in fig. 7.

At t = 0, one can see that current in a phase coil is at negative maximum, while b and c phases have positive current of 0.5 units.

Since main flux at the coil side is close to zero, there is very little torque produced from there. There is a tendency to move due to the $b'$ and c coil sides, but they are in opposite directions however. Hence there is no net torque on the rotor. This brings up another important conclusion the resistance of the rotor is an important part of torque production in the induction machine. While a high resistance rotor is better suited for torque production, it would also be loss

## Equivalent Circuit

It is often required to make quantitative predictions about the behavior of the induction machine, under various operating conditions. For this purpose, it is convenient to represent the machine as an equivalent circuit under sinusoidal steady state operating conditions. Since the operation is balanced, a single-phase equivalent circuit is sufficient for most purposes.

In order to derive the equivalent circuit, let us consider a machine with an open circuited rotor. Since no current can flow and as a consequence no torque can be produced, the situation is like a transformer open-circuited on the secondary (rotor). The equivalent circuit under this condition can be drawn as shown in fig. 11.

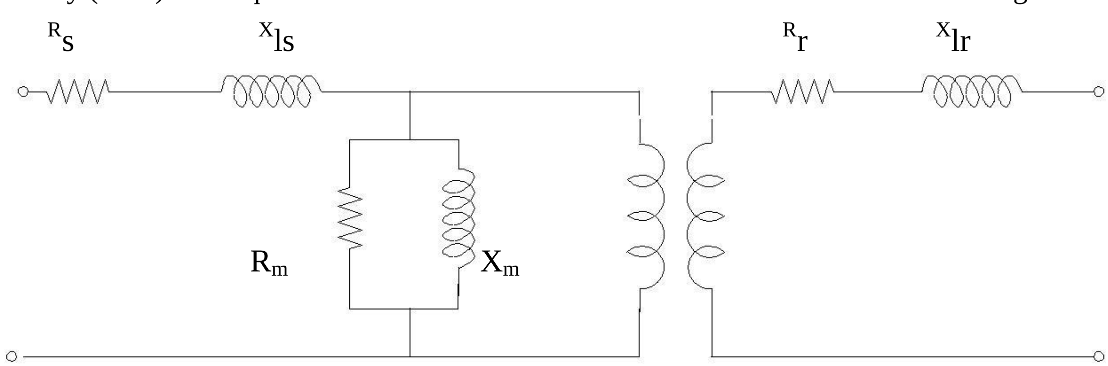



**Figure 11: Induction machine with the rotor open**

This is just the normal transformer equivalent circuit (why?). Measurements are generally made on the stator side and the rotor, in most circumstances, is shorted (if required, through some external circuitry). Since most of the electrical interaction is from the stator, it makes sense to refer all parameters to the stator.

Let us consider the rotor to be shorted. Let the steady speed attained by the rotor be $\omega_r$ and the synchronous speed be $\omega_s$. The induced voltage on the rotor is now proportional to the slip i.e., slip times the induced voltage under open circuit (why? ). Further, the voltage induced and the current that flows in the rotor is at a frequency equal to slip times the stator excitation frequency (why? ). The equivalent circuit can be made to represent this by shorting the secondary side and is shown in fig. 17.

$R_r'$ and $X_{lr}'$ refer to the rotor resistance and leakage resistance referred to the stator side (using the square of the turns ratio, as is done in transformer). The secondary side loop is excited by a voltage $sE_1$ , which is also at a frequency $sf_1$ . This is the reason why the rotor

leakage is $sX_{lr}'$ now .

This is then the per-phase equivalent circuit of the induction machine, also called as exact equivalent circuit. Note that the voltage coming across the magnetizing branch is the applied stator voltage, reduced by the stator impedance drop. Generally the stator impedance drop is only a small fraction of the applied voltage. This fact is taken to advantage and the magnetizing branch is shifted to be directly across the input terminals and is shown in fig. 19.

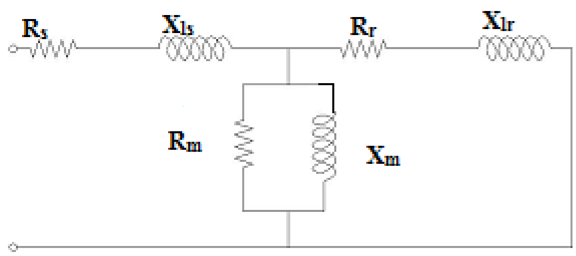



**Figure 18: The Exact equivalent circuit**

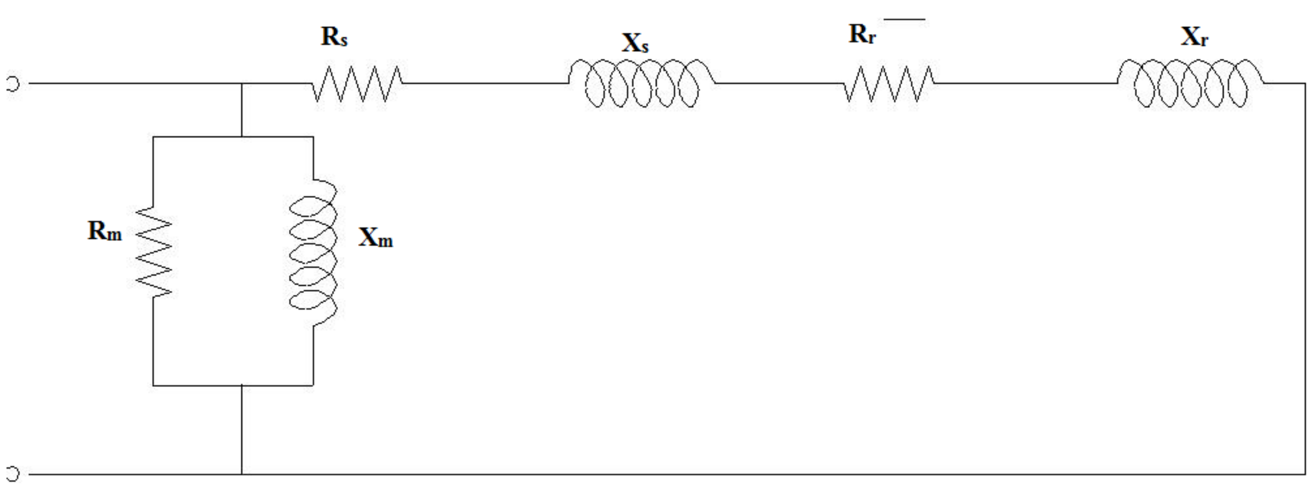


**Figure 19: The approximate equivalent circuit**

This circuit, called the approximate equivalent circuit, is simple to use for quick calculations.
With this equation the equivalent circuit can be modified as shown in fig. 20.

Dividing the equation for the rotor current by s and merging the two sides of the trans-former is not just a mathematical jugglery. The power dissipated in the rotor resistance (per phase) is obviously $I_2^{'2}R_r^{'}$ . From the equivalent circuit of fig. 20 one can see that the rotor current (referred to stator of course) flows through a resistance $R_r^{'}/s$ which has a component $R_r^{'}(1 - s)/s$ in addition to $R_r^{'}$ , which also dissipates power. What does this represent?

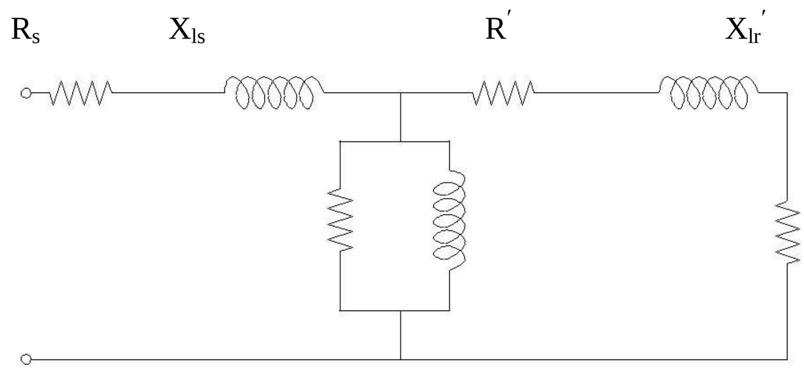



**Figure 20: The exact equivalent circuit - separation of rotor resistance**

From the equivalent circuit, one can see that the dissipation in $R_s$ represents the stator loss, and dissipation in $R_m$ represents the iron loss. Therefore, the power absorption indicated by the rotor part of the circuit must represent all other means of power consumption - the actual mechanical output, friction and windage loss components and the rotor copper loss components. Since the dissipation in $R_r^{'}$ is rotor copper loss, the power dissipation in $R_r^{'}(1 - s)/s$ is the sum total of the remaining. In standard terminology, dissipation in

- $R_r^{'}/s$ is called the air gap power.
- $R_r^{'}$ is the rotor copper loss.
- $R_r^{'}(1 - s)/s$ is the mechanical output.

In an ideal case where there are no mechanical losses, the last term would represent the actual output available at the shaft. Out of the power $P_g$ Transferred at the air gap, a fraction s is dissipated in the rotor and $(1 - s)$ is delivered as output at the shaft. If there are no mechanical losses like friction and windage, this represents the power available to the load.

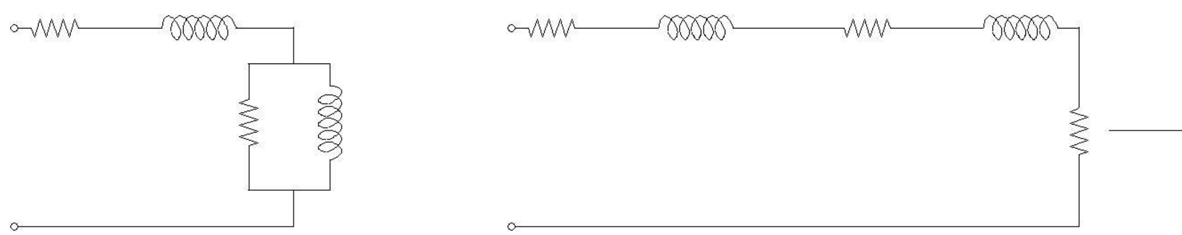
## Determination of Circuit Parameters

In order to find values for the various elements of the equivalent circuit, tests must be conducted on a particular machine, which is to be represented by the equivalent circuit. In order to do this, we note the following.

1. When the machine is run on no-load, there is very little torque developed by it. In an ideal case where there are no mechanical losses, there is no mechanical power developed at no-load. Recalling the explanations in the section on torque production, the flow of current in the rotor is indicative of the torque that is produced. If no torque is produced, one may conclude that no current would be flowing in the rotor either. The rotor branch acts like an open circuit. This conclusion may also be reached by reasoning that when there is no load, an ideal machine will run up to its synchronous speed where the slip is zero resulting in an infinite impedance in the rotor branch.

2. When the machine is prevented from rotation, and supply is given, the slip remains at unity. The elements representing the magnetizing branch $R_m$ &$X_m$ are high impedances much larger than $R_r'$ & $X_{lr}'$ in series. Thus, in the exact equivalent circuit of the induction machine, the magnetizing branch may be neglected.

From these considerations, we may reduce the induction machine exact equivalent circuit of fig.18 to those shown in fig. 21



(a) No-load equivalent (b) Blocked rotor equivalent

Figure 21: Reduced equivalent circuits

These two observations and the reduced equivalent circuits are used as the basis for the two most commonly used tests to find out the equivalent circuit parameters — the blocked rotor test and no load test. They are also referred to as the short circuit test and open circuit test respectively in conceptual analogy to the transformer.

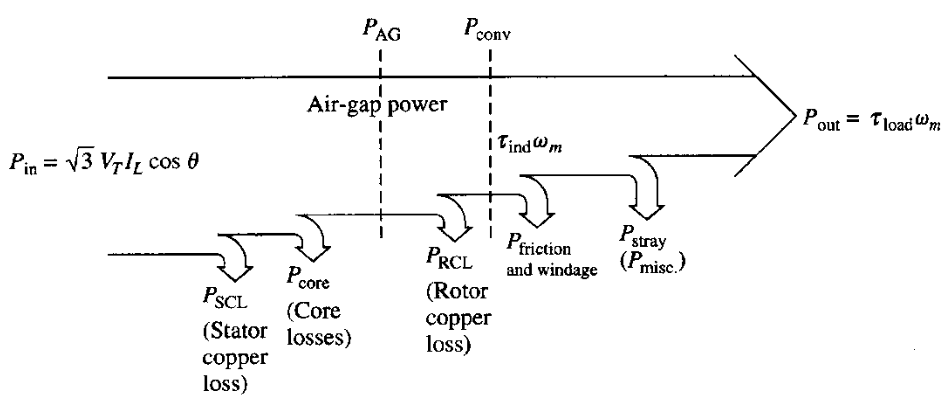
## Numerical problems on EMF and Slip

1. The EMF in the stator of an 8 pole induction motor has a frequency of 50 Hz and that in the rotor is 1.5Hz. At what speed the motor is running and what $I_{sc}$ the slip?
2. In case of an 8-pole induction motor the supply frequency was 50 Hz and the shaft speed was 735 rpm. Compute (i) Synchronous speed (ii) Slip speed per unit slip (iii) Percentage slip.
3. An 8 pole, 3phase alternator is coupled to a prime mover running at 750 rpm. It supplies an induction motor which has a full load speed of 960 rpm. Find the number of poles of induction motor and slip.
4. The frequency of stator EMF is 50 Hz for an 8-pole induction motor. If the rotor frequency is 2.5 Hz, calculate the slip and the actual speed of rotor.

## Copper losses

- Copper loss in the stator ($P_{SCL}$) = $I_1^2R_1$
- Copper loss in the rotor ($P_{RCL}$) = $I_2^2R_2$

- Core loss ($P_{core}$)
- Mechanical power loss due to friction and windage
- How these powers flow in the motor?




## <u>Power</u>

$$P_{in} = \sqrt{3}\, V_L I_L \cos\theta = 3 V_{ph} I_{ph} \cos\theta$$

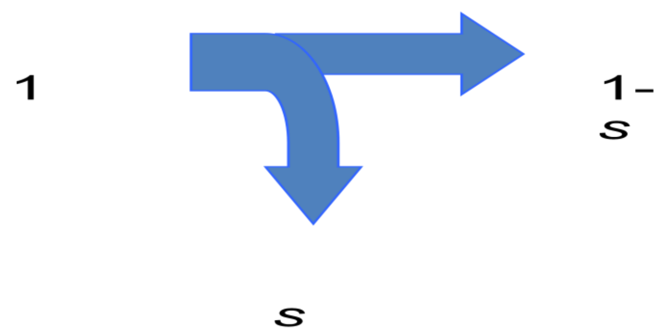
$$P_{SCL} = 3\, I_1^2 R_1$$

$$P_{AG} = P_{in} - (P_{SCL} + P_{core})$$

$$P_{RCL} = 3 I_2^2 R_2$$

$$P_{conv} = P_{AG} - P_{RCL}$$

$$P_{out} = P_{conv} - (P_{f+w} + P_{stray}) \qquad \tau_{ind} = \frac{P_{conv}}{\omega_m}$$



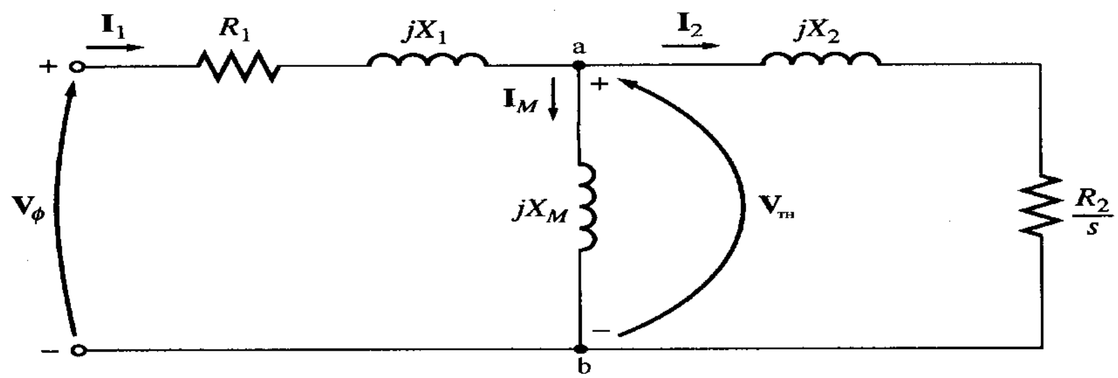

$$\boxed{\begin{array}{ccccc} P_{AG} & : & P_{RCL} & : & P_{conv} \\ 1 & : & s & : & 1-s \end{array}}$$

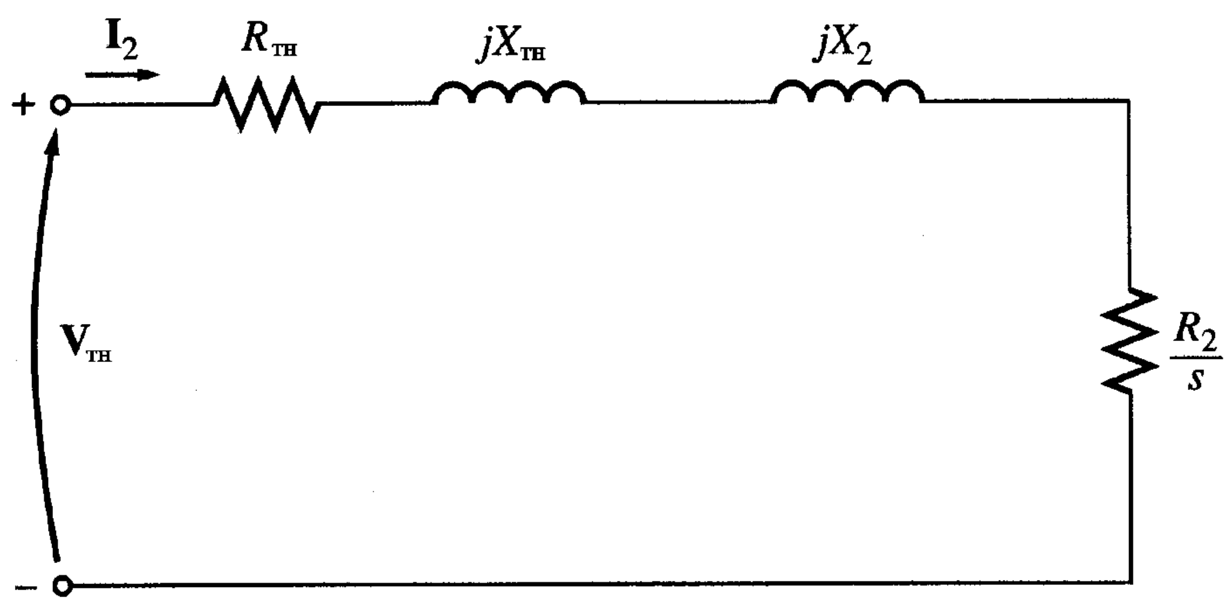
## Numerical Problems on Torque and Power developed

1. The power input to a 500V, 50Hz, 6-pole, 3-phase induction motor running at 975 rpm is 40 KW. The stator losses are 1KW and the friction and wind age losses total to 2KW, Calculate i) The slip ii) Rotor copper loss iii) Shaft power.
2. A 4 pole, 400 V, 3phase IM has a standstill rotor EMF of 100 V per phase. The rotor has resistance of 50 Ώ/ph and standstill reactance of 0.5 Ώ/ph. Calculate the maximum torque & slip at which it occurs. Neglect stator impedance.
3. An 8-pole, 50 Hz, 3 phase slip ring IM has effective resistance of 0.08 /phase the speed corresponding to maximum torque is 650 rpm. What is the value of resistance to be inserted in rotor circuit to obtain maximum torque at starting?
4. 500HP, 30, 440V, 50Hz induction motor has a speed of 950 rpm on full load. The machine has 6 poles. Calculate
   a) Slip and Speed of rotor field with respect to rotor.
   b) Speed of rotor field with respect to stator
   c) Complete alternations of rotor voltage per minute.

Relative speed between stator field with respect to rotors

- Thevenin's theorem can be used to transform the network to the left of points 'a' and 'b' into an equivalent voltage source $V_{TH}$ in series with equivalent resistance




$$V_{TH} = V_\phi \frac{jX_M}{R_1 + j(X_1 + X_M)} \qquad |V_{TH}| = |V_\phi| \frac{X_M}{\sqrt{R_1^2 + (X_1 + X_M)^2}}$$

$$R_{TH} + jX_{TH} = (R_1 + jX_1) // jX_M$$

Since $X_M >> X_1$ and $X_M >> R_1$

$$\boxed{V_{TH} \approx V_\phi \frac{X_M}{X_1 + X_M}}$$

Because $X_M >> X_1$ and $X_M + X_1 >> R_1$

$$\boxed{\begin{gathered} R_{TH} \approx R_1 \left( \frac{X_M}{X_1 + X_M} \right)^2 \\ X_{TH} \approx X_1 \end{gathered}}$$

$$I_2 = \frac{V_{TH}}{Z_T} = \frac{V_{TH}}{\sqrt{\left( R_{TH} + \frac{R_2}{s} \right)^2 + (X_{TH} + X_2)^2}}$$

Then the power converted to mechanical ($P_{\text{conv}}$)

$$P_{conv} = 3I_2^2 \frac{R_2(1-s)}{s}$$

And the internal mechanical torque ($T_{\text{conv}}$)

$$\tau_{ind} = \frac{P_{conv}}{\omega_m} = \frac{P_{conv}}{(1-s)\omega_s} = \frac{3I_2^2 \frac{R_2}{s}}{\omega_s} = \frac{P_{AG}}{\omega_s}$$

$$\tau_{ind} = \frac{3}{\omega_s} \left( \frac{V_{TH}}{\sqrt{\left( R_{TH} + \frac{R_2}{s} \right)^2 + (X_{TH} + X_2)^2}} \right)^2 \left( \frac{R_2}{s} \right)$$

$$\boxed{\tau_{ind} = \frac{1}{\omega_s} \frac{3V_{TH}^2 \left( \frac{R_2}{s} \right)}{\left( R_{TH} + \frac{R_2}{s} \right)^2 + (X_{TH} + X_2)^2}}$$

## The complete torque-speed characteristics

In order to estimate the speed torque characteristic let us suppose that a sinusoidal voltage is impressed on the machine. Recalling that the equivalent circuit is the per-phase representation of the machine. The mechanical power output was shown to be $(1 - s)P_g$ (power dissipated in $R_r'/s$). The torque is obtained by dividing this by the shaft speed $\omega_m$.Thus we have,

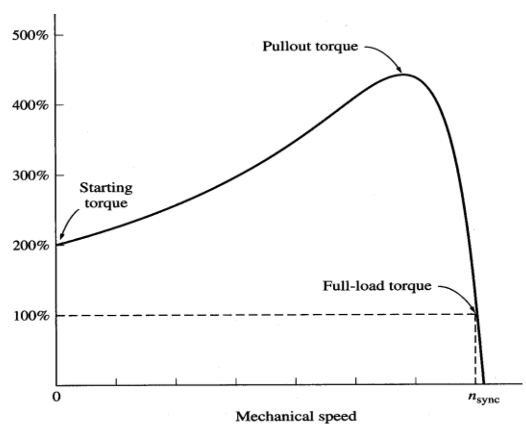
$$\frac{P_g(1-s)}{} = \frac{P_g(1-s)}{\omega_s(1-s)} = |I_s|^2 \frac{R_r'}{s\omega_s} \qquad (16)$$

where $\omega_s$ is the synchronous speed in radians per second and s is the slip. Further, this is the torque produced per phase. Hence the overall torque is given by

$$T_e = \frac{3}{\omega_s} \cdot \frac{V_s^2}{\left(R_s + \frac{R_2}{s}\right)^2 + \left(X_{ls} + X'_{lr}\right)} \cdot \frac{R'_r}{s}$$

The torque may be plotted as a function of ‘s’ and is called the torque-slip (or torque-speed, since slip indicates speed) characteristic — a very important characteristic of the induction machine. Eqn. 17 is valid for a two-pole (one pole pair) machine. In general, this expression should be multiplied by p, the number of pole-pairs. A typical torque-speed characteristic is shown in fig.

This plot corresponds to a 3 kW, 4 pole, 60 Hz machine..




We must note that the approximate equivalent circuit was used in deriving this relation. Readers with access to MATLAB or suitable equivalents (octave, scilab available free under GNU at the time of this writing) may find out the difference caused by using the ‘exact’ equivalent circuit by using the script found here. A comparison between the two is found in the plot of fig. 23. The plots correspond to a 3 kW, 4 pole, 50 Hz machine, with a rated speed of 1440 rpm. It can be seen that the approximate equivalent circuit is a good approximation in the operating speed range of the machine. Comparing fig. 22 with fig. 23, we can see that the slope and shape of the characteristics are dependent intimately on the machine parameters.

Further, this curve is obtained by varying slip with the applied voltage being held con-stant. Coupled with the fact that this is an equivalent circuit valid under steady state, it implies that if this characteristic is to be measured experimentally, we need to look at the torque for a given speed after all transients have died down. One cannot, for example, try to obtain this curve by directly starting the motor with full voltage applied to the terminals and measuring the torque and speed dynamically as it runs up to steady speed.

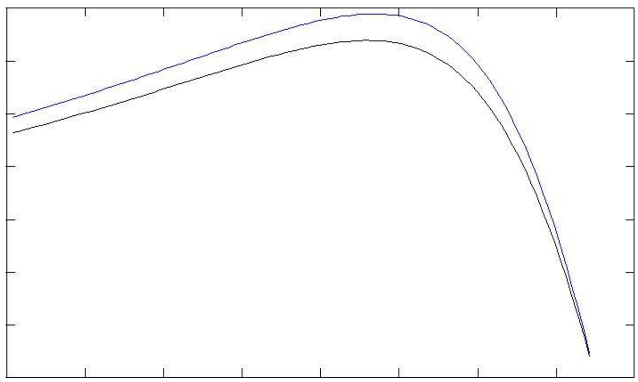


**Figure 15. Comparison of exact and approximate circuit predictions**

Another point to note is that the equivalent circuit and the values of torque predicted is valid when the applied voltage waveform is sinusoidal. With non-sinusoidal voltage wave-forms, the procedure is not as straightforward.

With respect to the direction of rotation of the air-gap flux, the rotor maybe driven to higher speeds by a prime mover or may also be rotated in the reverse direction. The torque-speed relation for the machine under the entire speed range is called the complete speed-torque characteristic. A typical curve is shown in fig. for a four-pole machine, the synchronous speed being 1500 rpm. Note that negative speeds correspond to slip values greater than 1, and speeds greater than 1500 rpm correspond to negative slip. The plot also shows the operating modes of the induction machine in various regions. The slip axis is also shown for convenience.

Restricting ourselves to positive values of slip, we see that the curve has a peak point. This is the maximum torque that the machine can produce, and is called as stalling torque. If the load torque is more than this value, the machine stops rotating or stalls. It occurs at a slip $\hat{s}$, which for the machine of fig is 0.38. At values of slip lower than $\hat{s}$, the curve falls steeply down to zero at $s = 0$.

The torque at synchronous speed is therefore zero. At values of slip higher than s = sˆ, the curve falls slowly to a minimum value at s = 1. The torque at s = 1 (speed = 0) is called the starting torque.

The value of the stalling torque may be obtained by differentiating the expression for torque with respect to zero and setting it to zero to find the value of sˆ. Using this method

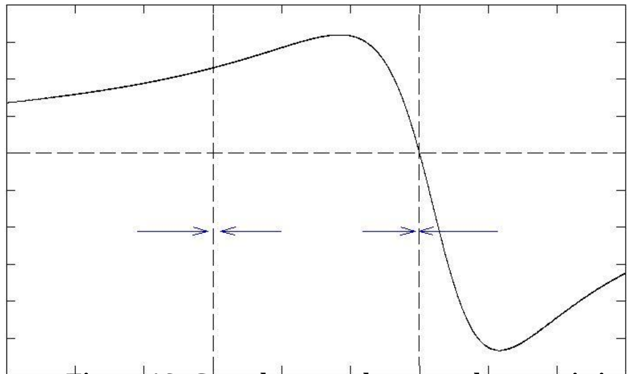


**Figure 16: Complete speed-torque characteristic**

This fact can be made use of conveniently to alter sˆ. If it is possible to change $R_r'$, then we can get a whole series of torque-speed characteristics, the maximum torque remaining constant all the while. But this is a subject to be discussed later.

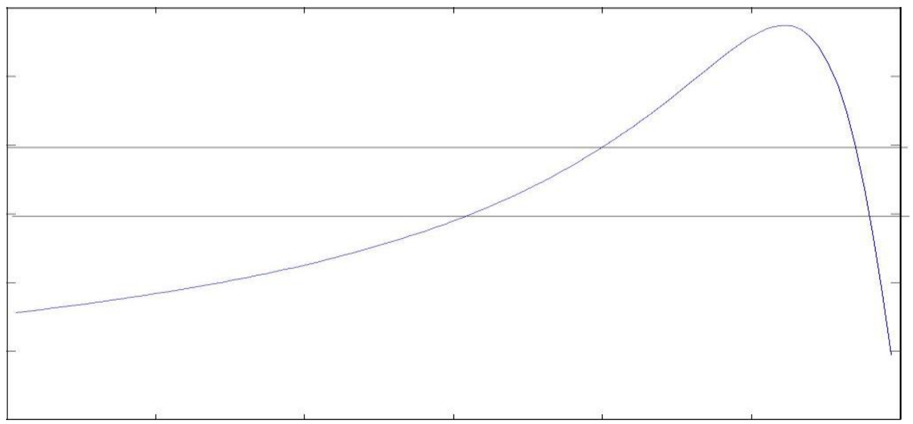
While considering the negative slip range, (generator mode) we note that the maximum torque is higher than in the positive slip region (motoring mode)

## Operating Point

Consider a speed torque characteristic shown in fig. 25 for an induction machine, having the load characteristic also superimposed on it. The load is a constant torque load i.e., the torque required for operation is fixed irrespective of speed.

The system consisting of the motor and load will operate at a point where the two characteristics meet. From the above plot, we note that there are two such points. We therefore need to find out which of these is the actual operating point.

To answer this we must note that, in practice, the characteristics are never fixed; they change slightly with time. It would be appropriate to consider a small band around the curve drawn where the actual points of the characteristic will lie. This being the case let us considers that the system is operating at point 1, and the load torque demand increases slightly. This is shown in fig. 26, where the change is exaggerated for clarity. This would shift the point of operation to a point $1'$ at which the slip would be less and the developed torque higher.



Speed, rpm

**Figure 17: Stability of operating point**

The difference in torque developed $\triangle T_e$, being positive will accelerate the machine. Any overshoot in speed as it approaches the point $1'$ will cause it to further accelerate since the developed torque is increasing. Similar arguments may be used to show that if for some reason the developed torque becomes smaller the speed would drop and the effect is cumulative. Therefore we may conclude that 1 is not a stable operating point.

Let us consider the point 2. If this point shifts to $2'$ , the slip is now higher (speed is lower) and the positive difference in torque will accelerate the machine. This behavior will tend to bring the operating point towards 2 once again. In other words, disturbances at point 2 will not cause a runaway effect. Similar arguments may be given for the case where the load characteristic shifts down. Therefore we conclude that point 2 is a stable operating point.

From the foregoing discussions, we can say that the entire region of the speed-torque characteristic from $s = 0$ to $s = \hat{s}$ is an unstable region, while the region from $s = \hat{s}$ to $s = 0$ is a stable region. Therefore the machine will always operate between $s = 0$
and $s = \hat{s}$

## Modes of Operation

Let us consider a situation where the machine has just been excited with three phase supply and the rotor has not yet started moving. A little reflection on the definition of the slip indicates that we are at the point $s = 1$. When the rotating magnetic field is set up due to stator currents, it is the induced emf that causes current in the rotor, and the interaction between the two causes torque. It has already been pointed out that it is the presence of the non-zero slip that causes a torque to be developed. Thus the region of the curve between $s = 0$ and $s = 1$ is the region where the machine produces torque to rotate a passive load and hence is called the motoring region. Note further that the direction of rotation of the rotor is the same as that of the air gap flux.

Suppose when the rotor is rotating, we change the phase sequence of excitation to the machine. This would cause the rotating stator field to reverse its direction — the rotating stator MMF and the rotor are now moving in opposite directions. If we adopt the convention that positive direction is the direction of the air gap flux, the rotor speed would then be a negative quantity. The slip would be a number greater than unity. Further, the rotor as we know should be"dragged along" by the stator field. Since the rotor is rotating in the opposite direction to that of the field, it would now tend to slow down, and reach zero speed. Therefore this region ($s > 1$) is called the braking region. (What would happen if the supply is not cut-OFF when the speed reaches zero?)

There is yet another situation. Consider a situation where the induction machine is operating from mains and is driving an active load (a load capable of producing rotation by itself). A typical example is that of a windmill, where the fan like blades of the wind mill are connected to the shaft of the induction machine. Rotation of the blades may be caused by the motoring action of the machine, or by wind blowing. Further suppose that both acting independently cause rotation in the same direction. Now when both grid and wind act, a strong wind may cause the rotor to rotate faster than the MMF produced by the stator excitation. A little reflection shows that slip is then negative. Further, the wind is rotating the rotor to a speed higher than what the electrical supply alone would cause. In order to do this it has to contend with an opposing torque generated by the machine preventing the speed build up. The torque generated is therefore negative. It is this action of the wind against the torque of the machine that enables wind-energy generation. The region of slip $s > 1$ is the generating mode of operation. Indeed this is at present the most commonly used approach in wind-energy generation. It may be noted from the torque expression of eqn. 17 that torque is negative for negative values of slip.

## Numerical Problems on Torque

1. A 25 hp, 6 poles, 50 Hz, 3-phase induction motor has stator/ rotor ratio of 6/5. The stator and rotor impedances per phase are (0.25+ j0.75) ohms and (0.173+j0.52) ohms respectively. Find the starting torque exerted by the motor when an external resistance of 1 ohm is inserted in each phase, the motor being started directly on the 400 V supply system. Assume star connection
2. A 40 kw, 3-phase slip-ring induction motor of negligible stator impedance runs at a speed of 0.96 time's synchronous speed at rated torque. The slip at maximum torque is 4 times the full load value. If the rotor resistance of the motor is increased by 5 times, determine: a) the speed,

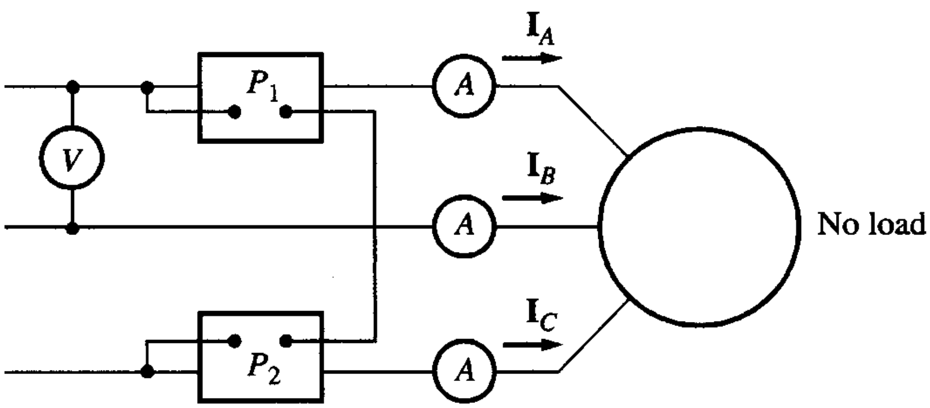
power output and rotor copper loss at rated torque b) The speed corresponding o maximum torque

3. The power input to a 6-pole, 50 Hz, 3-phase induction motor is 700 W at no load and 10 kw at full load. The no load copper loss may be assumed negligible whiles the full load stator and rotor copper losses are 295 W and 310 W respectively. Find the full load speed, shaft torque and efficiency of the motor assuming rotational and core losses to be equal.

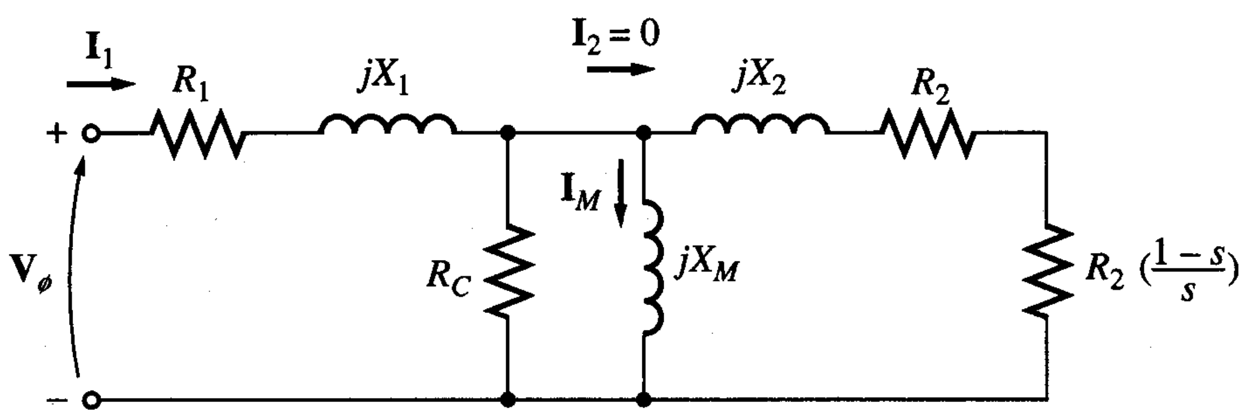
# UNIT-II

# Testing and Speed control of Induction machines

## No load Test



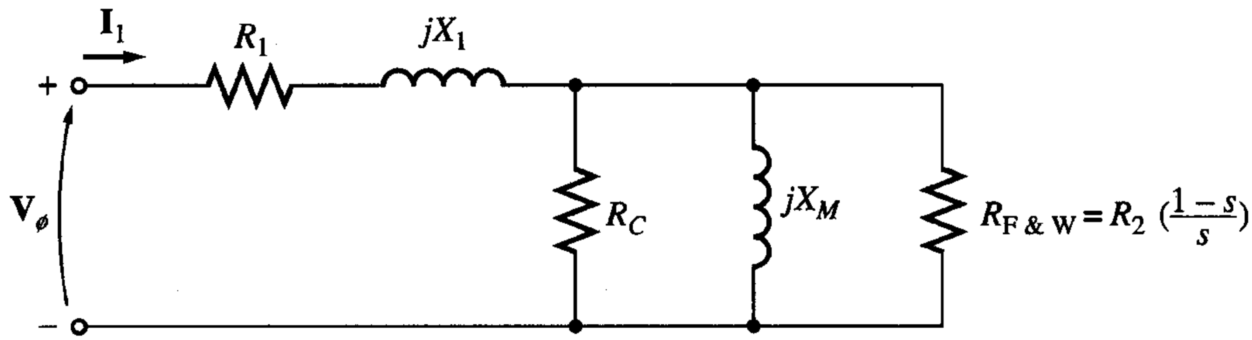

1. The motor is allowed to spin freely
2. The only load on the motor is the friction and windage losses, so all $P_{conv}$ is consumed by mechanical losses
3. The slip is very small



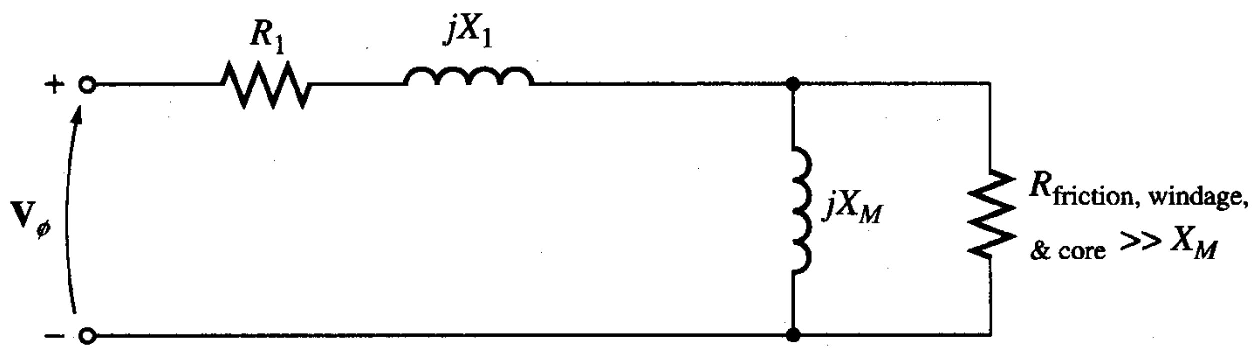

At this small slip

$$\frac{R_2(1-s)}{s} \gg R_2 \qquad \& \qquad \frac{R_2(1-s)}{s} \gg X_2$$

The equivalent circuit reduces to…




Combining $R_c$ & $R_{F+W}$ we get……




- At the no-load conditions, the input power measured by meters must equal the losses in the motor.
- The $P_{RCL}$ is negligible because $I_2$ is extremely small because $R_2(1\text{-}s)/s$ is very large.
- The input power equals

$$\begin{aligned} P_{in} &= P_{SCL} + P_{core} + P_{F\&W} \\ &= 3I_1^2 R_1 + P_{rot} \end{aligned}$$

Where

$$P_{rot} = P_{core} + P_{F\&W}$$

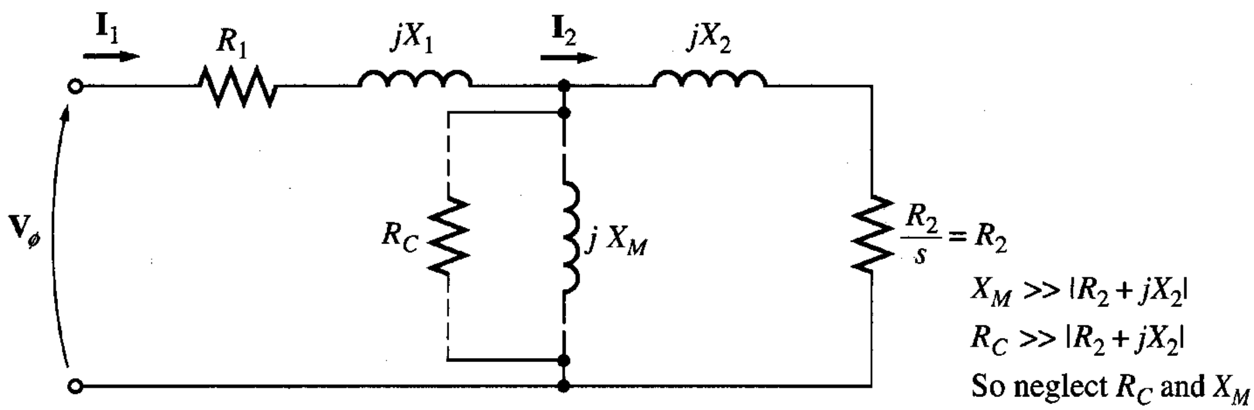
The equivalent input impedance is thus approximately

$$|Z_{eq}| = \frac{V_\phi}{I_{1,nl}} \approx X_1 + X_M$$

If $X_1$ can be found, in some other fashion, the magnetizing impedance $X_M$ will be known

## Blocked rotor test

- **In this test, the rotor is locked or blocked so that it cannot move, a voltage is applied to the motor, and the resulting voltage, current and power are measured.**




- The AC voltage applied to the stator is adjusted so that the current flow is approximately full-load value.

- The locked-rotor power factor can be found as

$$\boxed{PF = \cos\theta = \frac{P_{in}}{\sqrt{3}V_l I_l}}$$

- The magnitude of the total impedance

$$\boxed{|Z_{LR}| = \frac{V_\phi}{I}}$$

$$|Z_{LR}| = R_{LR} + jX'_{LR}$$
$$= |Z_{LR}|\cos\theta + j|Z_{LR}|\sin\theta$$

$$R_{LR} = R_1 + R_2$$
$$X'_{LR} = X'_1 + X'_2$$

Where $X'_1$ and $X'_2$ are the stator and rotor reactances at the test frequency respectively

$$R_2 = R_{LR} - R_1$$

$$X_{LR} = \frac{f_{rated}}{f_{test}} X'_{LR} = X_1 + X_2$$

| | $X_1$ and $X_2$ as function of $X_{LR}$ | |
|---|---|---|
| Rotor Design | $X_1$ | $X_2$ |
| Wound rotor | 0.5 $X_{LR}$ | 0.5 $X_{LR}$ |
| Design A | 0.5 $X_{LR}$ | 0.5 $X_{LR}$ |
| Design B | 0.4 $X_{LR}$ | 0.6 $X_{LR}$ |
| Design C | 0.3 $X_{LR}$ | 0.7 $X_{LR}$ |
| Design D | 0.5 $X_{LR}$ | 0.5 $X_{LR}$ |

## Numerical problems on circle diagram

1. Draw the circle diagram of a 20 hp, 400 V, 50 Hz, 3- phase star- connected induction motor from the following test data (line values)

| | | | |
|---|---|---|---|
| No load | 400 V | 9 A | Pf 0.2 |
| Blocked rotor | 200 V | 50 A | Pf 0.4 |

From the circle diagram find (i) line current and power factor at full load (ii) Maximum power output

2. Draw the circle diagram for a 5 hp, 200 V, 50 Hz, 4 pole, 3 – phase, star connected induction motor from the following data: (i) 200 V, 5A, 350 W (ii) 100 V, 26 A, 1700 W (iii) Rotor copper loss at standstill = half of the total copper loss. Estimate there from the full load current, power factor, speed and torque.

3. A 400 V, 3-Phase, 50 Hz, star connected induction motor has the following test results:

| | | | |
|---|---|---|---|
| No load test | 400 V | 8.5 A | 1100 W |
| Blocked rotor test | 180 V | 45 A | 5799 W |

Calculate the line current and power factor when operating at 4% slip. The stator resistance per phase is 0.5 Ω

4. Draw the circle diagram of a 100 hp, 400 V, 50 Hz, 3- phase star- connected induction motor from the following test data (line values)

| | | | |
|---|---|---|---|
| No load test | 400 V | 9 A | Pf 0.3 |
| Blocked rotor test | 200 V | 100 A | Pf 0.45 |

From the circle diagram find (i) line current and power factor at full load (ii) Maximum power output

5. Draw the circle diagram for a 20 hp, 200 V, 50 Hz, 4 pole, 3 – phase, star connected induction motor from the following data: (i) 200 V, 9A, 425 W (ii) 100 V, 59 A, 2100 W (iii) Rotor copper loss at standstill = Quarter of the total copper loss. Estimate there from the full load current, power factor, speed and torque.

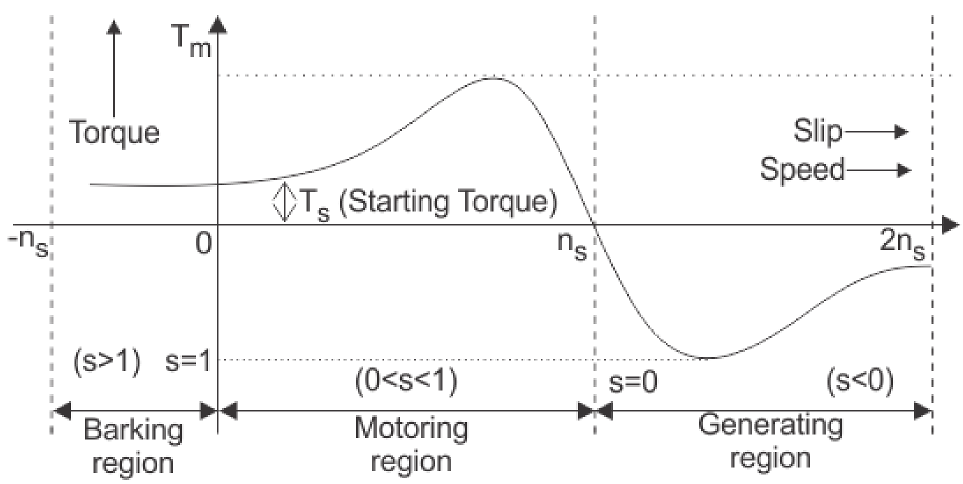
6. A 400 V, 3-Phase, 50 Hz, star connected induction motor has the following test results:

| | | | |
|---|---|---|---|
| No load test | 400 V | 9.5 A | 1150 W |
| Blocked rotor test | 195 V | 63A | 6899 W |

Calculate the line current and power factor when operating at 5% slip. The stator resistance per phase is 0.5 Ω

## Starting methods of three phase induction motor

In this article we are going to discuss various *methods of starting three phase induction motor*. Before we discuss this, it is very essential here to recall the torque slip characteristic of the three phase induction motor which is given below.




From the torque slip characteristic it is clear that at the slip equals to one we have some positive starting torque hence we can say that the three phase induction motor is self starting machine, then why there is a need of starters for three phase induction motor? The answer is very simple.

If we look at the equivalent circuit of the three phase induction motor at the time of starting, we can see the motor behaves like an electrical transformer with short circuited secondary winding, because at the time of starting, the rotor is stationary and the back emf due to the rotation is not developed yet hence the motor draws the high starting current. So the reason of using the starter is clear here. We use starters in order to limit the high starting current. We use different starters for both the type of three phase induction motors. Let us consider first squirrel cage type of induction motor. In order to choose a particular type of starting method for the squirrel cage type of induction motor, we have three main considerations and these are,

## Full Voltage Starting Method for Squirrel Cage Induction Motor

In this type we have only one method of starting.

### Direct on Line Starting Method

This method is also known as the DOL method for starting the three phase squirrel cage induction motor. In this method we directly switch the stator of the three phase squirrel cage induction motor on to the supply mains. The motor at the time of starting draws very high starting current (about 5 to 7 times the full load current) for the very short duration. The amount of current drawn by the motor depends upon its design and size. But such a high value of current does not harm the motor because of rugged construction of the squirrel cage induction motor.

Such a high value of current causes sudden undesirable voltage drop in the supply voltage. A live example of this sudden drop of voltage is the dimming of the tube lights and bulbs in our homes at the instant of starting of refrigerator motor. Now let us derive the expression for starting torque in terms of full load torque for the direct online starter. We have various quantities that involved in the expression for the starting torque are written below:

We define $T_s$ as starting torque

$T_f$ as full load torque

$I_f$ as per phase rotor current at full load

$I_s$ as per phase rotor current at the time of starting

$s_f$ as full load slip

$s_s$ as starting slip

$R_2$ as rotor resistance

$W_s$ as synchronous speed of the motor

Now we can directly write the expression for torque of induction motor as

$$T = \frac{1}{W_s} I^2 \frac{r}{s}$$

From the help of the above expression we write the ratio of starting torque to full load torque as

$$\frac{T_s}{T_f} = \left(\frac{I_s}{I_f}\right)^2 \times s_f$$

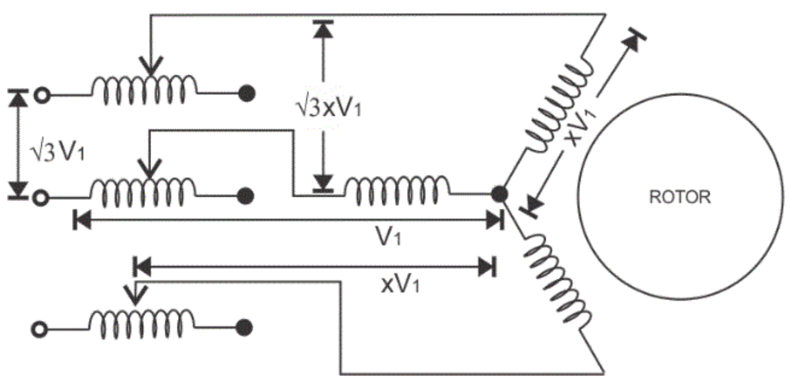
Here we have assumed that the rotor resistance is constant and it does not vary with the frequency of the rotor current.

## Reduced voltage method for starting squirrel cage induction motor

In reduced voltage method we have three different type of starting method and these are written below:

1. Stator resistor starting method
2. Auto transformer staring method
3. Star delta starting method
4. Now let us discuss each of these methods in detail.
5. Stator Resistor Starting Method

Given below is the figure for the starting resistor method:




In this method we add resistor or a reactor in each phase as shown in the diagram (between the motor terminal and the supply mains).Thus by adding resistor we can control the supply voltage. Only a fraction of the voltage (x) of the supply voltage is applied at the time of starting of the induction motor. The value of x is always less than one. Due to the drop in the voltage the starting torque also decreases. We will derive the expression for the starting torque in terms of the voltage fraction x in order to show the variation of the starting torque with the value of x. As the motor speeds up the reactor or resistor is cut out from the circuit and finally the resistors are short circuited when the motor reaches to its operating speed. Now let us derive the expression for starting torque in terms of full load torque for the stator resistor starting method. We have various quantities that involved in the expression for the starting torque is written below.

We define $T_s$ as starting torque

$T_f$ as full load torque $I_f$ as per phase rotor current at full load

$I_s$ as per phase rotor current at the time of starting

$s_f$ as full load slip

$s_s$ as starting slip

$R_2$ as rotor resistance

$W_s$ as synchronous speed of the motor

Now we can directly write the expression for torque of the induction motor as

$$T = \frac{1}{W_s} \times I^2 \frac{r}{s}$$

From the help of the above expression we write the ratio of starting torque to full load torque as

$$\frac{T_s}{T_f} = \left(\frac{I_s}{I_f}\right)^2 \times s_f \cdots\cdots (i)$$

Here we have assumed that the rotor resistance is constant and it does not vary with the frequency of the rotor current. From the above equation we can have the expression for the starting torque in terms of the full load torque. Now at the time of starting the per phase voltage is reduced to $xV_1$, the per phase starting current is also reduced to $xI_s$. On substituting the value of $I_s$ as $xI_s$ in equation 1.

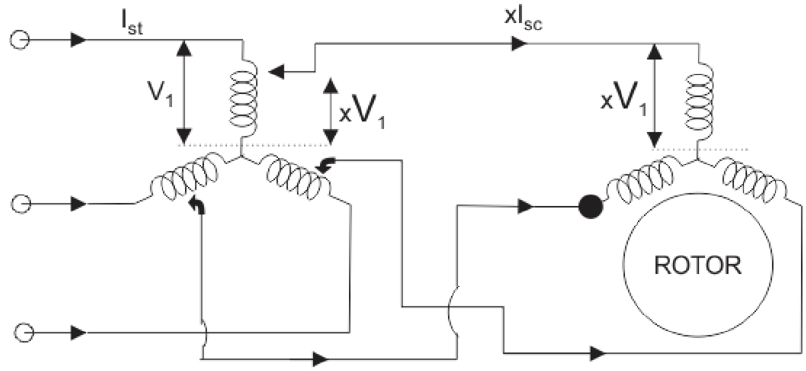
$$\frac{T_s}{T_f} = \left(\frac{xI_s}{I_f}\right)^2 \times s_f$$

$$\frac{T_s}{T_f} = \left(\frac{I_s}{I_f}\right)^2 \times s_f \times x^2$$

This shows the variation of the starting torque with the value of x. Now there are some considerations regarding this method. If we add series resistor then the energy losses are increased so it's better to use series reactor in place of resistor because it is more effective in reducing the voltage however series reactor is more costly than the series resistance.

**Auto Transformer Starting Method**

As the name suggests in this method we connect auto transformer in between the three phase power supply and the induction motor as shown in the given diagram:




Pertaining to Auto-Transfer Starting

The auto transformer is a step down transformer hence it reduces the per phase supply voltage from $V_1$ to $xV_1$.The reduction in voltage reduces current from $I_s$ to $xI_s$. After the motor reaches to its normal operating speed, the auto transformer is disconnected and then full line voltage is applied. Now let us derive the expression for starting torque in terms of full load torque for the auto transformer starting method. We have various quantities that involved in the expression for the starting torque are written below: We define $T_s$ as starting torque $T_f$ as full load torque $I_f$ as per phase rotor current at full load $I_s$ as per phase rotor current at the time of starting $s_f$ as full load slip $s_s$ as starting slip $R_2$ as rotor resistance $W_s$ as synchronous speed of the motor Now we can directly write the expression for torque of the induction motor as

$$T = \frac{1}{W_s} \times I^2 \frac{r}{s}$$

From the help of the above expression we write the ratio of starting torque to full load torque as

$$\frac{T_s}{T_f} = \left(\frac{I_s}{I_f}\right)^2 \times s_f \cdots\cdots (i)$$

Here we have assumed that the rotor resistance is constant and it does not vary with the frequency of the rotor current. From the above equation we can have the expression for the starting torque in terms of the full load torque. Now at the time of starting the per phase voltage is reduced to $xV_1$, the per phase starting current is also reduced to $xI_s$. On substituting the value of $I_s$ as $xI_s$ in equation 1. We have

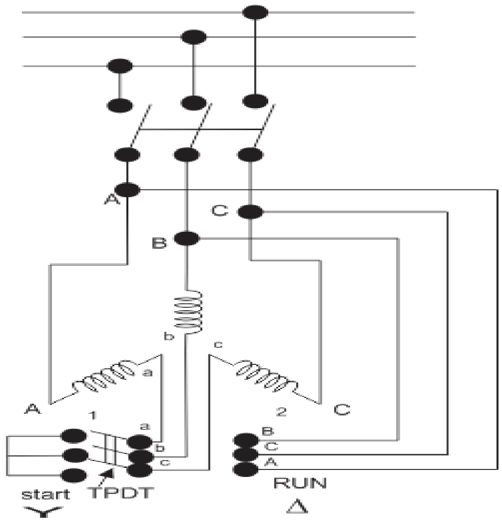
$$\frac{T_s}{T_f} = \left(\frac{xI_s}{I_f}\right)^2 \times s_f$$

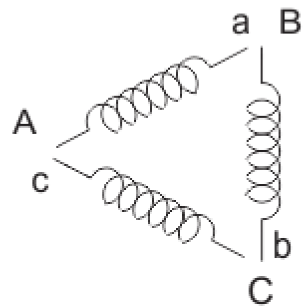
$$\frac{T_s}{T_f} = \left(\frac{I_s}{I_f}\right)^2 \times s_f \times x^2$$

This shows the variation of the starting torque with the value of x.

## Star-Delta Starting Method

Connection diagram is shown below for star delta method,




Star Delta

This method is used for the motors designed to operate in delta connected winding. The terminals are marked for the phases of the stator are shown above. Now let us see this method works. The stator phases are first connected to the star by the help of triple pole double throw switch (TPDT switch) in the diagram the position is marked as 1 then after this when the steady state speed is reached the switch is thrown to position 2 as shown in the above diagram. Now let analyze the working of the above circuit. In the first position the terminals of the motor are short circuited and in the second position from the diagram the terminal a, b and c are respectively connected to B, C and A. Now let us derive the expression for starting torque in terms of full load torque for the star delta starting method. Now we can directly write the expression for torque of the induction motor as

$$T = \frac{1}{W_s} \times I^2 \frac{r}{s}$$

From the help of the above expression we write the ratio of starting torque to full load torque as

$$\frac{T_s}{T_f} = \left(\frac{I_s}{I_f}\right)^2 \times s_f \cdots\cdots(ii)$$

Here we have assumed that the rotor resistance is constant and it does not vary with the frequency of the rotor current. Let us assume the line voltage to be $V_l$ then the per phase starting current when connected in star position is $I_{ss}$ which is given by

$$I_{ss} = \frac{V_l}{\sqrt{3} \times Z}$$

When stator is in delta connected position we have starting current

$$I_{sd} = \frac{V_l}{Z}\ clearly,\ I_{sd} = \sqrt{3} \times I_{ss}\ and\ I_{fd} = I_{ss}$$

From the above equation we have

$$\frac{T_s}{T_f} = \frac{1}{3}\left(\frac{I_{sd}}{I_{fd}}\right)^2 \times s_f \cdots\cdots(iii)$$

This shows that the reduced voltage method has an advantage of reducing the starting current but the disadvantage is that all these methods of reduced voltage causes the objectionable reduction in the starting torque.

### Starting Methods of Wound Rotor Motors

We can employ all the methods that we have discussed for starting of the squirrel cage induction motor in order to start the wound rotor motors. We will discuss the cheapest method of starting the wound rotors motor here.

### Addition of External Resistances in Rotor Circuit

This will decrease the starting current, increases the starting torque and also improves the power factor. The circuit diagram is shown below: In the circuit diagram, the three slip rings shown are connected to the rotor terminals of the wound rotor motor. At the time of starting of the motor, the entire external resistance is added in the rotor circuit. Then the external rotor resistance is decreased in steps as the rotor speeds up, however the motor torque remain maximum during the acceleration period of the motor. Under normal condition when the motor develops load torque the external resistance is removed. After completing this article, we are able to compare induction motor with synchronous motor. Point wise comparison between the induction motor and synchronous motor is written below; (a) Induction motor always operates at lagging power factor while the synchronous motor can operate at both lagging and leading power factor. (b) In an induction motor the value of maximum torque is directly proportional to the square of the supply voltage while in case of synchronous machine the maximum torque is directly proportional to the supply voltage. (c) In an induction motor we can easily control speed while with synchronous motor, in normal condition we cannot control speed of the motor. (d) Induction motor has inherent self starting torque while the synchronous motor has no inherent self starting torque. (e) We cannot use induction motor to improve the power factor of the supply system while with the use of synchronous motor we can improve the power factor of the supply system. (f) It is a singly excited machine means there is no requirement of dc excitation while the synchronous motor is doubly excited motor means there is requirement of separate dc excitation. (g) In case of induction motor on increasing the load the speed of the motor decreases while with the speed of the synchronous motor remains constant.

## Induction Generator

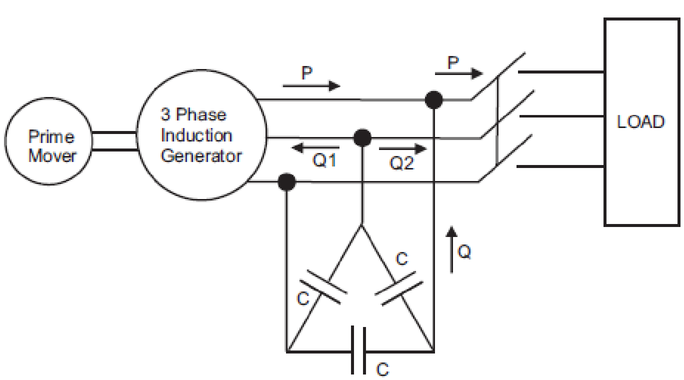
Induction machine is sometimes used as a generator. It is also called Asynchronous Generator. What are the conditions when the poly phase (here three phase) induction machine will behave as an induction generator**?** The following are conditions when the induction machine will behave as an *induction* generator are written below:

(a) Slip becomes negative due to this the rotor current and rotor emf attains negative value. (b) The prime mover torque becomes opposite to electric torque. Now let us discuss how we can achieve these conditions. Suppose that an induction machine is coupled with the prime mover whose speed can be controlled. If the speed of the prime mover is increased such that the slip becomes negative (i.e. speed of the prime mover becomes greater than the synchronous speed).

Due to this, all the conditions that we have mentioned above will become fulfilled and machine will behave like an induction generator. Now if the speed of the prime mover is further increased such that it exceeds the negative maximum value of the torque produced then the generating effect of the generator vanishes. Clearly the speed of the induction generator during the whole operation is not synchronous; therefore the induction generation is also called a synchronous generator. Induction generator is not a self excited machine therefore in order to develop the rotating magnetic field; it requires magnetizing current and reactive power. The induction generator obtains its magnetizing current and reactive power from the various sources like the supply mains or it may be another synchronous generator. The induction generator can't work in isolation because it continuously requires reactive power from the supply system. However we can have a self excited or isolated induction generation in one case if we will use capacitor bank for reactive power supply instead of AC supply system. So let us discuss isolated induction generator in detail,

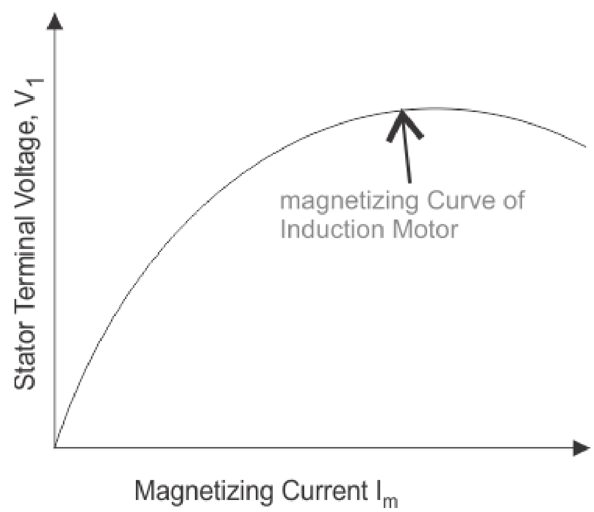
## Isolated Induction Generator

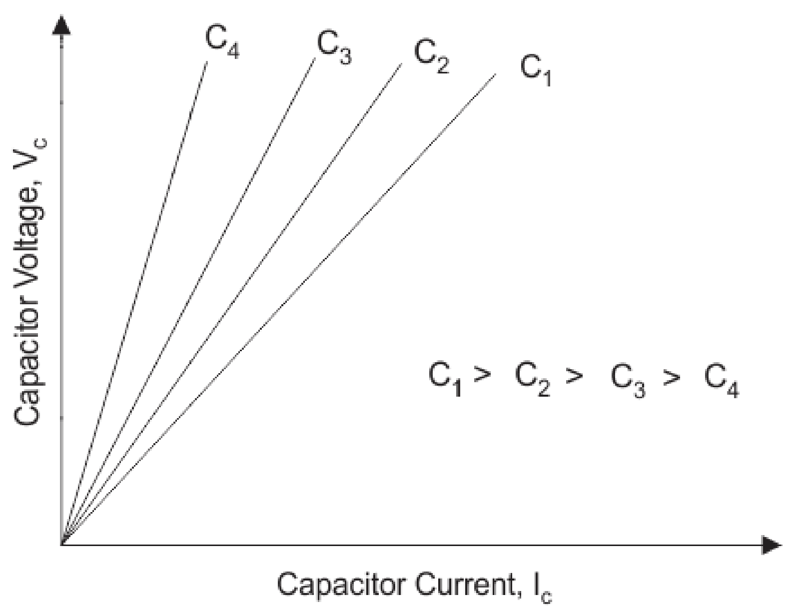
This type of generator is also known as self excited generator. Now why it is called self excited? It is because it uses capacitor bank which is connected across its stator terminals as shown in the diagram given below,

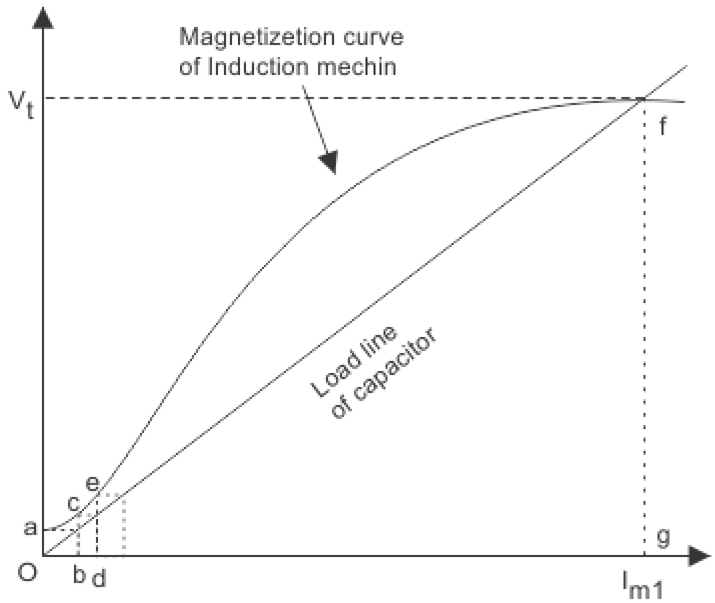


The function of the capacitor bank is to provide the lagging reactive power to the induction generator as well as load. So mathematically we can write total reactive power provided by the capacitor bank is equals to the summation of the reactive power consumed by the induction generator as well as the load. There is generation of small terminal voltage oa (as in figure given below) across the stator terminal due the residual magnetism when the rotor of the induction machine runs at the required speed. Due to this voltage oa the capacitor current ob is produced. The current bc sends current od which generates the voltage de.

The cumulative process of voltage generation continues till the saturation curve of the induction generator cuts the capacitor load line at some point. This point is marked as f in the given curve.






## Application of Induction Generator

Let us discuss application of induction generator: We have two types of induction generator let us discuss the application of each type of generator separately: Externally excited generators are widely used for regenerative breaking of hoists driven by the three phase induction motors. Self-excited generators are used in the wind mills. Thus this type of generator helps in converting the unconventional sources of energy into electrical energy. Now let us discuss some disadvantages of externally excited generator:

- The efficiency of the externally excited generator is not so good.
- We cannot use externally excited generator at lagging power factor which major drawback of this type of generator.
- The amount of reactive power used to run these types of generator required is quite large.

## Advantages of Induction Generator

1. It has robust construction requiring less maintenance. Also it is relatively cheaper.
2. It has small size per KW output power.
3. It runs in parallel without hunting
4. No synchronization to the supply line is required like a synchronous generator.

**Limitations:** It cannot generate reactive volt-amperes. It requires reactive volt-amperes from the supply line to furnish its excitation.

## Speed control of Induction Machines

We have seen the speed torque characteristic of the machine. In the stable region of operation in the motoring mode, the curve is rather steep and goes from zero torque at synchronous speed to the stall torque at a value of slip $s = \hat{s}$. Normally $\hat{s}$ may be such that stall torque is about three times that of the rated operating torque of the machine, and hence may be about 0.3 or less. This means that in the entire loading range of the machine, the speed change is quite small. The machine speed is quite stiff with respect to load changes. The entire speed variation is only in the range $n_s$ to $(1 - \hat{s})n_s$, $n_s$ being dependent on supply frequency and number of poles.

The foregoing discussion shows that the induction machine, when operating from mains is essentially a constant speed machine. Many industrial drives, typically for fan or pump applications, have typically constant speed requirements and hence the induction machine is ideally suited for these. However, the induction machine, especially the squirrel cage type, is quite rugged and has a simple construction. Therefore it is good candidate for variable speed applications if it can be achieved.

## Speed control by changing applied voltage

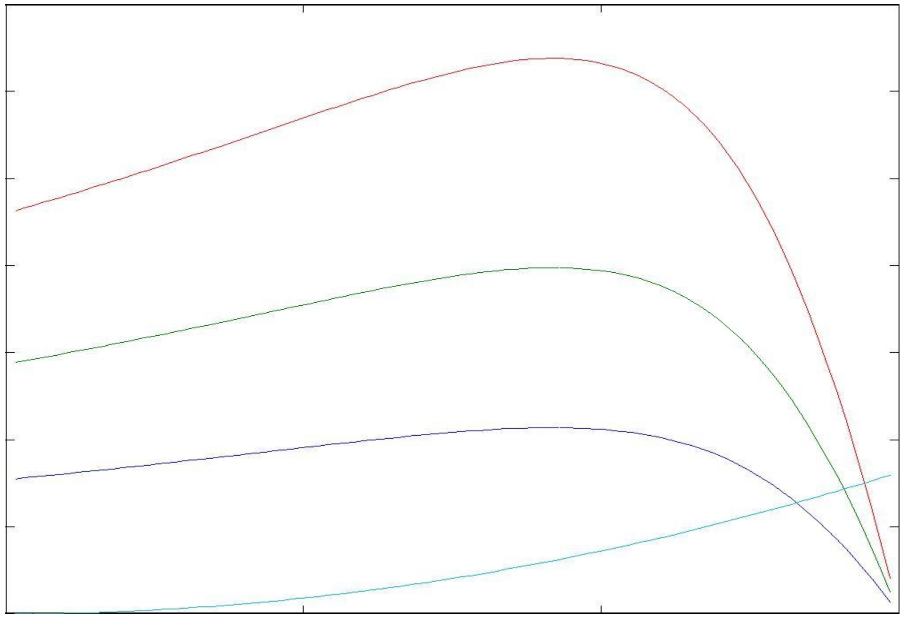
From the torque equation of the induction machine given in eqn.17, we can see that the torque depends on the square of the applied voltage. The variation of speed torque curves with respect to the applied voltage is shown in fig. 18. These curves show that the slip at maximum torque $\hat{s}$ remains same, while the value of stall torque comes down with decrease in applied voltage. The speed range for stable operation remains the same.

Further, we also note that the starting torque is also lower at lower voltages. Thus, even if a given voltage level is sufficient for achieving the running torque, the machine may not start. This method of trying to control the speed is best suited for loads that require very little starting torque, but their torque requirement may increase with speed.

Figure 18 also shows a load torque characteristic — one that is typical of a fan type of load. In a fan (blower) type of load, the variation of torque with speed is such that $T \propto \omega^2$. Here one can see that it may be possible to run the motor to lower speeds within the range $n_s$ to $(1 - \hat{s})n_s$. Further, since the load torque at zero speed is zero, the machine can start even at reduced voltages. This will not be possible with constant torque type of loads.

One may note that if the applied voltage is reduced, the voltage across the magnetizing branch also comes down. This in turn means that the magnetizing current and hence flux level are reduced. Reduction in the flux level in the machine impairs torque production

### Stator voltage variation



Speed, rpm

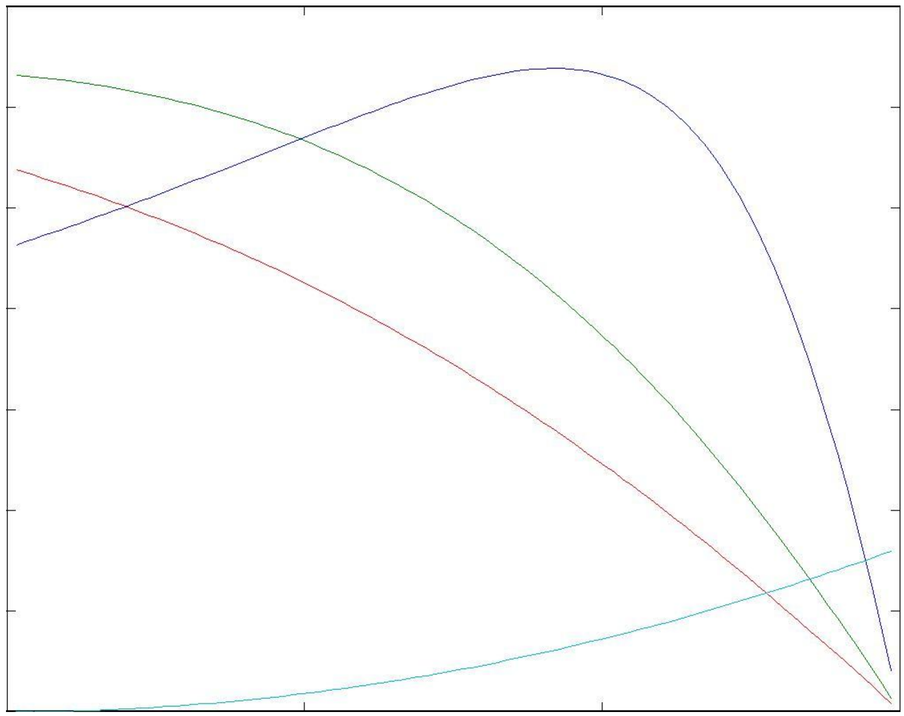
**Figure 18: Speed-torque curves: voltage variation**

If, however, the machine is running under lightly loaded conditions, then operating under rated flux levels is not required. Under such conditions, reduction in magnetizing current improves the power factor of operation. Some amount of energy saving may also be achieved.

Voltage control may be achieved by adding series resistors or a series inductor / autotransformer (a bulky solution) or a more modern solution using semiconductor devices. A typical solid state circuit used for this purpose is the AC voltage controller or AC chopper. Another use of voltage control is in the so-called 'soft-start' of the machine. This is discussed in the section on starting methods.

## Rotor resistance control

The reader may recall from eqn.17 the expression for the torque of the induction machine. Clearly, it is dependent on the rotor resistance. Further, eqn.19 shows that the maximum value is independent of the rotor resistance. The slip at maximum torque eqn.18 is dependent on the rotor resistance. Therefore, we may expect that if the rotor resistance is changed, the maximum torque point shifts to higher slip values, while retaining a constant torque. Figure 28 shows a family of torque-speed characteristic obtained by changing the rotor resistance.Rotor resistance variation



Speed, rpm

**Figure 19: Speed-torque curves: rotor resistance variation**

Note that while the maximum torque and synchronous speed remain constant, the slip at which maximum torque occurs increases with increase in rotor resistance, and so does the starting torque. Whether the load is of constant torque type or fan-type, it is evident that the speed control range is more with this method. Further, rotor resistance control could also be used as a means of generating high starting torque.

For all its advantages, the scheme has two serious drawbacks. Firstly, in order to vary the rotor resistance, it is necessary to connect external variable resistors (winding resistance itself cannot be changed). This therefore necessitates a slip-ring machine, since only in that case rotor terminals are available outside. For cage rotor machines, there are no rotor terminals. Secondly, the method is not very efficient since the additional resistance and operation at high slips entails dissipation.

The resistors connected to the slip-ring brushes should have good power dissipation capability. Water based rheostats may be used for this. A 'solid-state' alternative to a rheostat is a chopper controlled resistance where the duty ratio control of the chopper presents a variable resistance load to the rotor of the induction machine.

## Cascade control

The power drawn from the rotor terminals could be spent more usefully. Apart from using the heat generated in meaning full ways, the slip ring output could be connected to another induction machine. The stator of the second machine would carry slip frequency currents of the first machine which would generate some useful mechanical power. A still better option would be to mechanically couple the shafts of the two machines together. This sort of a connection is called cascade connection and it gives some measure of speed control as shown below.

Let the frequency of supply given to the first machine be $f_1$ , its number poles be $p_1$, and its slip of operation be $s_1$ . Let $f_2$, $p_2$ and $s_2$ be the corresponding quantities for the second machine. The frequency of currents flowing in the rotor of the first machine and hence in the stator of the second machine is $s_1f_1$. Therefore $f_2 = s_1f_1$. Since the machines are coupled at the shaft, the speed of the rotor is common for both. Hence, if n is the speed of the rotor in radians,

$$n = \frac{f_1}{p_1}(1 - s_1) = \pm \frac{s_1 f_1}{p_2}(1 - s_2). \qquad (20)$$

Note that while giving the rotor output of the first machine to the stator of the second, the resultant stator mmf of the second machine may set up an air-gap flux which rotates in the same direction as that of the rotor, or opposes it. This results in values for speed as

$$n = \frac{f_1}{p_1 + p_2} \quad \text{or} \quad n = \frac{f_1}{p_1 - p_2} \quad (s_2 \text{ negligible}) \qquad (21)$$

The latter expression is for the case where the second machine is connected in opposite phase sequence to the first. The cascade connected system can therefore run at two possible speeds

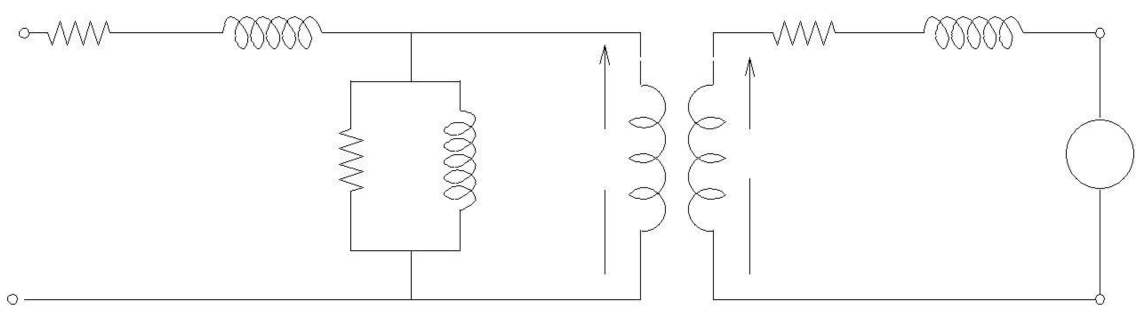


**Figure 19: Generalized rotor control**

Speed control through rotor terminals can be considered in a much more general way. Consider the induction machine equivalent circuit of fig. 29, where the rotor circuit has been terminated with a voltage source $E_r$ .

If the rotor terminals are shorted, it behaves like a normal induction machine. This is equivalent to saying that across the rotor terminals a voltage source of zero magnitude is connected. Different situations could then be considered if this voltage source $E_r$ had a non-zero magnitude. Let the power consumed by that source be $P_r$ . Then considering the rotor side circuit power dissipation per phase

$$sE_1I_2' \cos \varphi_2 = I_2' R_2' + P_r . \qquad (22)$$

Clearly now, the value of s can be changed by the value of $P_r$ . For $P_r = 0$, the machine is like a normal machine with a short circuited rotor. As $P_r$ becomes positive, for all other circuit conditions remaining constant, s increases or in the other words, speed reduces. As $P_r$ becomes negative,the right hand side of the equation and hence the slip decreases. The physical interpretation is that we now have an active source connected on the rotor side which is able to supply part of the rotor copper losses. When $P_r = -I_2'^2 R_2$ the entire copper loss is supplied by the external source. The RHS and hence the slip is zero. This corresponds to operation at synchronous speed. In general the circuitry connected to the rotor may not be a simple resistor or a machine but a power electronic circuit which can process this power requirement. This circuit may drive a machine or recover power back to the mains. Such circuits are called static kramer drives.

## Pole changing schemes

Sometimes induction machines have a special stator winding capable of being externally connected to form two different number of pole numbers. Since the synchronous speed of the induction machine is given by $n_s = f_s/p$ (in rev./s) where p is the number of pole pairs, this would correspond to changing the synchronous speed. With the slip now corresponding to the new synchronous speed, the operating speed is changed. This method of speed control is a stepped variation and generally restricted to two steps.

If the changes in stator winding connections are made so that the air gap flux remains constant, then at any winding connection, the same maximum torque is achievable. Such winding arrangements are therefore referred to as constant-torque connections. If however such connection changes result in air gap flux changes that are inversely proportional to the synchronous speeds, then such connections are called constant-horsepower type.
The following figure serves to illustrate the basic principle. Consider a magnetic pole structure consisting of four pole faces A, B, C, D as shown in fig. 20.

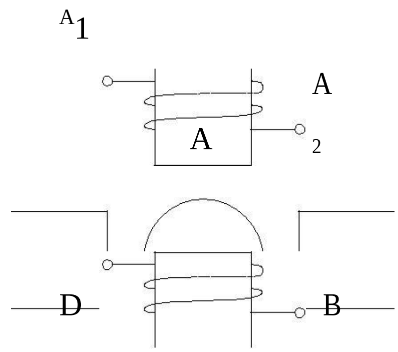



**Figure 20: Pole arrangement**

# UNIT-III

# Synchronous generators

## Introduction

Synchronous machines are principally used as alternating current generators. They supply the electric power used by all sectors of modern society. Synchronous machine is an important electromechanical energy converter. Synchronous generators usually operate in parallel forming a large power system supplying electrical power to consumers or loads. For these applications the synchronous generators are built in large units, their rating ranging from tens to hundreds of Megawatts. These synchronous machines can also be run as synchronous motors.

Synchronous machines are AC machines that have a field circuit supplied by an external DC source. Synchronous machines are having two major parts namely stationary part stator and a rotating field system called rotor.

In a synchronous generator, a DC current is applied to the rotor winding producing a rotor magnetic field. The rotor is then driven by external means producing a rotating magnetic field, which induces a 3-phase voltage within the stator winding.

Field windings are the windings producing the main magnetic field (rotor windings for synchronous machines); armature windings are the windings where the main voltage is induced (stator windings for synchronous machines).

## Types of synchronous machines

According to the arrangement of armature and field winding, the synchronous machines are classified as rotating armature type or rotating field type.

In rotating armature type the armature winding is on the rotor and the field winding is on the stator. The generated emf or current is brought to the load via the slip rings. These type of generators are built only in small units.

In case of rotating field type generators field windings are on the rotor and the armature windings are on the stator. Here the field current is supplied through a pair of slip rings and the induced emf or current is supplied to the load via the stationary terminals.

Based on the type of the prime movers employed the synchronous generators are classified as

1. Hydro generators: The generators which are driven by hydraulic turbines are called hydro generators. These are run at lower speeds less than 1000 rpm.

2. Turbo generators: These are the generators driven by steam turbines. These generators are run at very

high speed of 1500rpm or above.

3. Engine driven Generators: These are driven by IC engines. These are run at aspeed less than 1500 rpm.

## Construction of synchronous machines

1. Salient pole Machines: These type of machines have salient pole or projecting poles with concentrated field windings. This type of construction is for the machines which are driven by hydraulic turbines or Diesel engines.

2. Non salient pole or cylindrical rotor or Round rotor Machines: These machines are having cylindrical smooth rotor construction with distributed field winding in slots. This type of rotor construction is employed for the machine driven by steam turbines.

## Stator core:

The stator is the outer stationary part of the machine, which consists of

- The outer cylindrical frame called yoke**,** which is made either of welded sheet steel, cast iron.

- The magnetic path**,** which comprises a set of slotted steel laminations called stator core pressed into the cylindrical space inside the outer frame. The magnetic path is laminated to reduce eddy currents, reducing losses and heating. CRGO laminations of 0.5 mm thickness are used to reduce the iron losses.

A set of insulated electrical windings are placed inside the slots of the laminated stator. The cross-sectional area of these windings must be large enough for the power rating of the machine. For a 3-phase generator, 3 sets of windings are required, one for each phase connected in star. Fig. 1 shows one stator lamination of a synchronous generator. In case of generators where the diameter is too large stator lamination cannot be punched in on circular piece. In such cases the laminations are punched in segments. A number of segments are assembled together to form one circular laminations. All the laminations are insulated from each other by a thin layer of varnish.

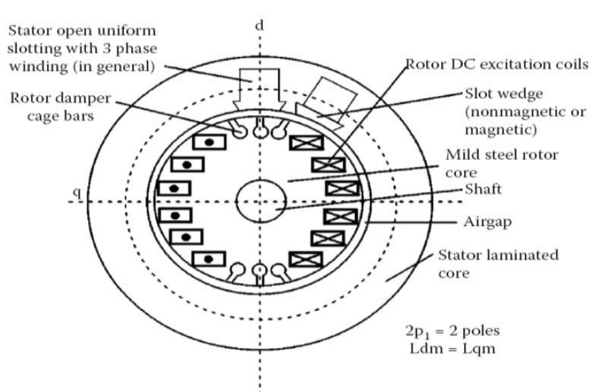



**Figure 21. Non Salient pole generator**

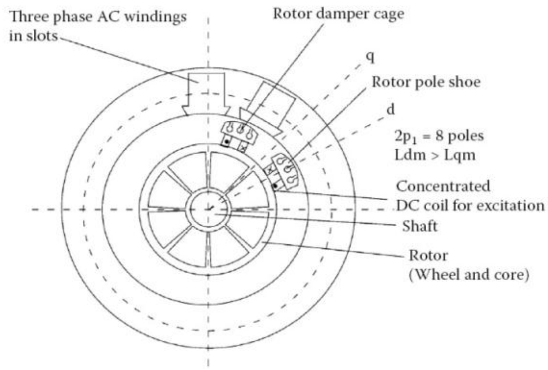



**Figure: 22. Salient pole generator**

Rotor of water wheel generator consists of salient poles. Poles are built with thin silicon steel laminations of 0.5mm to 0.8 mm thickness to reduce eddy current laminations. The laminations are clamped by heavy end plates and secured by studs or rivets. For low speed rotors poles have the bolted on construction for the machines with little higher peripheral speed poles have dove tailed construction as shown in Figs. Generally rectangular or round pole constructions are used for such type of alternators. However the round poles have the advantages over rectangular poles.

Generators driven by water wheel turbines are of either horizontal or vertical shaft type. Generators with fairly higher speeds are built with horizontal shaft and the generators with higher power ratings and low speeds are built with vertical shaft design. Vertical shaft generators are of two types of designs (i) Umbrella type where in the bearing is mounted below the rotor. (ii) Suspended type where in the bearing is mounted above the rotor.

In case of turbo alternator the rotors are manufactured form solid steel forging. The rotor is slotted to accommodate the field winding. Normally two third of the rotor periphery is slotted to accommodate the winding and the remaining one third unslotted portion acts as the pole. Rectangular slots with tapering teeth are milled in the rotor. Generally rectangular aluminum or copper strips are employed for filed windings. The field windings and the overhangs of the field windings are secured in place by steel retaining rings to protect against high centrifugal forces. Hard composition insulation materials are used in the slots which can with stand high forces, stresses and temperatures. Perfect balancing of the rotor is done for such type of rotors.

Damper windings are provided in the pole faces of salient pole alternators. Damper windings are nothing but the copper or aluminum bars housed in the slots of the pole faces. The ends of the damper bars are short circuited at the ends by short circuiting rings similar to end rings as in the case of squirrel cage rotors. These damper windings are serving the function of providing mechanical balance; provide damping effect, reduce the effect of over voltages and damp out hunting in case of alternators. In case of synchronous motors they act as rotor bars and help in self starting of the motor.

## Relation between Speed and Frequency:

In the previous course on induction motors it is established that the relation between speed and frequency and number of poles is given by

Frequency $f = P \times N / 120$ Hz

## Operation of Alternators:

Similar to the case of DC generator, the behavior of a Synchronous generator connected to an external load is different than that at no-load. In order to understand the performance of the Synchronous generator when it is loaded, consider the flux distributions in the machine when the armature also carries a current. Unlike in the DC machine in alternators the emf peak and the current peak will not occur in the same coil due to the effect of the power factor of the load. The current and the induced emf will be at their peaks in the same coil only for UPF loads. For zero power factor lagging loads, the current reaches its peak in a coil which falls behind that coil wherein the induced emf is at its peak by 90 electrical degrees or half a pole-pitch. Likewise for zero power factor leading loads, the current reaches its peak in a coil which is ahead of that coil wherein the induced emf is at its peak by 90 electrical degrees or half a pole-pitch. For simplicity, assume the resistance and leakage reactance of the stator windings to be negligible. Also assume the magnetic circuit to be linear i.e. the flux in the magnetic circuit is deemed to be proportional to the resultant ampere-turns - in other words the machine is operating in the linear portion of the magnetization characteristics. Thus the emf induced is the same as the terminal voltage, and the phase-angle between current and emf is determined only by the power factor (pf) of the external load connected to the synchronous generator.

## Windings in Alternators:

In case of three phase alternators the following types of windings are employed.

- Lap winding,
- wave winding and
- Mush winding.

Based on pitch of the coil

- full pitched
- short pitched windings

Based on number of layers

- Single layer
- Double layer

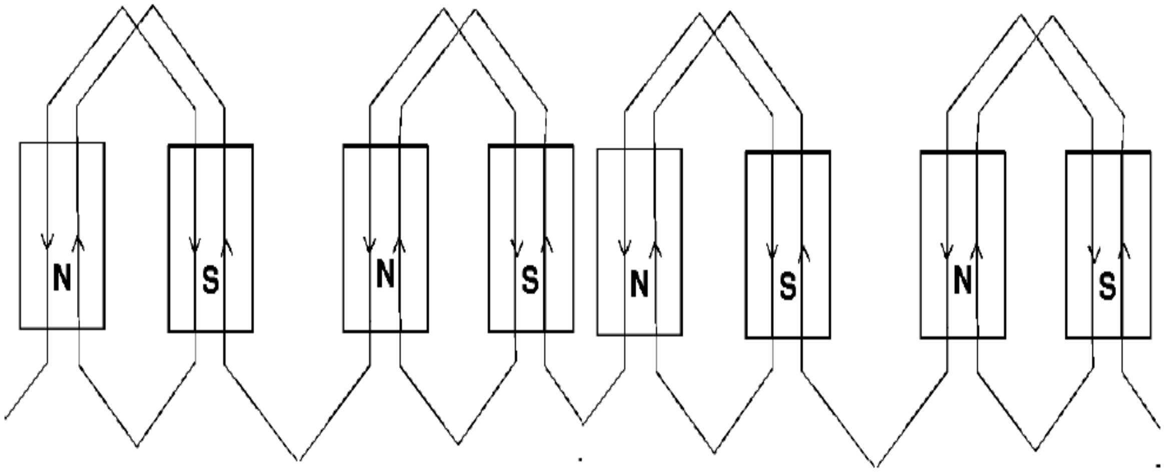



**Figure: 23. Single layer winding**

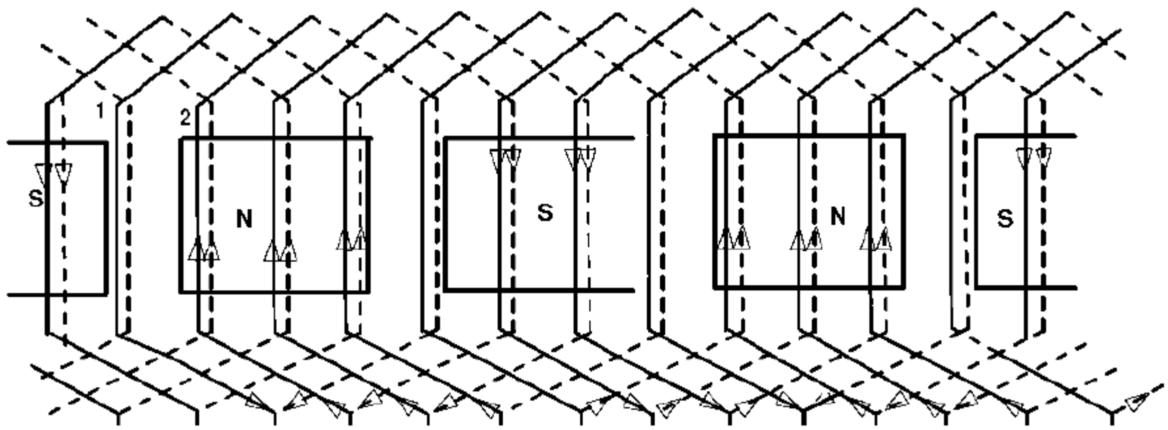



**Figure: 24. Double layer winding**

## EMF Equation of an alternator:

Consider the following $\Phi$ = flux per pole in wb

P = Number of poles

$N_s$ = Synchronous speed in rpm

f = frequency of induced EMF in Hz

Z = total number of stator conductors

$Z_{ph}$ = conductors per phase connected in series

$T_{ph}$ = Number of turns per phase

Assuming concentrated winding, considering one conductor placed in a slot

According to Faradays Law electromagnetic induction,

The average value of EMF induced per conductor in one revolution

$e_{avg}$ = d /dt $e_{avg}$ = Change of Flux in one revolution/ Time taken for one revolution

Change of Flux in one revolution = p

Time taken for one revolution = $60/N_s$ seconds

Hence $e_{avg}$ = (p) / ($60/N_s$) = $PN_s$ / 60

We know f = $PN_s$ /120

Hence $PN_s$ /60 = 2f

Hence $e_{avg}$ = 2f volts

Hence average EMF per turn = 2 x 2f volts = 4f volts

If there are $T_{ph,}$ number of turns per phase connected in series, then average emf induced in $T_{ph}$ turns is

$E_{ph,\,avg} = T_{ph} \times e_{avg} = 4\,f\Phi T_{ph}$ volts

Hence RMS value of EMF induced E = 1.11 x $E_{phavg}$

$$= 1.11 \times 4\,f\,\Phi T_{ph} \text{ volts}$$

$$= 4.44\,f\,\Phi T_{ph} \text{ volts}$$

This is the general EMF equation for the machine having concentrated and full pitched winding.

In practice, alternators will have short pitched winding and hence coil span will not be $180^0$, but one or two slots short than the full pitch.

**Pitch Factor:**

Pitch factor $K_p$= EMF induced in a short pitched coil/ EMF induced in a full pitched coil = (2E cos α/2)/ 2E

$$K_p = \cos \alpha/2$$

Where α is called chording angle.

**Distribution Factor:** Even though we assumed concentrated winding in deriving EMF equation, in practice an attempt is made to distribute the winding in all the slots coming under a pole. Such a winding is called distributed winding.

In concentrated winding the EMF induced in all the coil sides will be same in magnitude and in phase with each other. In case of distributed winding the magnitude of EMF will be same but the EMF s induced in each coil side will not be in phase with each other as they are distributed in the slots under a pole. Hence the total EMF will not be same as that in concentrated winding but will be equal to the vector sum of the EMF s induced. Hence it will be less than that in the concentrated winding. Now the factor by which the EMF induced in a distributed winding gets reduced is called distribution factor and defined as the ratio of EMF induced in a distributed winding to EMF induced in a concentrated winding.

Distribution factor $K_d$ = EMF induced in a distributed winding/ EMF induced in a concentrated winding = vector sum of the EMF / arithmetic sum of the EMF Let

E = EMF induced per coil side

m = number of slots per pole per phase,

n = number of slots per pole

$\beta$ = slot angle = 180/n

The EMF induced in concentrated winding with m slots per pole per phase = mE volts.

Fig below shows the method of calculating the vector sum of the voltages in a distributed winding having a mutual phase difference of $\beta$. When m is large curve ACEN will form the arc of a circle of radius r.

From the figure below AC = 2rsin $\beta/2$

Hence arithmetic sum = m2r sin$\beta/2$

Now the vector sum of the EMF s = 2rsin m$\beta/2$

Hence the distribution factor $K_d$ = vector sum of the EMF / arithmetic sum of the EMF = (2r sin m$\beta$/2) / (m x 2r sin $\beta$/2)

$$K_d = (\sin m\beta/2) / (m \sin \beta/2)$$

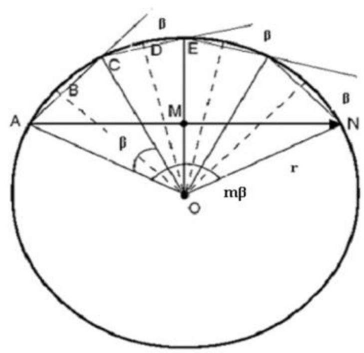



**Figure: 25 Calculation of vector sum**

In practical machines the windings will be generally short pitched and distributed over the periphery of the machine. Hence in deducing the EMF equation both pitch factor and distribution factor has to be considered. Hence the general EMF equation including pitch factor and distribution factor can be given as EMF induced per phase = 4.44 f $T_{ph}$ x $K_pK_d$ volts

$E_{ph}$ = 4.44 $K_pK_d$ f $T_{ph}$ vlolts

Hence the line Voltage $E_L = \sqrt{3}$ x phase voltage = $\sqrt{3}\, E_{ph}$

**Harmonics:** When the uniformly sinusoidal distributed air gap flux is cut by either the stationary or rotating armature sinusoidal EMF is induced in the alternator. Hence the nature of the waveform of induced EMF and current is sinusoidal. But when the alternator is loaded waveform will not continue to be sinusoidal or becomes non sinusoidal. Such non sinusoidal wave form is called complex wave form. By using Fourier series representation it is possible to represent complex non sinusoidal waveform in terms of series of sinusoidal components called harmonics, whose frequencies are integral multiples of fundamental wave. The fundamental wave form is one which is having the frequency same as that of complex wave.

The waveform which is of the frequency twice that of the fundamental is called second order harmonic. The one which is having the frequency three times that of the fundamental is called third harmonic and so on. These harmonic components can be represented as follows.

Fundamental: $e_1 = E_{m1} \sin(\omega t \pm \theta_1)$

2nd Harmonic $e_2 = E_{m2} \sin(2\omega t \pm \theta_2)$

3rd Harmonic $e_3 = E_{m3} \sin(3\omega t \pm \theta_3)$

5th Harmonic $e_5 = E_{m5} \sin(5\omega t \pm \theta_5)$ etc.

In case of alternators as the field system and the stator coils are symmetrical the induced EMF will also be symmetrical and hence the generated EMF in an alternator will not contain any even harmonics.

**Slot Harmonics:** As the armature or stator of an alternator is slotted, some harmonics are induced into the EMF which is called slot harmonics. The presence of slot in the stator makes the air gap reluctance at the surface of the stator non uniform. Since in case of alternators the poles are moving or there is a relative motion between the stator and rotor, the slots and the teeth alternately occupy any point in the air gap. Due to this the reluctance or the air gap will be continuously varying. Due to this variation of reluctance ripples will be formed in the air gap between the rotor and stator slots and teeth. This ripple formed in the air gap will induce ripple EMF called slot harmonics.

**Minimization of Harmonics:** To minimize the harmonics in the induced waveforms following methods are employed:

1. Distribution of stator winding.
2. Short Chording
3. Fractional slot winding
4. Skewing
5. Larger air gap length.

**Effect of Harmonics on induced EMF:**

The harmonics will affect both pitch factor and distribution factor and hence the induced EMF. In a well designed alternator the air gap flux density distribution will be symmetrical and hence can be represented in Fourier series as follows.

$B = B_{m1}\sin \omega t + B_{m3} \sin 3\omega t + B_{m5}\sin 5\omega t + \ldots\ldots\ldots\ldots\ldots\ldots$

The EMF induced by the above flux density distribution is given by

$e = E_{m1}\sin \omega t + E_{m3} \sin 3\omega t + E_{m5}\sin 5\omega t + \ldots\ldots\ldots\ldots\ldots\ldots$

The RMS value of the resultant voltage induced can be given as

$E_{ph} = \sqrt{[(E_1)^2 + (E_3)^2 + (E_5)^2 + \ldots\ldots\ldots\ldots\ldots (E_n)^2]}$

And line voltage $E_{Line} = \sqrt{3} \times E_{ph}$

**Effect of Harmonics of pitch and distribution Factor:**

The pitch factor is given by $K_p = \cos \alpha/2$, where α is the chording angle.

For any harmonic say $n^{th}$ harmonic the pitch factor is given by $K_{pn} = \cos n\alpha/2$

The distribution factor is given by $K_d = (\sin m\beta/2) / (m \sin \beta/2)$

For any harmonic say $n^{th}$ harmonic the distribution factor is given by $K_{dn} = (\sin m\, n\beta/2) / (m \sin n\beta/2)$

## Armature Reaction:

### Magnetic fluxes in alternators

There are three main fluxes associated with an alternator:

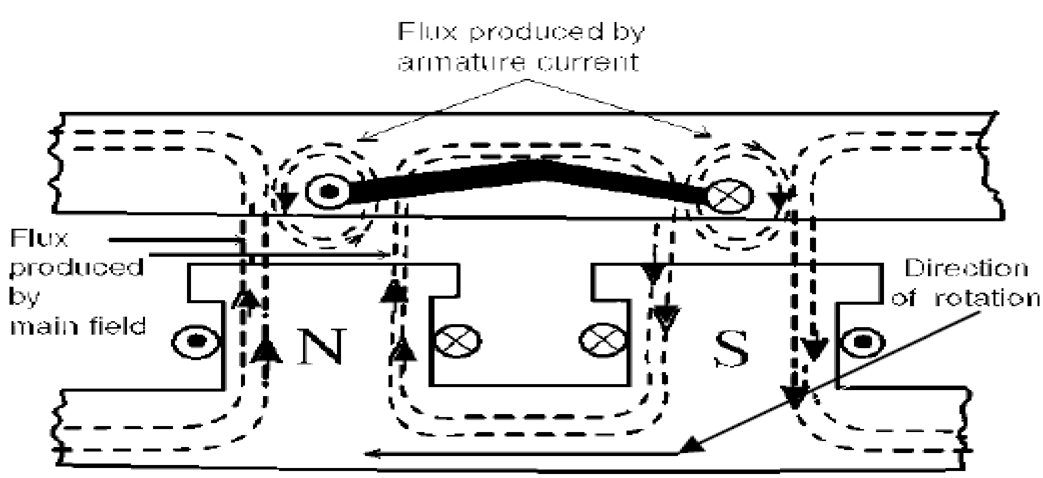
i) Main useful flux linked with both field & armature winding.

ii) Leakage flux linked only with armature winding.

iii) Leakage flux linked only with field winding.

The useful flux which links with both windings is due to combined MMF of the armature winding and field winding. When the armature winding of an alternator carries current then an MMF sets in armature. This armature MMF reacts with field MMF producing the resultant flux, which differs from flux of field winding alone. The effect of armature reaction depends on nature of load (power factor of load). At no load condition, the armature has no reaction due to absence of armature flux. When armature delivers current at unity power factor load, then the resultant flux is displaced along the air gap towards the trailing pole tip. Under this condition, armature reaction has distorting effect on MMF wave as shown in Figure. At zero lagging power factor loads the armature current is lagging by 90° with armature voltage. Under this condition, the position of armature conductor when inducing maximum EMF is the centre line of field MMF. Since there is no distortion but the two MMF are in opposition, the armature reaction is now purely demagnetizing as shown in Figure. Now at zero power factors leading, the armature current leads armature voltage by 90°. Under this condition, the MMF of armature as well as the field winding is in same phase and additive. The armature MMF has magnetizing effect due to leading armature current as shown in below figure.

### (a) Unity Power Factor



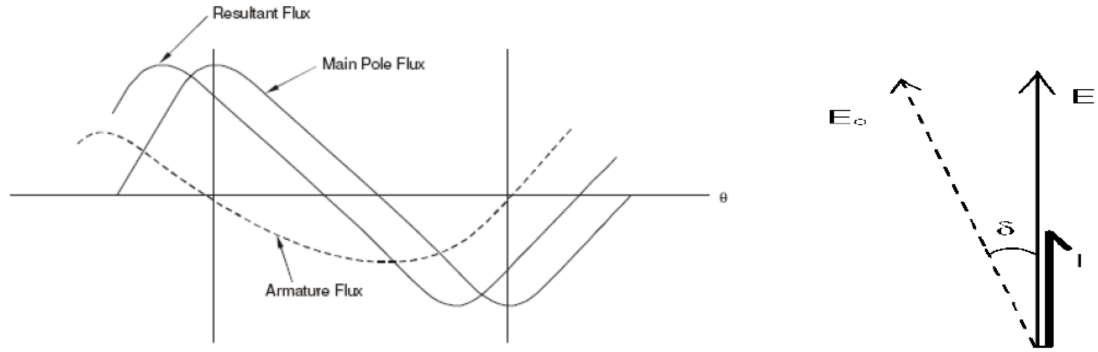



**Figure: 26 Distorting effect of armature reaction**

## (b) Zero Power Factor Lagging

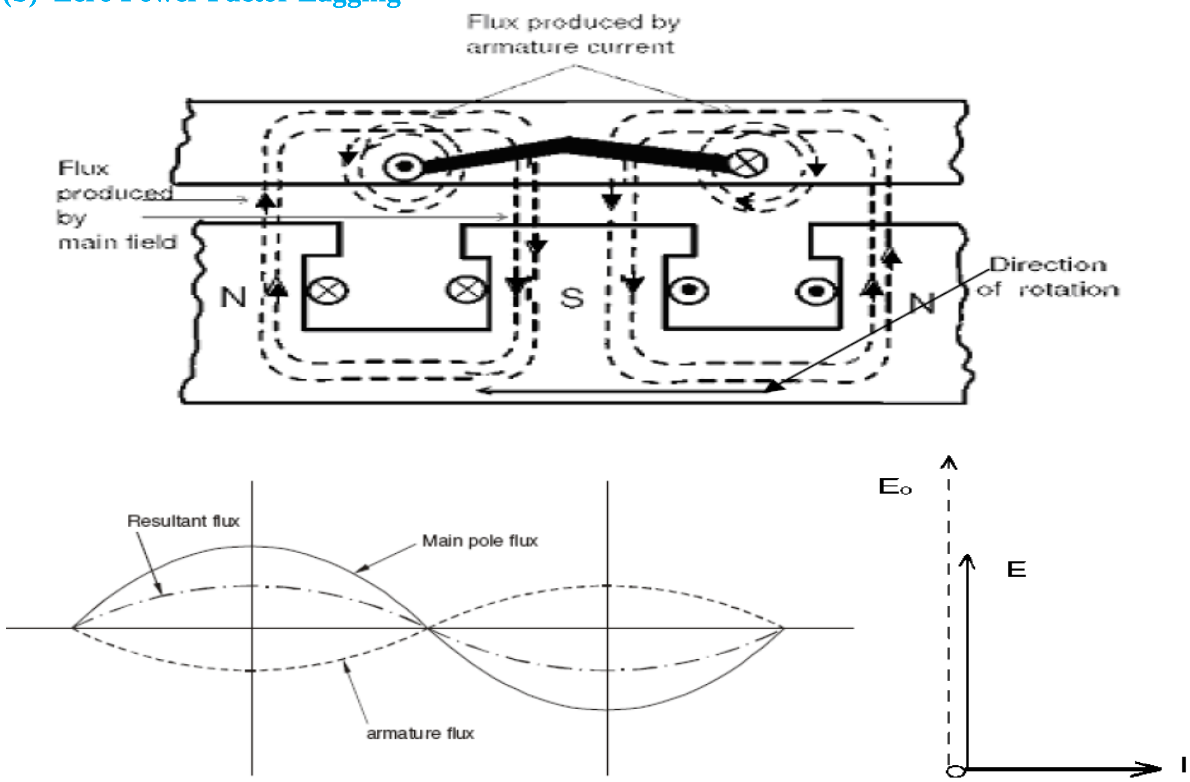



**Figure 27. Demagnetizing effect of armature reaction**

## (c) Zero Power Factor Leading

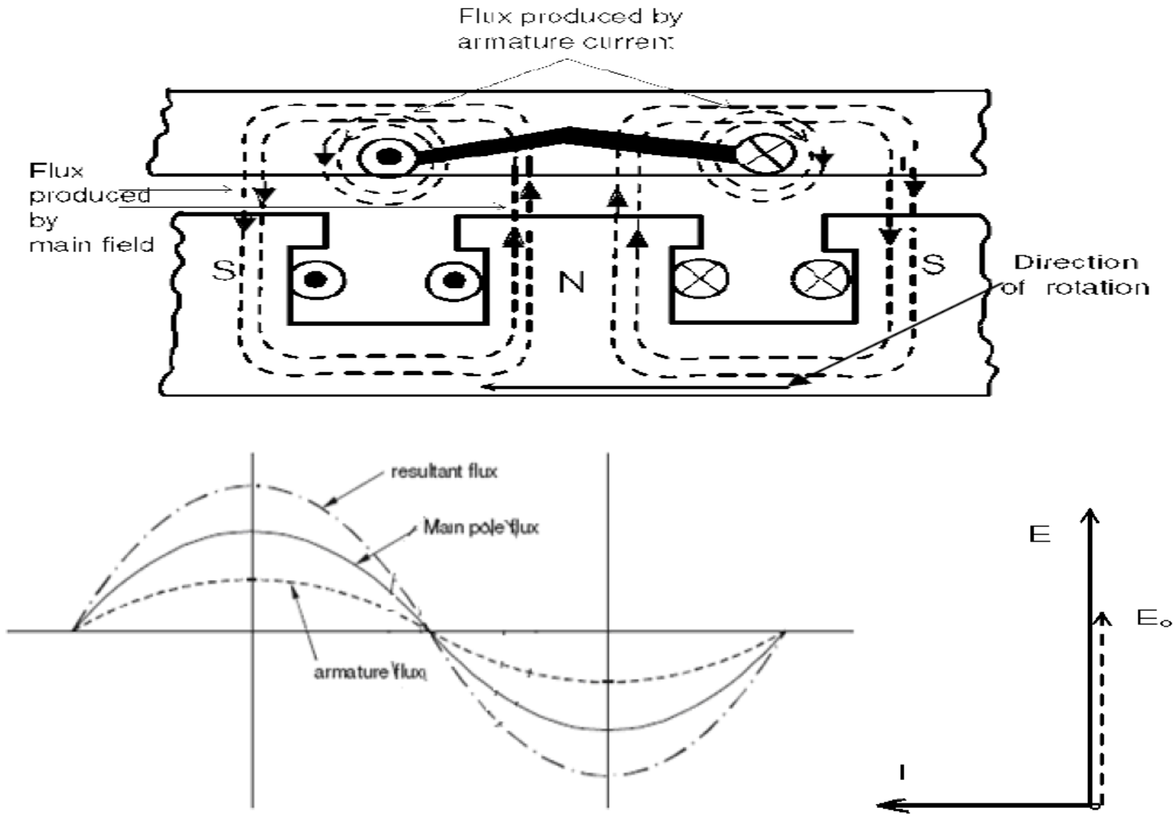


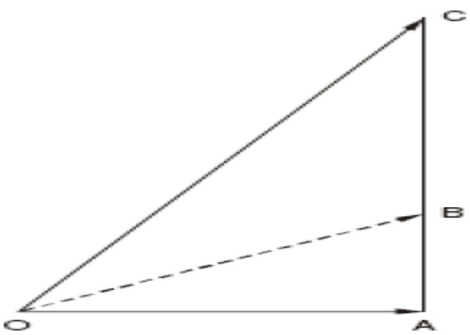

**Figure 28. Magnetizing effect of armature reaction**

When the rotor is run, a voltage E is induced in the stator windings. If a load is connected to the terminals of the generator, a current flows. The 3-phase stator current flow will produce a magnetic field of its own. This stator magnetic field will distort the original rotor magnetic field, changing the resulting phase voltage. This effect is called armature reaction because the armature (stator) current affects the magnetic field.

From the phasor diagrams of the armature reaction it can be seen that $E_0$ is the EMF induced under no load condition and E can be considered as the EMF under loaded condition. It can also be understood that the $E_0$ is the EMF induced due to the field winding acting alone and E is the EMF induced when both field winding and stator winding are acting in combination. Hence EMF E can be considered as sum of $E_0$ and another fictitious EMF $E_a$ proportional to the stator current. From the figures it can be seen that the EMF $E_a$ is always in quadrature with current. This resembles the EMF induced in an inductive reactance. Hence the effect of armature reaction is exactly same as if the stator has an additional reactance $x_a= E_a/I$. This is called the armature reaction reactance. The leakage reactance is the true reactance and the armature reaction reactance is a fictitious reactance.

## Synchronous Reactance and Synchronous Impedance

The synchronous reactance is an equivalent reactance the effects of which are supposed to reproduce the combined effects of both the armature leakage reactance and the armature reaction. The alternator is supposed to have no armature reaction at all, but is supposed to possess an armature reactance in excess of its true leakage reactance. When the synchronous reactance is combined vectorially with the armature resistance, a quantity called the synchronous impedance is obtained as shown in figure.




OA = Armature Resistance

AB = Leakage Reactance

BC = Equivalent Reactance of Armature Reaction

AC = Synchronous Reactance

OC = Synchronous Impedance

The armature winding has one more reactance called armature reaction reactance in addition to leakage reactance and resistance. Considering all the three parameters the equivalent circuit of a synchronous generator can be written as shown below. The sum of leakage reactance and armature reaction reactance is called synchronous reactance $X_s$. Under this condition impedance of the armature winding is called the synchronous impedance $Z_s$.

Hence synchronous reactance $\mathbf{X_s = X_l + X_a}$ per phase and

synchronous impedance $\mathbf{Z_s = R_a + j\, X_s}$ per phase

As the armature reaction reactance is dependent on armature current so is synchronous reactance and hence synchronous impedance is dependent on armature current or load current.

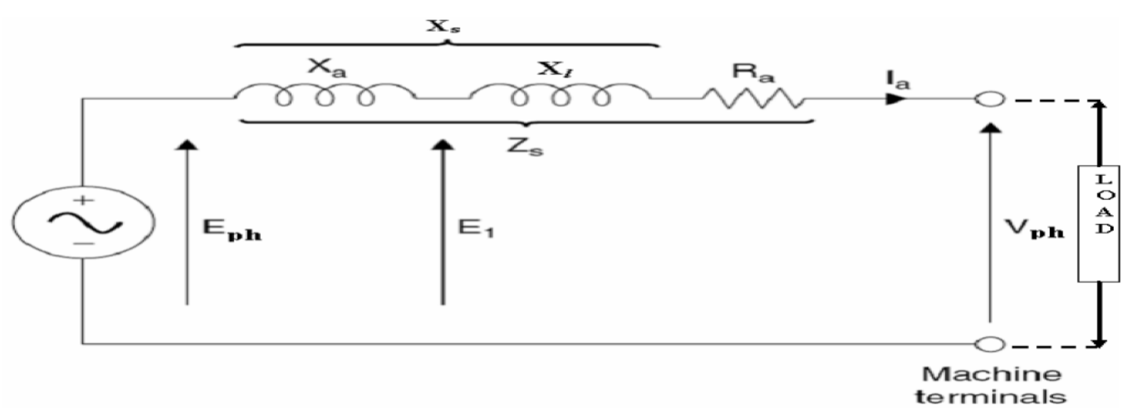



**Figure: 29. Equivalent circuit of alternator**

## Phasor diagram

In the phasor diagrams E is the induced EMF /phase = $E_{ph}$ and V is the terminal voltage /phase = $V_{ph}$. From each of the phasor diagrams the expression for the induced EMF $E_{ph}$ can be expressed in terms of $V_{ph}$, armature current, resistance, reactance's and impedance of the machine as follows.

### (i) Unity power factor load

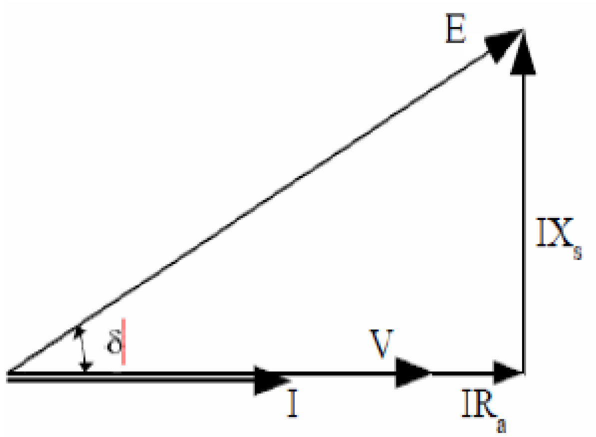



**Figure: 30. Phasor diagram at Unity power factor load**

Under unity power factor load: $E_{ph} = (V + IR_a) + j\,(IX_S)$

$$E_{ph} = \sqrt{[\,(V + IR_a)^2 + (IX_S)^2]}$$

### (ii) Zero power factor lagging

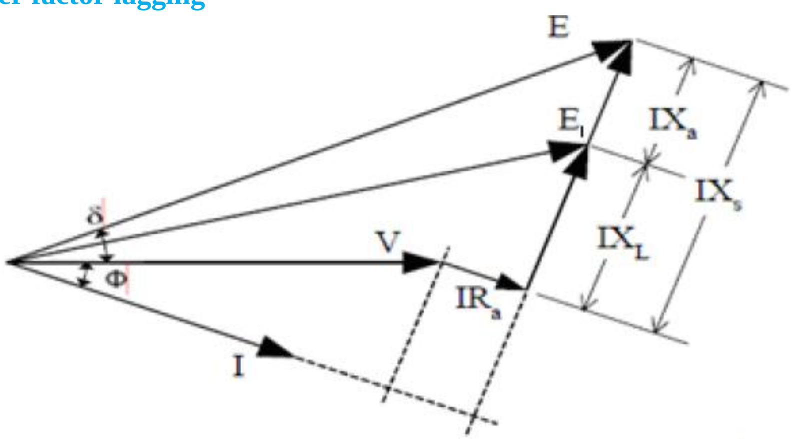



**Figure: 31. Phasor diagram at zero power factor lagging**

Under zero power factor lagging: $E_{ph} = V + (IR_a + j\ IX_S) = V + I(R_a + j\ X_S)$

The above expression can also be written as $E_{ph} = \sqrt{[}\ (V \cos\ \ + IR_a)^2 + (V \sin\ \ + IX_S)^2]$

### a) Zero power factor leading

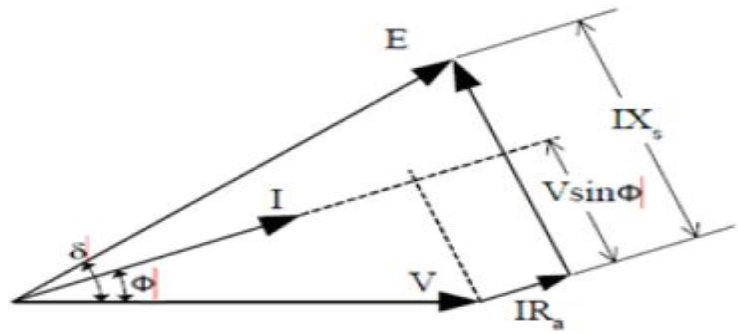



**Figure: 32. Phasor diagram at zero power factor leading.**

## Numerical problems:

**1.** A 3Φ, 50 Hz, star connected salient pole alternator has 216 slots with 5 conductors per slot. All the conductors of each phase are connected in series; the winding is distributed and full pitched. The flux per pole is 30 mwb and the alternator runs at 250 rpm. Determine the phase and line voltages of EMF induced.

**Slon:** $N_S = 250$ rpm, f = 50 Hz,

$P = 120 \times f/N_S = 120 \times 50/250 = 24$ poles

m = number of slots/pole/phase = 216/(24 x 3) = 3

$\beta = 180^0$ / number of slots/pole $= 180^0$ / (216/24) $= 20^0$ Hence distribution factor $K_d$ = ( sin mβ/2) / (m sin β/2)

= ( sin 3 x 20 / 2) / (3 sin 20/2)

= 0.9597

Pitch factor $K_p = 1$ for full pitched winding. We have emf induced per conductor

$T_{ph} = Z_{ph}/2$ ; $Z_{ph} = Z/3$

Z = conductor/ slot x number of slots $T_{ph} = Z/6 = 216 \times 5 /6 = 180$

Therefore $E_{ph} = 4.44\ K_pK_d\ f\ \text{8}\ T_{ph}$ vlolts

$= 4.44 \times 1 \times 0.9597 \times 50 \times 30 \times 10^{-3} \times 180 = 1150.488$ volts

Hence the line Voltage $E_L = \sqrt{3}$ x phase voltage $= \sqrt{3}\ E_{ph} = \sqrt{3} \times 1150.488 = 1992.65$ volts

**2.** A 3Φ, 16 pole, star connected salient pole alternator has 144 slots with 10 conductors per slot. The alternator is run at 375 rpm. The terminal voltage of the generator found to be 2.657 KV. Determine the frequency of the induced EMF and the flux per pole.

**Soln:** $N_S$ = 375 rpm, p =16, slots = 144, Total no. of conductors = 144 x 10 = 1440 $E_L$ = 2.657 kV,

f = P $N_S$/120 = 16 x 375/120 = 50 Hz

Assuming full pitched winding $k_p$ = 1

Number of slots per pole per phase = 144/(16 x 3) = 3

Slot angle β = $180^0$ / number of slots/pole = $180^0$ /9 = $20^0$

Hence distribution factor $K_d$ = ( sin mβ/2) / (m sin β/2)

= ( sin 3 x 20 / 2) / (3 sin 20/2)

= 0.9597

Turns per phase $T_{ph}$ = 144 x 10/ 6 = 240 $E_{ph}$ = $E_L$/√3 = 2.657/√3 = 1.534 kV

$E_{ph}$ = 4.44 $K_pK_d$ f 8 $T_{ph}$ vlolts

1534.0 = 4.44 x 1 x 0.9597 x 50 x 8 x 240

8 = 0.03 wb = 30 mwb

**3.** A 4 pole, 3 phase, 50 Hz, star connected alternator has 60 slots with 4 conductors per slot. The coils are short pitched by 3 slots. If the phase spread is $60^0$, find the line voltage induced for a flux per pole of 0.943 wb.

**Slon:** p = 4, f = 50 Hz, Slots = 60, cond/slot = 4 , short pitched by 3 slots, phase spread = $60^0$, Φ = 0.943 wb

Number of slots/pole/phase m = 60/(4 x 3) = 5

Slot angle β = phase spread/ number of slots per pole/phase = 60/5 = 12

Distribution factor $k_d$ = (sin mβ/2) / (m sinβ/2)

= sin ( 5 x 12/2) / 5 sin(12/2)

= 0.957

Pitch factor = cos α/2

Coils are short chorded by 3 slots Slot angle = 180/number of slots/pole = 180/15 = 12

Therefore coil is short pitched by α = 3 x slot angle = 3 x 12 = $36^0$

Hence pitch factor $k_p$ = cos α/2 = cos 36/2 = 0.95

Number of turns per phase $T_{ph}$ = $Z_{ph}$/2 = (Z/3)/2 = Z /6 = 60 x 4 /6 = 40 EMF induced per phase $E_{ph}$ = 4.44 $k_p$ $k_d$ f Φ $T_{ph}$ volts

= 4.44 x 0.95 x 0.957 x 50 x 0.943 x 40

$= 7613$ volts

Line voltage $E_L = \sqrt{3} \times E_{ph}$

$= \sqrt{3} \times 7613 = 13185$ volts

**4.** In a 3 phase star connected alternator, there are 2 coil sides per slot and 16 turns per coil. The stator has 288 slots. When run at 250 rpm the line voltage is 6600 volts at 50 Hz. The coils are shot pitched by 2 slots. Calculate the flux per pole.

**Slon:** $N_S = 250$ rpm, f = 50 Hz, slots = 288, $E_L = 6600$ volts, 2 coilsides/slot, 16 turns /coil Short pitched by 2 slots

Number of poles = 120f/ $N_S$ = 120 x 50/250 = 24

Number of slots /pole/phase m = 288 / ( 24 x 3) = 4

Number of slots /pole = 288 / 24 = 12

Slot angle β = 180/ number of slots per pole = 180 / 12 = $15^0$

Distribution factor $k_d = (\sin m\beta/2) / (m \sin\beta/2)$

$= \sin(4 \times 15/2) / 4 \sin(15/2)$

$= 0.9576$

Coils are short chorded by 2 slots Slot angle = 15

Therefore coil is short pitched by α = 2 x slot angle = 2 x 15 = $30^0$ Hence pitch factor $k_p = \cos \alpha/2 = \cos 30/2 = 0.9659$

Two coil sides per slot and 16 turns per coil

Total number of conductors per slot = 2 x 16 = 32 turns Total conductors = 32 x 288

Turns per phase = 32 x 288 / 6 = 1536

$E_{ph} = 6600 / \sqrt{3} = 3810.51$ volts,

We have EMF induced per phase $E_{ph} = 4.44\, k_p\, k_d\, f\, \Phi\, T_{ph}$ volts

$3810.51 = 4.44 \times 0.9659 \times 0.9576 \times 50 \times \Phi \times 1536$

$\Phi = 0.02$ wb

**5.** A 10 pole, 600 rpm, 50Hz, alternator has the following sinusoidal flux density distribution. $B = \sin\theta + 0.4 \sin 3\theta + 0.2 \sin 5\theta$ wb/m$^2$. The alternator has 180 slots with 2 layer 3 turn coils with a coil span of 15 slots. The coils are connected in $60^0$ groups. If the armature diameter is 1.2 m and core length is 0.4 m, calculate (a) the expression for instantaneous emf/conductor (b) the expression for instantaneous emf/coil (c) the phase and line voltages if the machine is star connected.

**Slon:** Area under one pole pitch = π DL/p = π x 1.2 x 0.4/10 = 0.1508 m$^2$ Fundamental flux/pole, $\Phi_1$ = average flux density x area

$= 2/\pi \times 1 \times 0.1508$

$= 0.096$ wb

(a) rms value of emf induced/conductor = $2.22 f \Phi_1 = 2.22 \times 50 \times 0.096 = 10.656$ volts

maximum value of emf/conductor = $\sqrt{2} \times 10.656 = 15.07$ volts

$3^{rd}$ harmonic voltage = $0.4 \times 15.07 = 6.02$ volts

$5^{th}$ harmonic voltage = $0.2 \times 15.07 = 3.01$ volts

The expression for instantaneous emf/conductor $e = 15.07 \sin\theta + 6.02 \sin 3\theta + 3.01 \sin 5\theta$ volts

(b) conductors/slot = 6 = conductors/coil, slots = 180, coil span = 15 slots slots/pole = 18

slot angle $\beta$ = 180/number of slots/ pole = 180/ 18 = $10^0$ coil is short chorded by 3 slots hence $\alpha = 30^0$

Pitch factor $k_{pn} = \cos n\alpha/2$

$k_{p1} = \cos\alpha/2 = \cos 30/2 = 0.9659$ $k_{p3} = \cos 3 \times 30/2 = 0.707$

$k_{p5} = \cos 5 \times 30/2 = 0.2588$

Fundamental rms value of emf induced/coil = $2.22\, k_p\, f\, \Phi_1\, Z$

$= 2.22 \times 0.9659 \times 50 \times 0.096 \times 6$

$= 61.76$ volts

Maximum value of emf induced/coil = $\sqrt{2} \times 61.76 = 87.34$ volts Similarly

$3^{rd}$ harmonic voltage = 25.53 volts

$5^{th}$ harmonic voltage = 4.677 volts

expression for instantaneous emf/coil $e = 87.34 \sin\theta + 25.53 \sin 3\theta + 4.677 \sin 5\theta$ volts

slot angle $\beta$ = 180/number of slots/ pole = 180/ 18 = $10^0$

number of slots/pole/phase = 180/(10 x 3) = 6 Distribution factor $k_{dn} = (\sin m n\beta/2) / (m \sin n\beta/2)$

$k_{d1} = \sin(6 \times 10/2) / 6 \sin(10/2) = 0.956$

$k_{d3} = \sin(6 \times 3 \times 10/2) / 6 \sin(3 \times 10/2) = 0.644$

$k_{d5} = \sin(6 \times 5 \times 10/2) / 6 \sin(5 \times 10/2) = 0.197$

Turns/phase $T_{ph} = 180 \times 6/6 = 180$

rms value of emf induced = $4.44\, k_{pn}\, k_{dn}\, (nf)\, \Phi_n\, T_{ph}$ for any nth harmonic

fundamental voltage $E_{ph1} = 4.44\, k_{p1}\, k_{d1}\, f\, \Phi_1\, T_{ph}$

$= 4.44 \times 0.9659 \times 0.956 \times 50 \times 0.096 \times 180$

$= 3542.68$ volts

Similarly 3rd harmonic voltage $E_{ph3}$ = 697.65 volts

5th harmonic voltage $E_{ph5}$ = 39.09 volts

Phase voltage = $\sqrt{(E^2_{ph1} + E^2_{ph3} + E^2_{ph5})}$

$= \sqrt{(3542.68^2 + 697.65^2 + 39.09^2)}$

= 3610.93 volts

Line voltage = $\sqrt{3} \times \sqrt{(E^2_{ph1} + E^2_{ph5})}$

$= \sqrt{3} \times \sqrt{(3542.68^2 + 39.09^2)}$

= 6136.48 volts

**6.** A three phase 600 kVA, 400 volts, delta connected alternator is reconnected in star. Calculate its new ratings in terms of voltage, current and volt-ampere.

**Slon:** (i) when the machine is delta connected $V_L = V_{ph}$ = 400 volts

Volt-ampere = $\sqrt{3} \times V_L \times I_L$ = 600 kVA Hence $I_L$ = 600 kVA/ $\sqrt{3}$ x 400 = 866 amps

and $I_{ph} = I_L / \sqrt{3}$ = 866 /$\sqrt{3}$ = 500 amps

When it is reconnected in star phase voltage and phase current will remain same, as

$E_{ph} = 4.44\, k_p\, k_d\, f\, \Phi\, T_{ph}$ and $I_{ph} = V_{ph} / Z_{ph}$

(ii) When star connected

$V_{ph}$ = 400 volts and $V_L = \sqrt{3} \times V_{ph} = \sqrt{3} \times 400$ = 692.8 volts

$I_L = I_{ph}$ = 500 amps

Hence VA rating = $\sqrt{3} \times V_L \times I_L = \sqrt{3} \times 692.8 \times 500$ = 600 kVA

Irrespective of the type of connection the power output of the alternator remains same. Only line voltage and line currents will change.

## Voltage Regulation:

When an alternator is subjected to a varying load, the voltage at the armature terminals varies to a certain extent, and the amount of this variation determines the regulation of the machine. When the alternator is loaded the terminal voltage decreases as the drops in the machine stars increasing and hence it will always be different than the induced EMF.

Voltage regulation of an alternator is defined as the change in terminal voltage from no load to full load expressed as a percentage of rated voltage when the load at a given power factor is removed with out change in speed and excitation. Or The numerical value of the regulation is defined as the percentage rise in voltage when full load at the specified power-factor is switched off with speed and field current remaining unchanged expressed as a percentage of rated voltage.

Hence regulation can be expressed as

% Regulation = $\frac{Eph - Vph}{Vph} \times 100$

Where $E_{ph}$ = Induced EMF /phase, $V_{ph}$ = Rated terminal voltage/phase

## Methods of finding Voltage Regulation:

The voltage regulation of an alternator can be determined by different methods. In case of small generators it can be determined by direct loading whereas in case of large generators it cannot determined by direct loading but will be usually predetermined by different methods. Following are the different methods used for predetermination of regulation of alternators.

1) Direct loading method
2) EMF method or Synchronous impedance method
3) MMF method or Ampere turns method
4) ASA modified MMF method
5) ZPF method or Potier triangle method

All the above methods other than direct loading are valid for non salient pole machines only. As the alternators are manufactured in large capacity direct loading of alternators is not employed for determination of regulation. Other methods can be employed for predetermination of regulation. Hence the other methods of determination of regulations will be discussed in the following sections.

## EMF method:

This method is also known as synchronous impedance method. Here the magnetic circuit is assumed to be unsaturated. In this method the MMFs (fluxes) produced by rotor and stator are replaced by their equivalent EMF, and hence called EMF method.

To predetermine the regulation by this method the following information's are to be determined. Armature resistance /phase of the alternator, open circuit and short circuit characteristics of the alternator.

## OC & SC test on alternator:

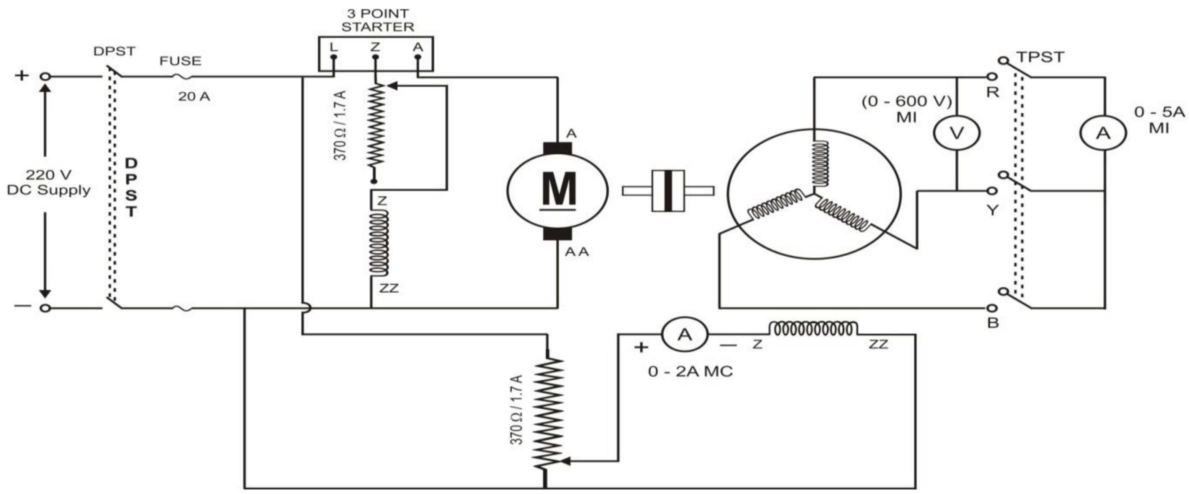



**Figure: 33 OC & SC test on alternator**

## Open Circuit Characteristic (O.C.C.):

The open-circuit characteristic or magnetization curve is really the B-H curve of the complete magnetic circuit of the alternator. Indeed, in large turbo-alternators, where the air gap is relatively long, the curve shows a gradual bend. It is determined by inserting resistance in the field circuit and measuring corresponding value of terminal voltage and field current. Two voltmeters are connected across the armature terminals. The machine is run at rated speed and field current is increased gradually to $I_{f1}$ till armature voltage reaches rated value or even 25% more than the rated voltage. Figure 32 illustrates a typical circuit for OC and SC test and figure 33 illustrates OC and SC curve. The major portion of the exciting ampere-turns is required to force the flux across the air gap, the reluctance of which is assumed to be constant. A straight line called the air gap line can therefore be drawn as shown, dividing the excitation for any voltage into two portions, (a) that required to force the flux across the air gap, and (b) that required to force it through the remainder of the magnetic circuit. The shorter the air gap, the steeper is the air gap line.

## Procedure to conduct OC test:

i) Start the prime mover and adjust the speed to the synchronous speed of the alternator.

ii) Keep the field circuit rheostat in cut in position and switch on DC supply.

iii) Keep the TPST switch of the stator circuit in open position.

iv) Vary the field current from minimum in steps and take the readings of field current and stator terminal voltage, till the voltage read by the voltmeter reaches up to 110% of rated voltage. Reduce the field current and stop the machine.

i) Plot of terminal voltage/ phase Vs field current gives the OC curve.

## Short Circuit Characteristic (S.C.C.):

The short-circuit characteristic, as its name implies, refers to the behavior of the alternator when its armature is short-circuited. In a single-phase machine the armature terminals are short-circuited through an ammeter, but in a three-phase machine all three phases must be short-circuited. An ammeter is connected in series with each armature terminal, the three remaining ammeter terminals being short-circuited. The machine is run at rated speed and field current is increased gradually to $I_{f2}$ till armature current reaches rated value. The armature short-circuit current and the field current are found to be proportional to each other over a wide range, as shown in Figure 33, so that the short-circuit characteristic is a straight line. Under short-circuit conditions the armature current is almost 90° out of phase with the voltage, and the armature MMF has a direct demagnetizing action on the field. The resultant ampere − turns inducing the armature EMF are, therefore, very small and is equal to the difference between the field and the armature ampere − turns. This results in low MMF in the magnetic circuit, which remains in unsaturated condition and hence the small value of induced EMF increases linearly with field current. This small induced armature EMF is equal to the voltage drop in the winding itself, since the terminal voltage is zero by assumption. It is the voltage required to circulate the short-circuit current through the armature windings. The armature resistance is usually small compared with the reactance.

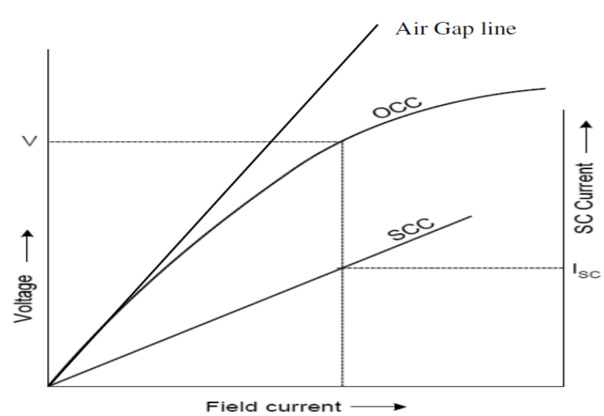



**Figure: 34 OCC & SCC of an alternator**

## Short-Circuit Ratio:

The short-circuit ratio is defined as the ratio of the field current required to produce rated volts on open circuit to field current required to circulate full-load current with the armature short-circuited.

Short-circuit ratio = $I_{f1}/I_{f2}$

## Determination of synchronous impedance $Z_s$:

As the terminals of the stator are short circuited in SC test, the short circuit current is circulated against the impedance of the stator called the synchronous impedance. This impedance can be estimated form the OC and SC characteristics.

The ratio of open circuit voltage to the short circuit current at a particular field current, or at a field current responsible for circulating the rated current is called the synchronous impedance.

Synchronous impedance $Z_s = \frac{\text{open circuit voltage per phase}}{\text{short circuit current per phase}}$ for same Field current.

Armature resistance $R_a$ of the stator can be measured using Voltmeter – Ammeter method. Using synchronous impedance and armature resistance synchronous reactance and hence regulation can be calculated as follows using EMF method.

$Z_s = \sqrt{Ra^2 + Xs^2}$ and Synchronous reactance $X_s = \sqrt{Zs^2 - Ra^2}$

Hence induced EMF per phase can be found as $E_{ph} = \sqrt{(V\,Cos\Phi + \mathrm{IaRa})^2 + (VSin\Phi \pm \mathrm{IaXs})^2}$

In the above expression in second term + sign is for lagging power factor and – sign is for leading power factor.

% Regulation = $\frac{Eph - Vph}{Vph} \times 100$

Where $E_{ph}$ = induced EMF /phase, $V_{ph}$ = rated terminal voltage/phase

Synchronous impedance method is easy but it gives approximate results. This method gives the value of regulation which is greater (poor) than the actual value and hence this method is called pessimistic method. The complete phasor diagram for the EMF method is shown in figure

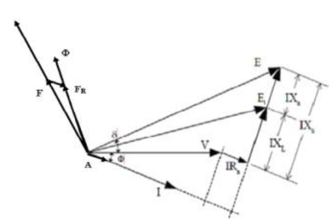


**Figure: 35 Phasor diagram of alternator**

## Numerical Problems on EMF method:

**1.** A 1200 kVA, 3300 volts, 50 Hz, three phase star connected alternator has an armature resistance of 0.25 Ω per phase. A field current of 40 Amps produces a short circuit current of 200 Amps and an open circuit emf of 1100 volts line to line. Find the % regulation at full load 0.8 pf lagging and leading by using emf method.

**Soln:** Full load current = $1200 \times 10^3 / (\sqrt{3} \times 3300) = 210$ amps; Voltage per phase $V_{ph} = 3300/\sqrt{3} = 1905$ volts

Synchronous impedance $Z_S$ = oc voltage per phase/ sc current per phase …….. for same excitation

$$= (1100/\sqrt{3}) / 200 = \mathbf{3.17\ \Omega}$$

Synchronous reactance = $X_S = \sqrt{[(Z_S)^2 - (R_a)^2]} = \sqrt{(3.17)^2 + (0.25)^2} = 3.16\ \Omega$

**0.8 pf lagging:** referring to the phasor diagram

$$E_{ph} = \sqrt{[(V \cos 8 + IR_a)^2 + (V \sin 8 + IX_S)^2]}$$

$$= \sqrt{[(1905 \times 0.8 + 210 \times 0.25)^2 + (1905 \times 0.6 + 210 \times 3.16)^2}$$

$$= 2398 \text{ volts}$$

Voltage regulation = $[(E_{ph} - V_{ph})/ V_{ph}] \times 100$

$$= [(2398 - 1905) / 1905] \times 100$$

$$= \mathbf{25.9\ \%}$$

**0.8 pf leading:** $E_{ph} = \sqrt{[(V \cos 8 + IR_a)^2 + (V \sin 8 - IX_S)^2]}$

$$= \sqrt{[(1905 \times 0.8 + 210 \times 0.25)^2 + (1905 \times 0.6 - 210 \times 3.16)^2}$$

$$= \mathbf{1647\ volts}$$

Voltage regulation = $[(E_{ph} - V_{ph} / V_{ph})] \times 100$

$$= [(1647 - 1905) / 1905] \times 100$$

$$= \mathbf{-13.54\ \%}$$

**2.** A 10 MVA 6.6 kV, 3phase star connected alternator gave open circuit and short circuit data as follows.

| | | | | | | |
|---|---|---|---|---|---|---|
| Field current in amps: | 25 | 50 | 75 | 100 | 125 | 150 |
| OC voltage in kV (L-L): | 2.4 | 4.8 | 6.1 | 7.1 | 7.6 | 7.9 |
| SC Current in Amps: | 288 | 528 | 875 | - | - | - |

Find the voltage regulation at full load 0.8 pf lagging by emf method. Armature resistance per phase =0.13 Ω.

**Soln:** Full load current = $10 \times 10^6 / (\sqrt{3} \times 6600)$ = 875 amps;

Voltage per phase $V_{ph} = 6600/\sqrt{3} = 3810$volts

Corresponding to the full load current of 875 amps oc voltage from the oc and sc characteristics is 6100 volts

Hence synchronous impedance $Z_S$ = oc voltage per phase/ sc current per phase

$= (6100/\sqrt{3})/ 875$

$= \mathbf{4.02\ \Omega}$

**pf lagging:** $E_{ph} = \sqrt{[ (V \cos 8 + IR_a)^2 + (V \sin 8 + IX_S)^2]}$

$=\sqrt{[(3810 \times 0.8 + 875 \times 0.13)^2 + ( 3810 \times 0.6 – 875 \times 4.01789)^2}$

= 6607.26 volts

Voltage regulation = $[(E_{ph} – V_{ph} / V_{ph} )] \times 100$

= [(6607.26 – 3810) / 3810] x 100

= **73.42%**

## MMF Method

This method of determining the regulation of an alternator is also called Ampere-turn method or Rothert's MMF method. The method is based on the results of open circuit test and short circuit test on an alternator.

For any synchronous generator i.e. alternator, it requires MMF which is product of field current and turns of field winding for two separate purposes.

1. It must have an MMF necessary to induce the rated terminal voltage on open circuit.

2. It must have an MMF equal and opposite to that of armature reaction MMF

**Note**: In most of the cases as number of turns on the field winding is not known, the MMF is calculate and expressed i terms of the field current itself.

The field MMF required to induce the rated terminal voltage on open circuit can be obtained from open circuit test results and open circuit characteristics. This is denoted as $F_O$.

We know that the synchronous impedance has two components, armature resistance and synchronous reactance. Now synchronous reactance also has two components, armature leakage reactance and armature reaction reactance. In short circuit test, field MMF is necessary to overcome drop across armature resistance and leakage reactance and also to overcome effect of armature reaction. But drop across armature resistance and also to overcome effect of armature reaction. But drop across armature resistance and leakage reactance is very small and can be neglected. Thus in short circuit test, field MMF circulates the full load current balancing the armature reaction effect. The value of ampere-turns required to circulate full load current can be obtained from short circuit characteristics. This is denoted as $F_{AR}$.

Under short circuit condition as resistance and leakage reactance of armature do not play any significant role, the armature reaction reactance is dominating and hence the power factor of such purely reactive circuit is zero lagging. Hence $F_{AR}$ gives demagnetizing ampere turns. Thus the field MMF is entirely used to overcome the armature reaction which is wholly demagnetizing in nature.

The two components of total field MMF which are $F_O$ and $F_{AR}$ are indicated in O.C.C. (open circuit characteristics) and S.C.C. (short circuit characteristics) as shown in the Fig.36

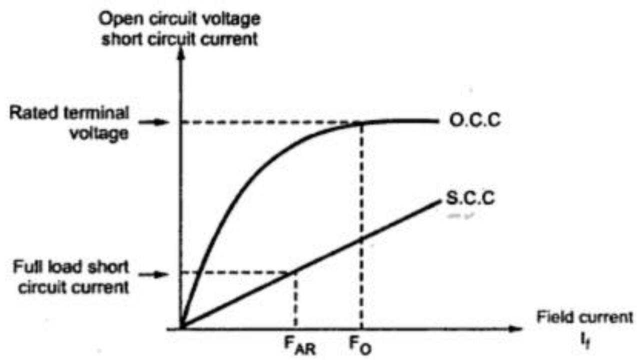



**Fig. 36 OC and SC Curves**

If the alternator is supplying full load, then total field MMF is the vector sum of its two components $F_O$ and $F_{AR}$. This depends on the power factor of the load which alternator is supplying. The resultant field MMF is denoted as $F_R$. Let us consider the various power factors and the resultant $F_R$.

**Zero lagging P.f:** As long as power factor is zero lagging, the armature reaction is completely demagnetizing. Hence the resultant $F_R$ is the algebraic sum of the two components $F_O$ and $F_{AR}$. Field MMF is not only required to produce rated terminal voltage but also required to overcome completely demagnetizing armature reaction effect.

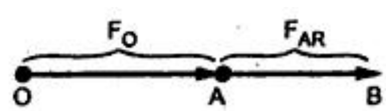



**Fig. 37 Vector**

This is shown in the Fig. 37.

$OA = F_O$

$AB = F_{AR}$ demagnetizing

$OB = F_R = F_O + F_{AR}$

Total field MMF is greater than $F_O$.

**Zero leading P.f:** When the power factor is zero leading then the armature reaction is totally magnetizing and helps main flux to induce rated terminal voltage. Hence net field MMF required is less than that required to induce rated voltage normally, as part of its function is done by magnetizing armature reaction component. The net field MMF is the algebraic difference between the two components $F_O$ and $F_{AR}$. This is shown in the Fig. 3.

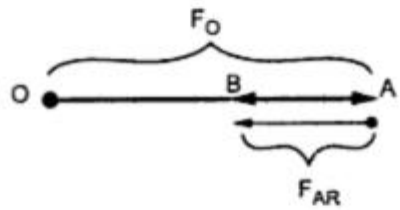



**Fig. 38 Vector**

$OA = F_O$

$AB = F_{AR}$ magnetizing

$OB = F_O - F_{AR} = F_R$

Total MMF is less than $F_O$

**Unity P.f:** Under unity power factor condition, the armature reaction is cross magnetizing and its effect is to distort the main flux. Thus and F are at right angles to each other and hence resultant MMFis the vector sum of $F_O$ and $F_{AR}$. This is shown in the Fig.39

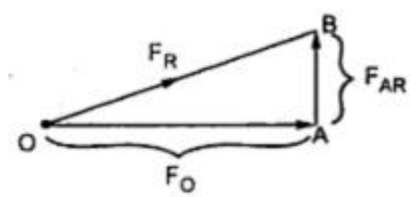



**Fig. 39 Vector**

$OA = F_O$

$AB = F_{AR}$ cross magnetizing

$$OB = F_R = \bar{F}_O + \bar{F}_{AR}$$

**General Case:** Now consider that the load power factor is cos Φ. In such case, the resultant MMF is to be determined by vector addition of $F_O$ and $F_{AR}$.

**Lagging P.f:** When the load P.f. is cosΦ lagging, the phase current $I_{aph}$ lags $V_{ph}$ by angle Φ. The component $F_O$ is at right angles to $V_{ph}$ while $F_{AR}$ is in phase with the current $I_{aph}$. This is because the armature current $I_{aph}$ decides the armature reaction. The armature reaction $F_{AR}$ due to current $I_{aph}$ is to be overcome by field MMF. Hence while finding resultant field MMF, - $F_{AR}$ should be added to vectorially. This is because resultant field MMF tries to counterbalance armature reaction to produce rated terminal voltage. The phasor diagram is shown in the Fig. 5.

From the phasor diagram the various magnitude are,

$OA = F_O$, $AB = F_{AR}$, $OB = F_R$

Consider triangle OCB which is right angle triangle. The $F_{AR}$ is split into two parts as,

$AC = F_{AR}\ \sin\Phi$ and $BC = F_{AR}\cos\Phi$

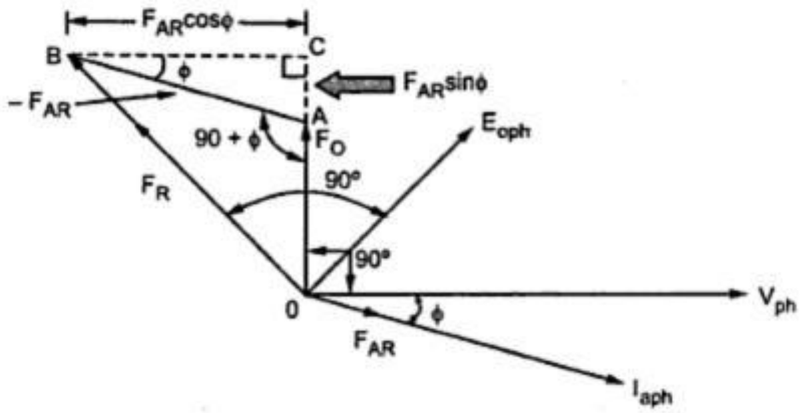



**Fig. 40 Vector diagram for lagging power factor**

$\therefore\quad (F_R)^2 = (F_O + F_{AR}\ \sin\Phi)^2 + (F_{AR}\cos\Phi)^2$ ................ (1)

From this relation (1), $F_R$ can be determined.

**Leading p.f:** When the load p.f. is cosΦ leading, the phase current $I_{aph}$ leads $V_{ph}$ by Φ. The component $F_O$ is at right angles to $V_{ph}$ and $F_{AR}$ is in phase with $I_{aph}$. The resultant $F_R$ can be obtained by adding - $F_{AR}$ to $F_O$. The phasor diagram is shown in the Fig.41.

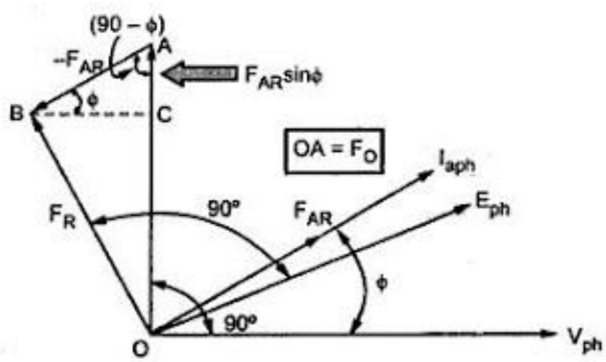



**Fig. 41 Vector diagram for lagging power factor**

From the phasor diagram, various magnitudes are,

$AC = F_{AR}\ \sin\Phi$ and $BC = F_{AR}\ \cos\Phi$

$OA = F_O$, $AB = F_{AR}$ and $OB = F_R$

Consider triangle OCB which is right angles triangle.

$\therefore \quad (OB)^2 = (OC)^2 + (BC)^2$

$\therefore \quad (F_R)^2 = (F_O - F_{AR}\ \sin\Phi)^2 + (F_{AR} \cos\Phi)$ .................... (2)

From the relation (2), $F_R$ can be obtained.

Using relations (1) and (2), resultant field MMF$F_R$ for any P.f. load condition can be obtained.

Once $F_R$ is known, obtain corresponding voltage which is induced EMF $E_{ph}$, required to get rated terminal voltage $V_{ph}$. This is possible from open circuit characteristics drawn.

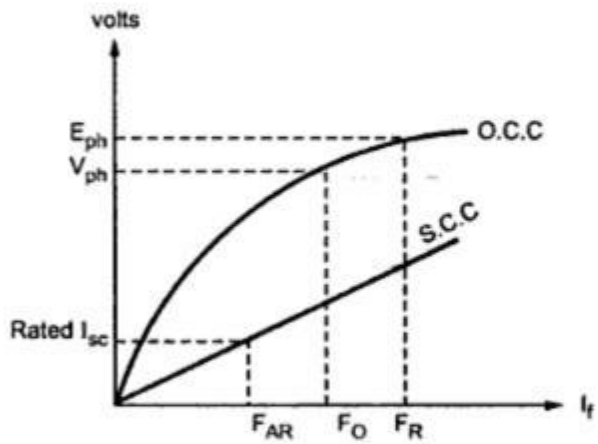



**Fig. 42 OC and SC curves**

Once $E_{ph}$ is known then the regulation can be obtained as,

$$\% R = \frac{E_{ph} - V_{ph}}{V_{ph}} \times 100$$

**Note**: To obtain $E_{ph}$ corresponding to $F_R$, O.C.C. must be drawn to the scale, from the open circuit test readings.

**Note**: This ampere-turn method gives the regulation of an alternator which is lower than the actually observed. Hence the method is called optimistic method.

**Important note**: When the armature resistance is neglected then $F_O$ is field MMF required to produce rated $V_{ph}$ at the output terminals. But if the effective armature resistance is given then $F_O$ is to be calculated from O.C.C. such that $F_O$ represents the excitation (field current) required a voltage of $V_{ph} + I_{aph} R_{aph} \cos\Phi$ where

$V_{ph}$ = rated voltage per phase

$I_{aph}$ = full load current per phase

$R_a$ = armature resistance per phase

$\cos\Phi$ = power factor of the load

It can also be noted that, $F_R$ can be obtained using the cosine rule to the triangle formed by $F_O$, $F_{AR}$ and $F_O$ as shown in the Fig. 8.

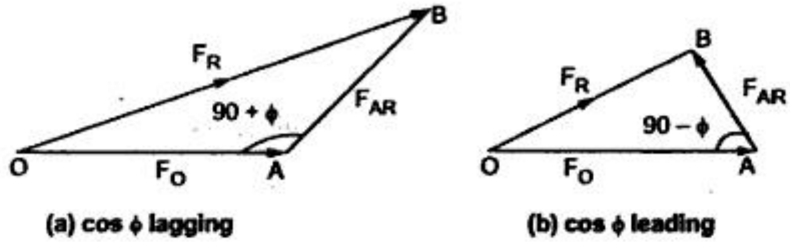



**Fig. 43 Vector diagrams for leading and lagging power factor**

Using cosine rule to triangle OAB,

$$(F_R)^2 = (F_O)^2 + (F_{AR})^2 - 2\, F_O\, F_{AR} \cos (F_O \hat{\ } F_{AR})$$

$$F_O \hat{\ } F_{AR} = 90 + \phi \text{ if } \phi \text{ is lagging} \qquad \dots (3)$$

$$= 90 - \phi \text{ if } \phi \text{ is leading}$$

Students can use equations 1, 2 or 3 to calculate $F_R$.

the angle between $E_o$ and $V_{ph}$ is denoted as δ and is called power angle. Neglecting $R_a$ we can write,

$I_a X_s \cos\Phi = E_o \sin\delta$

$P_d = V_{ph} I_a \cos\Phi$ = internal power of machine

$$P_d = \frac{V_{ph} E_o}{X_s} \sin \delta$$

**Note**: This equation shows that the internal power of the machine is proportional to sin δ.

## Numerical Problems

1. A 3.5 MVA, 50 Hz, star connected alternator rated at 4160 volts gave the following results on oc test.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Field current: amps | 50 | 100 | 150 | 200 | 250 | 300 | 350 | 400 |
| OC voltage (L-L) : | 1620 | 3150 | 4160 | 4750 | 5130 | 5370 | 5550 | 5650 |

A filed current of 200 amps was found necessary to circulate full load current on short circuit of the alternator. Calculate voltage regulation by mmf method at 0.8 pf lagging. Neglect stator resistance.

**Soln:** Draw oc and sc characteristics as shown in figure below

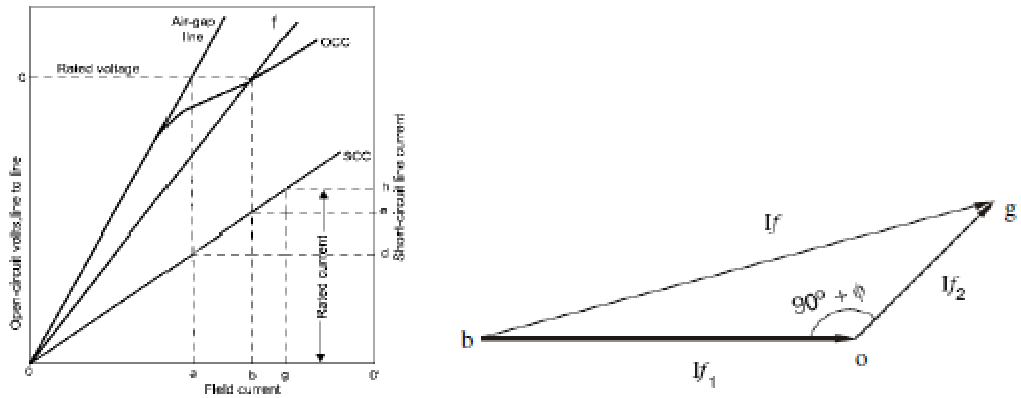



Figure 38

Full load current = $3.5 \times 10^6 / (\sqrt{3} \times 4160) = 486$ amps.
From occ the field current required to produce rated voltage of 4160 volts is 150 amps. From the characteristics it is ob ($I_{f1}$). The field current required to circulate full load current on short circuit is og ($I_{f2}$), from the characteristics and is equal to 200 amps. This filed current is drawn at an angle of 90+Φ w r t ob. The two field currents can be vectorially added as shown in the vector diagram above.
From the above phasor diagram the total field current bg can be computed using cosine rule as

$$bg = \sqrt{(I_{f1})^2 + (I_{f2})^2 + (I_{f1}) \times (I_{f2}) \times \cos(180 - (90+\Phi))}$$

$$= \sqrt{(150)^2 + (200)^2 + (150) \times (200) \times \cos(180 - (90+36.86))}$$

= 313.8 volts.

Corresponding to this filed current of 313.8 amps the induced emf $E_0$ form the occ is 3140 volts.

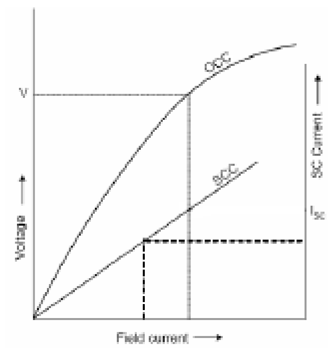
Hence % regulation = (3140 – 2401)/ 2401 x 100 = 30.77%

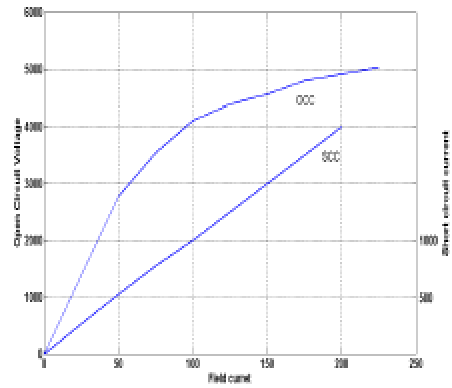
2. A 10 MVA, 50 Hz, 6.6 kV, 3-phase star connected alternator has the following oc and sc test data

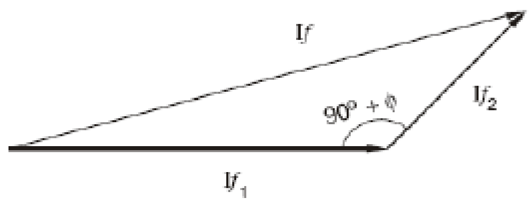
| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Field current: amps | 25 | 50 | 75 | 100 | 125 | 150 | 175 | 200 | 225 |
| OC voltage (L-L) : | 2400 | 4800 | 6100 | 7100 | 7600 | 7900 | 8300 | 8500 | 8700 |
| SC Current amps : | 288 | 582 | 875 | | | | | | |

Calculate the voltage regulation of the alternator by emf and mmf method at a pf of 0.8 lagging. The armature resistance is 0.13 Ω per phase.
**Soln:** Draw oc and sc characteristics as shown in figure below( already solved by emf method)
Full Load current $I_a = 10 \times 10^6 / (\sqrt{3} \times 6.6 \times 10^3) = 875$ amps
Phase voltage = 6600/√3 = 3.81 kV
MMF Method: Normal voltage including resistive drop = $V + I_aR_a\cos\phi$
= 3810 + 875 x 0.13 x 0.8
= 3901 volts

From OCC the field current required to produce this normal voltage is 98 amps and is represented as $I_{f1}$ as shown in the phasor diagram. The field current required to produce the rated current of 875 amps on short circuit is 75 amps and is drawn at an angle of 90+ϕ as *If2* as shown. The total field current required to obtain the EMF E0 is $I_f$.




Using cosine rule

$$I_f = \sqrt{(I_{f1})^2 + (I_{f2})^2 + (I_{f1}) \times (I_{f2}) \times \cos(180 - (90+\Phi))}$$

$$= \sqrt{(98 + (75 + (98\text{x} (75\text{x} \cos(180 - (90+36.86))}$$

= 155 amps.

Corresponding to this filed current of 155 amps the induced emf $E_0$ form the occ is 4619 volts.

Hence % regulation = (4619 – 3810)/ 3810 x 100 = 21.2 %

## Zero Power Factor (ZPF) method Potier Triangle Method:

During the operation of the alternator, resistance voltage drop $I_aR_a$ and armature leakage reactance drop $I_aX_L$ are actually EMF quantities and the armature reaction reactance is a MMF quantity. To determine the regulation of the alternator by this method OCC, SCC and ZPF test details and characteristics are required. AS explained earlier OC and sc tests are conducted and OCC and SCC are drawn. ZPF test is conducted by connecting the alternator to ZPF load and exciting the alternator in such way that the alternator supplies the rated current at rated voltage running at rated speed. To plot ZPF characteristics only two points are required. One point is corresponding to the zero voltage and rated current that can be obtained from SCC and the other at rated voltage and rated current under ZPF load. This zero power factor curve appears like *OCC* but shifted by a factor *IXL* vertically and horizontally by armature reaction MMF as shown below in figure. Following are the steps to draw ZPF characteristics.

By suitable tests plot OCC and SCC. Draw air gap line. Conduct ZPF test at full load for rated voltage and fix the point B. Draw the line BH with length equal to field current required to produce full load current on short circuit. Draw HD parallel to the air gap line so as to cut the OCC. Draw DE perpendicular to HB or parallel to voltage axis. Now, DE represents voltage drop *IXL* and BE represents the field current required to overcome the effect of armature reaction.

Triangle BDE is called Potier triangle and XL is the Potier reactance. Find E from *V*, *IRa*, *IXL* and Φ. Use the expression $E = \sqrt{(V\cos\Phi + IRa)^2 + (V\sin\Phi) + IXL)^2}$ to compute E. Find field current corresponding to *E*. Draw FG with magnitude equal to BE at angle (90+Ψ) from field current axis, where Ψ is the phase angle of current from voltage vector *E* (internal phase angle).

The resultant field current is given by OG. Mark this length on field current axis. From OCC find the corresponding *E0*. Find the regulation.

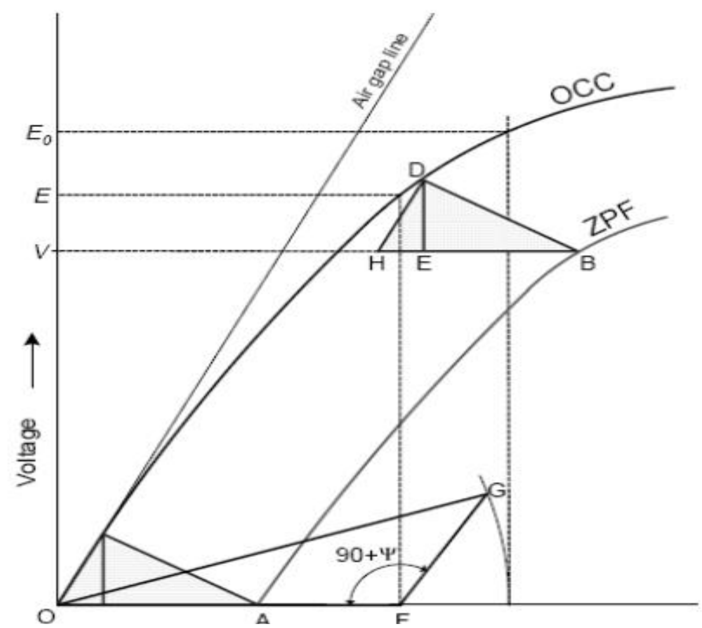


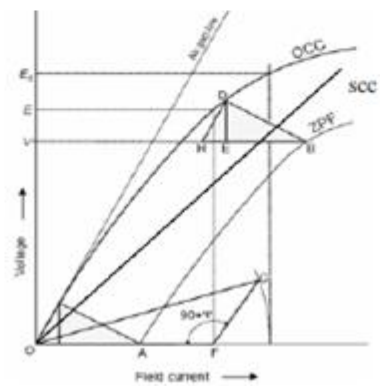

**Figure: 44 ZPF method characteristics**

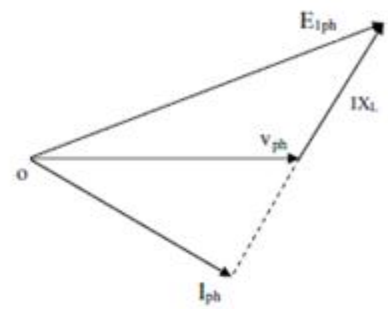
**1.** A 10 kVA, 440 volts, 50 Hz, 3 phase, star connected, alternator has the open circuit characteristics as below.

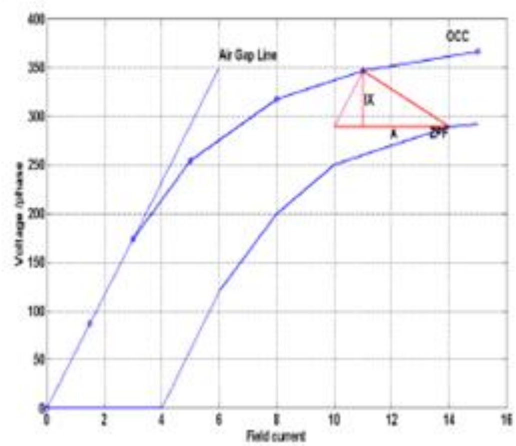
| | | | | | | |
|---|---|---|---|---|---|---|
| Field current (amps) | 1.5 | 3 | 5 | 8 | 11 | 15 |
| OC voltage (Line) | 150 | 300 | 440 | 550 | 600 | 635 |

With full load zero power factor, the excitation required is 14 amps to produce 500 volts terminal voltage. On short circuit 4 amps excitation is required to produce full load current. Determine the full load voltage regulation at 0.8 P.f lagging and leading.

**Sol:** Draw OC, SC and ZPF characteristics to scale as shown. OC characteristics are drawn from the details given above. For sc characteristics 4 amps field current gives full load current. For ZPF characteristics two points are sufficient, one is 4 amps corresponding voltage of 0 volts, and the other is 14 amps corresponding to 500 vols.



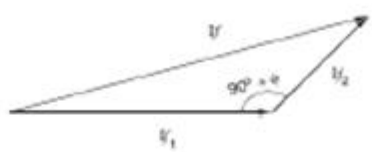



From the potier triangle BDE, armature leakage reactance DE is 55 volts.
As armature resistance is negligible $V_{ph}$ and $IX_L$ drop are to be added.

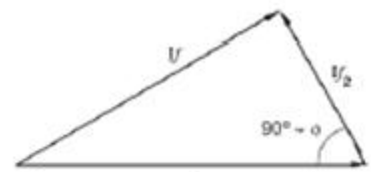
(i) lagging PF
$V_{ph} = 440/\sqrt{3} = 254$ volts. Full load current $10000/(3 \times 254) = 13.123$ amps
Adding $V_{ph}$ and $IX_L$ drop vectorially, as shown in figure above.

$$E_{1ph} = \sqrt{(V_{ph}\cos\Phi)^2 + (V_{ph}\sin\Phi + IX_L)^2}$$
$$= \sqrt{(254 \times 0.8)^2 + (254 \times 0.8 + 55)^2}$$
$$= 290.4 \text{ volts}$$

Corresponding to this voltage find the field current F1 from occ is 6.1 amps, ($I_{f1}$)
From potier triangle the filed current required to balance the armature reaction is BE is 3.1 amps ($I_{f2}$)



For lagging P.F. load



For leading P.F. load

Adding the above two currents vectorially , $I_f = 8.337$ amps.
Corresponding to this field current the emf $E_{ph}$ from OCC is 328 volts
Hence regulation = (328 – 254)/254 x 100 = 29.11 %
(ii) leading PF
For the leading pf
Adding $V_{ph}$ and $IX_L$ drop vectorially,

$$E_{1ph} = \sqrt{(V_{ph}\cos\Phi)^2 + (V_{ph}\sin\Phi - IX_L)^2}$$
$$= \sqrt{(254 \times 0.8)^2 + (254 \times 0.8 - 55)^2}$$
$$= 225.4 \text{ volts}$$

Corresponding to this voltage find the field current , $I_{f1}$ from occ is 4.1 amps
From potier triangle the filed current ($I_{f2}$) required to balance the armature reaction BE is 3.1 amps
Adding the above two currents vectorially,(by cosine rule) $I_f = 3.34$ amps.
Corresponding to this field current the emf $E_{ph}$ from OCC is 90 volts
Hence regulation = (90 – 254)/254 x 100 = -25.2 %

**Ex.** A 11 kv , 1000 kVA, 3 phase star connected alternator has a resistance of 2 Ω per phase. The open circuit and full load ZPF characteristics are given below. Determine the full load voltage regulation at 0.8 pf lagging by Potier triangle method.

| | | | | | | |
|---|---|---|---|---|---|---|
| Field current (amps): | 40 | 50 | 80 | 110 | 140 | 180 |
| OC voltage (L-L): | 5800 | 7000 | 10100 | 12500 | 13750 | 15000 |
| ZPF voltage(L-L): | 0 | 1500 | 5200 | 8500 | 10500 | 12200 |

Draw the OCC and ZPF characteristics as shown in figure.
Phase voltage = 11000 = 6350 volts. Rated current per phase = $1000 \times 10^3 / (\sqrt{3} \times 11000) = 52.48$ A
Draw OCC ZPF and the Potier triangle as shown.

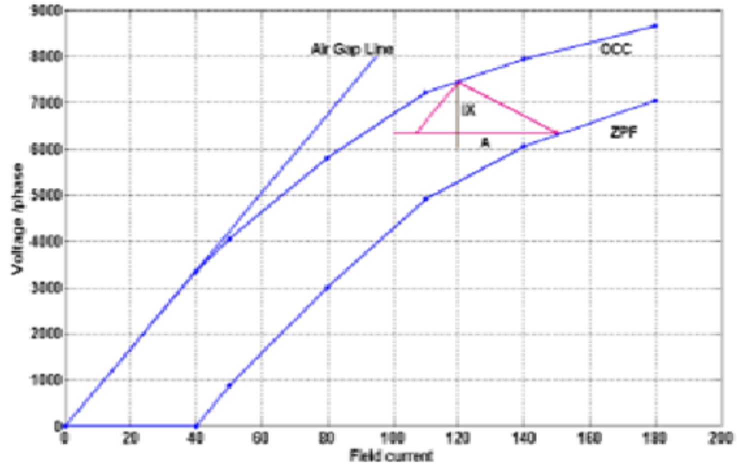
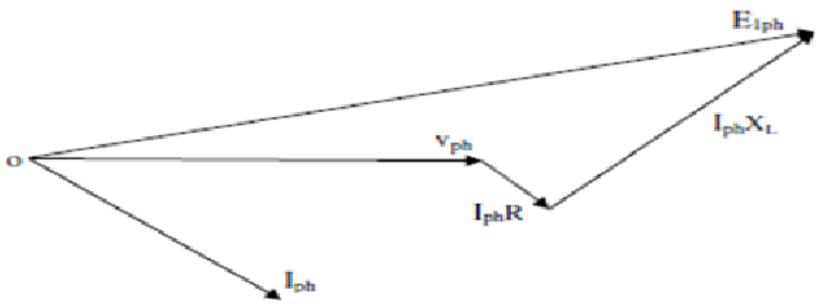



Figure.45

From the Potier triangle $IX_L = 1000$ volts





From the phasor diagram taking current as reference

$E_{1ph} = V \angle \varphi + I\,(R + j\,X_L)$

$= 6350 \angle 36.86 + 105 + j1000$

$= 7072 \angle 42.8$ volts.

Corresponding to this voltage of 7072 volts the field current ($I_{f1}$) required 111 amps

From potier triangle the filed current ($I_{f2}$) required to balance the armature reaction is 28 amps

Adding both the field currents $I_{f1}$ and $I_{f2}$ vectorially by cosine rule $I_f = 130$ amps.

Corresponding to this field current oc voltage from occ is 7650 volts

Hence % regulation = (7650 – 6350)/6350 x 100 = 20.5 %

3. A 5000kVA, 6.6kV, 3 phase Y connected alternator has an effective resistance of 0.075 Ω per phase. Estimate by zpf method the regulation for a load of 500A at p.f (i) unity (ii) 0.9 leading (iii) 0.71 lagging from the following OCC and zpf FL curves.

| | | | | | |
|---|---|---|---|---|---|
| **If(A)** | **32** | **50** | **75** | **100** | **140** |
| **Voc(KV)** | **3100** | **4900** | **6600** | **7500** | **8300** |
| **V (KV)for ZPF** | **0** | **1850** | **4250** | **5800** | **7000** |

**4.** A 2500kVA, 6.6kV, 3 phase Y connected alternator has an effective resistance of 0.08 Ω per phase. Estimate by ZPF method the regulation for a load of 250A at P.f (i) unity (ii) 0.85 leading (iii) 0.92 lagging from the following OCC and ZPF FL curves

| **If(A)** | **32** | **50** | **75** | **100** | **140** |
|---|---|---|---|---|---|
| **Voc(V)** | **3200** | **5000** | **6500** | **8500** | **9000** |
| **V (V)for ZPF** | **0** | **1925** | **3800** | **4800** | **6000** |

## ASA Method

Tests:

Conduct tests to find OCC (up to 125% of rated voltage)

SCC (for rated current)

ZPF (for rated current and rated voltage)

Armature Resistance (if required) Steps: 1. Follow steps 1 to 7 as in ZPF method.

2. Find If1 corresponding to terminal voltage V using air gap line (OF1 in figure).

3. Draw If2 with length equal to field current required to circulate rated current during short circuit condition at an angle (90+Φ) from $I_{f1}$. The resultant of If1 and $I_{f2}$ gives $I_f$ (OF2 in figure).

4. Extend OF2 up to F so that $F_2F$ accounts for the additional field current accounting for the effect of saturation. $F_2F$ is found for voltage E as shown.

5. Project total field current OF to the field current axis and find corresponding voltage E0 using OCC.

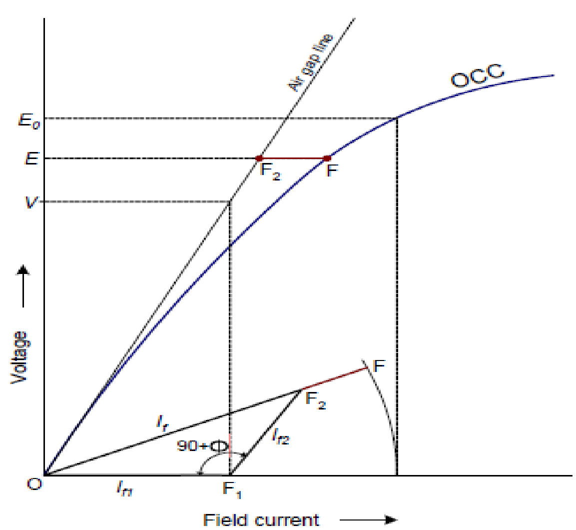



**Fig 36 ASA Method Characteristics**

## Numerical Problems on ASA method:

1. A 5000kVA, 6.6kV, 3 phase Y connected alternator has an effective resistance of 0.075 Ω per phase. Estimate the regulation by ASA method for a load of 500A at P.f (i) unity (ii) 0.9 leading (iii) 0.71 lagging from the following OCC and ZPF F.L curves.

| If(A) | 32 | 50 | 75 | 100 | 140 |
|---|---|---|---|---|---|
| Voc(KV) | 3100 | 4900 | 6600 | 7500 | 8300 |
| V (KV)for ZPF | 0 | 1850 | 4250 | 5800 | 7000 |

**2.** A 2500kVA, 6.6kV, 3 phase Y connected alternator has an effective resistance of 0.08 Ω per phase. Estimate the regulation by ASA method for a load of 250A at P.f (i) unity (ii) 0.85 leading (iii) 0.92 lagging from the following OCC and ZPF F.L curves

| If(A) | 32 | 50 | 75 | 100 | 140 |
|---|---|---|---|---|---|
| Voc(V) | 3200 | 5000 | 6500 | 8500 | 9000 |
| V (V)for ZPF | 0 | 1925 | 3800 | 4800 | 6000 |

## Salient pole alternators and Blondel's Two reaction Theory:

The details of synchronous generators developed so far is applicable to only round rotor or non salient pole alternators. In such machines the air gap is uniform throughout and hence the effect of MMF will be same whether it acts along the pole axis or the inter polar axis. Hence reactance of the stator is same throughout and hence it is called synchronous reactance. But in case salient pole machines the air gap is non uniform and it is smaller along pole axis and is larger along the inter polar axis. These axes are called direct axis or d-axis and quadrature axis or q-axis. Hence the effect of MMF when acting along direct axis will be different than that when it is acting along quadrature axis. Hence the reactance of the stator cannot be same when the MMF is acting along d – axis and q- axis. As the length of the air gap is small along direct axis reluctance of the magnetic circuit is less and the air gap along the q – axis is larger and hence the along the quadrature axis will be comparatively higher. Hence along d-axis more flux is produced than q-axis. Therefore the reactance due to armature reaction will be different along d-axis and q-axis. This reactance's are

$X_{ad}$ = direct axis reactance; $X_{aq}$ = quadrature axis reactance

Hence the effect of armature reaction in the case of a salient pole synchronous machine can be taken as two components - one acting along the direct axis (coinciding with the main field pole axis) and the other acting along the quadrature axis (inter-polar region or magnetic neutral axis) - and as such the MMF components of armature-reaction in a salient-pole machine cannot be considered as acting on the same magnetic circuit. Hence the effect of the armature reaction cannot be taken into account by considering only the synchronous reactance, in the case of a salient pole synchronous machine.

In fact, the direct-axis component Fad acts over a magnetic circuit identical with that of the main field system and produces a comparable effect while the quadrature-axis component $F_{aq}$ acts along the inter polar axis, resulting in an altogether smaller effect and, in addition, a flux distribution totally different from that of Fad or the main field MMF. This explains why the application of cylindrical-rotor theory to salient-pole machines for predicting the performance gives results not conforming to the performance obtained from an actual test.

Blondel's two-reaction theory considers the effects of the quadrature and direct-axis components of the armature reaction separately. Neglecting saturation, their different effects are considered by assigning to each an appropriate value of armature-reaction "reactance," respectively $X_{aq}$ and $X_{aq}$. The effects of armature resistance and true leakage reactance ($X_L$) may be treated separately, or may be added to the armature reaction coefficients on the assumption that they are the same, for either the direct-axis or quadrature-axis components of the armature current (which is almost true). Thus the combined reactance values can be expressed as: $X_{sd} = X_{aq} + X_L$ *and* $X_{aq} = Xaq + X_L$ for the direct- and cross-reaction axes respectively.

In a salient-pole machine, $X_{aq}$, the quadrature-axis reactance is smaller than $X_{ad}$, the direct-axis reactance, since the flux produced by a given current component in that axis is smaller as the reluctance of the magnetic path consists mostly of the inter polar spaces. It is essential to clearly note the difference between the quadrature and direct-axis components $I_{aq}$, and $I_{ad}$ of the armature current Ia, and the reactive and active components $I_{aa}$ and $I_{ar}$. Although both pairs are represented by phasor in phase quadrature, the former are related to the induced EMF Et while the latter are referred to the terminal voltage V. These phasor are clearly indicated with reference to the phasor diagram of a (salient pole) synchronous generator supplying a lagging power factor (p.f) load, shown in Fig3.5

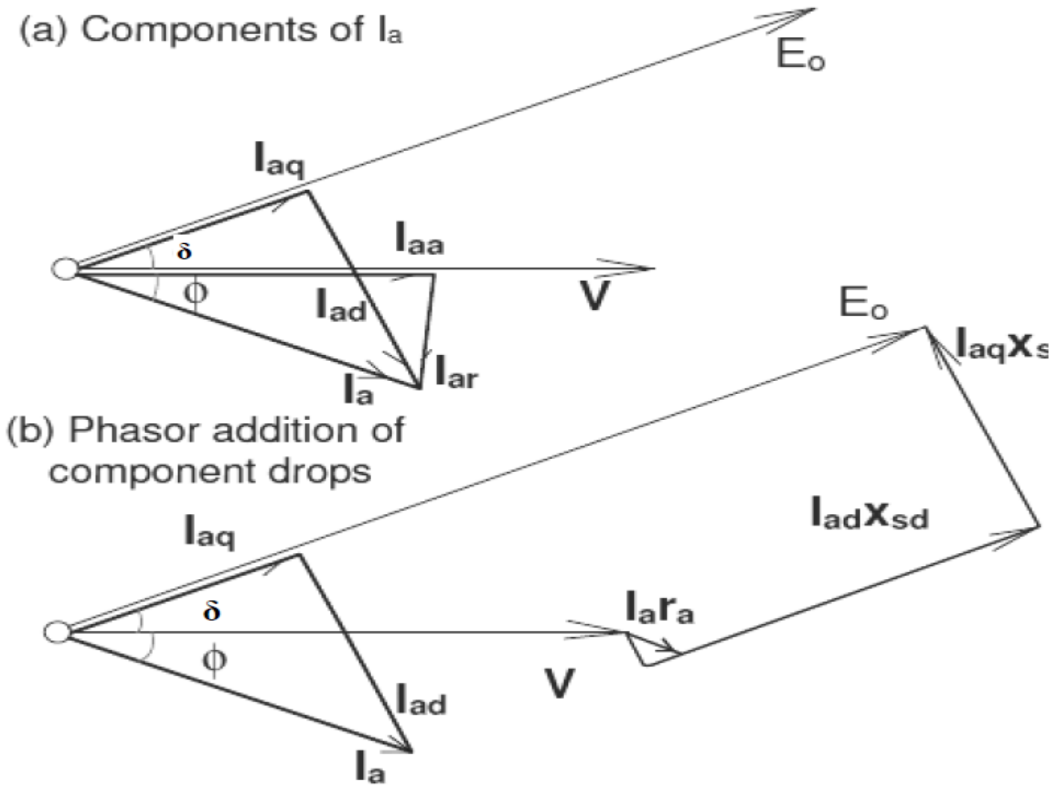



**Figure: 37. Phasor diagram of salient pole alternator**

$I_{aq} = I_a \cos(\delta+\varphi)$; $I_{ad} = I_a \sin(\delta+\varphi)$; and $I_a = \sqrt{(Iaq)^2 + (Iad)^2}$

$I_{aa} = I_a \cos \varphi$; $I_{ar} = Ia \sin \varphi$; and $I_a = \sqrt{(Iaa)^2 + (Iar)^2}$

Where $\delta$ = torque or power angle and $\varphi$ = the p.f. angle of the load.

The phasor diagram shows the two reactance voltage components $I_{aq} \times X_{sq}$ and $I_{ad} \times X_{sd}$ which are in quadrature with their respective components of the armature current. The resistance drop $I_a \times Ra$ is added in phase with $I_a$ although we could take it as $I_{aq} \times Ra$ and $I_{ad} \times Ra$ separately, which is unnecessary as $I_a = I_{ad} + jI_{aq}$.

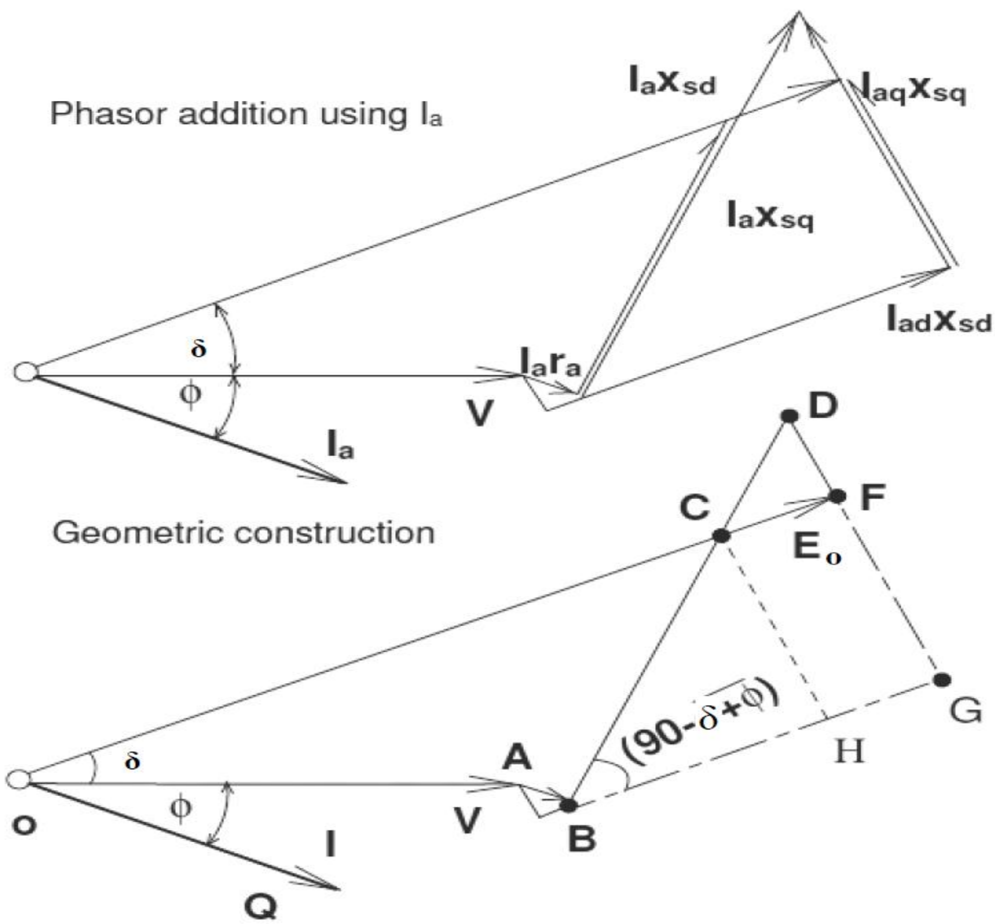



**Figure: 38. Phasor diagram of salient pole alternator**

## Power output of a Salient Pole Synchronous Machine

Neglecting the armature winding resistance, the power output of the generator is given by:

$$P = V \times I_a \cos \varphi$$

This can be expressed in terms of δ, by noting that

$I_a \cos \varphi = I_{aq} \cos \delta + I_{ad} \sin \delta$

$V \cos \delta = E_o - I_{ad} X_{sd}$ and $V \sin \delta = I_{aq} X_{sq}$

Substituting the above expressions for power we get

$$P = [\frac{\mathrm{V \sin \delta}}{\mathrm{Xsd}} \cos \delta + \frac{\mathrm{Eo - Vcos\ \delta}}{\mathrm{Xsd}} \sin \delta]$$

On simplification we get

$$P = [\frac{\mathrm{VEo}}{\mathrm{Xsd}} \sin\delta \ + V^2 \frac{(\mathrm{Xsd - Xsq})}{2\mathrm{Xsq}} \sin\delta\ ]$$

The above expression for power can also be written as

$$P = [\frac{VEo}{Xd}\sin\delta + V^2\frac{(Xd - Xq)}{2Xd \times Xq}\sin 2\delta]$$

The above expression for power consists of two terms first is called electromagnetic power and the second is called reluctance power.

It is clear from the above expression that the power is a little more than that for a cylindrical rotor synchronous machine, as the first term alone represents the power for a cylindrical rotor synchronous machine. A term in (sin 2δ) is added into the power – angle characteristic of a non-salient pole synchronous machine. This also shows that it is possible to generate an MMF even if the excitation E0 is zero. However this magnitude is quite less compared with that obtained with a finite E0. Likewise it can be shown that the machine develops a torque - called the reluctance torque - as this torque is developed due to the variation of the reluctance in the magnetic circuit even if the excitation E0 is zero.

**Determination of $X_d$ and $X_q$ by slip test:**

The direct and quadrature axis reactance's $X_d$ and $X_q$ can be of a synchronous machine can be experimentally determined by a simple test known as slip test. Basic circuit diagram for conducting this test is shown in figure. Here the armature terminals are supplied with a subnormal voltage of rated frequency with field circuit left open. The generator is driven by a prime mover at a slip speed which is slightly more or less than the synchronous speed. This is equivalent to the condition in which the armature MMF remains stationary and rotor rotates at a slip speed with respect to the armature MMF. As the rotor poles slip through the armature MMF the armature MMF will be in line with direct axis and quadrature axis alternately. When it is in line with the direct axis the armature MMF directly acts on the magnetic circuit and at this instant the voltage applied divided by armature current gives the direct axis synchronous reactance. When the armature MMF coincides with the quadrature axis then the voltage impressed divided by armature current gives the quadrature axis synchronous reactance. Since $X_d > X_q$ the pointers of the ammeter reading the armature current will oscillate from a minimum to a maximum. Similarly the terminal voltage will also oscillate between the minimum and maximum.

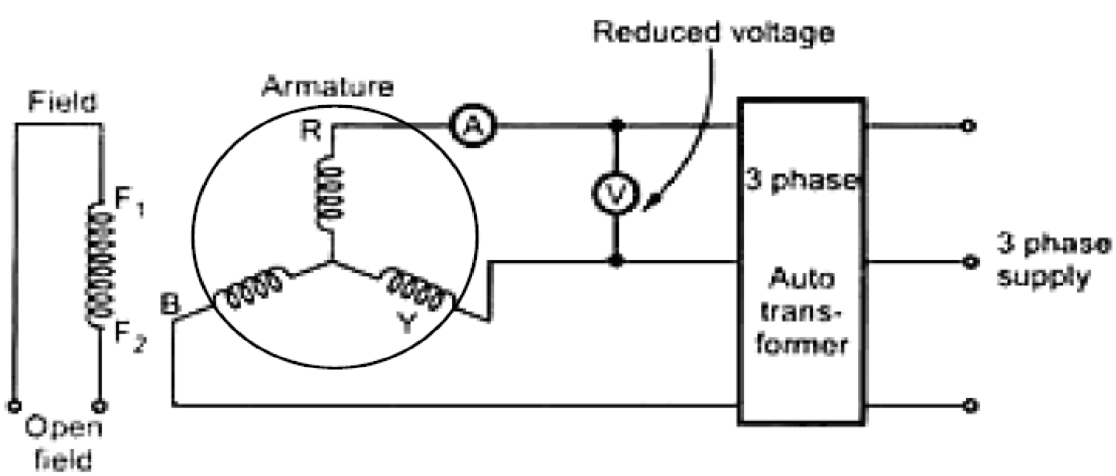


**Figure: 39 Slip test**

$$Xd = \frac{\text{Maximum voltage}}{\text{Minimum current}}$$

$$Xq = \frac{\text{Minimum voltage}}{\text{Maximum current}}$$

$$Eo = Vcos\delta - \text{IqRe} - \text{IdXd}$$

$$\%Voltage\ regulation = \frac{Eo - V}{V}$$

**Numerical Problems on two reaction analysis:**

1. A 3- phase star connected salient pole synchronous generator is driven at a speed near synchronous with the field circuit open, and the stator is supplied from a balanced 3- phase supply. Voltmeter connected across the line gave minimum and maximum readings of 2,800 and 2,820 volts. The line current fluctuated between 360 and 275 amperes. Find the direct and quadrature axis synchronous reactance's per phase. Neglect armature resistance

Sol:

$$Xd = \frac{Maximum\ voltage}{Minimum\ current}$$

$$Xq = \frac{Minimum\ voltage}{Maximum\ current}$$

$$Xd = \frac{2{,}820}{275} = 10.25\ \Omega$$

$$Xq = \frac{2{,}800}{360} = 7.77\Omega$$

2. A 3- phase synchronous generator has per phase a direct axis reactance of 1.0 p.u and quadrature axis reactance of 0.65 p.u. Draw a phasor diagram of the machine when operating at full load at a p.f of 0.8 lagging and estimate from there i) the load angle and ii) p.u no load EMF. Neglect armature resistance

3. A 5 KVA, 220 V, 3- phase, star connected salient pole alternator with direct and quadrature axis reactance's of 12 Ω and 7Ω respectively, delivers full load current at unity power factor.

4. A 10 KVA, 380 V, 50 HZ, 3- phase, star connected salient pole alternator has direct and quadrature axis reactance's of 12 Ω and 8Ω respectively. The armature resistance of 1 Ω per phase. The generator

delivers rated load at 0.8 p.f lagging with the terminal voltage being maintained at rated value, If the load angle is $16.15^{o}$. Determine i) d- axis and q- axis components of armature current ii) Excitation voltage of generator

## PARALLEL OPERATION OF SYNCHRONOUS GENERATORS:

**Synchronizing of alternators:**

The operation of connecting two alternators in parallel is known as synchronizing. Certain conditions must be fulfilled before this can be affected. The incoming machine must have its voltage and frequency equal to that of the bus bars and, should be in same phase with bus bar voltage. The instruments or apparatus for determining when these conditions are fulfilled are called synchroscopes. Synchronizing can be done with the help of (i) dark lamp method or (ii) by using synchroscope.

**Reasons for operating in parallel:**

a) Handling larger loads.
b) Maintenance can be done without power disruption.
c) Increasing system reliability.
d) Increased efficiency.

**Conditions required for Paralleling:**

The figure below shows a synchronous generator G1 supplying power to a load, with another generator G2 about to be paralleled with G1 by closing switch S1. What conditions must be met before the switch can be closed and the 2 generators connected in parallel?

Paralleling 2 or more generators must be done carefully as to avoid generator or other system component damage. Conditions to be satisfied are as follows:

a) RMS line voltages must be equal.
b) The generators to be paralleled must have the same phase sequence.
c) The oncoming generator (the new generator) must have the same operating frequency as compared to the system frequency.

### Advantages of Parallel Operating Alternators

a) When there is maintenance or an inspection, one machine can be taken out from service and the other alternators can keep up for the continuity of supply.
b) Load supply can be increased.
c) During light loads, more than one alternator can be shut down while the other will operate in nearly full load.

d) High efficiency.

e) The operating cost is reduced.

f) Ensures the protection of supply and enables cost-effective generation.

g) The generation cost is reduced.

h) Break down of a generator does not cause any interruption in the supply.

i) Reliability of the whole power system increases.

**General Procedure for Paralleling Generators:**

Consider the figure shown below. Suppose that generator G2 is to be connected to the running system as shown below:

1. Using Voltmeters, the field current of the oncoming generator should be adjusted until its terminal voltage is equal to the line voltage of the running system.

2. Check and verify phase sequence to be identical to the system phase sequence. There are 2 methods to do this:

    i. One way is using the 3 lamp method, where the lamps are stretched across the open terminals of the switch connecting the generator to the system (as shown in the figure below). As the phase changes between the 2 systems, the lamps first get bright (large phase difference) and then get dim (small phase difference). If all 3 lamps get bright and dark together, then the systems have the same phase sequence. If the lamps brighten in succession, then the systems have the opposite phase sequence, and one of the sequences must be reversed.

    ii. Using a Synchroscope – a meter that measures the difference in phase angles (it does not check phase sequences only phase angles).

3. Check and verify generator frequency is same as that of the system frequency. This is done by watching a frequency of brightening and dimming of the lamps until the frequencies are close by making them to change very slowly.

4. Once the frequencies are nearly equal, the voltages in the 2 systems will change phase with respect to each other very slowly. The phase changes are observed, and when the phase angles are equal, the switch connecting the 2 systems is closed.

## Achieving Synchronization

To synchronize AC generators, several important factors must be checked.

1. The phase rotation of both generator systems must be the same. Check this with lights as described later or use a phase rotation meter to determine ABC or ACB rotation.

2. The AC voltages of both generators should be equal. In practice the voltage of the on-coming generator is usually 1-2 volts higher than that of the other operating generator.

3. The frequencies of the on-coming generators must match when synchronized. In practice the frequency of the on-coming generator is 1-2 hertz higher than that of the on-line generator. This can be observed with lights or by using a synchroscope.

The speed and output voltage of the on-coming generator are slightly higher to pre vent it from becoming a load to the system when it's connected.

Two methods of synchronization using lights are described below.

## Three Dark Lamps Method:

The following describes the method of synchronizing two alternators using the three-dark method.

Fig. 3.8 illustrates a circuit used to parallel two three-phase alternators. Alternator G2 is connected to the load circuit. Alternator G1 is to be paralleled with alternator G2 Three lamps rated at double the output voltage to the load is connected between alternator G2 and the load circuit as shown. When both machines are operating, one of two effects will be observed:

1. The three lamps will light and go out in unison at a rate which depends on the difference in frequency between the two alternators.

2. The three lamps will light and go out at a rate which depends on the difference in frequency between the two machines, but not in unison. In this case, the machines are not connected in the proper phase sequence and are said to be out of phase. To correct this, it's necessary to interchange any two leads to alternator G1. The machines are not paralleled until all lamps light and go out in unison. The lamp method is shown for greater simplicity of operation.

By making slight adjustments in the speed of alternator G1 the frequency of the machines can be equalized so that the synchronizing lamps will light and go out at the lowest possible rate. When the three lamps are out, the instantaneous electrical polarity of the three leads from G1 is the same as that of G2 At this instant, the voltage of G1 is equal to and in phase with that of G2 Now the paralleling switch can be closed so that both alternators supply power to the load. The two alternators are in synchronism, according to the three dark lamps method

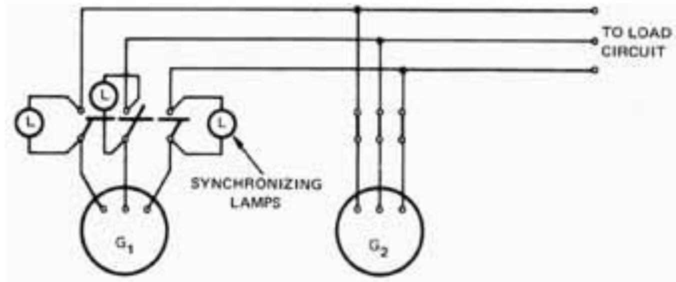



**Fig 40 Synchronization of alternators**

The three dark method has certain disadvantages and is seldom used. A large voltage may be present across an incandescent lamp even though it's dark (burned out). As a result, it's possible to close the paralleling connection while there is still a large voltage and phase difference between the machines. For small capacity machines operating at low speed, the phase difference may not affect the operation of the machines. However, when large capacity units having low armature reactance operate at high speed, a considerable amount of damage may result if there is a large phase difference and an attempt is made to parallel the units.

**Two Bright, One Dark Lamp Method:**

Another method of synchronizing alternators is the two bright, one dark method. In this method, any two connections from the synchronizing lamps are crossed after the alternators are connected and tested for the proper phase rotation. (The alternators are tested by the three dark method Fig. 41(A) shows the connections for establishing the proper phase rotation by the three dark method. Fig. 41(B) shows the lamp connections required to synchronize the alternator by the two bright, one dark method.

When the alternators are synchronized, lamps 1 and 2 are bright and lamp 3 is dark. Since two of the lamps are becoming brighter as one is dimming, it's easier to determine the moment when the paralleling switch can be closed. Furthermore, by observing the sequence of lamp brightness, it's possible to tell whether the speed of the alternator being synchronized is too slow or too fast.

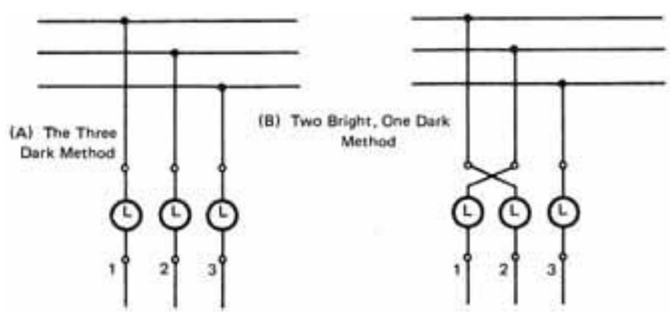



**Fig. 41 Methods of synchronizing alternators. (A) The Three-Dark Method; (B) Two-Bright, One-Dark Method**

**Synchroscope:**

A synchroscope is recommended for synchronizing two alternators since it shows very accurately the exact instant of synchronism (Fig. 42). The pointer rotates clock wise when an alternator is running fast and counterclockwise when an alternator is running slow. When the pointer is stationary, pointing upward, the alternators are synchronized. The synchroscope is connected across one phase only. For this reason it cannot be used safely until the alternators have been tested and connected together for the proper phase rotation. Synchronizing lamps or other means must be used to determine the phase rotation. In commercial applications, the alternator connections to a three-phase bus through a paralleling switch are permanent. This means that tests for phase rotation are not necessary. As a result, a synchroscope is the only instrument required to bring the machines into synchronization and thus parallel them; however, a set of lights is often used as a double-check system.

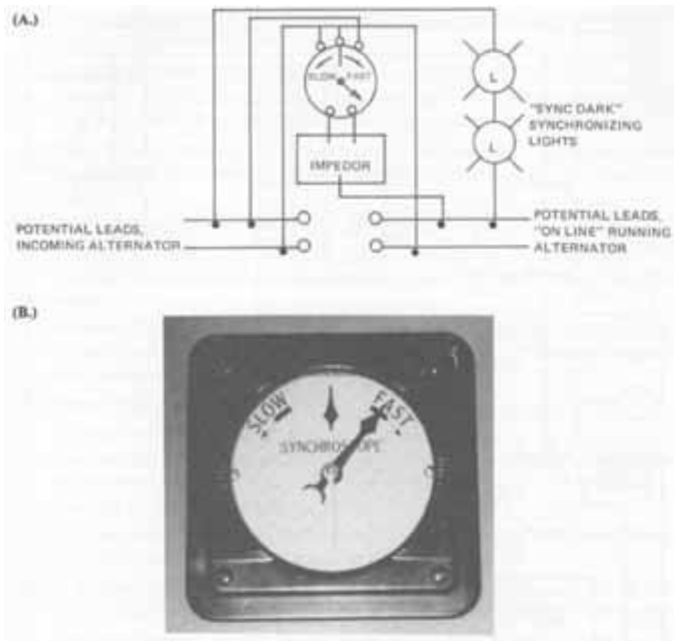



**Fig. 42 A) Diagram of synchroscope connection B) Photo of synchroscope meter face and synchronizing lig***hts*

## Prime Movers:

In industrial applications, alternators are driven by various types of prime movers such as steam turbines, water turbines, and internal combustion engines. For applications on ships, alternators often are driven by dc motors. Regardless of how alternators are driven, speed variation is a factor in paralleling the machines. Thus, the electrician should have knowledge of speed governors and other speed regulating devices. This text, however, does not detail the operation of these mechanical devices.

## Paralleling Alternators:

Since apprentices are likely to be required to parallel alternators driven by dc motors sometime in their instruction, the following steps outline the procedure for paralleling these machines. Fig. 431illustrates a typical circuit for paralleling two three-phase alternators.

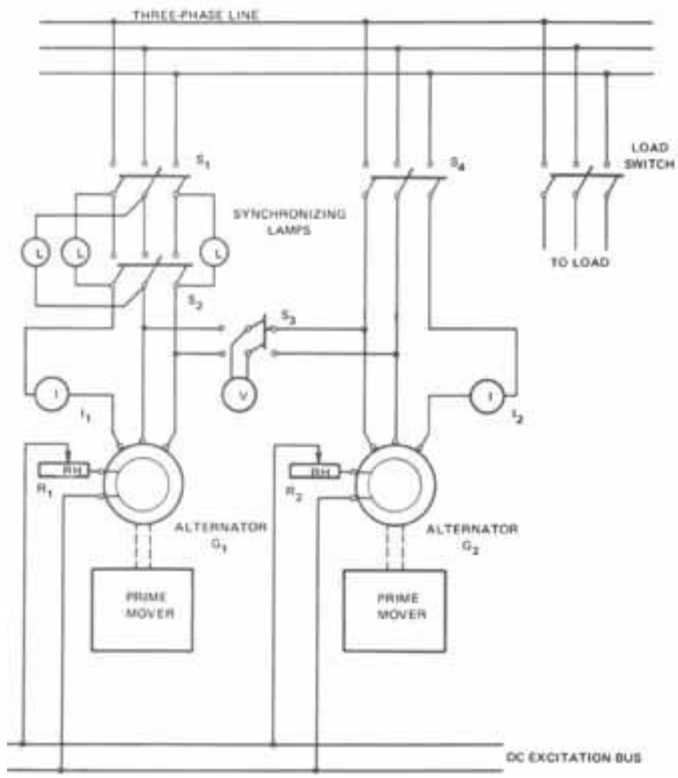


**Fig. 43 Parallel operation of alternators**

**Procedure:**

1. Set the field rheostat R2 of alternator G2 to the maximum resistance position.

2. Knowing the number of field poles in alternator G2 determine the speed required to generate the desired frequency.

3. Energize the prime mover to bring alternator G2 up to the required speed.

4. Set Switch S to read the ac voltage across one phase of G2 Adjust field rheostat R2 until the output voltage is equal to the rated voltage of the load circuit.

5. Close the load switch and switch S4 to feed the load circuit. Readjust the speed of the prime mover to maintain the predetermined speed required for the desired frequency.

6. Readjust R2 to obtain the rated ac voltage of the load circuit.

7. Energize the prime mover to drive the second alternator, G1 Adjust the speed of the alternator to the approximate value required to match the frequencies of the alternators.

8. Set switche S3 to measure the ac voltage across one phase of G1. Adjust field rheostat R1 until the ac voltage is equal at either position of switch S3. The voltage output of both alternators is now equal.

9. Phase Rotation

With paralleling switch S2 open, close switch S1.

The three sets of lamps across the terminals of the open switch will respond in one of two ways:

a. The three lamps will brighten and then dim in unison.

b. Two lamps will brighten in unison as the remaining lamp dims. Then the two bright lamps will dim as the dark lamp brightens.

10. If the lamps respond as in 9a, the alternators are connected for the proper phase rotation. The operator then may proceed to the next step in synchronizing the alternators.

11. If the lamps respond as in 9b, the alternators are not in the proper phase rotation. To correct the condition, interchange any two alternator leads at the terminals of switch S2. All three lamps should dim together and brighten together. No attempt to parallel the alternators should be made until the lamps respond in this manner.

12. The three lamp sets will flicker (dim and brighten) at a rate equal to the frequency difference between the two alternators. Adjust the speed control of prime mover M1 to make the lamps flicker at the lowest possible rate.

13. Interchange two lamp set leads (not alternator leads) at the terminals of switch S2 so that the alternators can be synchronized using the two bright, one dark method.

14. Again adjust the field rheostat of alternator G1 until both alternators have the same output voltage as measured at either position of the voltmeter switch S3.

15. With one hand on switch S2 watch the lamps. Close the switch at the exact instant that two lamps are at their brightest and the other lamp is out. This operation shunts out the synchronizing lamps and parallels the alternators.

16. Ammeters I1 and I2 indicate the amount of load current carried by each alternator. If the load circuit has a unity power factor, then the sum of the ammeter readings should equal the reading of the ammeter in the load circuit.

17. Note that a change in the field excitation of either alternator does not appreciably change the amount of current supplied to the system. Such a change in field excitation does, however, affect the power factor of the specific alternator. The field rheostat of each machine should be adjusted to the highest power factor as indicated by the lowest value of current from the individual machine. Increasing or decreasing the mechanical power to either alternator will increase or decrease the load current of that machine. As a result, the division of the load between the alternators can be changed by slight changes in the alternator speed.

## Synchronizing Power and Torque Coefficient

Definition: – Synchronizing Power is defined as the varying of the synchronous power P on varying in the load angle δ. It is also called Stiffness of Coupling, Stability or Rigidity factor. It is represented as $P_{syn}$. A synchronous machine, whether a generator or a motor, when synchronized to infinite Bus bars has an inherent tendency to remain in Synchronism.

Consider asynchronous generator transferring a steady power $P_a$ at a steady load angle $\delta_0$. Suppose that, due to a transient disturbance, the rotor of the generator accelerates, resulting from an increase in the load angle by dδ.

The operating point of the machine shifts to a new constant power line and the load on the machine increases to $P_a + \delta P$. The steady power input of the machine does not change, and the additional load which is added decreases the speed of the machine and brings it back to synchronism.

Similarly, if due to a transient disturbance, the rotor of the machine retards resulting a decrease in the load angle. The operating point of the machine shifts to a new constant power line and the load on the machine decreases to $(P_a – \delta P)$. Since the input remains unchanged, the reduction in load accelerates the rotor. The machine again comes in synchronism.

The effectiveness of this correcting action depends on the change in power transfer for a given change in load angle. The measure of effectiveness is given by **Synchronizing Power Coefficient**.

$$P_{syn} = \frac{dP}{d\delta} \text{ ------------------ (1)}$$

Power output per phase of the cylindrical rotor generator

$$P = \frac{V}{Zs}[E\cos(\theta_z - \delta) - V\cos\theta_z] \text{ -------------- (2)}$$

$$P_{syn} = \frac{dP}{d\delta} = \frac{VE}{Zs}\sin(\theta_z - \delta) \text{ ---------------- (3)}$$

The synchronizing torque coefficient

$$T_{syn} = \frac{dT}{d\delta} = \frac{1}{2\pi ns}\frac{dP}{d\delta} \text{ ------------------ (4)}$$

$$T_{syn} = \frac{VE}{2\pi ns}\sin(\theta_z - \delta) \text{ ------------------------- (5)}$$

In many synchronous machines Xs>> R, therefore, for a cylindrical rotor machine, neglecting saturation and stator resistance equation (3) and (5) becomes

$$P_{syn} = \frac{VE}{Xs}\cos\delta \text{ ------------------------ (6)}$$

$$T_{syn} = \frac{VE}{2\times \text{Input power} \times ns \times Xs}\sin(\theta_z - \delta) \text{ ---------------- (7)}$$

For a salient pole machine

$P = \frac{VE}{Xs}\sin\delta + \frac{1}{2}\ V^2(\frac{1}{Xd} - \frac{1}{Xq})\sin 2\delta$ --------------- (8)

$\mathbf{P_{syn}} == \frac{VE}{Xs}\cos\delta + V^2((\frac{1}{Xd} - \frac{1}{Xq})\cos 2\delta)$ ------------ (9)

## Unit of Synchronizing Power Coefficient $P_{syn}$

The **synchronizing Power Coefficient** is expressed in watts per electrical radian.

Therefore,

$P_{syn} = \frac{VE}{Xs}\cos\delta$ W/ Electrical radian -------------- (10)

Since, $\pi$ radians = $180^0$

1 radian = $180/\pi$ degrees

$P_{syn2} = \frac{dP}{d\delta}\ \frac{\pi}{180}$ W/ Electrical radian ------------- (11)

If P is the total number of poles of the machine

$\theta_{electrical} = \frac{P}{2}\theta mechanical$

Synchronizing Power Coefficient per mechanical radian is given by the equation shown below.

$P_{syn3} = P\ \frac{dP}{d\delta}$ Watts ---------------- (12)

Synchronizing Power Coefficient per mechanical degree is given as

$P_{syn3} = \frac{P\pi}{180}\frac{dP}{d\delta}$ Watts ------------------ (13)

## Synchronizing Torque Coefficient

**Synchronizing Torque Coefficient** gives rise to the synchronizing torque coefficient at synchronous speed. That is, the Synchronizing Torque is the torque which at synchronous speed gives the synchronizing power. If $\tau_{syn}$ is the synchronizing torque coefficient than the equation is given as shown below

$$T_{syn} = \frac{1}{\omega_s} m \frac{dP}{d\delta} \text{ Nm/ Electrical radian ------------------- (14)}$$

$$T_{syn} = \frac{1}{\omega_s} m \frac{dP}{d\delta} \times \frac{P\pi}{180} \text{ Nm/ Electrical radian --------------- (15)}$$

Where,

- m is the number of phases of the machine
- $\omega_s = 2 \pi n_s$
- $n_s$ is the synchronous speed in revolution per second

$$T_{syn} = \frac{P_{syn}}{\omega_s} = \frac{P_{syn}}{2\pi n_s}$$

## Significance of Synchronous Power Coefficient

The **Synchronous Power Coefficien**t $P_{syn}$ is the measure of the stiffness between the rotor and the stator coupling. A large value of $P_{syn}$ indicates that the coupling is stiff or rigid. Too rigid a coupling means and the machine will be subjected to shock, with the change of load or supply. These shocks may damage the rotor or the windings. We have,

$$P_{syn} = \frac{3VE}{Xs} \cos\delta \text{ -------------- (17)}$$

$$T_{syn} = \frac{3}{2\pi ns} \frac{VE}{Xs} \cos\delta \text{ ----------- (18)}$$

The above two equations (17) and (18) show that $P_{syn}$ is inversely proportional to the synchronous reactance. Machines with large air gaps have relatively small reactance. The synchronous machine with the larger air gap is stiffer than a machine with a smaller air gap. Since $P_{syn}$ is directly proportional to $E_f$, an overexcited machine is stiffer than an under the excited machine.

The restoring action is great when $\delta = 0$, that is at no load. When the value of $\delta = \pm 90^0$, the restoring action is zero. At this condition, the machine is in unstable equilibrium and at steady state limit of stability. Therefore, it is impossible to run a machine at the steady state limit of stability since its ability to resist small changes is zero unless the machine provided with special fast acting excitation system.

## Load Sharing

When several alternators are required to run in parallel, it probably happens that their rated outputs differ. In such cases it is usual to divide the total load between them in such a way that each alternator takes the load in the same proportion of its rated load in total rated outputs. The total load is not divided equally. Alternatively, it may be desired to run one large alternator permanently on full load, the fluctuations in load being borne by one or more of the others.

If the alternators are sharing the load equally the power triangles are as shown in figure below.

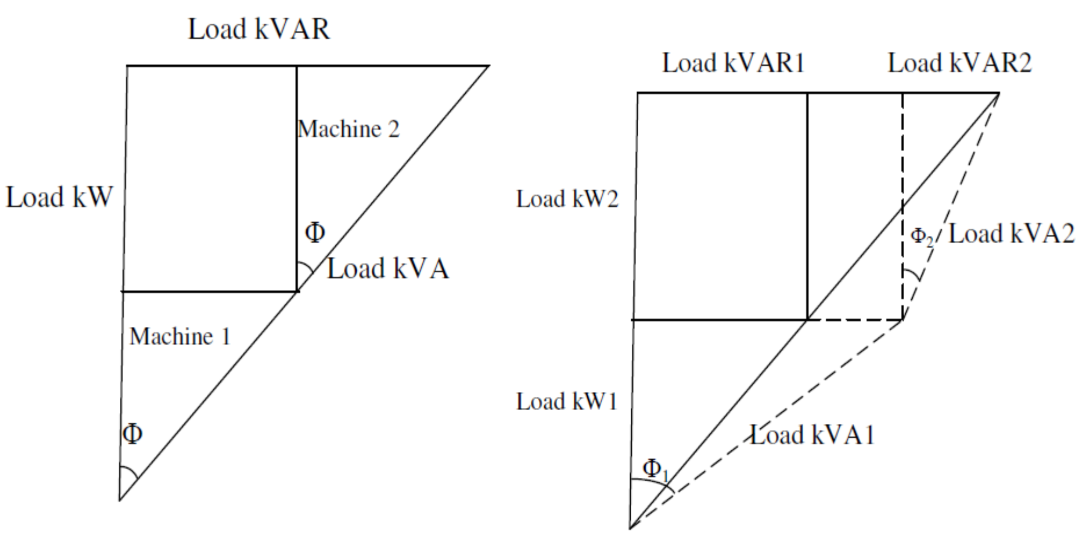



**Figure: 44. Load sharing of alternators.**

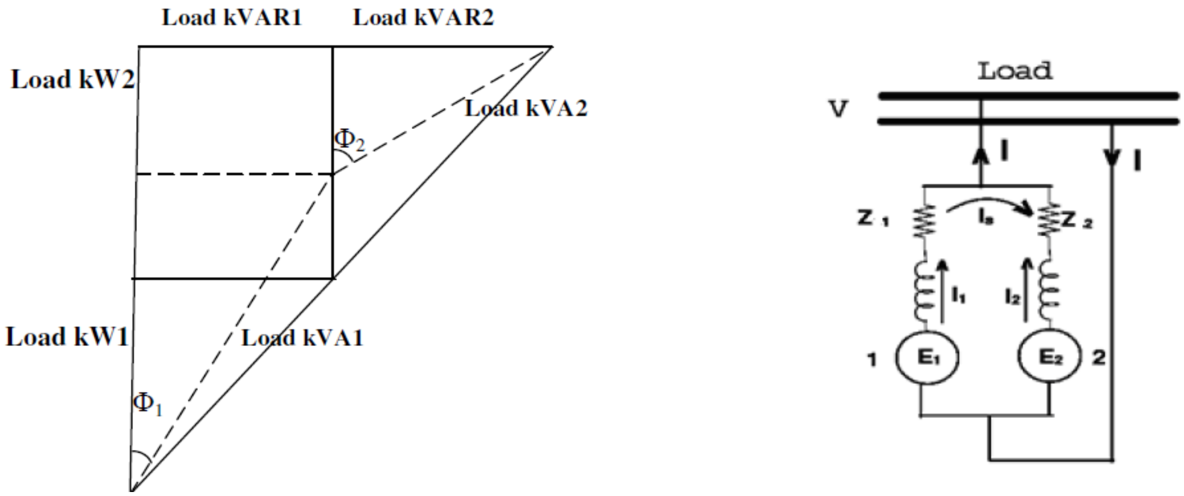



**Figure: 45. Load sharing of alternators.**

## Sharing of load when two alternators are in parallel

Consider two alternators with identical speed load characteristics connected in parallel as shown in figure above.

Let $E_1$, $E_2$ be the induced EMF per phase,

$Z_1$, $Z_2$ be the impedances per phase

$I_1$, $I_2$ be the current supplied by each machine per phase

Z be the load impedance per phase,

V be the terminal voltage per phase

From the circuit we have $V = E_1 - I_1Z_1 = E_2 - I_2Z_2$ and hence

$$I_1 = \frac{E1 - V}{Z1} \quad ,$$

$$I2 = \frac{E2 - V}{Z2}$$

And also $V = (I_1 + I_2)\, Z = IZ$ solving above equations

$$I_1 = \frac{[\frac{(E1-E2)}{Z} + E1Z2]}{[Z(Z1+Z2)+Z1Z2}$$

$$I_2 = \frac{[\frac{(E2-E1)}{Z} + E2Z1]}{[Z(Z1+Z2)+Z1Z2}$$

Then total current is given by

$$I = I_1 + I_2 = \frac{[E1Z2+E2Z1]}{[Z(Z1+Z2)+Z1Z2}$$

And the circulating current or synchronizing current

$$Isy = \frac{[E1 - E2]}{(Z1 + Z2)}$$

## Numerical Problems on Load sharing

1. Two alternators working in parallel supply following loads i) Lighting load of 500 KW ii) 1000 KW at 0.9 p.f. lagging iii) 500 KW at 0.95 p.f. lead iv) 800 KW at 0.8 lagging. One alternator is supplying 1500 KW at 0.95 p.f. lagging. Calculate the load on the other machine.

Sol: The KW and KVAR components of each load are as follows. For lagging loads KVAR is considered positive whereas for leading loads KVAR is considered negative.

| **Load** | **KW** | **cos∅** | **tan∅** | **KVAR = KW × tan∅** |
|---|---|---|---|---|
| 1 | 500 | 1 | 0 | 0 |
| 2 | 1000 | 0.9 lag | 0.4843 | 100 × 0.4843 = 484.30 |
| 3 | 500 | 0.9 lead | 0.4843 | 500× 0.4843 = - 242.15 |
| 4 | 800 | 0.8 lab | 0.75 | 800×0.75 = 600 |

Total KW = 500 + 1000 + 500 + 800 = 2800

Total KVAR = 0 + 484.30 – 242.15 + 600 = 842.15

If Φsc is total p.f. angle of combined load

$$\tan\Phi sc = \frac{KVAR}{KW} = \frac{842.15}{280} = 0.3007$$

One alternator is supplying 1500 KW at 0.95 p.f. lagging

$\cos\Phi_1 = 0.95$, $\tan\Phi_1 = 0.3286$

Reactive component (KVAR) of machine 1 = KW × tan $\Phi_1$

= 1500 × 0.3286

= 492.9 KVAR

Active component of machine 2 = Total KW component − Active component of machine 1

= 2800 – 1500

= 1300 KW

Reactive component of machine 2 = Total KVAR component – Reactive component of machine 1

= 842.15 – 492.9

= 349.25 KVAR

The p.f. of machine 2 can be calculated as,

$$\tan\Phi_1 = \frac{\text{KVAR Component of machine 2}}{\text{KW Component of machine 2}}$$

$\Phi_2 = 15.03^0$

Power factor = cos $\Phi_2$ = 0.9657 lag

Output of machine 2 = 1300 KW

Power factor of machine 2 = 0.9657 lagging

2. Two exactly similar turbo alternators are rated at 25 MW each. They are running in parallel. The speed load characteristics of the driving turbines are such that the frequency of alternator 1 drops uniformly from 50 Hz on no load to 48 Hz on full load and that of alternator 2 from 50 Hz to 48.5 Hz. How will they share a total load of 30 MW
3. Two identical 400 KVA alternators operate in parallel. The governor of the prime mover of first machine is such that the frequency drops uniformly from 50Hz on no load to 48 Hz on full load. The corresponding speed drop of the second machine is 50 Hz to 47.5 Hz. Find i) how will the two machines share a load of 6000 KW? ii) What is the maximum load of unity power factor that can be delivered without overloading either machine?
4. Two single phase alternators operating in parallel have induced EMFs on open circuit of $210\angle0^0$ V and $210\angle10^0$ V and respective reactance's of j5Ω and j7Ω. Calculate i) Terminal voltage ii) Currents and iii) Power delivered by each of the alternators to a load of resistance 9Ω.

5. Two alternators running in parallel supply lighting load of 2500 KW and a motor load of 5000 KW at 0.870 P.F. one machine is loaded to 4500 KW at a P.F. of 0.79 lagging. What is the KW output and P.F. of the other machine?

6. Two single phase alternators operate in parallel and supply a load impedance of (6+ j7) Ω. If the impedance of the machine is (0.4 + j5) and EMFs are (220+j0) and (220+j0) volts respectively determine for each machine. i) Terminal voltage ii) Power factor and iii) Output

7. Two exactly similar turbo alternators are rated at 25 MW each. They are running in parallel. The speed load characteristics of the driving turbines are such that the frequency of alternator 1 drops uniformly from 50 Hz

on no load to 49 Hz on full load and that of alternator 2 from 50 Hz to 47 Hz. How will they share a total load of 40 MW

8. Two identical 400 KVA alternators operate in parallel. The governor of the prime mover of first machine is such that the frequency drops uniformly from 50Hz on no load to 48 Hz on full load. The corresponding speed drop of the second machine is 50 Hz to 47.5 Hz. Find i) how will the two machines share a load of 7000 KW? ii) What is the maximum load of unity power factor that can be delivered without overloading either machine?

9. Two single phase alternators operating in parallel have induced EMFs on open circuit of $220\angle 0^0$ V and $220\angle 10^0$ V and respective reactance's of j3Ω and j4Ω. Calculate i) Terminal voltage ii) Currents and iii) Power delivered by each of the alternators to a load of resistance 6Ω.

Sol: Impedance of alternators 1, $Z_1 = j3\Omega$

Impedance of alternator 2, $Z_2 = j4\Omega$

$E_1 = 220\angle 0^0$ V

$E_2 = 220\angle 10^0$ V

Current $I_1$ is given by,

$$I_1 = \frac{(E1-E2)Z+E1Z2}{Z(Z1+Z2)+Z1Z2}$$

$$I_1 = \frac{(220\angle 0-220\angle 10)6+(220\angle 10)j4}{6(j3+j4)+(j3)(j4)} = 14.90\angle 17.71^0$$

$$I_2 = \frac{(E2-E1)Z+E2Z1}{Z(Z1+Z2)+Z1Z2} = \frac{(220\angle 10-220\angle 0)6+(220\angle 10)j3}{6(j3+j4)+(j3)(j4)}$$

$$I_2 = 20.36\angle -7.23^0 \text{ A}$$

$I = I_1 + I_2$

$= (14.19 - j\,4.53) + (20.19 - j\,2.56)$

$= 35.10\angle -11.65^0$

Now Voltage $V = IZ = (35.10\angle -11.65^0) \times (6\angle 0^0)$

$= 210.6\angle -11.65^0$

$P_1 = VI_1 \cos\Phi_1 = 210.6 \times 14.90 \times \cos 7.23^0$

$= 2989.22$ Watts

$P_2 = VI_2 \cos\Phi_2 = 210.6 \times 20.36 \times \cos 17.71^0$

$= 4253.72$ Watts

10. Two alternators running in parallel supply lighting load of 2500 KW and a motor load of 5000 KW at 0.707 P.F. one machine is loaded to 4000 KW at a P.F. of 0.8 lagging. What is the KW output and P.F. of the other machine?

11. Two single phase alternators operate in parallel and supply a load impedance of (3+ j4) Ω. If the impedance of the machine is (0.2 + j2) and EMFs are (220+j0) and (220+j0) volts respectively determine for each machine. i) Terminal voltage ii) Power factor and iii) Output

# UNIT-IV

# Synchronous motors

## Principle of operation

In order to understand the principle of operation of a synchronous motor, assume that the armature winding (laid out in the stator) of a 3-phase synchronous machine is connected to a suitable balanced 3-phase source and the field winding to a D.C source of rated voltage. The current flowing through the field coils will set up stationary magnetic poles of alternate North and South. On the other hand, the 3-phase currents flowing in the armature winding produce a rotating magnetic field rotating at synchronous speed. In other words there will be moving North and South poles established in the stator due to the 3-phase currents i.e. at any location in the stator there will be a North Pole at some instant of time and it will become a South Pole after a time period corresponding to half a cycle. (After a time = 1/2f , where f = frequency of the supply). Assume that the stationary South pole in the rotor is aligned with the North pole in the stator moving in clockwise direction at a particular instant of time, as shown in Figure below. These two poles get attracted and try to maintain this alignment (as per Lenz's law) and hence the rotor pole tries to follow the stator pole as the conditions are suitable for the production of torque in the clockwise direction. However, the rotor cannot move instantaneously due to its mechanical inertia, and so it needs some time to move.

In the mean time, the stator pole would quickly (a time duration corresponding to half a cycle) change its polarity and becomes a South Pole. So the force of attraction will no longer be present and instead the like poles experience a force of Repulsion as shown in Figure below. In other words, the conditions are now suitable for the production of torque in the anticlockwise direction. Even this condition will not last longer as the stator pole.

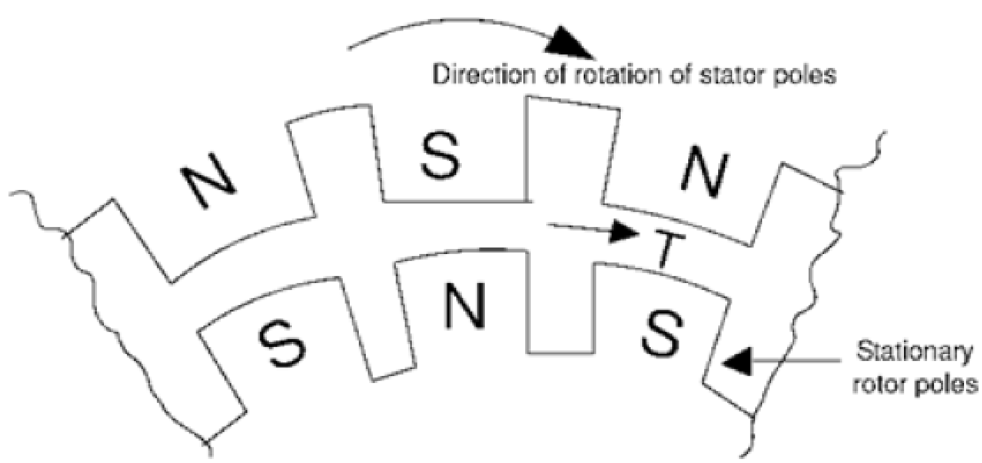


**Figure: 46. Force of attraction between stator poles and rotor poles - resulting in production of torque in clockwise direction**.

Would again change to North Pole after a time of 1/2f. Thus the rotor will experience an alternating force which tries to move it clockwise and anticlockwise at twice the frequency of the supply, i.e. at intervals corresponding to 1/2f seconds. As this duration is quite small compared to the mechanical time constant of the rotor, the rotor cannot respond and move in any direction. The rotor continues to be stationary only.

On the contrary if the rotor is brought to near synchronous speed by some external device say a small motor mounted on the same shaft as that of the rotor, the rotor poles get locked to the unlike poles in the stator and the rotor continues to run at the synchronous speed even if the supply to the motor is disconnected. Thus the synchronous rotor cannot start rotating on its own when the rotor and stator are supplied with rated voltage and frequency and hence the synchronous motor has no starting torque. So, some special provision has to be made either inside the machine or outside of the machine so that the rotor is brought to near about its synchronous speed. At that time, if the armature is supplied with electrical power, the rotor can pull into step and continue to run at its synchronous speed. Some of the commonly used methods for starting synchronous rotor are described in the following paragraph.

Would again change to North Pole after a time of 1/2f. Thus the rotor will experience an alternating force which tries to move it clockwise and anticlockwise at twice the frequency of the supply, i.e. at intervals corresponding to 1/2f seconds. As this duration is quite small compared to the mechanical time constant of the rotor, the rotor cannot respond and move in any direction. The rotor continues to be stationary only.

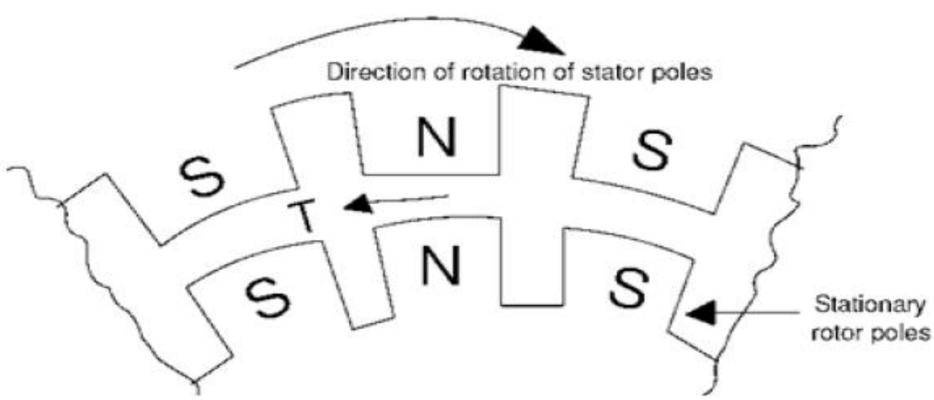



**Figure: 47. Force of repulsion between stator poles and rotor poles - resulting in production of torque in anticlockwise direction**

## Working of Synchronous Motor

The stator is wound for the similar number of poles as that of rotor, and fed with three phase AC supply. The 3 phase AC supply produces rotating magnetic field in stator. The rotor winding is fed with DC supply which magnetizes the rotor.

- Now, the stator poles are revolving with synchronous speed (let's say clockwise). If the rotor position is such that, N pole of the rotor is near the N pole of the stator (as shown in first schematic of above figure), then the poles of the stator and rotor will repel each other, and the torque produced will be anticlockwise.
- The stator poles are rotating with synchronous speed, and they rotate around very fast and interchange their position. But at this very soon, rotor cannot rotate with the same angle (due to inertia), and the next position will be likely the second schematic in above figure. In this case, poles of the stator will attract the poles of rotor, and the torque produced will be clockwise.
- Hence, the rotor will undergo to a rapidly reversing torque, and the motor will not start.

But, if the rotor is rotated up to the synchronous speed of the stator by means of an external force (in the direction of revolving field of the stator), and the rotor field is excited near the synchronous speed, the poles of stator will keep attracting the opposite poles of the rotor (as the rotor is also, now, rotating with it and the position of the poles will be similar throughout the cycle). Now, the rotor will undergo unidirectional torque. The opposite poles of the stator and rotor will get locked with each other, and the rotor will rotate at the synchronous speed.

## Characteristic features of a synchronous motor

- Synchronous motor will run either at synchronous speed or will not run at all.
- The only way to change its speed is to change its supply frequency. (As $N_s = 120f / P$)
- Synchronous motors are not self starting. They need some external force to bring them near to the synchronous speed.
- They can operate under any power factor, lagging as well as leading. Hence, synchronous motors can be used for power factor improvement.

As seen earlier, synchronous motor is not self starting. It is necessary to rotate the rotor at a speed very near to synchronous speed. This is possible by various methods in practice. The various methods to start the synchronous motor are,

1. Using pony motors
2. Using damper winding
3. As a slip ring induction motor
4. Using small DC. Machine coupled to it.

## 1. Using pony motors

In this method, the rotor is brought to the synchronous speed with the help of some external device like small induction motor. Such an external device is called 'pony motor'.

Once the rotor attains the synchronous speed, the d.c. excitation to the rotor is switched on. Once the synchronism is established pony motor is decoupled. The motor then continues to rotate as synchronous motor.

## 2. Using Damper Winding

In a synchronous motor, in addition to the normal field winding, the additional winding consisting of copper bars placed in the slots in the pole faces. The bars are short circuited with the help of end rings. Such an additional winding on the rotor is called damper winding. This winding as short circuited, acts as a squirrel cage rotor winding of an induction motor. The schematic representation of such damper winding is shown in the Fig.48.

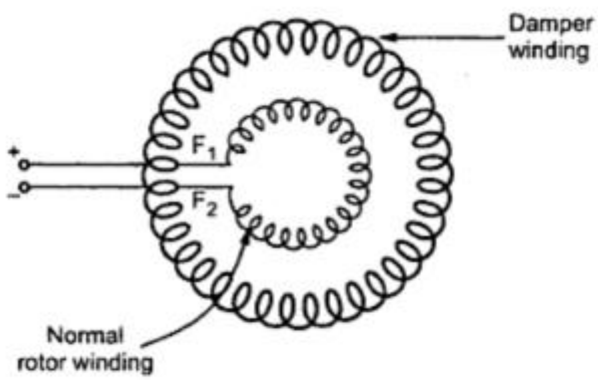



**Fig . 48 Starting as a squirrel cage I.M.**

Once the rotor is excited by a three phase supply, the motors starts rotating as an induction motor at sub synchronous speed. Then DC supply is given to the field winding. At a particular instant motor gets pulled into synchronism and starts rotating at a synchronous speed. As rotor rotates at synchronous speed, the relative motion between damper winding and the rotating magnetic field is zero. Hence when motor is running as synchronous motor, there cannot be any induced EMF in the damper winding. So damper winding is active only at start, to run the motor as an induction motor at start. Afterwards it is out of the circuit. As damper winding is short circuited and motor gets started as induction motor, it draws high current at start so induction motor starters like star-delta, autotransformer etc. used to start the synchronous motor as an induction motor.

## 3. As a Slip Ring Induction Motor

The above method of starting synchronous motor as a squirrel cage induction motor does not provide high starting torque. So to achieve this, instead of shorting the damper winding, it is designed to a form a three phase star or delta connected winding. The three ends of this winding are brought out through slip rings. An external rheostat then can be introduced in series with the rotor circuit. So when stator is excited, the motor starts as a slip ring induction motor and due to resistance added in the rotor provides high starting torque. The resistance is then gradually cut off, as motor gathers speed. When motor attains speed near synchronous. DC excitation is provided to the rotor, then motors gets pulled into synchronism and starts rotating at synchronous speed. The damper winding is shorted by shorting the slip rings. The initial resistance added in the rotor not only provides high starting torque but also limits high inrush of starting current. Hence it acts as a motor resistance starter.

The synchronous motor started by this method is called a slip ring induction motor is shown in the Fig.49.

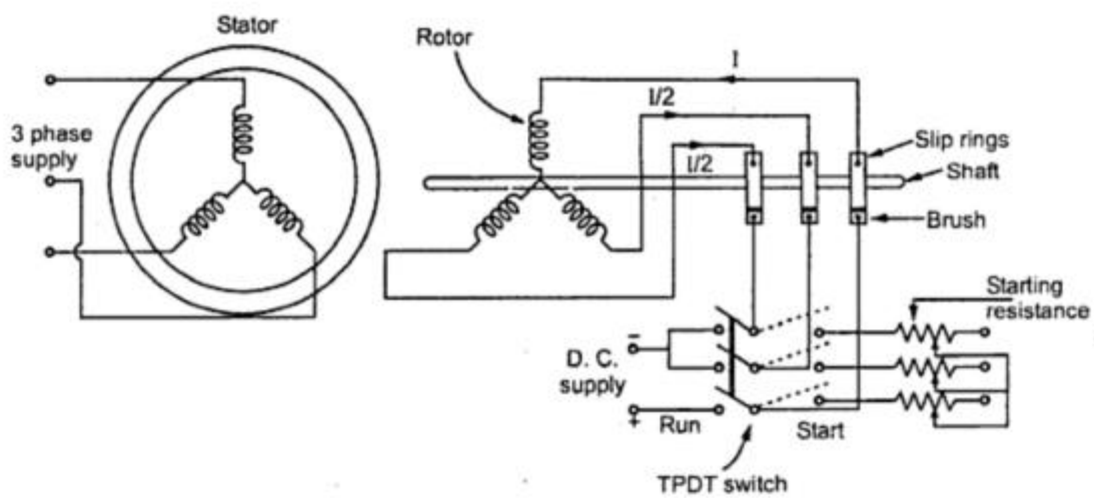



**Fig. 49 Starting as a slip ring I.M.**

It can be observed from the Fig. 49 that the same three phase rotor winding acts as a normal rotor winding by shorting two of the phases. From the positive terminal, current 'I' flows in one of the phases, which divides into two other phases at start point as 1/2 through each, when switch is thrown on d.c. supply side.

### 4. Using Small D.C. Machine

Many a times a large synchronous motor is provided with a coupled DC machine. This machine is used as a DC motor to rotate the synchronous motor at a synchronous speed. Then the excitation to the rotor is provided. Once motor starts running as a synchronous motor, the same DC machine acts as a DC generator called exciter. The field of the synchronous motor is then excited by this exciter itself.

### Effect of changes in load on, $I_a$, $\delta$, and p. f. of synchronous motor:

The effects of changes in mechanical or shaft load on armature current, power angle, and power factor can be seen from the phasor diagram shown in Figure below; as already stated, the applied stator voltage, frequency, and field excitation are assumed, constant. The initial load conditions are represented by the thick lines. The effect of increasing the shaft load to twice its initial value is represented by the light lines indicating the new steady state conditions. While drawing the phasor diagrams to show new steady-state conditions, the line of action of the new

$jI_aX_s$ phasor must be perpendicular to the new $I_a$ phasor. Furthermore, as shown in figure if the excitation is not changed, increasing the shaft load causes the locus of the $E_f$ phasor to follow a circular arc, thereby increasing its phase angle with increasing shaft load. Note also that an increase in shaft load is also accompanied by a decrease in $\Phi_i$; resulting in an increase in power factor.

As additional load is placed on the machine, the rotor continues to increase its angle of lag relative to the rotating magnetic field, thereby increasing both the angle of lag of the counter EMF phasor and the magnitude of the stator current. It is interesting to note that during all this load variation; however, except for the duration of transient conditions whereby the rotor assumes a new position in relation to the rotating magnetic field, the average speed of the machine does not change. As the load is being increased, a final point is reached at which a further increase in δ fails to cause a corresponding increase in motor torque, and the rotor pulls out of synchronism. In fact as stated earlier, the rotor poles at this point, will fall behind the stator poles such that they now come under the influence of like poles and the force of attraction no longer exists. Thus, the point of maximum torque occurs at a power angle of approximately 90◦ for a cylindrical-rotor machine. This maximum value of torque that causes a synchronous motor to pull out of synchronism is called the pull-out torque. In actual practice, the motor will never be operated at power angles close to $90^0$ as armature current will be many times its rated value at this load.

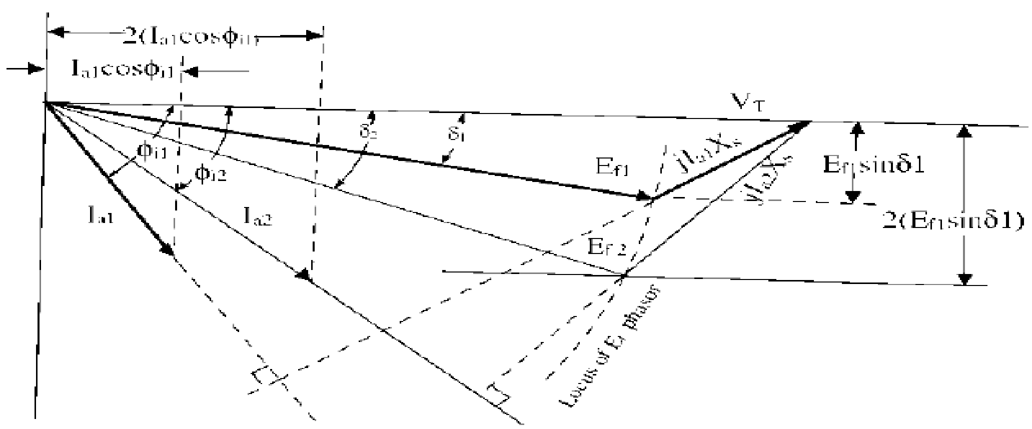



**Figure: 50. Effect of changes in load on, $I_a$, δ, and p. f**

## Effect of changes in excitation on the performance synchronous motor

Increasing the strength of the magnets will increase the magnetic attraction, and thereby cause the rotor magnets to have a closer alignment with the corresponding opposite poles of the rotating magnetic poles of the stator. This will obviously result in a smaller power angle. This fact can also be seen from power angle equation. When the shaft load is assumed to be constant, the steady-state value of $E_f$ sinδ must also be constant. An increase in $E_f$ will cause a transient increase in $E_f$ sinδ, and the rotor will accelerate. As the rotor changes its angular position, δ decreases until $E_f$ sinδ has the same steady-state value as before, at which time the rotor is again operating at synchronous speed, as it should run only at the synchronous speed. This change in angular position of the rotor magnets relative to the poles of rotating magnetic field of the stator occurs in a fraction of a second. The effect of changes in field excitation on armature current, power angle, and power factor of a synchronous motor operating with a constant shaft load, from a constant voltage, constant frequency supply, is illustrated in figure below.

$E_{f1} \sin \delta_1 = E_{f2} \sin \delta_2 = E_{f3} \sin \delta_3 = E_f \sin \delta$

This is shown in Figure below, where the locus of the tip of the Ef phasor is a straight line parallel to the VT phasor. Similarly,

$I_{a1} \cos \Phi_{i1} = I_{a2} \cos \Phi_{i2} = I_{a3} \cos \Phi_{i3} = I_a \cos \Phi_i$

This is also shown in Figure below, where the locus of the tip of the Ia phasor is a line perpendicular to the phasor VT.

Note that increasing the excitation from Ef1 to Ef3 caused the phase angle of the current phasor with respect to the terminal voltage VT (and hence the power factor) to go from lagging to leading. The value of field excitation that results in unity power factor is called normal excitation. Excitation greater than normal is called over excitation, and excitation less than normal is called under excitation.

Further, as indicated in Figure when operating in the overexcited mode, $|E_f| > |V_T|$.A synchronous motor operating under over excited condition is called a synchronous condenser.

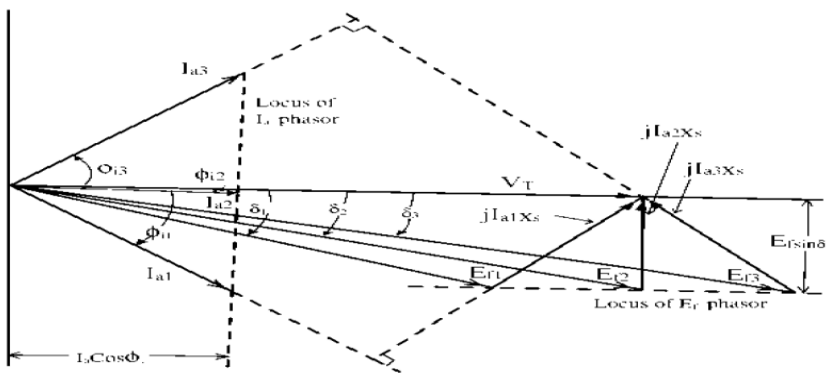
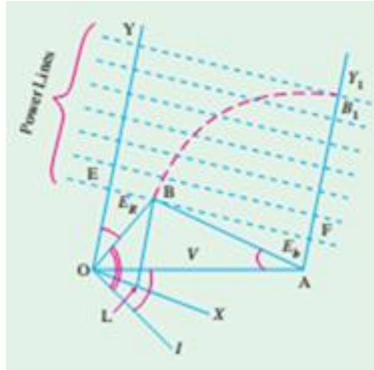



**Figure: 51. Phasor diagram showing effect of changes in field excitation on armature current, power angle and power factor of a synchronous motor**

## Constant-power Lines

In below fig, *OA* represents applied voltage / phase of the motor and *AB* is the back EMF./ phase, $E_b$. *OB* is their resultant voltage *ER*. The armature *B* current is *OI* lagging behind *ER* by an angle q = *ER* or vector *OB* represents (to some suitable scale) the main current *I*. *OY* is drawn at an angle f with *OB* (or at an angle q with *C A*). *BL* is drawn perpendicular to *OX* which is at right angles to *O Y*. Vector *OB*, when referred to *O Y*, also represents, on a different scale, the current both in magnitude and phase.




As V is constant, power input is dependent on B L. If motor is working with a constant intake, then locus of B is a straight line || to OX and ^ to OY i.e. line EF for which BL is constant. Hence, EF represents a constant-power input line for a given voltage but varying excitation. Similarly, a series of such parallel lines can be drawn each representing a definite power intake of the motor. As regards these constant-power lines, it is to be noted that

**1. F**or equal increase in intake, the power lines are parallel and equally-spaced

**2. Z**ero power line runs along OX

**3.** The perpendicular distance from B to OX (or zero power line) represents the motor intake

**4.** If excitation is fixed i.e. AB is constant in length, then as the load on motor is increased, increases. In other words, locus of B is a circle with radius = AB and centre at A. With increasing load, B goes on to lines of higher power till point B1 is reached. Any further increase in load on the motor will bring point B down to a lower line. It means that as load increases beyond the value corresponding to point B1, the motor intake decreases which is impossible. The area to the right of AY 1 represents unstable conditions. For a given voltage and excitation, the maximum power the motor can develop, is determined by the location of point B1 beyond which the motor pulls out of synchronism.

The Blondel diagram of a synchronous motor is an extension of a simple phasor diagram of a synchronous motor.

For a synchronous motor, the power input to the motor per phase is given by,

Pin $= V_{ph} I_{ph} \cos\Phi$? ………… per phase

The gross mechanical power developed per phase will be equal to the difference between Pin per phase and the per phase copper losses of the winding.

Copper loss per phase = $(Iaph)^2\ R_a$

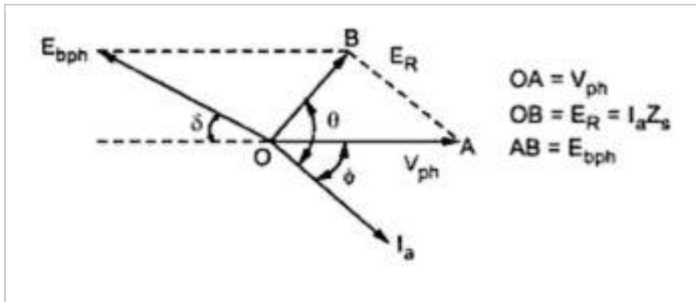
... Pm $= V_{ph} I_{ph}$ cos? – $(I_{aph})^2$ Ra ………. per phase

For mathematical convenience let Vph = V and $I_{aph} = I$, Pm = VI cosΦ – I2 Ra

... $I^2$ Ra – VI cosΦ + Pm = 0

$$\therefore \quad I^2 - \frac{V I \cos\phi}{R_a} + \frac{P_m}{R_a} = 0 \qquad \ldots(1)$$

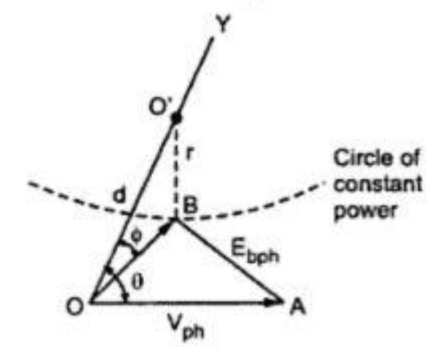
Now consider the phasor diagram as shown in the Fig. 1.




The equation (1) represents polar equation to a circle. To obtain this circle in a phasor diagram, draw a line OY at an angle ? with respect to OA.

$$\therefore \quad \angle YOA = \theta$$

$$\therefore \quad \angle YOB = \phi$$




The circle represented by equation (1) has a centre at some point O’ on the line OY. The circle drawn with centre as O’ and radius as O’B represents circle of constant power. This is called Blondel diagram, shown in the Fig. 2.

Thus if excitation is varied while the power is kept constant, then working point B while move along the circle of constant power.

Let O’B = Radius of circle = r

OO’ = Distant d

Applying cosine rule to triangle OBO’,

Now OB represents resultant ER which is Ia Zs. Thus OB is proportional to current and when referred to OY represents the current in both magnitude and phase.

OB = Ia = I say

Substituting various values in equation (2) we get,

r2 = I2 + d2 – 2dI cos?

$$\therefore \quad I^2 - 2d\,I\cos\phi + (d^2 - r^2) = 0 \qquad \ldots (3)$$

Comparing equations (1) and (3) we get,

$$OO' = d = \frac{V}{2R_a} \qquad \ldots (4)$$

Thus the point O’ is independent of power Pm and is a constant for a give motor operating at a fixed applied voltage V.

Comparing last term of equations (1) and (3),

$$d^2 - r^2 = \frac{P_m}{R_a}$$

$$\therefore \quad r^2 = \left(d^2 - \frac{P_m}{R_a}\right)$$

$$\therefore \quad r = \sqrt{d^2 - \frac{P_m}{R_a}}$$

$$= \sqrt{\left(\frac{V}{2R_a}\right)^2 - \frac{P_m}{R_a}}$$

$$\therefore \quad \boxed{r = \frac{1}{2R_a}\sqrt{V^2 - 4P_m R_a}} \qquad \ldots (5)$$

The equation shows that as power Pm must be real, then 4Pm Ra ? V2 . The maximum possible power per phase is,

$$4\,(P_m)_{max}\,R_a = V^2$$

$$\therefore \quad \boxed{(P_m)_{max} = \frac{V^2}{4R_a}} \qquad \ldots (6)$$

And the radius of the circle for maximum power is zero. Thus at the time of maximum power, the circles becomes a point O’. While when the power Pm = O, then r = V/2Ra = OO’

This shows that the circle of zero power passes through the points O and A.

The radius for any power Pm is given by,

$$r = \frac{V}{2R_a}\sqrt{1-\frac{4P_m R_a}{V^2}}$$

but $$(P_m)_{max} = \frac{V^2}{4R_a}, \text{ substituting above}$$

$$r = \frac{V}{2R_a}\sqrt{1-\frac{P_m}{(P_m)_{max}}}$$

$$\therefore \quad r = \frac{V}{2R_a}\sqrt{1-m}$$

where $$m = \frac{P_m}{(P_m)_{max}}$$

We know, $$OO' = d = \frac{V}{2R_a}$$

$$\therefore \quad r = d\sqrt{1-m} \qquad \ldots (7)$$

This is generalized expression for the radius for any power.

## Power Flow in Synchronous Motor:

The figure below gives the details regarding the power flow in synchronous motor.

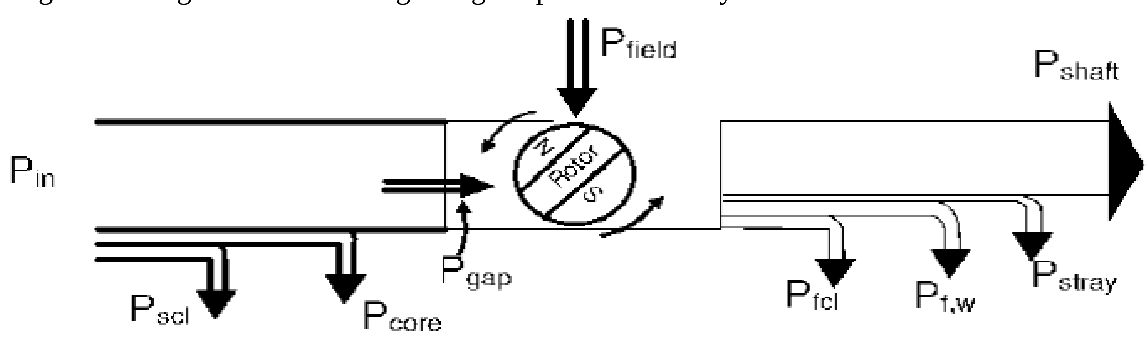



Figure: 4.8. Power stages in synchronous motor.

Where

$P_{in}$ = Power input to the motor

$P_{scl}$ = Power loss as stator copper loss

$P_{core}$= Power loss as core loss

$P_{gap}$= Power in the air gap

$P_{fcl}$ = Power loss as field copper loss

$P_{fw}$= Power loss as friction and windage loss $P_{stray}$

= Power loss as stray loss

$P_{shaft}$ = Shaft output of the machine

Power input to a synchronous motor is given by $P = 3V_{ph}I_{ph}\cos\Phi = \sqrt{3}V_L I_L \cos\Phi$. In stator as per the diagram there will be core loss and copper losses taking place. The remaining power will be converted to gross mechanical power.

Hence $P_m$= Power input to the motor – Total losses in stator.

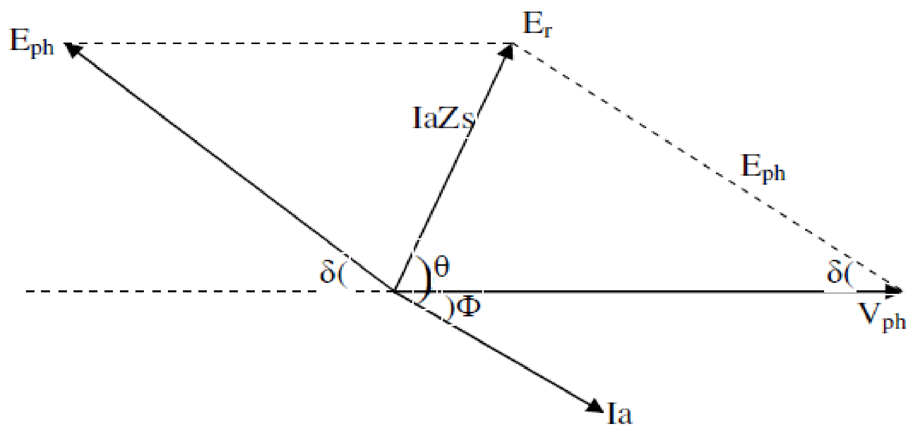



Figure: Phasor diagram

From the phasor diagram we can write Power input /phase $P_i = V_{ph}I_{ph}\cos\Phi$

Mechanical power developed by the motor $P_m$= $E_bI_a$ cos ∟ $E_b$ & $I_a$= $E_bI_a\cos(\delta - \Phi)$

Assuming iron losses as negligible stator cu losses = $P_i - P_m$

Power output /phase = $P_m$ – (field cu loss + friction & windage loss +stray loss)

## Torque developed in Motor:

Mechanical power is given by $P_m = 2\pi N_s T_g/60$ where Ns is the synchronous speed and the Tg is the gross torque developed.

$$P_m = 2\pi N_s T_g /60$$

$$\text{Hence } T_g = 60\, P_m/2\pi N_s$$

$$T_g = 9.55\, P_m/N_s \text{ N-m}$$

$$\text{Shaft output torque } T_{sh} = 60 \times P_{out}/2\pi N_s$$

$$T_{sh} = 9.55\, P_{out}/N_s \text{ N-m}$$

## Numerical problems on power developed and torque:

1. A 75 KW 3phase Y connected, 50 Hz, 440V cylindrical rotor synchronous motor operates at rated condition with 0.8pf leading. The motor efficiency excluding field and stator losses, is 95% and Xs=2.5Ω calculate (i) mechanical power developed (ii) armature current (iii) back EMF (iv) power angle and (v)max or pull out toque of the motor.

2. A 50 Hz, 6-pole, 3-Φ, and star-connected synchronous motor has a synchronous reactance of 12.2Ω/phase and negligible armature resistance. The excitation is such as to give an open-circuit voltage of 13.4kv.the motor is connected to 11.9KV, 50 Hz supply. What maximum load can the motor supply before losing synchronism? What is the corresponding motor torque, line current and power factor?

3. A 50 Hz, 8-pole, 3-Φ, and star-connected synchronous motor has a synchronous reactance of 10Ω/phase and negligible armature resistance. The excitation is such as to give an open-circuit voltage of 14 KV the motor is connected to 12.5 KV, 50 Hz supply. What maximum load can the motor supply before losing synchronism? What is the corresponding motor torque, line current and power factor?

## V and inverted V curve of synchronous motor:

Graphs of armature current vs. field current of synchronous motors are called V curves and are shown in Figure below for typical values of synchronous motor loads. The curves are related to the phasor diagram shown in figure below, and illustrate the effect of the variation of field excitation on armature current and power factor. It can be easily noted from these curves that an increase in shaft loads require an increase in field excitation in order to maintain the power factor at unity.

The points marked *a*, *b*, and *c* on the upper curve corresponds to the operating conditions of the phasor diagrams shown. Note that for $P = 0$, the lagging power factor operation is electrically equivalent to an inductor and the leading power factor operation is electrically equivalent to a capacitor. Leading power factor operation with $P = 0$ is sometimes referred to as synchronous condenser or synchronous capacitor operation. Typically, the synchronous machine V-curves are provided by the manufacturer so that the user can determine the resulting operation under a given set of conditions.

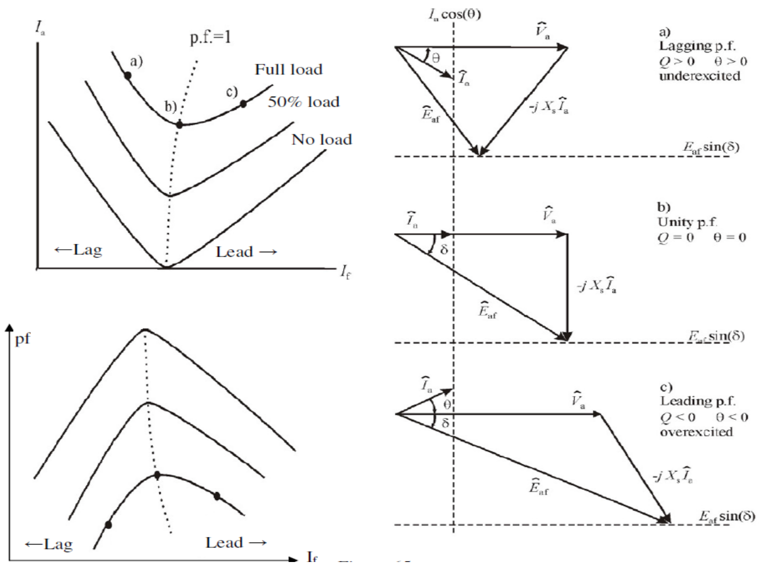



Figure: 52 Plots of power factor vs. field current of synchronous motors are called inverted V curves and are shown in Figure above for different values of synchronous motor loads.

## Numerical problems on Power output:

1. The excitation of a 415V, 3-phase, and mesh connected synchronous motor is such that the induced EMF is520V.the impedance per phase is (0.5+j4.0) Ω. If the friction and iron losses are constant at 1000watts, calculate the power output, line current, power factor and efficiency for maximum power output?
2. The excitation of a 415V, 3-phase, and mesh connected synchronous motor is such that the induced EMF is520V.the impedance per phase is (0.5+j4.0) Ω. If the friction and iron losses are constant at 1000watts, calculate the power output, line current, power factor and efficiency for maximum power output?
3. The excitation of a 415V, 3-phase, and mesh connected synchronous motor is such that the induced EMF is520V.the impedance per phase is (0.85+j6.0) Ω. If the friction and iron losses are constant at 800 watts, calculate the power output, line current, power factor and efficiency for maximum power output?

## Hunting:

Sudden changes of load on synchronous motors may sometimes set up oscillations that are superimposed upon the normal rotation, resulting in periodic variations of a very low frequency in speed. This effect is known as hunting or phase-swinging. Occasionally, the trouble is aggravated by the motor having a natural period of oscillation approximately equal to the hunting period. When the synchronous motor phase-swings into the unstable region, the motor may fall out of synchronism

### Damper winding:

The tendency of hunting can be minimized by the use of a damper winding. Damper windings are placed in the pole faces. No EMFs are induced in the damper bars and no current flows in the damper winding, which is not operative. Whenever any irregularity takes place in the speed of rotation, however, the polar flux moves from side to side of the pole, this movement causing the flux to move backwards and forwards across the damper bars. EMFs are induced in the damper bars forwards across the damper winding. These tend to damp out the superimposed oscillatory motion by absorbing its energy. The damper winding, thus, has no effect upon the normal average speed, it merely tends to damp out the oscillations in the speed, acting as a kind of electrical flywheel. In the case of a three-phase synchronous motor the stator currents set up a rotating MMF rotating at uniform speed and if the rotor is rotating at uniform speed, no EMFs are induced in the damper bars.

### Synchronous Condenser:

An over excited synchronous motor operates at unity or leading power factor. Generally, in large industrial plants the load power factor will be lagging. The specially designed synchronous motor running at zero load, taking leading current, approximately equal to 90$^o$. When it is connected in parallel with inductive loads to improve power factor, it is known as synchronous condenser. Compared to static capacitor the power factor can improve easily by variation of field excitation of motor. Phasor diagram of a synchronous condenser connected in parallel with an inductive load is given below.

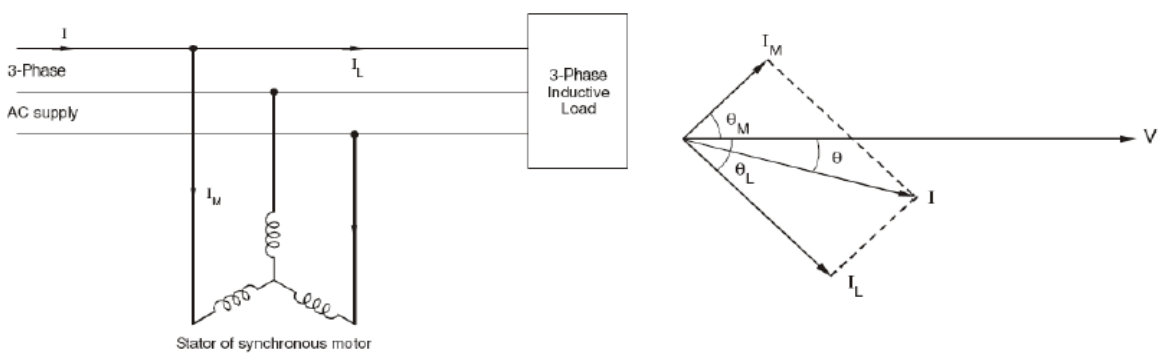



Figure: 53 Synchronous condenser and phasor diagram

# UNIT-V

# Single phase induction motor

## SINGLE PHASE MOTORS & SPECIAL MACHINES

### Introduction

The characteristics of single phase induction motors are identical to 3-phase induction motors except that single phase induction motor has no inherent starting torque and some special arrangements have to be made for making itself starting. It follows that during starting period the single phase induction motor must be converted to a type which is not a single phase induction motor in the sense in which the term is ordinarily used and it becomes a true single phase induction motor when it is running and after the speed and torque have been raised to a point beyond which the additional device may be dispensed with. For these reasons, it is necessary to distinguish clearly between the starting period when the motor is not a single phase induction motor and the normal running condition when it is a single phase induction motor. The starting device adds to the cost of the motor and also requires more space. For the same output a 1-phase motor is about 30% larger than a corresponding 3-phase motor.

The single phase induction motor in its simplest form is structurally the same as a poly- phase induction motor having a squirrel cage rotor, the only difference is that the single phase induction motor has single winding on the stator which produces MMF stationary in space but alternating in time, a poly phase stator winding carrying balanced currents produces MMF rotating in space around the air gap and constant in time with respect to an observer moving with the MMF. The stator winding of the single phase motor is disposed in slots around the inner periphery of a laminated ring similar to the 3-phase motor.

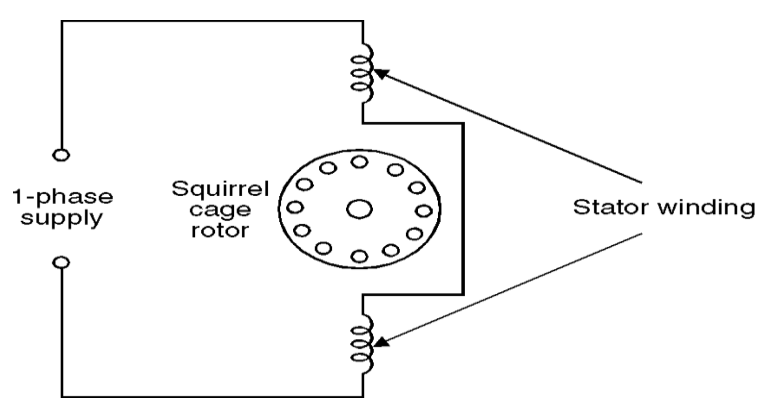



**Figure: 54. Elementary single phase induction m o t o r**

An induction motor with a cage rotor and single phase stator winding is shown schematically in Fig.

54.The actual stator winding as mentioned earlier is distributed in slots so as to produce an approximately sinusoidal space distribution of MMF.

## Principle of Operation

Suppose the rotor is at rest and 1-phase supply is given to stator winding. The current flowing in the stator winding gives rise to an MMF whose axis is along the winding and it is a pulsating MMF, stationary in space and varying in magnitude, as a function of time, varying from positive maxi- mum to zero to negative maximum and this pulsating MMF induces currents in the short-circuited rotor of the motor which gives rise to an MMF. The currents in the rotor are induced due to transformer action and the direction of the currents is such that the MMF so developed opposes the stator MMF. The axis of the rotor MMF is same as that of the stator MMF. Since the torque developed is proportional to sine of the angle between the two MMF and since the angle is zero, the net torque acting on the rotor is zero and hence the rotor remains stationary.

For analytical purposes a pulsating field can be resolved into two revolving fields of constant magnitude and rotating in opposite directions as shown in Fig. 55 and each field has a magnitude equal to half the maximum length of the original pulsating phasor.

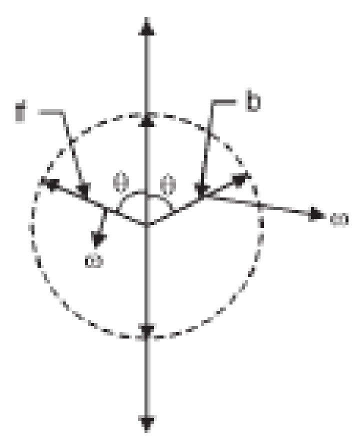



**Figure: 55. Representation of the pulsating field by space phasors**

These component waves rotate in opposite direction at synchronous speed. The forward (anticlockwise) and backward-rotating (clockwise) MMF waves f and b are shown in Fig. 55.. In case of 3-phase induction motor there is only one forward rotating magnetic field and hence torque

is developed and the motor is self-starting. However, in single phase induction motor each of this component MMF waves produces induction motor action but the corresponding torques are in opposite direction. With the rotor at rest the forward and backward field produce equal torques but opposite in direction and hence no net torque is developed on the motor and the motor remains stationary. If the forward and backward air gap fields remained equal when the rotor is revolving, each of the component fields would produce a torque-speed characteristic similar to that of a

Poly phase induction motor with negligible leakage impedance as shown by the dashed curves f

And b in Fig. 56

The resultant torque-speed characteristic which is the algebraic sum of the two component curves shows that if the motor were started by auxiliary means it would produce torque in what- ever direction it was started.

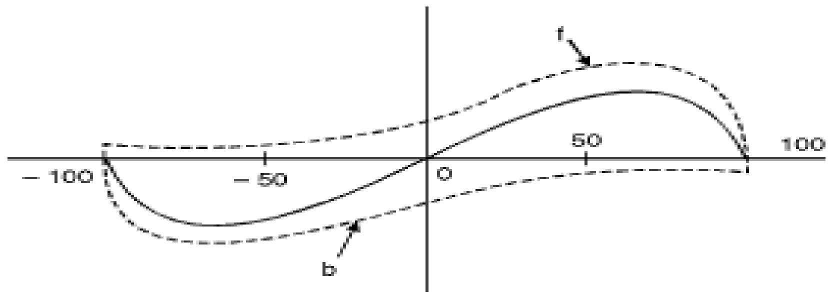



**Figure: 56. Torque-speed characteristic of a 1-phase induction motor based on constant forward and backward flux waves.**

In reality the two fields, forward and backward do not remain constant in the air gap and also the effect of stator leakage impedance can't be ignored. In the above qualitative analysis the effects of induced rotor currents have not been properly accounted for.

When single phase supply is connected to the stator and the rotor is given a push along the forward rotating field, the relative speed between the rotor and the forward rotating magnetic field goes on decreasing and hence the magnitude of induced currents also decreases and hence the MMF due to the induced current in the rotor decreases and its opposing effect to the forward rotating field decreases which means the forward rotating field becomes stronger as the rotor speeds up. However for the backward rotating field the relative speed between the rotor and the backward field increases as the rotor rotates and hence the rotor EMF increases and hence the MMF due to this component of current increases and its opposing effect to the backward rotating field increases and the net backward rotating field weakens as the rotor rotates along the forward rotating field. However, the sum of the two fields remains constant since it must induce the stator counter EMF which is approximately constant if the stator leakage impedance drop is negligible. Hence, with the rotor in motion the torque of the forward field is greater and that of the backward field is less than what is shown in Fig. 56. The true situation being as is shown in Fig. 57.

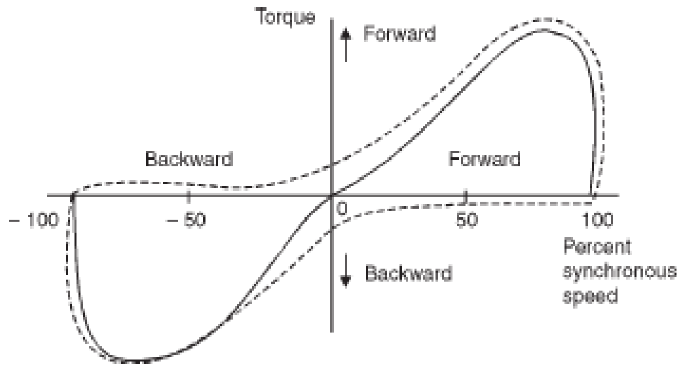



**Figure: 57. Torque-speed characteristic of a 1-phase induction motor taking into account changes in the flux waves.**

In the normal running region at a few per cent slip the forward field is several times stronger than the backward field and the flux wave does not differ materially from the constant

Amplitude revolving field in the air gap of a balanced poly phase motor. Therefore, in the normal running range of the motor, the torque-speed characteristic of a single phase motor is not very much different from that of a poly phase motor having the same rotor and operating with the same maximum air gap flux density.

In addition to the torque shown in Fig. 57, double-stator frequency torque pulsation are produced by the interaction of the oppositely rotating flux and MMF waves which move past each other at twice synchronous speed. These double frequency torques produce no average torque as these pulsations are sinusoidal and over the complete cycle the average torque is zero. However, sometimes these are additive to the main torque and for another half a cycle these are subtractive and therefore a variable torque acts on the shaft of the motor which makes the motor noisier as compared to a poly phase induction motor where the total torque is constant. Such torque pulsations are unavoidable in single phase circuits. Mathematically

$$T \alpha I^2$$

$$I = I_m \sin wt$$

$$T = K I_m^2 \sin$$

$$= K I_m^2 (1-\cos 2wt)/2$$

## Starting Of Single Phase Induction Motors

The single phase induction motors are classified based on the method of starting method and in fact are known by the same name descriptive of the method. Appropriate selection of these motors depends upon the starting and running torque requirements of the load, the duty cycle and limitations on starting and running current drawn from the supply by these motors. The cost of single phase induction motor increases with the size of the motor and with the performance such as starting torque to current ratio (higher ratio is desirable), hence, the user will like to go in for a smaller size (hp) motor with minimum cost, of course, meeting all the operational requirements. However, if a very large no. of fractional horsepower motors are required, a specific design can always be worked out which might give minimum cost for a given performance requirements. Following are the starting methods.

(a) Split-phase induction motor. The stator of a split phase induction motor has two windings, the main winding and the auxiliary winding. These windings are displaced in space by 90 electrical degrees as shown in Fig. 9.5 (a). The auxiliary winding is made of thin wire (super enamel copper wire) so that it has a high R/X ratio as compared to the main winding which has thick super enamel copper wire. Since the two windings are connected across the supply the

Torque is developed and the motor becomes a self-starting motor. After the motor starts, the auxiliary winding is disconnected usually by means of centrifugal switch that operates at about t5 per cent of synchronous speed. Finally the motor runs because of the main winding. Since this being single phase some level of humming noise is always associated with the motor during
Running. A typical torque speed characteristic is shown.It is to be noted that the direction of rotation of the motor can be reversed by reversing the connection to either the main winding or the auxiliary windings.

Current $I_m$ and $I_a$ in the main winding and auxiliary winding lag behind the supply voltage V, $I_a$ leading the current $I_m$ This means the current through auxiliary winding reaches maximum value first and the MMF or flux due to $I_a$ lies along the axis of the auxiliary winding and after some time the current $I_m$ reaches maximum value and the MMF or flux due to $I_m$ lies along the main winding axis. Thus the motor becomes a 2-phase unbalanced motor. It is unbalanced
Since the two currents are not exactly 90 degrees apart. Because of these two fields a starting

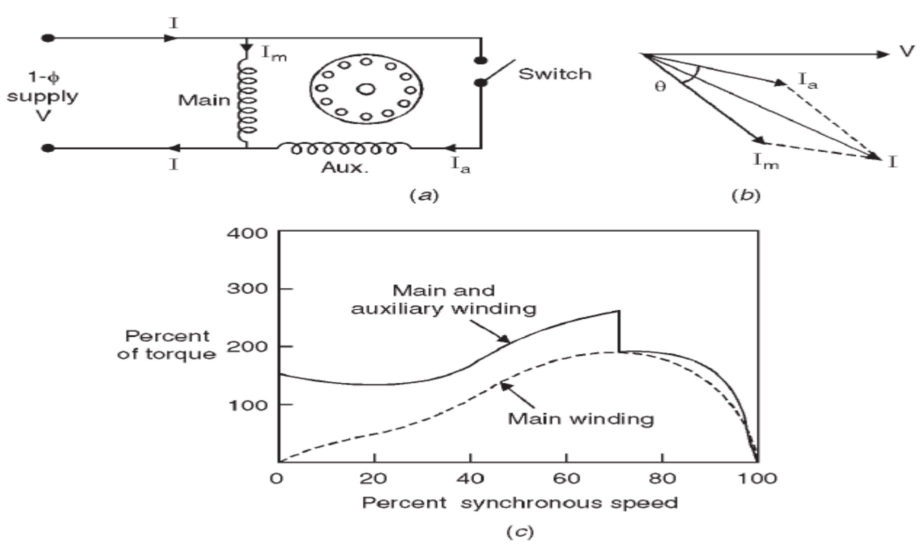



Figure: 58. Split phase induction motor (a) Connection

(b) Phasor diagram at starting (c) typical torque-speed characteristic.

(b) **Capacitor starts induction motor**: Capacitors are used to improve the starting and running performance of the single phase inductions motors.

The capacitor start induction motor is also a split phase motor. The capacitor of suitable
value is connected in series with the auxiliary coil through a switch such that $I_a$ the current in the auxiliary coil leads the current $I_m$ in the main coil by 90 electrical degrees in time phase so that the starting

torque is maximum for certain values of $I_a$ and $I_m$. This becomes a balanced 2- phase motor if the magnitude of $I_a$ and $I_m$ are equal and are displaced in time phase by 90° electrical degrees. Since the two windings are displaced in space by 90 electrical degrees as shown
in Fig. 9.6 maximum torque is developed at start. However, the auxiliary winding and capacitor are disconnected after the motor has picked up t5 per cent of the synchronous speed. The motor will start without any humming noise. However, after the auxiliary winding is disconnected, there will be some humming noise.

Since the auxiliary winding and capacitor are to be used intermittently, these can be designed for minimum cost. However, it is found that the best compromise among the factors of starting torque, starting current and costs results with a phase angle somewhat less than 90°
between $I_m$ and $I_a$. A typical torque-speed characteristic is shown in Fig. 5.6 (c) high starting torque being an outstanding feature.

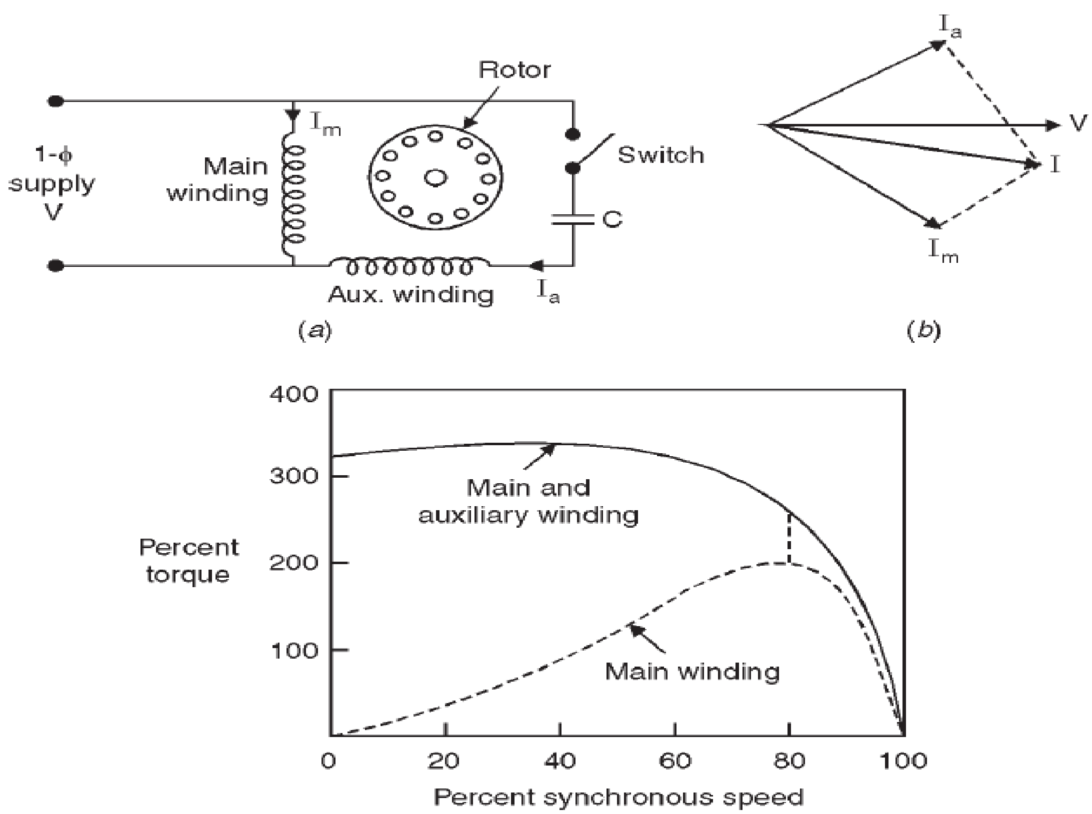



Figure: 5.6. Capacitor start motor (a) Connection

(b) Phasor diagram at start (c) Speed torque curve.

(c) Permanent-split capacitor motor. In this motor the auxiliary winding and capacitor are not disconnected from the motor after starting, thus the construction is simplified by the omission of the switch as shown in Fig. 5.7(a).

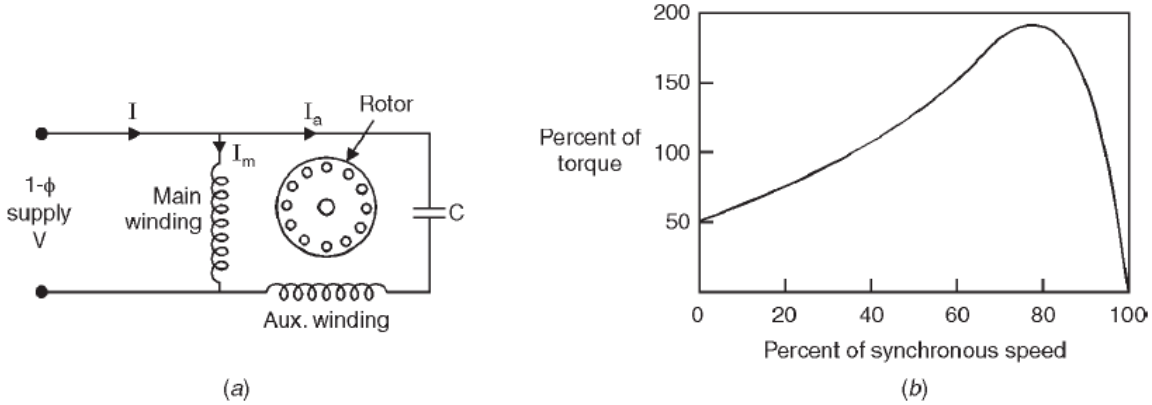



Figure: 5.7. Permanent split capacitor motor (a) Connection (b) Torque-speed characteristic.

Here the auxiliary winding and capacitor could be so designed that the motor works as a perfect 2-phase motor at anyone desired load. With this the backward rotating magnetic field would be completely eliminated. The double stator frequency torque pulsations would also be eliminated, thereby the motor starts and runs as a noise free motor. With this there is improve- ment in p.f. and efficiency of the motor. However, the starting torque must be sacrificed as the capacitance is necessarily a compromise between the best starting and running characteristics.

The torque-speed characteristic of the motor is shown in Fig. 9.7 (b).

(c) Capacitor start capacitor rum motor. If two capacitors are used with the auxiliary winding as shown in Fig. 5.8 (a), one for starting and other during the start and run, theoretically optimum starting and running performance can both be achieved.

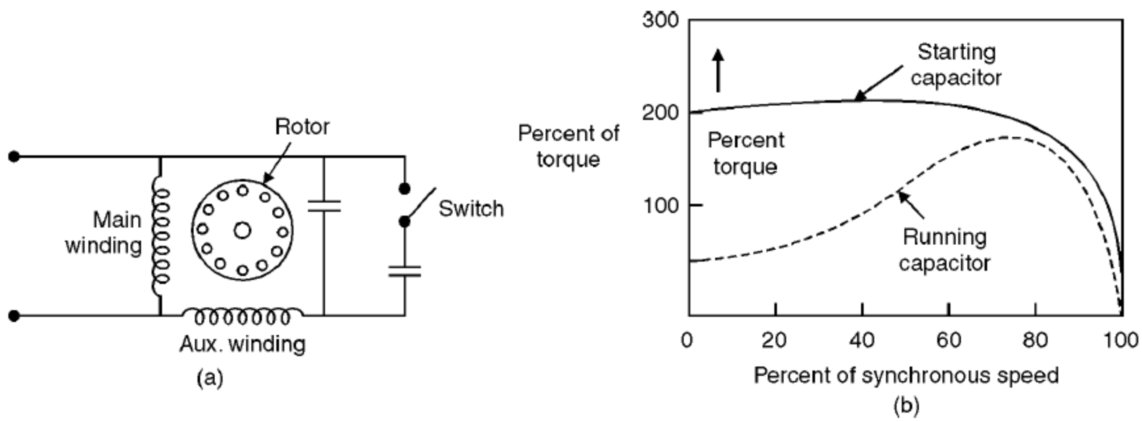



Fig. 5.8. (a) Capacitor start capacitor run motor (b) Torque-speed characteristic.

The small value capacitor required for optimum running conditions is permanently con- nected in series with the auxiliary winding and the much larger value required for starting is obtained by a capacitor connected in parallel with the running capacitor. The starting capacitor is disconnected after the motor starts.

The value of the capacitor for a capacitor start motor is about 300 μF for 1/2 hp motor Since

This capacitor must carry current for a short starting period; the capacitor is a special compact ac Electrolytic type made for motor starting duty. However, the capacitor permanently connected has a typical rating of 40 μF ; since it is connected permanently, the capacitor is an ac paper, foil and oil type. The cost of the motor is related to the performance; the permanent capacitor motor is the lowest cost, the capacitor start motor next and the capacitor start capacitor run has the highest cost.

## Shaded pole induction motor:

Fig. 58 (a) shows schematic diagram of shaded pole induction motor. The stator has salient poles with one portion of each pole surrounded by a short- circuited turn of copper called a shading coil. Induced currents in the shading coil (acts as an inductor) cause the flux in the shaded portion of the pole to lag the flux in the other portion. Hence the flux under the un shaded pole leads the flux under the shaded pole which results in a rotating field moving in the direction from un shaded to the shaded portion of the pole and a low starting torque is produced which rotates the rotor in the direction from un shaded to the shaded pole. A typical torque speed characteristic is shown in Fig. 58 (b). The efficiency is low. These motors are the least expensive type of fractional horse power motor and are built up to about 1/20 hp. Since the rotation of the motor is in the direction from un-shaded towards the shaded part of the pole, a shaded pole motor can be reversed only by providing two sets of shading coils which may be opened and closed or it may be reversed permanently by inverting the core.

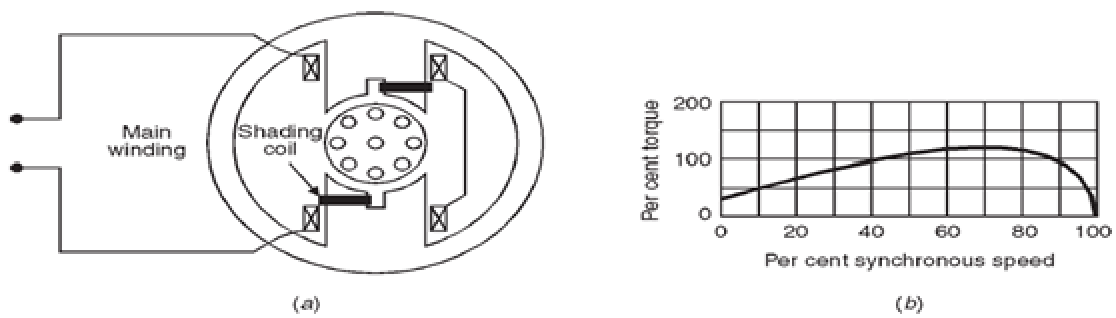



Figure: 58. Shaded-pole motor and typical torque-speed characteristic

## Numerical Problems:

1. 2-winding single-phase motor has the main auxiliary winding currents $I_m$=15 A and $I_a$=7.5 A at stand-still. The auxiliary winding current leads the main winding current by $\alpha=45^0$ electrical. The two winding are in space quadrature and the effective number of turns are $N_m$=80 and $N_a$=100. Compute the amplitudes of the forward and backward stator MMF waves. Also determine the magnitude of the auxiliary current and its phase angle difference α with the main winding current if only the backward field is to be present.

2. The following data pertains to a 230 V, 50 Hz capacitor start single phase induction motor at stand still. Main winding excited alone=100V, 2A, 40W, Auxiliary winding excited alone= 80 V, 1 A, 50 W. Determine the value of capacitance for determining the maximum torque

3. Find the mechanical power output of 185kw, 4 pole, 110V, 50Hz single phase induction motor, whose constants are given below at a slip of 0.05.$R_1$=1.86Ω, $X_1$=2.56 Ω, $X_\phi$=53.5 Ω, $R_2$=3.56 Ω $X_2$=2.56 Ω core loss 3.5w, friction and wind age loss 13.5w

4. 2. A 250w, 230V, 50Hz capacitor start motor has the following constants for the main and auxiliary windings: main winding, $Z_m$= (4.5+3.7j) Ω. Auxiliary winding $Z_a$= (9.5+3.5i) Ω. Determine the value of the starting capacitor that will place the main and auxiliary winding currents in quadrature at starting.

5. 3. A single phase induction motor has stator windings in space quadrature and is supplied with a single phase voltage of 200V at 50Hz. The standstill impedance of the main winding is (5.2+10.1i) and the auxiliary winding is (19.7+14.2j). Find the value of capacitance to be inserted in the auxiliary winding for maximum starting torque.

6. What is the motor torque $T_m$ required to accelerate an initial load of $3*10^{-4}$ kg $m^2$ from $f_1$ = 1500 Hz to $f_2$ = 2500 Hz during 100 ms. The frictional torque $T_f$ is 0.05 N-m and the step angle is $1.7^\circ$.