Direct-Current Motors

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Introduction

- The Brushed DC motor is not as widely used today as in the past, but it is still being used, especially at the low-power level.
- Focus is on topics of interest to the mechatronics engineer
  - Shunt-connected dc motor
  - Permanent-magnet dc motor

  similar operating characteristics
- A simplified method of analysis is used rather than an analysis wherein commutation is treated in detail. As a result the dc motor is the most straightforward to analyze of all the electromechanical devices.

Elementary Direct-Current Machine

- It is instructive to discuss the elementary two-pole dc machine prior to a formal analysis of the performance of a practical dc machine.
- The two-pole elementary machine is equipped with:
  - Field winding wound on stator poles
  - Rotor (or armature) coil
  - Commutator
    - Two semicircular copper segments mounted on the shaft at the end of the rotor and insulated from one another as well as from the iron of the rotor.
    - Each terminal of the rotor coil is connected to a copper segment.

- Stationary carbon brushes ride upon the copper segments whereby the rotor coil is connected to a stationary circuit by a near frictionless contact.

- The voltage equations for the field winding and rotor coil are:

$$v_f = r_f i_f + \frac{d\lambda_f}{dt}$$

$$v_{a-a'} = r_a i_{a-a'} + \frac{d\lambda_{a-a'}}{dt}$$

- The flux linkages can be expressed as:

$$\lambda_f = L_{ff} i_f + L_{fa} i_{a-a'}$$

$$\lambda_{a-a'} = L_{aa} i_{a-a'} + L_{af} i_f$$

- As a first approximation, the mutual inductance between the field winding and an armature coil is:

$$L_{af} = L_{fa} = -L\cos\theta_r \qquad \text{(L is a constant)}$$

- **Commutation**
  - As the rotor revolves, the action of the commutator is to switch the stationary terminals from one terminal of the rotor coil to the other. This switching occurs at: $\theta_r = 0, \pi, 2\pi, \ldots$.
  - At the instant of switching, each brush is in contact with both copper segments whereupon the rotor coil is short-circuited. It is desirable to commutate (short-circuit) the rotor coil at the instant the induced voltage is a minimum.
  - The waveform of the voltage induced in the open-circuited armature coil, during constant-speed operation with a constant field winding current, may be determined by setting $i_{a-a'} = 0$ and i_f = constant.

– Substitution:

$$\left.\begin{aligned} \lambda_{a-a'} &= L_{aa} i_{a-a'} + L_{af} i_f \\ L_{af} &= L_{fa} = -L\cos\theta_r \end{aligned}\right\} \Rightarrow \quad v_{a-a'} = r_a i_{a-a'} + \frac{d\lambda_{a-a'}}{dt}$$

– Result is an expression for the open-circuit voltage of coil a-a´ with the field current i_f a constant = I_f :

$$v_{a-a'} = \frac{d\theta_r}{dt} L I_f \sin\theta_r = \omega_r L I_f \sin\theta_r \qquad \omega_r = \text{rotor speed}$$

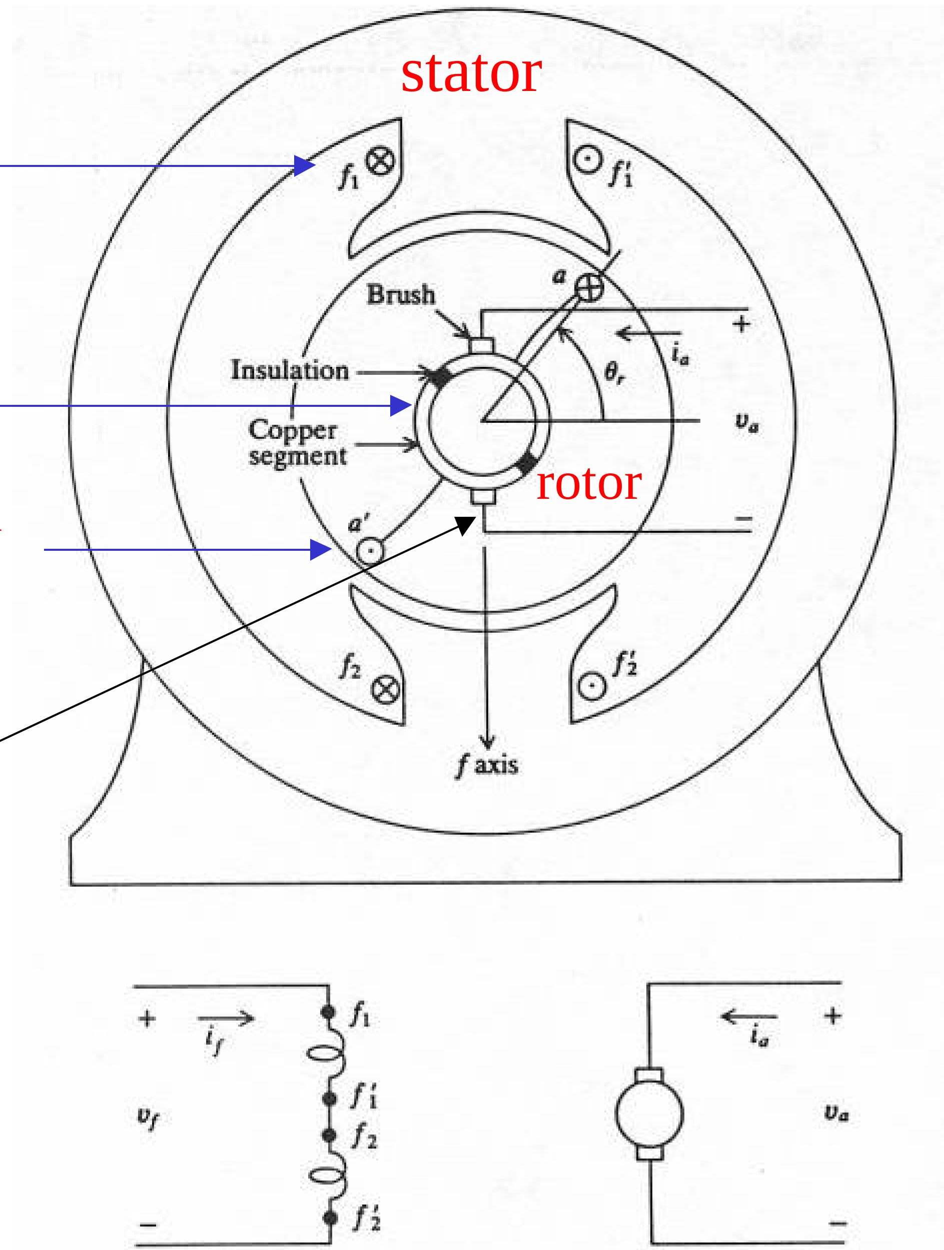
– This open-circuit coil voltage is zero at $\theta_r = 0, \pi, 2\pi, \ldots$ which is the rotor position during commutation.

field winding wound on stator poles with resistance r_f

commutator

rotor (armature) coil with resistance r_a

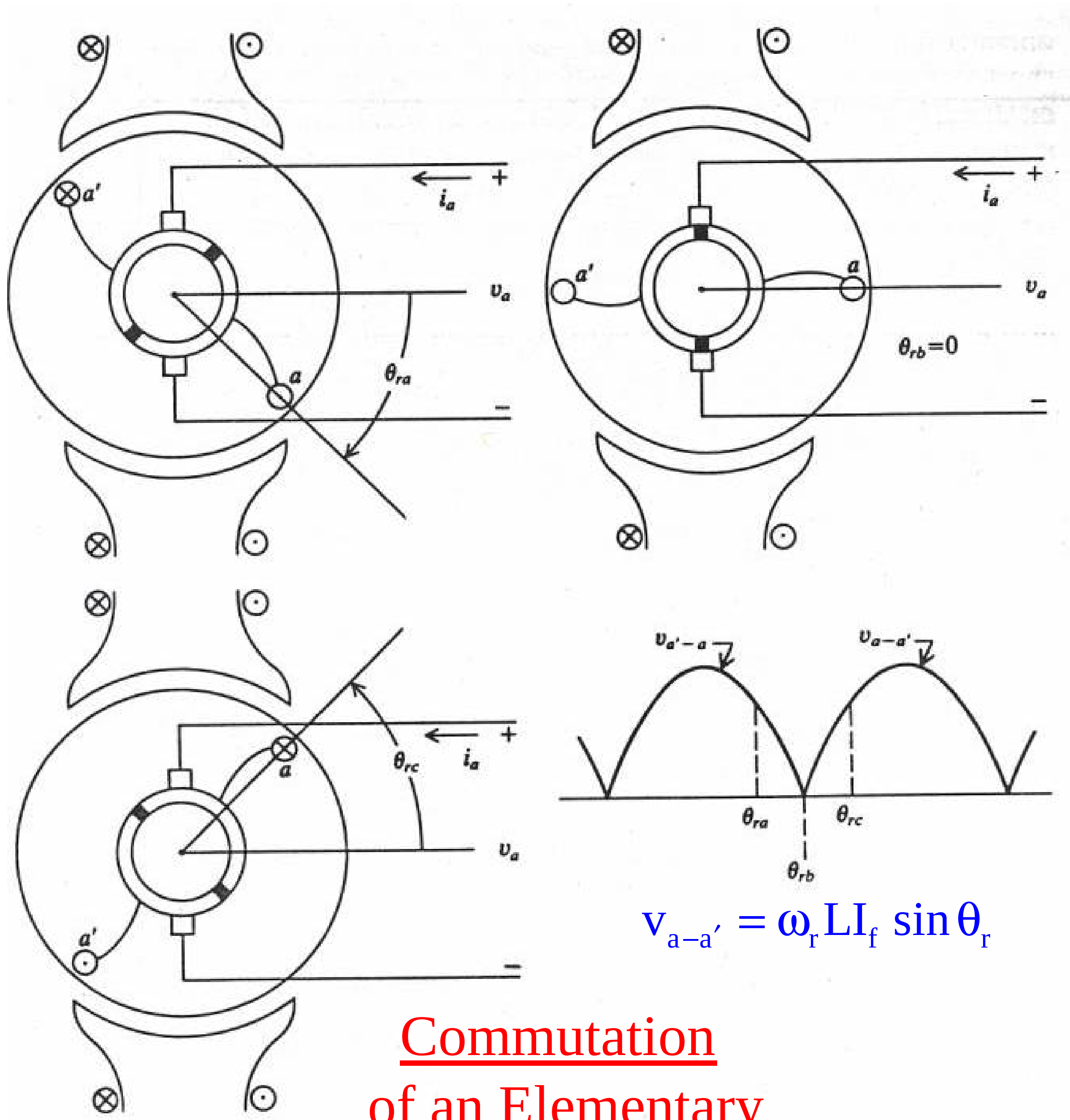
Brushes are stationary and nearly frictionless



Elementary Two-Pole DC Machine

$\pi < \theta_r < 2\pi$
i_a positive:
down coil side *a′*
and out of coil side *a*

$0 < \theta_r < \pi$
i_a positive:
down coil side *a*
and out of coil side *a′*



$$v_{a-a'} = \omega_r L I_f \sin\theta_r$$

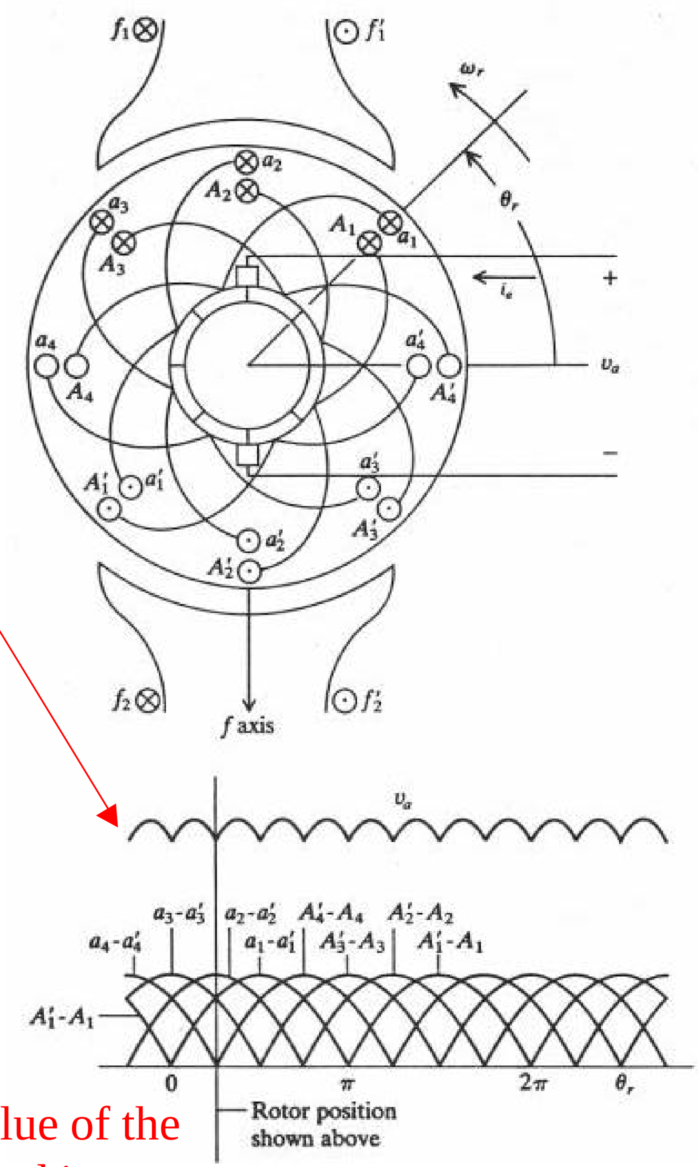
Commutation of an Elementary DC Machine

- The machine just analyzed is not a practicable machine. It could not be operated effectively as a motor supplied from a voltage source owing to the short-circuiting of the armature coil at each commutation.
- A practicable *dc* machine is shown with the rotor equipped with an *a* winding and an *A* winding. This requires some explanation.
- Here the parallel windings consist of only four coils. Usually the number of rotor coils is substantially more than four, thereby reducing the harmonic content of the open-circuit armature voltage.

- In this case, the rotor coils may be approximated as a uniformly distributed winding. Therein the rotor winding is considered as current sheets which are fixed in space due to the action of the commutator and which establish a magnetic axis positioned orthogonal to the magnetic axis of the field winding.

Follow the path of current through one of the parallel paths from one brush to the other.

The open-circuit or induced armature voltage is shown (plotted for one of the two parallel paths).



Positive current direction:

Into the paper in a_1, A_1; a_2 A_2; …

Out of the paper in a_1' A_1'; a_2' A_2'; …

Coils a_4-a_4' and A_4-A_4' are being commutated

Bottom brush short circuits a_4-a_4' coil

Top brush short circuits A_4-A_4' coil

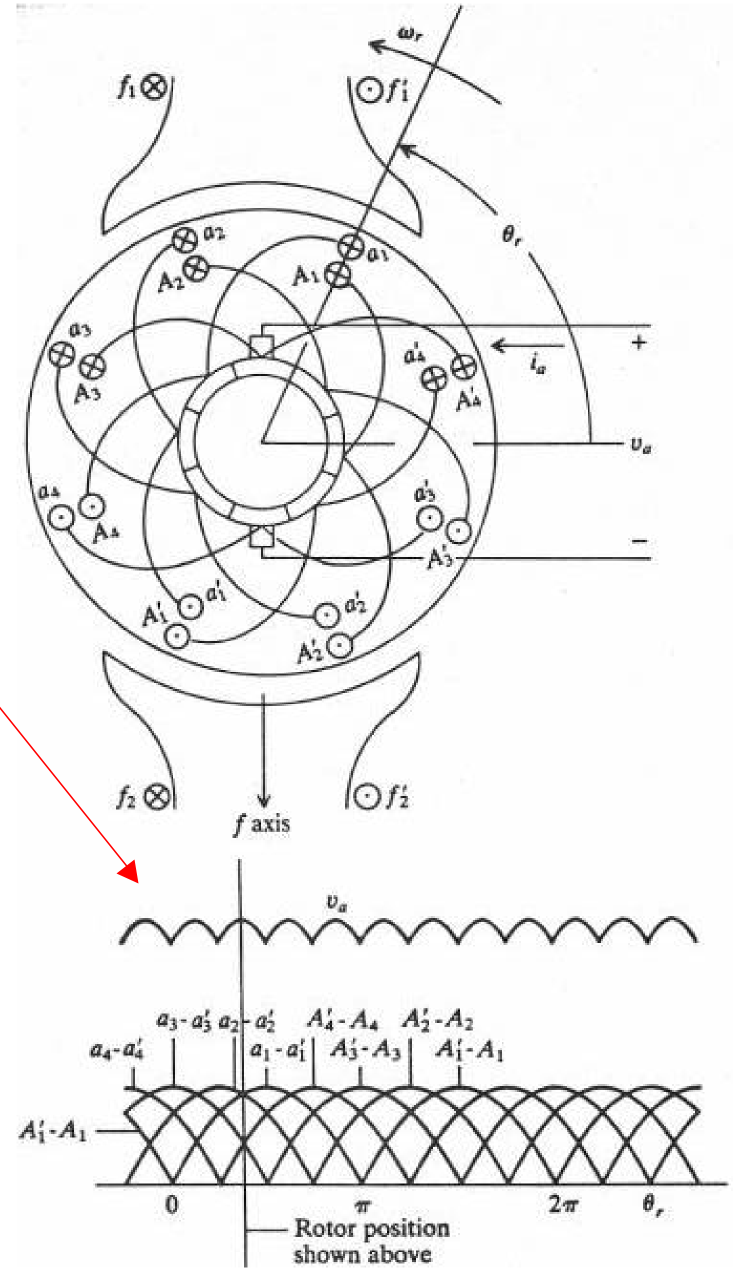
A DC Machine with Parallel Armature Windings

If the peak value of the voltage induced in one coil is 1 V, what is the max and min of v_a ?

The open-circuit or induced armature voltage is shown (plotted for one of the two parallel paths).

Top brush rides only on segment connecting A_3 and A_4'
Bottom brush rides only on segment connecting a_4 and a_3'

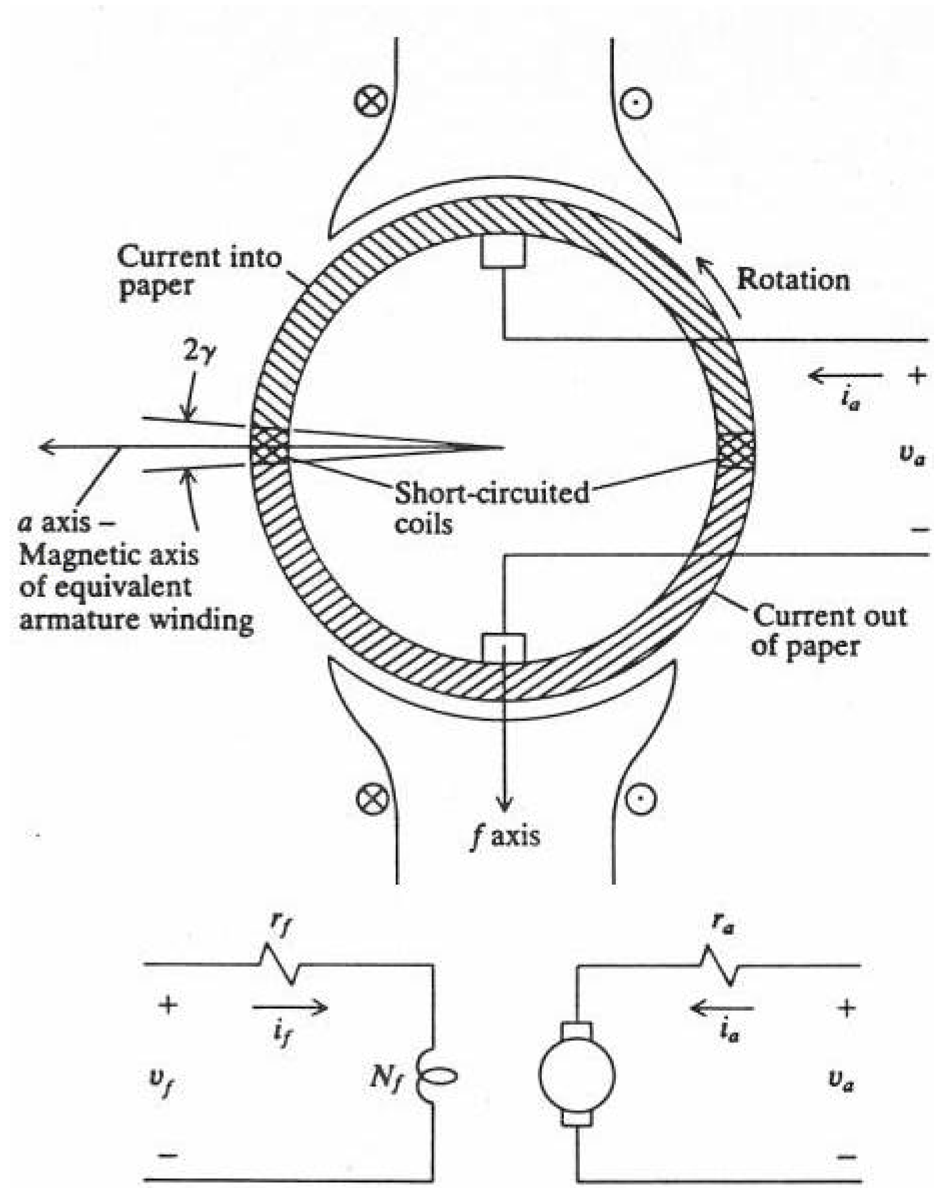
Current flows down the coils in the upper one half of the rotor and out of the coil sides in the bottom one half



A DC Machine with Parallel Armature Windings

Rotor advanced approximately 22.5º CCW

The brushes are shown depicted on the current sheet for the purpose of depicting commutation. The small angular displacement (2λ) designates the region of commutation wherein the coils are short-circuited.



Idealized DC Machine with Uniformly-Distributed Rotor Winding

- It is instructive to take a look at the arrangement of the armature windings and the method of commutation used in many of the low-power permanent-magnet dc motors.
- Small dc motors used in low-power control systems are often the permanent-magnet type, wherein a constant field flux is established by a permanent magnet rather than by a current flowing in a field winding.
- Three rotor positions of a typical low-power permanent-magnet dc motor are shown.

- Although we realize that in some cases it may be a rather crude approximation, we will consider the permanent-magnet dc motor as having current sheets on the armature with orthogonal armature and field magnetic axes.

Commutation of a Permanent-Magnet DC Motor

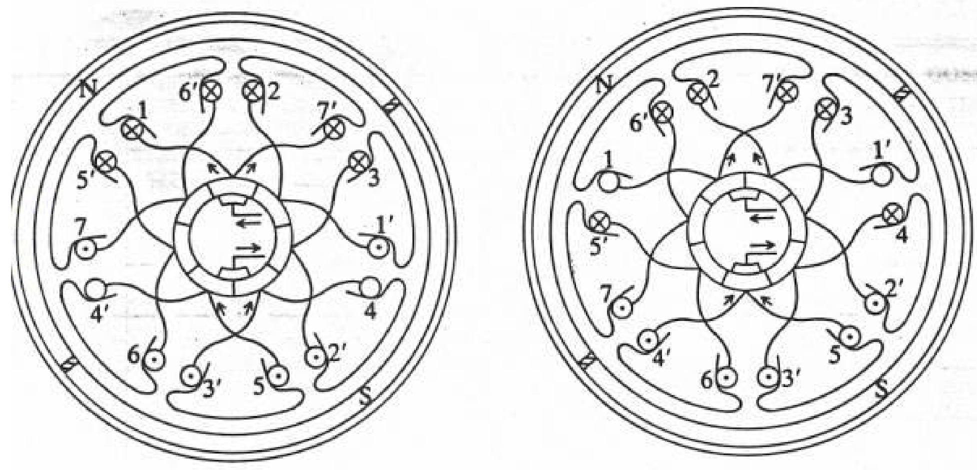
Figures:

(a) Winding 4 is being commutated

(b) Winding 1 is being commutated

(c) Winding 5 is being commutated

Rotor is turning in CCW direction.

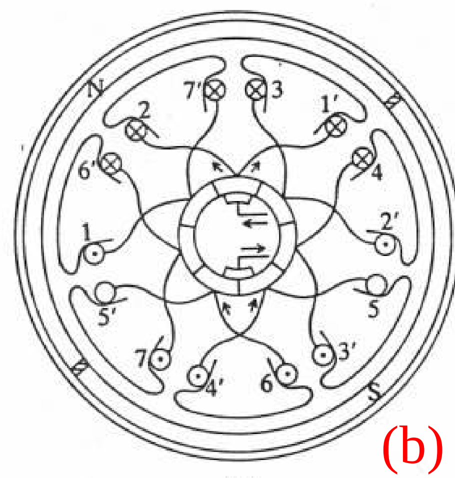


(a) (b)

(a) → (b):
4 and 1 are being commutated

The armature windings consist of a large number of turns of fine wire, hence, each circle represents many conductors.

Note that the position of the brushes is shifted approximately 40° relative to the line drawn between the center of the N and S poles. This shift in the brushes was probably determined experimentally by minimizing brush arcing for normal load conditions.



(c)

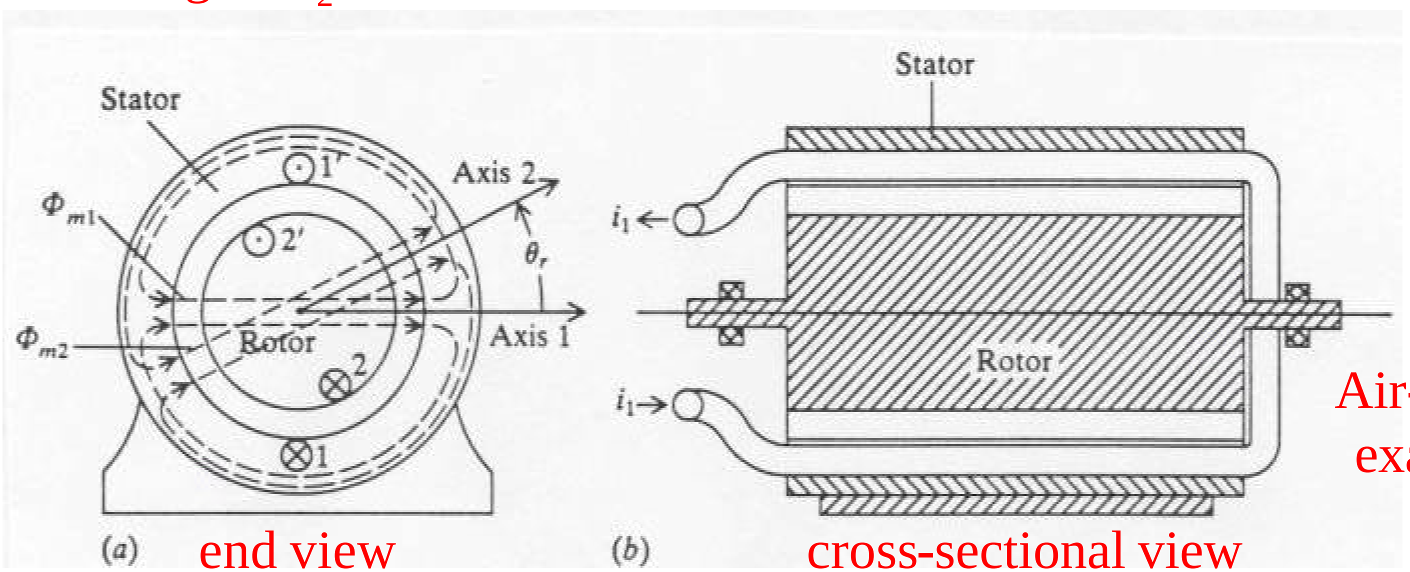
(b) → (c):
1 and 5
are being commutated

Review: Windings in Relative Motion

- The rotational device shown will be used to illustrate windings in relative motion.

Winding 1: N_1 turns on stator
Winding 2: N_2 turns on rotor

Assume that the turns are concentrated in one position.



end view

cross-sectional view

Air-gap size is exaggerated.

$$v_1 = r_1 i_1 + \frac{d\lambda_1}{dt}$$
$$v_2 = r_2 i_2 + \frac{d\lambda_2}{dt}$$

voltage equations

$$\lambda_1 = L_{11} i_i + L_{12} i_2$$
$$\lambda_2 = L_{21} i_i + L_{22} i_2$$

The magnetic system is assumed linear.

$$L_{11} = L_{\ell 1} + L_{m1} = \frac{N_1^2}{\Re_{\ell 1}} + \frac{N_1^2}{\Re_m}$$
$$L_{22} = L_{\ell 2} + L_{m2} = \frac{N_2^2}{\Re_{\ell 2}} + \frac{N_2^2}{\Re_m}$$

The self-inductances L_{11} and L_{22} are constants and may be expressed in terms of leakage and magnetizing inductances.

$\Re_m$ is the reluctance of the complete magnetic path of φ_{m1} and φ_{m2} , which is through the rotor and stator iron and twice across the air gap.

Let's now consider L_{12}.

$$\theta_r = \text{ angular displacement}$$

$$\omega_r = \text{ angular velocity}$$

$$\theta_r = \int_0^t \omega_r(\xi)\, d\xi + \theta_r(0)$$

When θ_r is zero, then the coupling between windings 1 and 2 is maximum. The magnetic system of winding 1 aids that of winding 2 with positive currents assumed. Hence the mutual inductance is positive.

$$L_{12}(0) = \frac{N_1 N_2}{\Re_m}$$

When θ_r is $\pi/2$, the windings are orthogonal. The mutual coupling is zero.

$$L_{12}\left(\frac{\pi}{2}\right) = 0$$

Assume that the mutual inductance may be adequately predicted by:

$$\begin{cases} L_{12}\left(\theta_r\right) = L_{sr}\cos\left(\theta_r\right) \\ L_{sr} = \dfrac{N_1 N_2}{\Re_m} \end{cases}$$

$$\boxed{\begin{aligned} v_1 &= r_1 i_1 + \frac{d\lambda_1}{dt} \\ v_2 &= r_2 i_2 + \frac{d\lambda_2}{dt} \end{aligned}}$$

L_{sr} is the amplitude of the sinusoidal mutual inductance between the stator and rotor windings.

In writing the voltage equations, the total derivative of the flux linkages is required.

$$\lambda_1 = L_{11} i_1 + \left(L_{sr}\cos\theta_r\right) i_2$$

$$\lambda_2 = L_{22} i_2 + \left(L_{sr}\cos\theta_r\right) i_1$$

$$\boxed{\begin{aligned} v_1 &= r_1 i_1 + L_{11}\frac{di_1}{dt} + L_{sr}\cos\theta_r \frac{di_2}{dt} - i_2\omega_r L_{sr}\sin\theta_r \\ v_2 &= r_2 i_2 + L_{22}\frac{di_2}{dt} + L_{sr}\cos\theta_r \frac{di_1}{dt} - i_1\omega_r L_{sr}\sin\theta_r \end{aligned}}$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}$$

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} L_{\ell 1} + L_{m1} & L_{sr} \cos\theta_r \\ L_{sr} \cos\theta_r & L_{\ell 2} + L_{m2} \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \end{bmatrix}$$

Since the magnetic system is assumed to be linear:

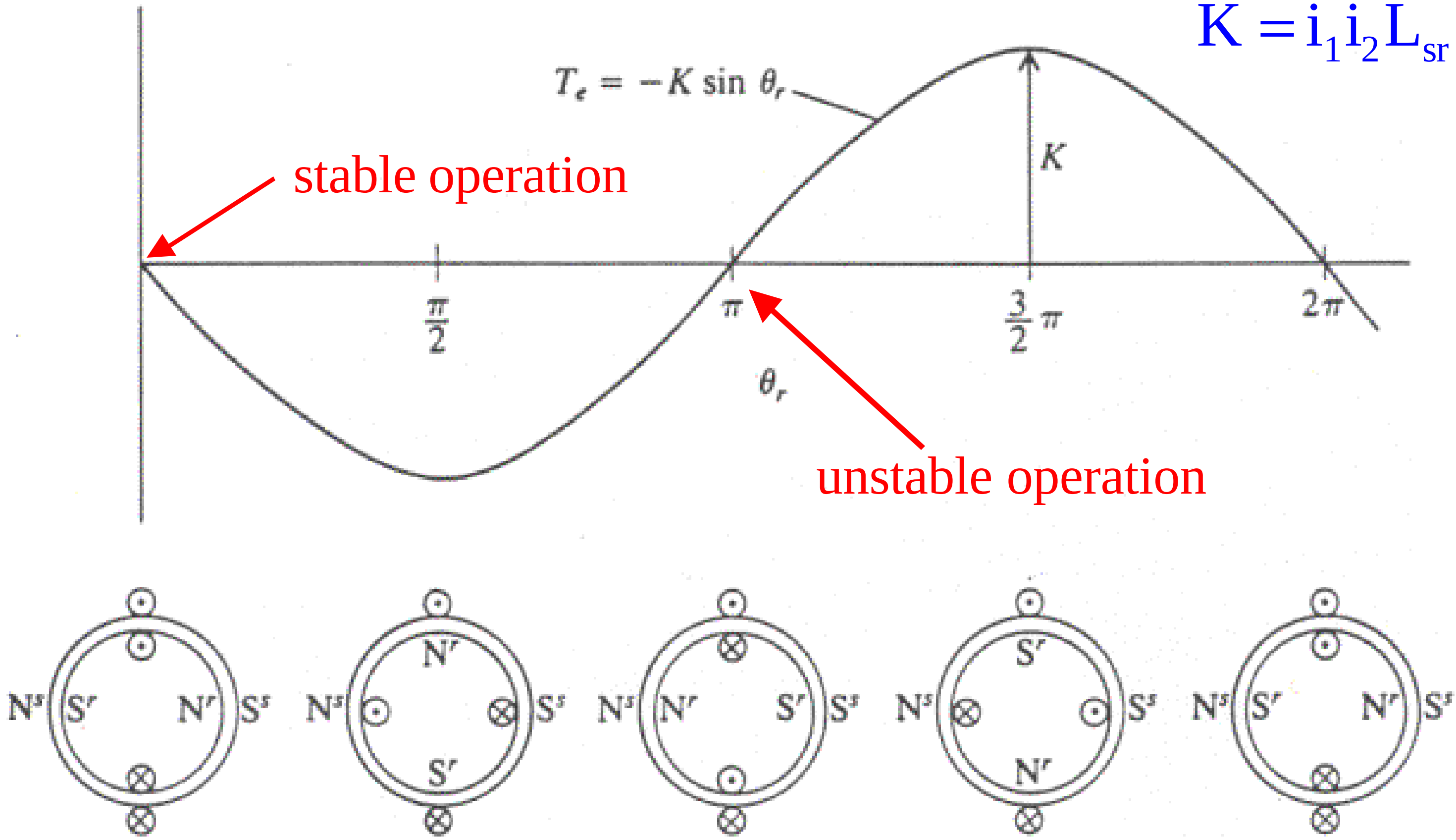
$$W_f\left(i_1, i_2, \theta_r\right) = \frac{1}{2} L_{11} i_1^2 + L_{12} i_1 i_2 + \frac{1}{2} L_{22} i_2^2 = W_c\left(i_1, i_2, \theta_r\right)$$

$$\left.\begin{aligned} T_e\left(\vec{i}, \theta\right) &= \sum_{j=1}^{J} \left[i_j \frac{\partial \lambda_j\left(\vec{i}, \theta\right)}{\partial \theta} \right] - \frac{\partial W_f\left(\vec{i}, \theta\right)}{\partial \theta} \\ T_e\left(\vec{i}, \theta\right) &= \frac{\partial W_c\left(\vec{i}, \theta\right)}{\partial \theta} \end{aligned}\right\} \quad T_e\left(i_1, i_2, \theta_r\right) = -i_1 i_2 L_{sr} \sin\theta_r$$

- Consider the case where i_1 and i_2 are both positive and constant:

$$T_e = -K \sin \theta_r$$

$$K = i_1 i_2 L_{sr}$$



Electromagnetic torque versus angular displacement with constant winding currents

- In the diagram showing the positions of the poles of the magnetic system created by constant positive current flowing in the windings, it may appear at first that the N^s and S^s poles produced by positive current flowing in the stator winding are positioned incorrectly.
- Recall that flux issues from a north pole of a magnet into the air. Since the stator and rotor windings must each be considered as creating separate magnetic systems, we realize, by the right-hand rule, that flux issues from the N pole of the magnetic system established by the stator winding into the air gap. Similarly, flux produced by positive current in the rotor winding enters the air gap from the N pole of the magnetic system of the rotor.

- Here electromagnetic torque is produced in an attempt to align the magnetic systems established by currents flowing in the stator and rotor windings; in other words, to align the 1- and 2-axes.

- Although operation with constant winding currents is somewhat impracticable, it does illustrate the principle of positioning of stepper motors with a permanent-magnet rotor which, in many respects, is analogous to holding i_2 constant on the elementary device considered here.

Voltage and Torque Equations

- Rigorous derivations are possible, but they are lengthy!
- The armature coils revolve in a magnetic field established by a current flowing in the field winding.
- Voltage is induced in these coils by virtue of this rotation. However, the action of the commutator causes the armature coils to appear as a stationary winding with its magnetic axis orthogonal to the magnetic axis of the field winding.
- Consequently, voltages are not induced in one winding due to the time rate of change of the current flowing in the other (transformer action).

- <u>Field and armature voltage equations</u>:

$$\begin{bmatrix} v_f \\ v_a \end{bmatrix} = \begin{bmatrix} r_f + DL_{FF} & 0 \\ \omega_r L_{AF} & r_a + DL_{AA} \end{bmatrix} \begin{bmatrix} i_f \\ i_a \end{bmatrix}$$

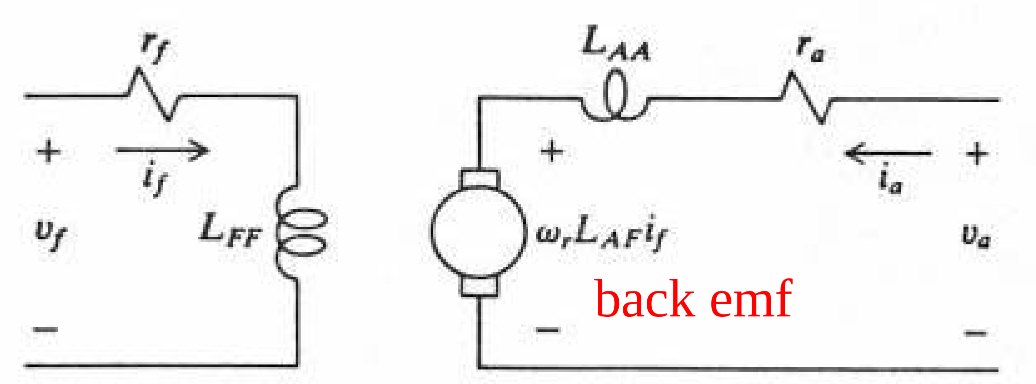
$$D \equiv \frac{d}{dt}$$

$$\boxed{\begin{aligned} v_f &= r_f i_f + L_{ff} \frac{di_f}{dt} \\ v_a &= r_a i_a + L_{AA} \frac{di_a}{dt} + i_f \omega_r L_{AF} \end{aligned}}$$

L_{FF}	self-inductance of the field windings
L_{AA}	self-inductance of the armature windings
ω_r	rotor speed
L_{AF}	mutual inductance between the field and the rotating armature coils

<u>Equivalent Circuit</u>

(suggested by above equations)



- The voltage induced in the armature circuit, $w_r L_{AF} i_f$, is commonly referred to as the counter or back emf. It also represents the open-circuit armature voltage.
- There are several other forms in which the field and armature voltage equations are often expressed. For example: $L_{AF} = \dfrac{N_a N_f}{\Re}$

N_a = equivalent turns of armature windings
N_f = equivalent turns of field windings

$$L_{AF} i_f = N_a \frac{N_f i_f}{\Re} = N_a \Phi_f$$

Φ = field flux per pole

$$L_{AF} i_f = k_v$$

frequently used

  – Even though a permanent magnet dc machine has no field circuit, the constant field flux produced by the permanent magnet is analogous to a dc machine with a constant k_v.

- The expression for the electromagnetic torque is:

$$T_e(i_1, i_2, \theta_r) = -i_1 i_2 L_{sr} \sin\theta_r$$

$$\theta_r = -\frac{\pi}{2}$$

$$T_e = L_{AF} i_f i_a = k_v i_a$$

To account for rotational losses, sometimes k_v is multiplied by a factor < 1

- The field winding produces a stationary mmf and, owing to commutation, the armature winding also produces a stationary mmf which is displaced ½π electrical degrees from the mmf produced by the field winding. It follows then that the interaction of the two mmf's produces the electromagnetic torque.
- The torque and rotor speed are related by:

$$T_e = J\frac{d\omega_r}{dt} + B_m\omega_r + T_L$$

- J is the inertia of the rotor and, in some cases, the connected mechanical load (kg-m^2)
- A positive electromagnetic torque T_e acts to turn the rotor in the direction of increasing θ_r
- The load torque T_L is positive for a torque, on the shaft of the rotor, which opposes a positive electromagnetic torque T_e
- The constant B_m is a damping coefficient associated with the mechanical rotational system of the machine (N-m-s) and it is generally small and often neglected

Physical Modeling

Note change in variable names from previous equations:

$R_f = r_f$

$L_f = L_{FF}$

$L_a = L_{AA}$

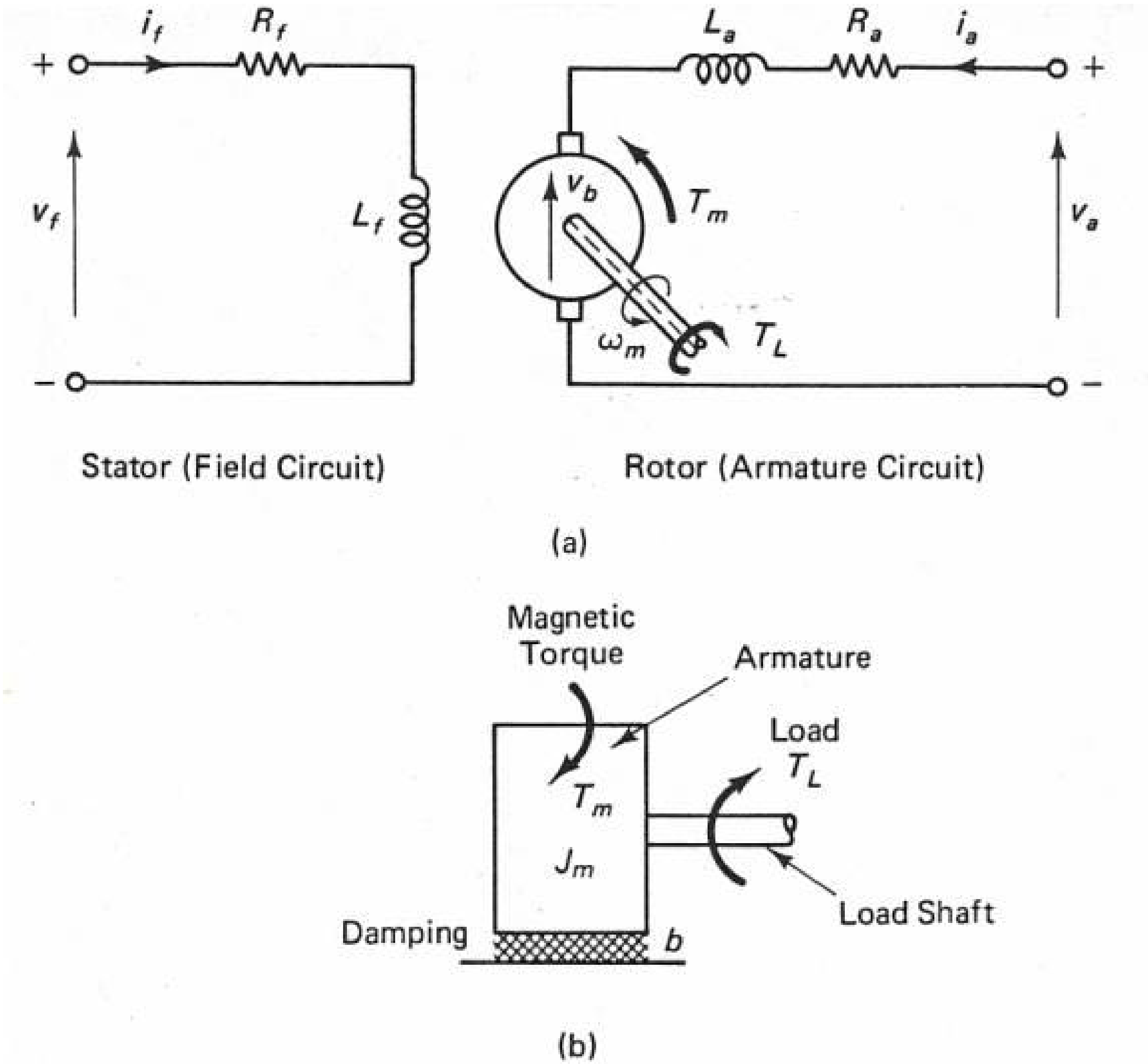
$R_a = r_a$

$\omega_m = \omega_r$

$b = B_m$

$T_m = T_e$

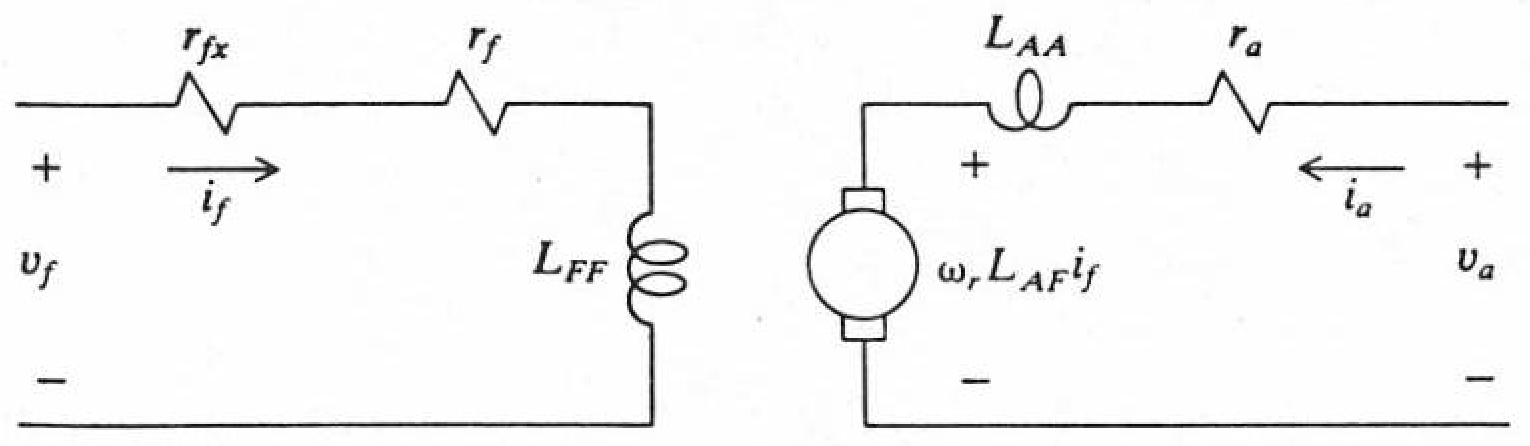
$J_m = J$



Basic Types of Direct-Current Machines

- The field and armature windings may be excited from separate sources or from the same source with the windings connected differently to form the basic types of dc machines.
  - Separate Winding Excitation (includes permanent magnet)
  - Shunt-Connected DC Machine
  - Series-Connected DC Machine
  - Compound-Connected DC Machine
- Here we give equivalent circuits for each of these machines along with an analysis and discussion of their steady-state operating characteristics.

- Separate Winding Excitation



Equivalent Circuit for Separate Field and Armature Excitation

- When the field and armature windings are supplied from separate voltage sources, the device may operate as either a motor or a generator.
- An external resistance r_{fx}, often referred to as a field rheostat, is connected in series with the field winding. It is used to adjust the field current if the field voltage is supplied from a constant source.
- The steady-state voltage equations are:

$$V_f = \left(r_{fx} + r_f\right) I_f = R_f I_f$$

$$V_a = r_a I_a + \omega_r L_{AF} I_f$$

- Note: capital letters are used to denote steady-state voltages and currents.

– In the steady-state, if B_m is assumed to be zero, then:

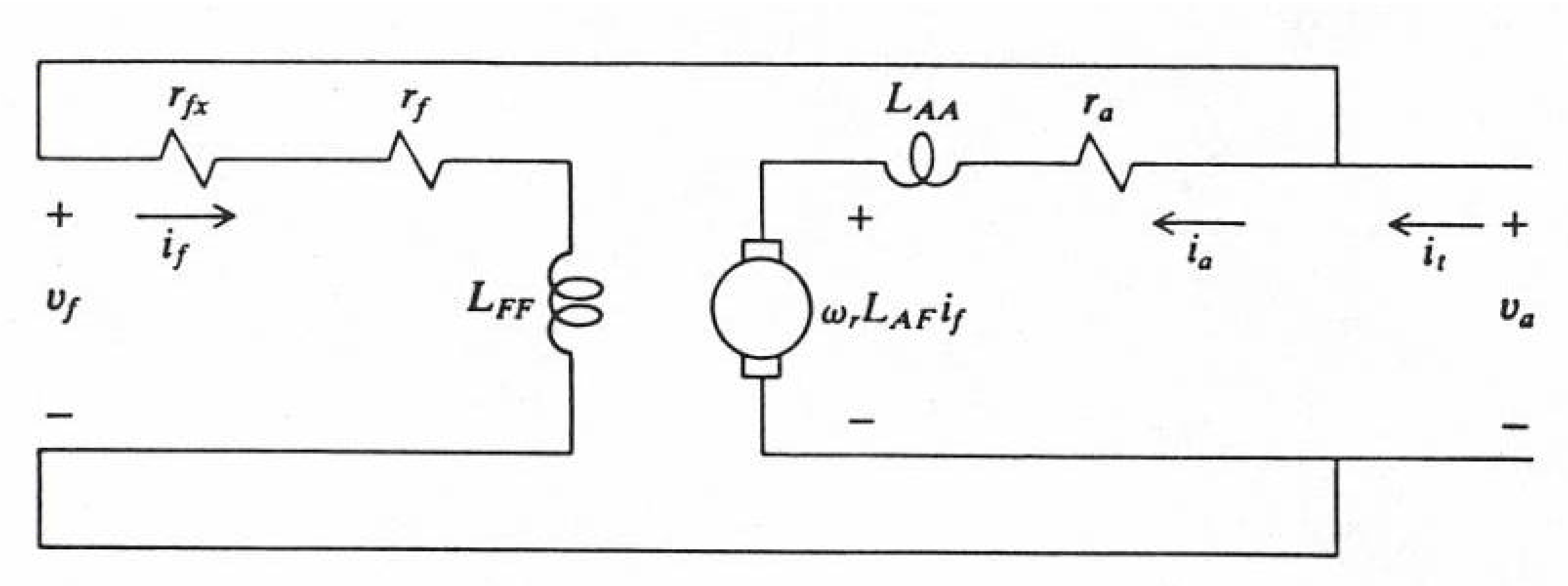
$$T_e = J\frac{d\omega_r}{dt} + B_m\omega_r + T_L = T_L$$

– Analysis of steady-state performance is straightforward.

– A permanent-magnet dc machine fits into this class of dc machines. The field flux is established in these devices by a permanent magnet. The voltage equation for the field winding is eliminated and $L_{AF}i_f$ is replaced by constant k_v, which can be measured if not given.

– Most small, hand-held, fractional-horsepower dc motors are of this type, and speed control is achieved by controlling the amplitude of the applied armature voltage.

- ## Shunt-Connected DC Machine



Equivalent Circuit of a Shunt-Connected DC Machine

- With the connection shown, the machine may operate either as a motor or a generator.
- Since the field winding is connected between the armature terminals, this winding arrangement is commonly referred to as a shunt-connected dc machine or simply a shunt machine.
- During steady-state operation:

$$V_a = r_a I_a + \omega_r L_{AF} I_f$$

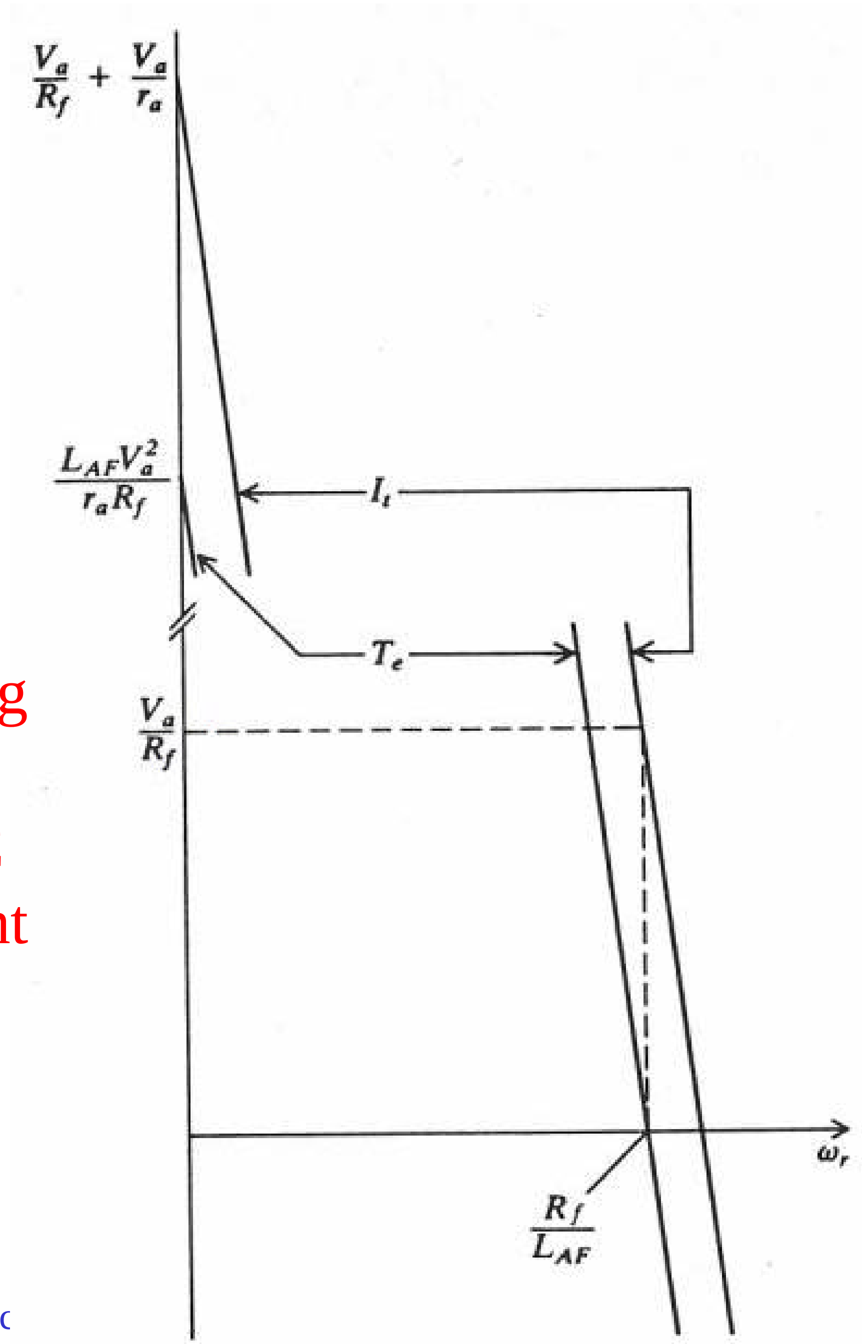
$$V_a = I_f R_f$$

- The total current is: $I_t = I_f + I_a$
- The steady-state electromagnetic torque is found:

$$\left.\begin{aligned} V_a &= r_a I_a + \omega_r L_{AF} I_f \\ V_f &= I_f R_f \end{aligned}\right\} \Rightarrow T_e = L_{AF} i_f i_a$$

$$T_e = \frac{L_{AF} V_a^2}{r_a R_f}\left(1 - \frac{L_{AF}}{R_f}\omega_r\right)$$

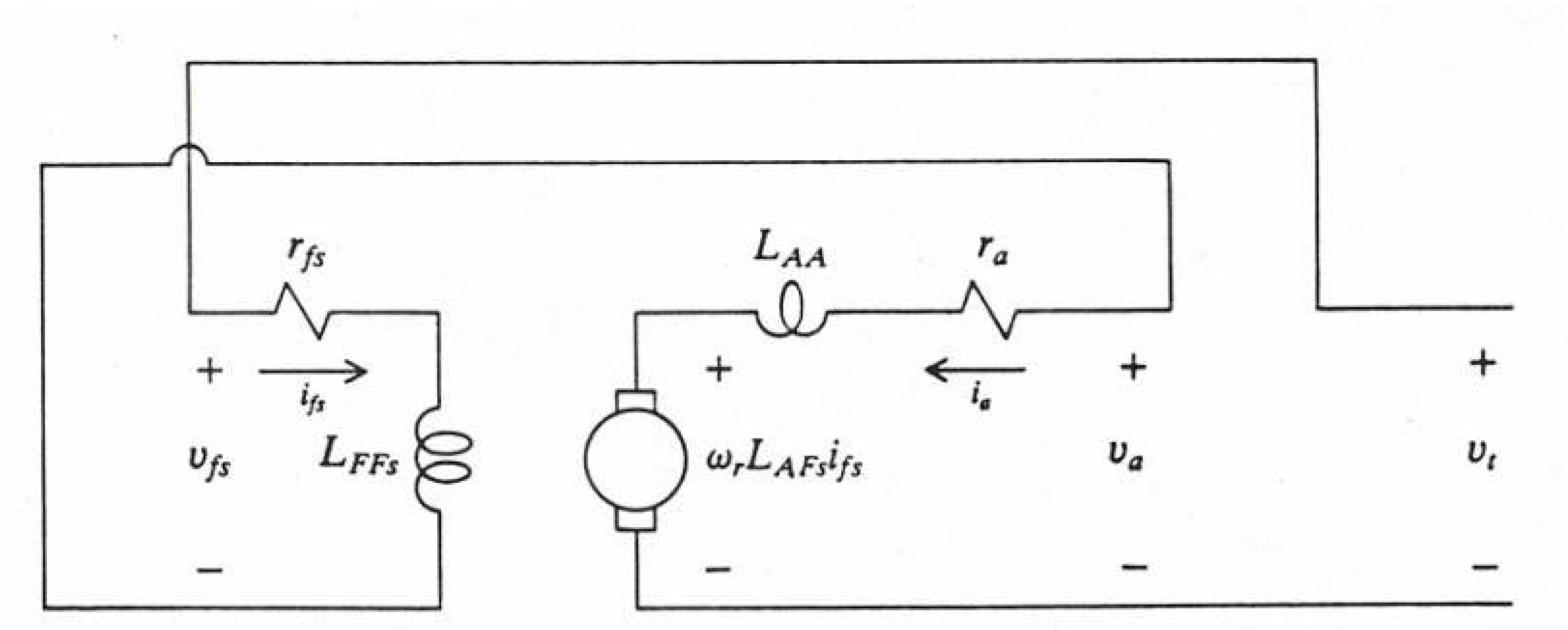
Steady-State Operating Characteristics of a Shunt-Connected DC Machine with Constant Source Voltage



- Several features of the steady-state operating characteristics of a shunt-connected dc machine with a constant voltage source warrant discussion:
  - At stall ($\omega_r = 0$), the steady-state armature current I_a is limited only by the armature resistance. In the case of small, permanent-magnet motors, the armature resistance is quite large and the starting armature current, which results when rated voltage is applied, is generally not damaging. However, large-horsepower machines are designed with a small armature resistance. Therefore, an excessively high armature current will occur during the starting period if rated voltage is applied to the armature terminals.

– To prevent high starting current, resistance may be inserted into the armature circuit at stall and decreased either manually or automatically to zero as the machine accelerates to normal operating speed.
– Other features of the shunt machine with a small armature resistance are the steep torque versus speed characteristics. In other words, the speed of the shunt machine does not change appreciably as the load torque is varied from zero to rated.

- Series-Connected DC Machine



Equivalent Circuit of a Series-Connected DC Machine

- When the field is connected in series with the armature circuit, the machine is referred to as a series-connected dc machine or a series machine.
- It is important to mention the physical difference between the field winding of a shunt machine and that of a series machine:
  - For a shunt-connected field winding, a large number of turns of small-diameter wire are used, making the resistance of the field winding quite large.
  - Since a series-connected field winding is in series with the armature, it is designed so as to minimize the voltage drop across it; thus, the winding is wound with a few turns of low-resistance wire.
- A series machine does not have wide application. However, a series field is often used in conjunction with a shunt field to form the more common compound-connected dc machine.

– For the series machine:

$$v_t = v_{fs} + v_a$$

$$i_a = i_{fs}$$

Subscript *s* stands for series field.

– The steady-state performance of the series-connected dc machine may be described by:

$$V_t = \left(r_a + r_{fs} + L_{AFs}\omega_r\right) I_a$$

$$T_e = L_{AF} i_f i_a = L_{AFs} I_a^2$$

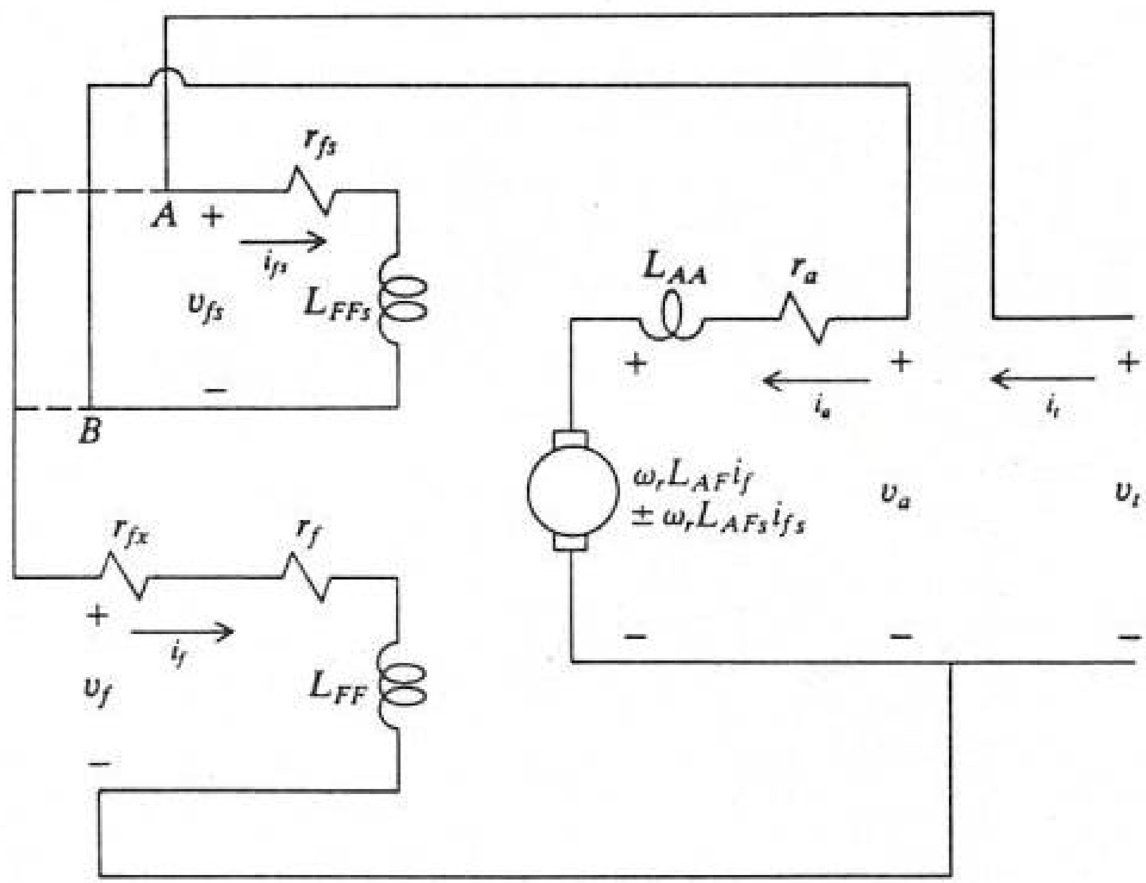
$$= \frac{L_{AFs} V_t^2}{\left(r_a + r_{fs} + L_{AFs}\omega_r\right)^2}$$

– Stall torque:

$$T_e = \frac{L_{AFs} V_t^2}{\left(r_a + r_{fs}\right)^2}$$

- The stall torque is quite high since it is proportional to the square of the armature current for a linear magnetic system. However, saturation of the magnetic system due to large armature currents will cause the torque to be less than calculated.
- At high rotor speeds, the torque decreases less rapidly with increasing speed. If the load torque is small, the series motor may accelerate to speeds large enough to cause damage to the machine.
- Consequently, the series motor is used in applications where a high starting torque is required and an appreciable load torque exists under normal operation.

• Compound-Connected DC Machine



Equivalent Circuit of a Compound-Connected DC Machine

- A compound-connected or compound dc machine is equipped with both a shunt and a series field winding.
- In most compound machines, the shunt field dominates the operating characteristics while the series field, which consists of a few turns of low-resistance wire, has a secondary influence. It may be connected so as to aid or oppose the flux produced by the shunt field.
- Depending upon the strength of the series field, this type of connection can produce a "flat" terminal voltage versus load current characteristic, whereupon a near-constant terminal voltage is achieved from no load to full load. In this case, the machine is said to be "flat-compounded."
- An "over-compounded" machine occurs when the strength of the series field causes the terminal voltage at full load to be larger than at no load.

- The meaning of an "under-compounded" machine is obvious.
- In the case of compound dc motors, the series field is often connected to oppose the flux produced by the shunt field (differential compounding). If properly designed, this type of connection can provide a near-constant speed from no-load to full-load torque.
- The voltage equations for a compound dc machine may be written as:

$$\begin{bmatrix} v_f \\ v_t \end{bmatrix} = \begin{bmatrix} R_f + DL_{FF} & \pm DL_{FS} & 0 \\ \omega_r L_{AF} \pm DL_{FS} & \pm\omega_r L_{AFs} + r_{fs} + DL_{FFs} & r_a + DL_{AA} \end{bmatrix} \begin{bmatrix} i_f \\ i_{fs} \\ i_a \end{bmatrix}$$

L_{FS} is the mutual inductance between the shunt and series fields and the plus and minus signs are used so that either a cumulative or a differential connection may be described.

- The shunt field may be connected ahead of the series field (long-shunt connection) or behind the series field (short-shunt connection), as shown by A and B, respectively, in the figure. The long-shunt connection is commonly used. In this case:

$$v_t = v_f = v_{fs} + v_a$$

$$i_t = i_f + i_{fs} \qquad \text{where } i_{fs} = i_a$$

- The steady-state performance of a long-shunt-connected compound machine may be described by the following equations:

$$V_t = \left[\frac{r_a + r_{fs} \pm L_{AFs}\omega_r}{1 - \frac{L_{AF}}{R_f}\omega_r} \right] I_a$$

– The torque for the long-shunt connection may be obtained by employing $T_e = L_{AF} i_f i_a$ for each field winding.

– In particular:

$$
\begin{aligned}
T_e &= L_{AF} I_f I_a \pm L_{AFs} I_{fs} I_a \\
&= \frac{L_{AF} V_t^2 \left[1 - \dfrac{L_{AF}}{R_f} \omega_r \right]}{R_f \left(r_a + r_{fs} \pm L_{AFs} \omega_r \right)} \pm \frac{L_{AFs} V_t^2 \left[1 - \dfrac{L_{AF}}{R_f} \omega_r \right]^2}{\left(r_a + r_{fs} \pm L_{AFs} \omega_r \right)^2}
\end{aligned}
$$

- Example Problem
  - A permanent-magnet dc motor is rated at 6V with the following parameters: $r_a = 7\ \Omega$, $L_{AA} = 120$ mH, $k_T = 2$ oz-in/A, $J = 150\ \mu$ oz-in-s^2. According to the motor information sheet, the no-load speed is approximately 3350 rpm and the no-load current is approximately 0.15 A. Interpret this information.
  - This permanent-magnet dc machine is operating with rated applied voltage and a load torque T_L of 0.5 oz-in. Determine the percent efficiency, i.e., (power output / power input)(100).

Time-Domain Block Diagrams, State Equations, and Transfer Functions

- Let's consider the time-domain block diagrams and state equations for the shunt-connected dc machine and the permanent-magnet dc machine.
- <u>Shunt-Connected DC machine</u>
  - The field and armature voltage equations and the relationship between torque and rotor speed are given as:

$$\begin{bmatrix} v_f \\ v_a \end{bmatrix} = \begin{bmatrix} r_f + DL_{FF} & 0 \\ \omega_r L_{AF} & r_a + DL_{AA} \end{bmatrix} \begin{bmatrix} i_f \\ i_a \end{bmatrix}$$

$$T_e = J\frac{d\omega_r}{dt} + B_m \omega_r + T_L$$

– These may be written as:

$$v_f = R_f\left[1+\frac{L_{FF}}{R_f}D\right]i_f = R_f\left[1+\tau_f D\right]i_f$$

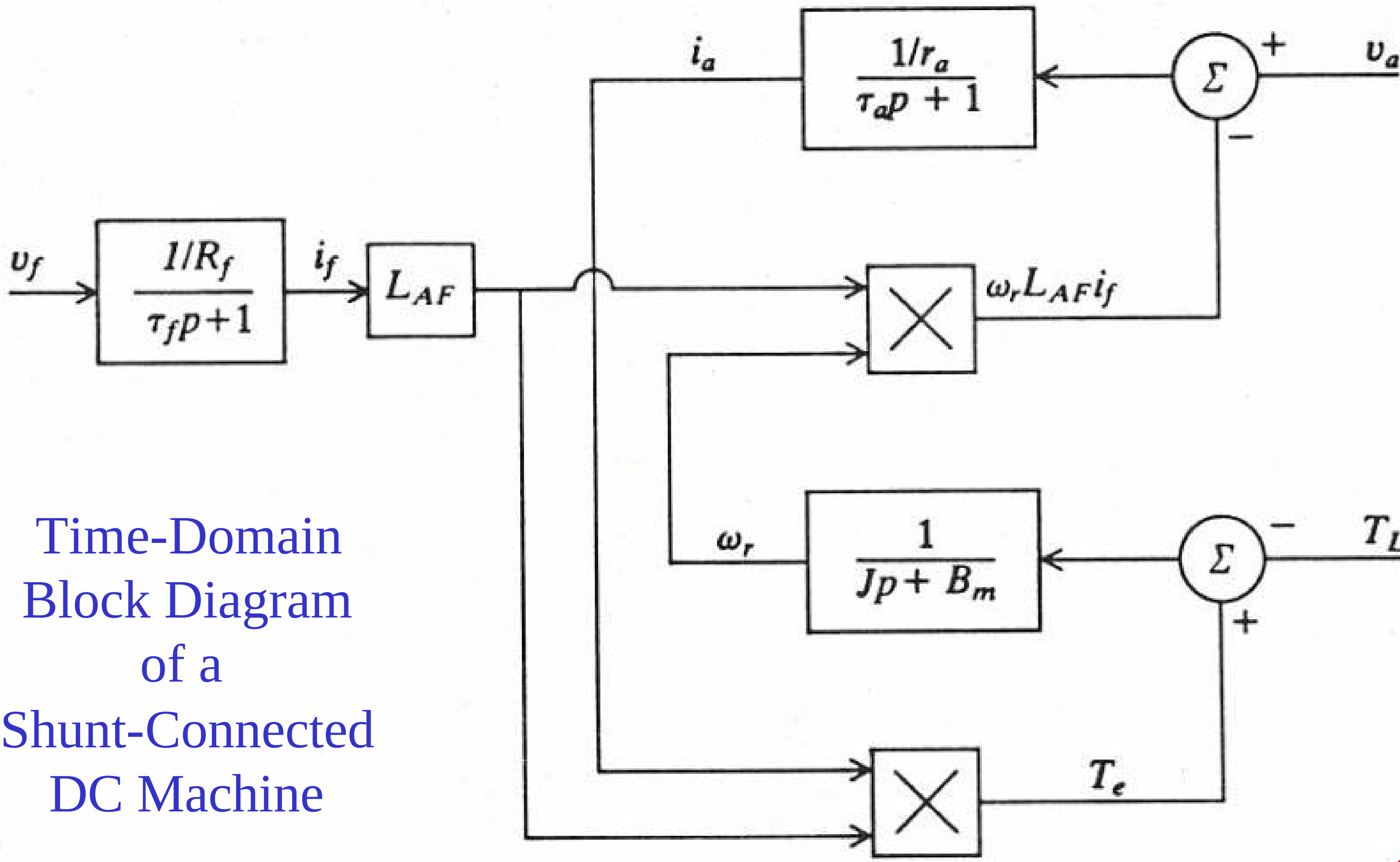
$$v_a = r_a\left[1+\frac{L_{AA}}{r_a}D\right]i_a + \omega_r L_{AF} i_f = r_a\left[1+\tau_a D\right]i_a + \omega_r L_{AF} i_f$$

$$T_e - T_L = \left(B_m + JD\right)\omega_r$$

– $\tau_f = L_{FF}/R_f$ is the field time constant and $\tau_a = L_{AA}/r_a$ is the armature time constant.

$$i_f = \frac{\frac{1}{R_f}}{\tau_f D + 1} v_f \qquad i_a = \frac{\frac{1}{r_a}}{\tau_a D + 1}\left(v_a - \omega_r L_{AF} i_f\right)$$

$$\omega_r = \frac{1}{JD + B_m}\left(T_e - T_L\right) = \frac{1}{JD + B_m}\left(L_{AF} i_f i_a - T_L\right)$$



Time-Domain
Block Diagram
of a
Shunt-Connected
DC Machine

$$p \triangleq D \triangleq \frac{d}{dt}$$

– The state variables of a system are defined as a minimal set of variables such that knowledge of these variables at any initial time t_0 plus information on the input excitation subsequently applied as sufficient to determine the state of the system at any time $t > t_0$.

– In the case of dc machines, the field current i_f, the armature current i_a, the rotor speed ω_r, and the rotor position θ_r are the state variables. However, since θ_r can be established from ω_r by using

$$\theta_r = \text{ angular displacement}$$

$$\omega_r = \text{ angular velocity}$$

$$\theta_r = \int_0^t \omega_r(\xi)\, d\xi + \theta_r(0)$$

and since θ_r is considered a state variable only when the shaft position is a controlled variable, we omit θ_r here.

– The state-variable equations (nonlinear) are:

$$\frac{di_f}{dt} = -\frac{R_f}{L_{FF}} i_f + \frac{1}{L_{FF}} v_f$$

$$\frac{di_a}{dt} = -\frac{r_a}{L_{AA}} i_a - \frac{L_{AF}}{L_{AA}} i_f \omega_r + \frac{1}{L_{AA}} v_a$$

$$\frac{d\omega_r}{dt} = -\frac{B_m}{J} \omega_r + \frac{L_{AF}}{J} i_f i_a - \frac{1}{J} T_L$$

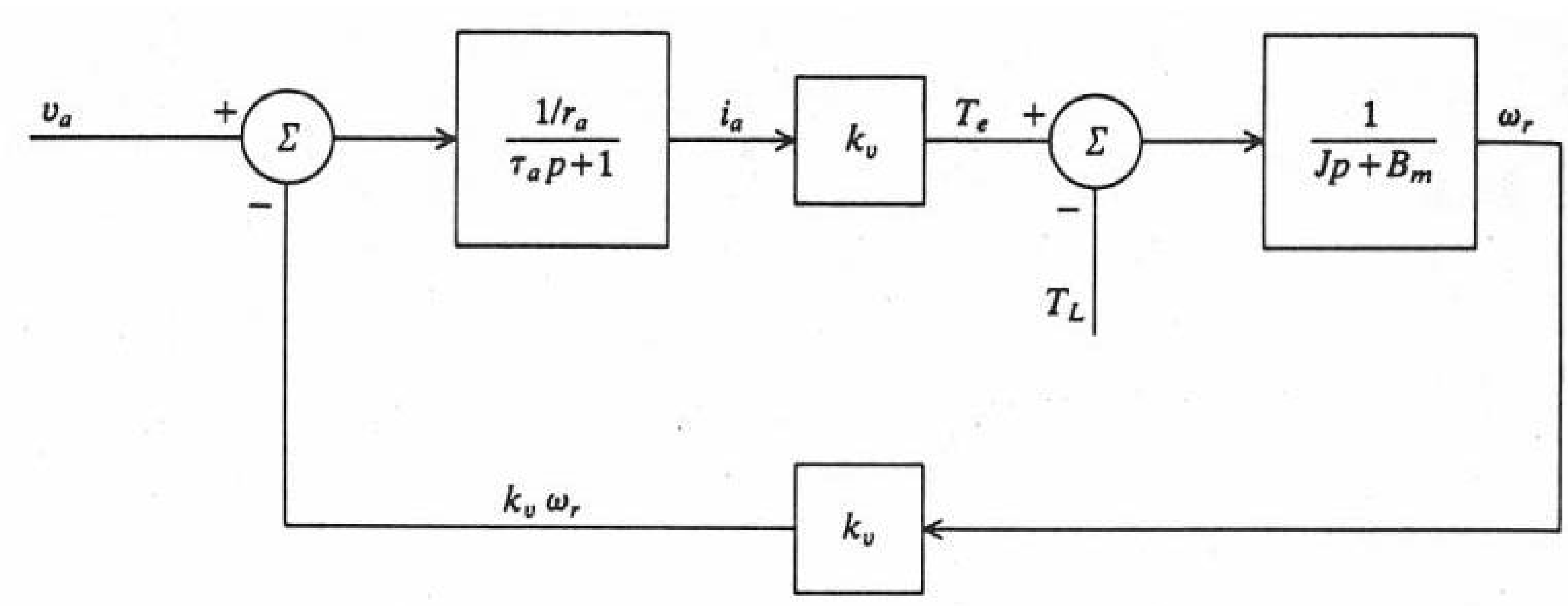
– In matrix form:

$$\frac{d}{dt}\begin{bmatrix} i_f \\ i_a \\ \omega_r \end{bmatrix} = \begin{bmatrix} -\frac{R_f}{L_{ff}} & 0 & 0 \\ 0 & -\frac{r_a}{L_{AA}} & 0 \\ 0 & 0 & -\frac{B_m}{J} \end{bmatrix}\begin{bmatrix} i_f \\ i_a \\ \omega_r \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{L_{AF}}{L_{AA}} i_f \omega_r \\ \frac{L_{AF}}{J} i_f i_a \end{bmatrix} + \begin{bmatrix} \frac{1}{L_{FF}} & 0 & 0 \\ 0 & \frac{1}{L_{AA}} & 0 \\ 0 & 0 & -\frac{1}{J} \end{bmatrix}\begin{bmatrix} v_f \\ v_a \\ T_L \end{bmatrix}$$

- Permanent-Magnet DC Machine
  - The equations which describe the operation of a permanent-magnet dc machine are identical to those of a shunt-connected dc machine with the field current held constant. So the work here applies to both.
  - For the permanent-magnet machine, $L_{AF}i_f$ is replaced by k_v, which is a constant determined by the strength of the magnet, the reluctance of the iron, and the number of turns of the armature winding.
  - The time-domain block diagram may be developed by using the following equations:

$$v_a = r_a\left[1+\frac{L_{AA}}{r_a}D\right]i_a + \omega_r L_{AF} i_f = r_a\left[1+\tau_a D\right]i_a + \omega_r k_v$$

$$T_e - T_L = \left(B_m + JD\right)\omega_r$$



Time-Domain Block Diagram of a Permanent-Magnet DC Machine

$$p \triangleq D \triangleq \frac{d}{dt}$$

– Since k_v is constant, the state variables are now i_a and ω_r. The state-variable equations are:

$$\frac{di_a}{dt} = -\frac{r_a}{L_{AA}} i_a - \frac{k_v}{L_{AA}} \omega_r + \frac{1}{L_{AA}} v_a$$

$$\frac{d\omega_r}{dt} = -\frac{B_m}{J} \omega_r + \frac{k_v}{J} i_a - \frac{1}{J} T_L$$

– In matrix form:

$$\frac{d}{dt}\begin{bmatrix} i_a \\ \omega_r \end{bmatrix} = \begin{bmatrix} -\dfrac{r_a}{L_{AA}} & -\dfrac{k_v}{L_{AA}} \\ \dfrac{k_v}{J} & -\dfrac{B_m}{J} \end{bmatrix} \begin{bmatrix} i_a \\ \omega_r \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L_{AA}} & 0 \\ 0 & -\dfrac{1}{J} \end{bmatrix} \begin{bmatrix} v_a \\ T_L \end{bmatrix}$$

- Transfer Functions for a Permanent-Magnet DC Motor
  - Once the permanent-magnet dc motor is portrayed in block diagram from, it is often advantageous, for control design purposes, to express transfer functions between state and input variables.
  - Here the state variables are i_a and ω_r and the input variables are v_a and T_L.
  - From the block diagram, we have the relations:

$$i_a = \frac{\frac{1}{r_a}}{\tau_a D + 1}\left(v_a - k_v \omega_r\right)$$

$$\omega_r = \frac{1}{JD + B_m}\left(k_v i_a - T_L\right)$$

– After substitution and considerable work, we find:

$$\omega_r = \frac{\left(\dfrac{1}{k_v \tau_a \tau_m}\right) v_a - \left(\dfrac{1}{J}\right)\left(D + \dfrac{1}{\tau_a}\right) T_L}{D^2 + \left(\dfrac{1}{\tau_a} + \dfrac{B_m}{J}\right) D + \left(\dfrac{1}{\tau_a}\right)\left(\dfrac{1}{\tau_m} + \dfrac{B_m}{J}\right)} \qquad \text{where} \quad \tau_m = \frac{J r_a}{k_v^2}$$

– τ_m is called the inertia time constant

$$i_a = \frac{\left(\dfrac{1}{\tau_a r_a}\right)\left(D + \dfrac{B_m}{J}\right) v_a - \left(\dfrac{1}{k_v \tau_a \tau_m}\right) T_L}{D^2 + \left(\dfrac{1}{\tau_a} + \dfrac{B_m}{J}\right) D + \left(\dfrac{1}{\tau_a}\right)\left(\dfrac{1}{\tau_m} + \dfrac{B_m}{J}\right)}$$

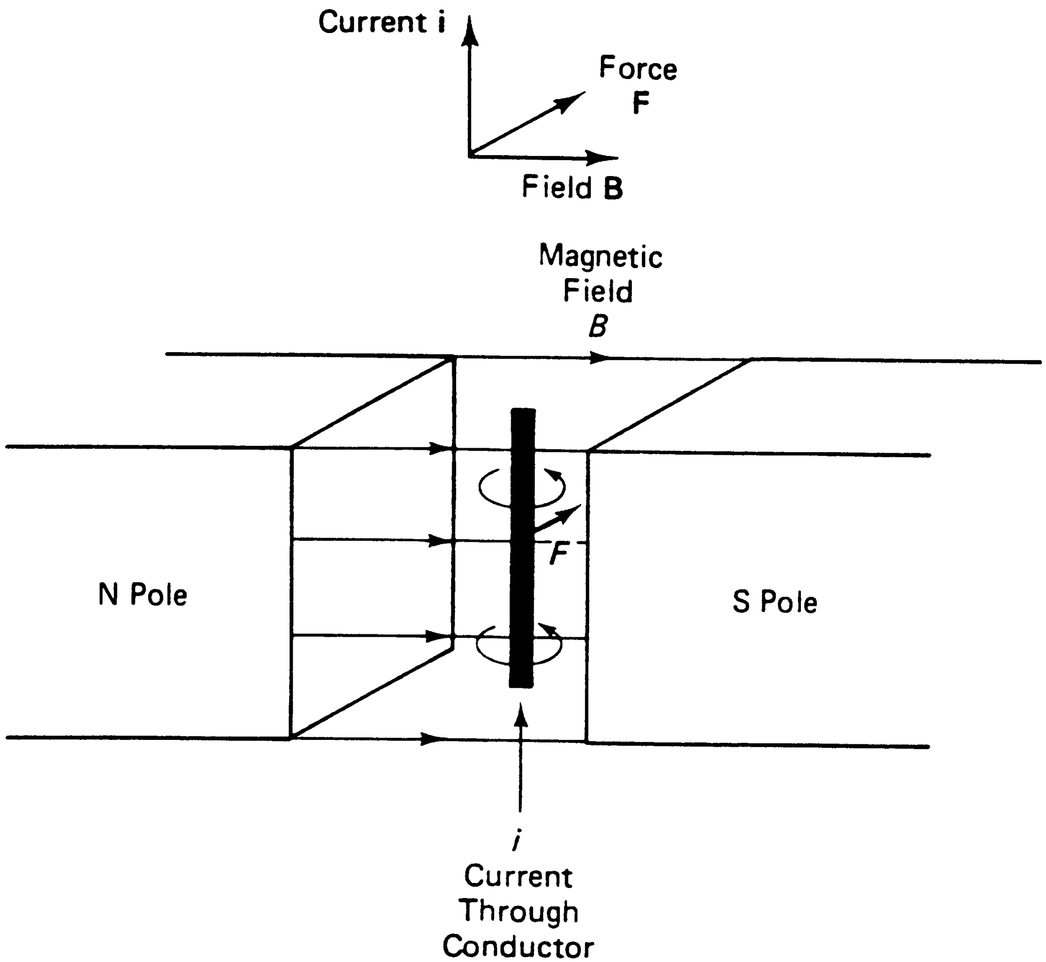
Summary

- The dc machine is unique in that it exerts a torque on the rotating member as a result of the interaction of two stationary, orthogonal magnetic systems.
- One is produced by current flowing in the windings of the stationary member (field) and the other is caused by the current flowing in the windings of the rotating member (armature).
- The permanent-magnet dc motor is still used quite widely in low-power control systems.
- However, brushless dc motors are rapidly replacing the permanent-magnet dc motor; the equations which describe these two devices are very similar.

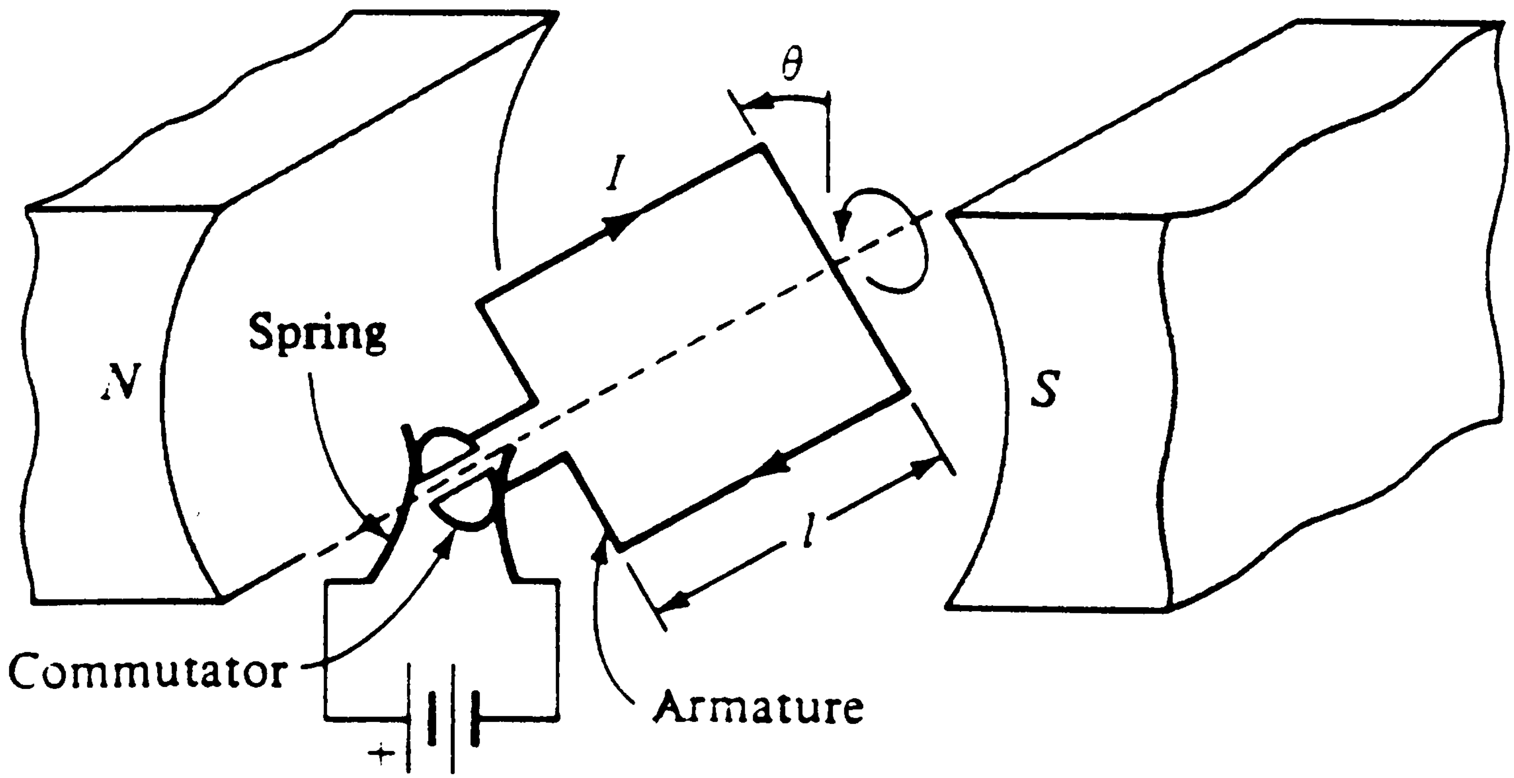
Elementary Approach to Permanent-Magnet DC Motor Modeling

$$\vec{F} = \oint i\overrightarrow{d\ell} \times \vec{B} = Bi\ell$$

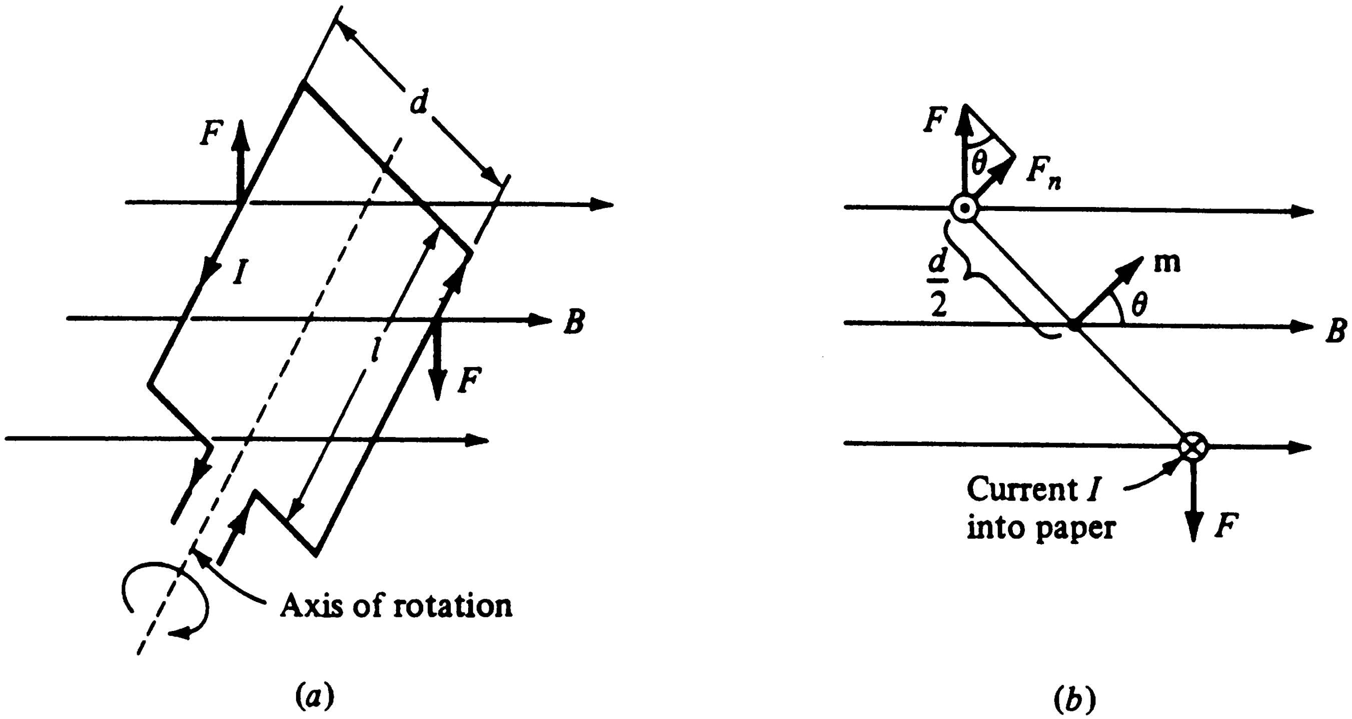
$$V_b = \int \vec{v} \times \vec{B} \bullet \overrightarrow{d\ell} = B\ell v$$



Elements of a Simple DC Motor



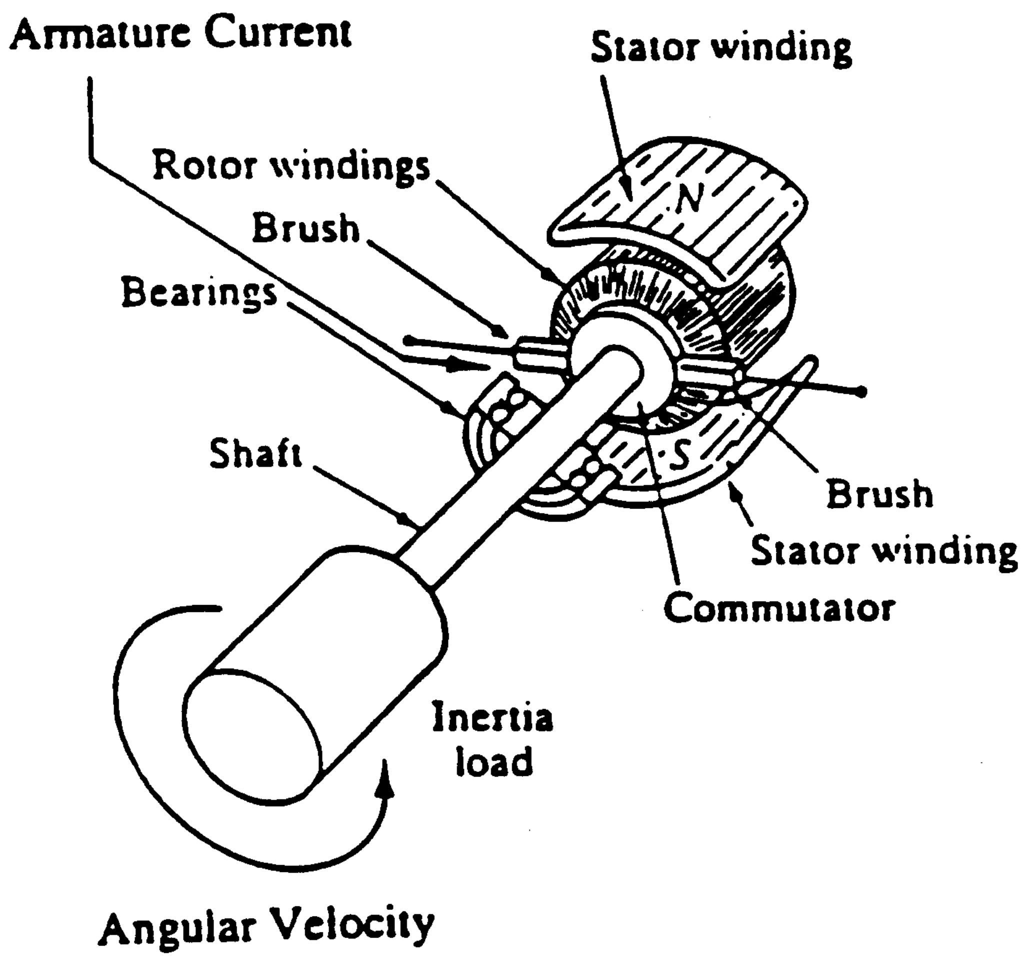
Torque of a DC Motor



$$T = 2F_n\left(\frac{d}{2}\right)N = \left(iB\ell\sin\theta\right)dN = iABN\sin\theta = mBN\sin\theta$$

$$\vec{T} = N\left[\vec{m}\times\vec{B}\right]$$

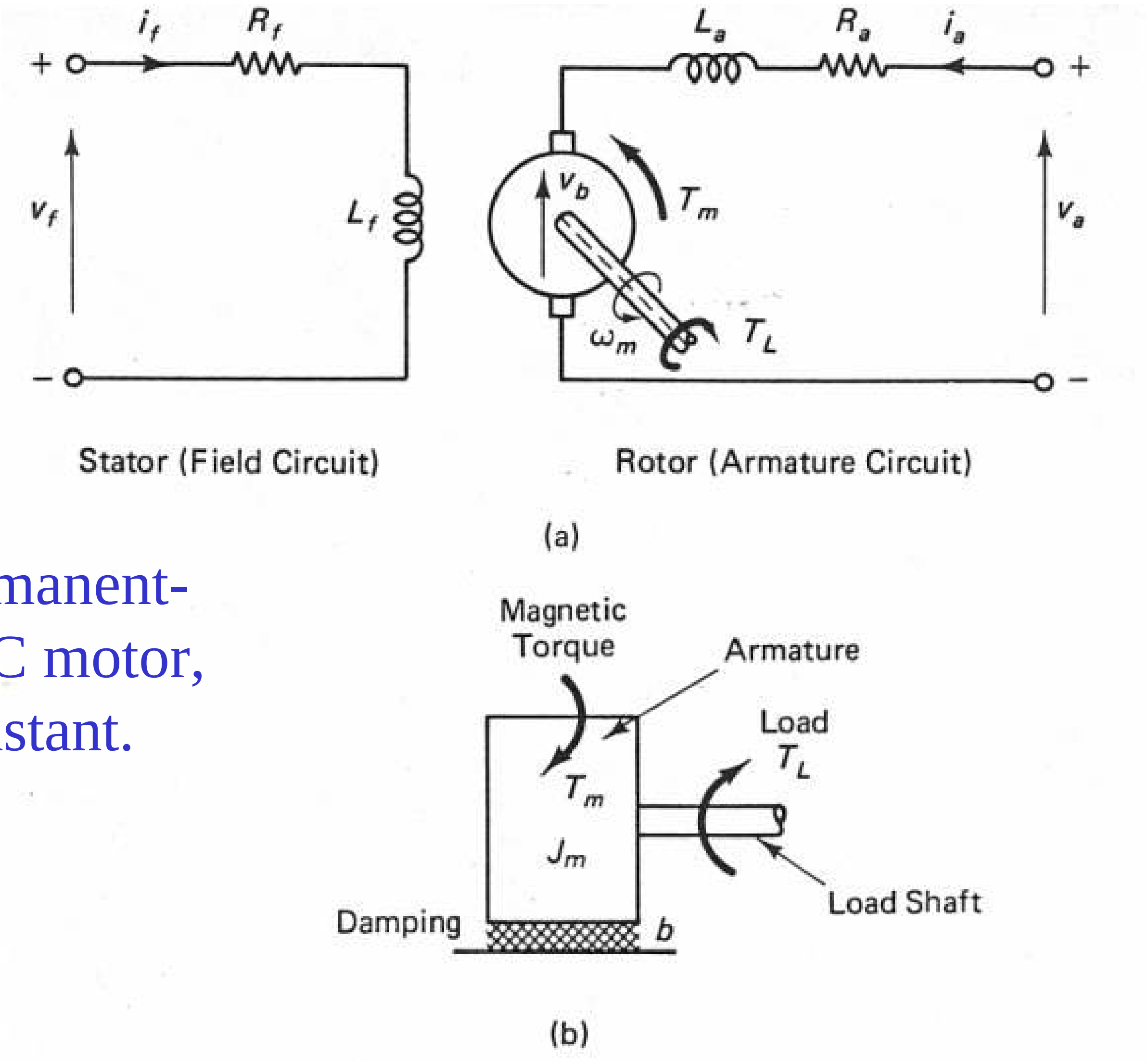
Schematic of a Brushed DC Motor



- <u>Modeling Assumptions</u>
  - The copper armature windings in the motor are treated as a resistance and inductance in series. The distributed inductance and resistance is lumped into two characteristic quantities, L and R.
  - The commutation of the motor is neglected. The system is treated as a single electrical network which is continuously energized.
  - The compliance of the shaft connecting the load to the motor is negligible. The shaft is treated as a rigid member. Similarly, the coupling between the tachometer and motor is also considered to be rigid.
  - The total inertia J is a single lumped inertia, equal to the sum of the inertias of the rotor, the tachometer, and the driven load.

- There exists motion only about the axis of rotation of the motor, i.e., a one-degree-of-freedom system.
- The parameters of the system are constant, i.e., they do not change over time.
- The damping in the mechanical system is modeled as viscous damping *B*, i.e., all stiction and dry friction are neglected.
- Neglect noise on either the sensor or command signal.
- The amplifier dynamics are assumed to be fast relative to the motor dynamics. The unit is modeled by its DC gain, K_{amp}.
- The tachometer dynamics are assumed to be fast relative to the motor dynamics. The unit is modeled by its DC gain, K_{tach}.

Physical Modeling



For a permanent-magnet DC motor, i_f = constant.

- <u>Mathematical Modeling</u>
- The steps in mathematical modeling are as follows:
  - Define System, System Boundary, System Inputs and Outputs
  - Define Through and Across Variables
  - Write Physical Relations for Each Element
  - Write System Relations of Equilibrium and/or Compatibility
  - Combine System Relations and Physical Relations to Generate the Mathematical Model for the System

Physical Relations

$$V_L = L\frac{di_L}{dt} \qquad V_R = Ri_R \qquad T_B = B\omega$$

$$T_J = J\alpha = J\dot{\omega} \qquad J \equiv J_{motor} + J_{tachometer} + J_{load}$$

$$T_m = K_t i_m \qquad V_b = K_b\omega$$

$$P_{out} = T_m\omega = K_t i_m \omega \qquad P_{in} = V_b i_m = K_b \omega i_m$$

$$\frac{P_{out}}{P_{in}} = \frac{K_t}{K_b}$$

$$P_{out} = P_{in}$$

$$K_t = K_b \equiv K_m$$

$$K_t(\text{oz}-\text{in}/\text{A}) = 1.3524 K_b(\text{V}/\text{krpm})$$

$$K_t(\text{Nm}/\text{A}) = 9.5493\times 10^{-3} K_b(\text{V}/\text{krpm})$$

$$K_t(\text{Nm}/\text{A}) = K_b(\text{V}-\text{s}/\text{rad})$$

System Relations + Equations of Motion

$$V_{in} - V_R - V_L - V_b = 0$$

$$T_m - T_B - T_J = 0$$

$$i_R = i_L = i_m \equiv i$$

$$V_{in} - Ri - L\frac{di}{dt} - K_b\omega = 0 \qquad\qquad J\dot{\omega} + B\omega - K_t i = 0$$

$$\begin{bmatrix} \dot{\omega} \\ \dot{i} \end{bmatrix} = \begin{bmatrix} -B/J & K_t/J \\ -K_b/L & -R/L \end{bmatrix} \begin{bmatrix} \omega \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L \end{bmatrix} V_{in}$$

Steady-State Conditions

$$V_{in} - Ri - L\frac{di}{dt} - K_b\omega = 0$$

$$V_{in} - R\left(\frac{T}{K_t}\right) - K_b\omega = 0$$

$$T = \frac{K_t}{R}V_{in} - \frac{K_t K_b}{R}\omega$$

$$T_s = \frac{K_t}{R}V_{in}$$ Stall Torque

$$\omega_0 = \frac{V_{in}}{K_b}$$ No-Load Speed

Transfer Functions

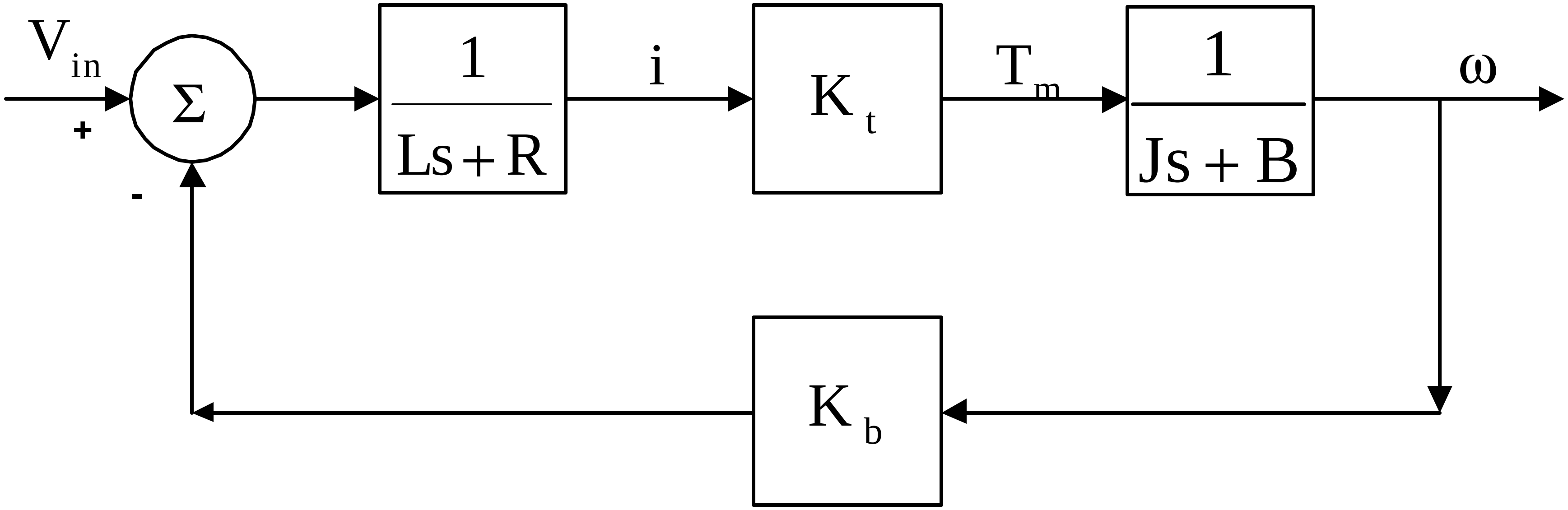
$$V_{in} - Ri - L\frac{di}{dt} - K_b\omega = 0 \qquad J\dot{\omega} + B\omega - K_t i = 0$$

$$V_{in}(s) - (Ls + R)I(s) - K_b\Omega(s) = 0 \qquad (Js + B)\Omega(s) - K_t I(s) = 0$$

$$\frac{\Omega(s)}{V_{in}(s)} = \frac{K_t}{(Js + B)(Ls + R) + K_t K_b} = \frac{K_t}{JLs^2 + (BL + JR)s + (BR + K_t K_b)}$$

$$= \frac{\dfrac{K_t}{JL}}{s^2 + \left(\dfrac{B}{J} + \dfrac{R}{L}\right)s + \left(\dfrac{BR}{JL} + \dfrac{K_t K_b}{JL}\right)}$$

Block Diagram



Simplification

$$\tau_m = \frac{J}{B} \quad << \quad \tau_e = \frac{L}{R}$$

$$V_{in} - Ri - K_b\omega = 0 \qquad\qquad J\dot{\omega} + B\omega - K_t i = 0$$

$$J\dot{\omega} + B\omega = K_t i = K_t\left(\frac{1}{R}\left(V_{in} - K_b\omega\right)\right) = \frac{K_t}{R}\left(V_{in} - K_b\omega\right)$$

$$\dot{\omega} + \left(\frac{K_t K_b}{RJ} + \frac{B}{J}\right)\omega = \frac{K_t}{RJ}V_{in}$$

$$\dot{\omega} + \left(\frac{1}{\tau_{motor}} + \frac{1}{\tau_m}\right)\omega = \frac{K_t}{RJ}V_{in}$$

$$\dot{\omega} + \left(\frac{1}{\tau_{motor}}\right)\omega = \frac{K_t}{RJ}V_{in} \qquad \text{since } \tau_m >> \tau_{motor}$$

Control of DC Motors

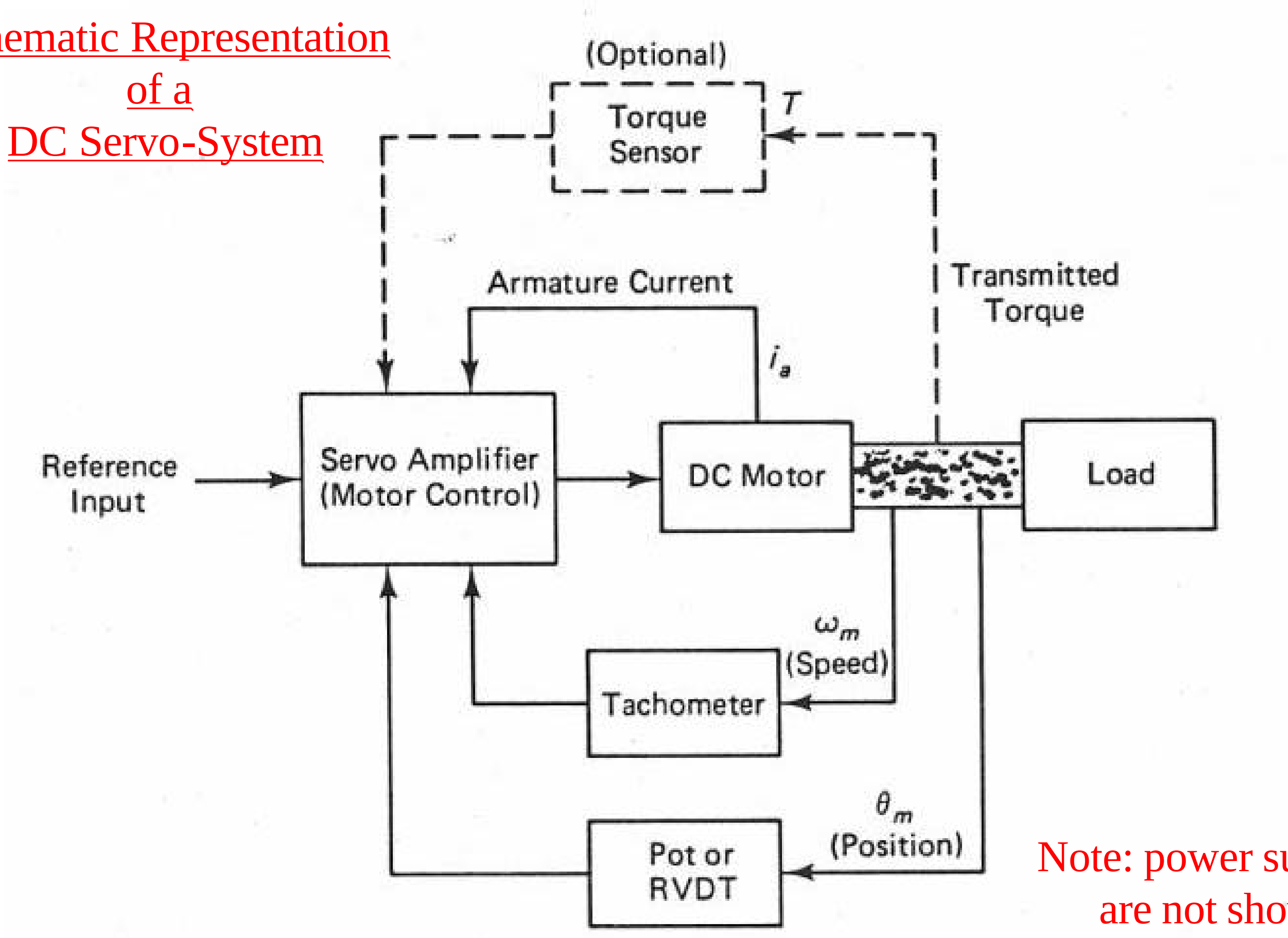
- DC Motors can be operated over a wide range of speeds and torques and are particularly suited as variable-drive actuators.
- The function of a conventional servo system that uses a DC motor as the actuator is almost exclusively motion control (position and speed control). There are applications that require torque control and they usually require more sophisticated control techniques.
- Two methods of control of a DC motor are:
  - Armature Control
  - Field Control

- <u>Armature Control</u>
  - Here the field current in the stator circuit is kept constant. The input voltage v_a to the rotor circuit is varied in order to achieve a desired performance. The motor torque can be kept constant simply by keeping the armature current constant because the field current is virtually constant in the case of armature control. Since v_a directly determines the motor back emf after allowance is made for the impedance drop due to resistance and inductance of the armature circuit, it follows that armature control is particularly suitable for speed manipulation over a wide range of speeds.

- <u>Field Control</u>
  - Here the armature voltage (and current) is kept constant. The input voltage v_f to the field circuit is varied. Since i_a is kept more or less constant, the torque will vary in proportion to the field current i_f. Since the armature voltage is kept constant, the back emf will remain virtually unchanged. Hence the speed will be inversely proportional to i_f. Therefore, by increasing the field voltage, the motor torque can be increased while the motor speed is decreased, so that the output power will remain more or less constant in field control. Field control is particularly suitable for constant-power drives under varying torque-speed conditions, e.g., tape-transport mechanisms.

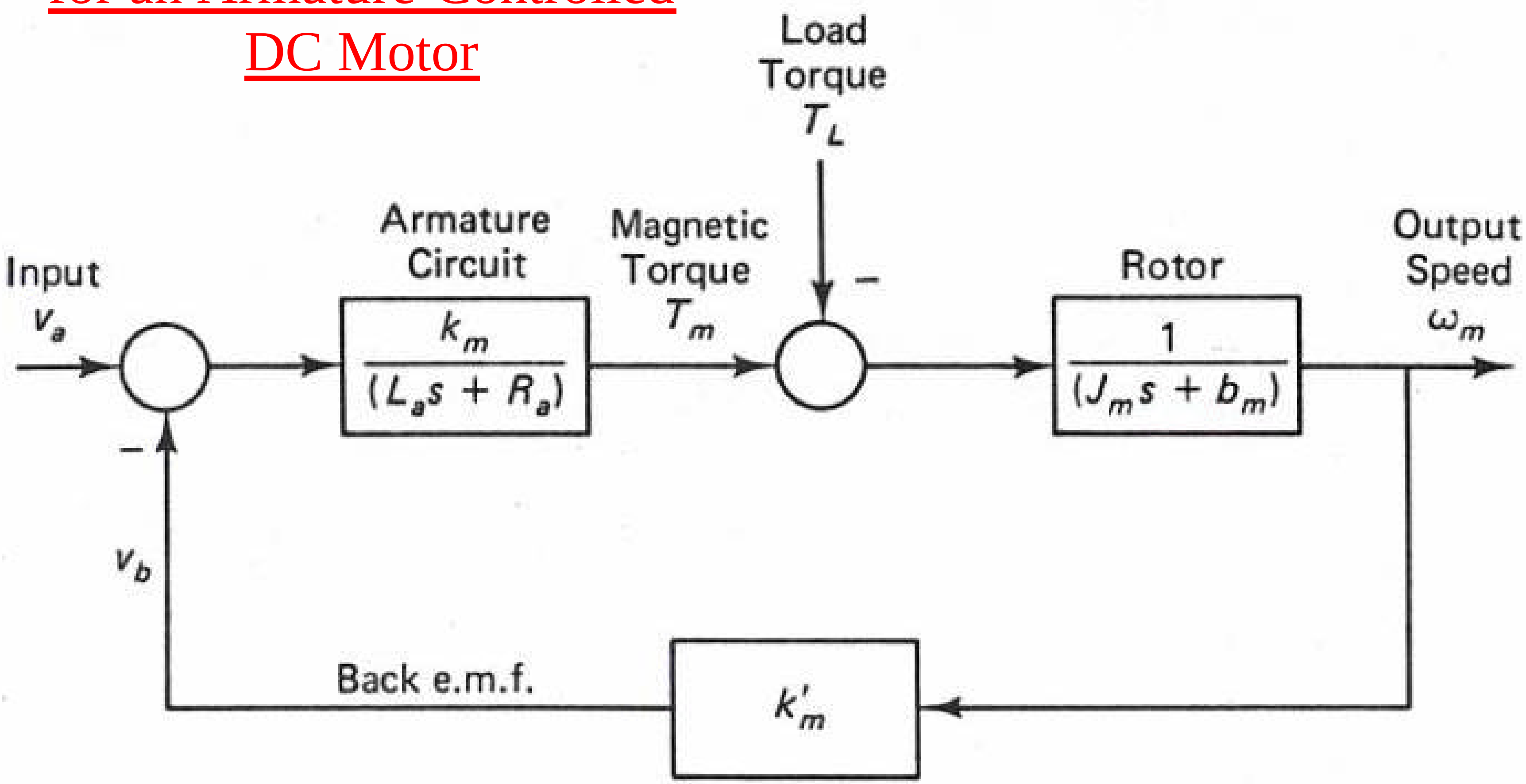
- DC Servomotors
  - DC servo-systems normally employ both velocity feedback and position feedback for accurate position control and also for accurate velocity control. Motion control requires indirect control of motor torque. In applications where torque itself is a primary output (e.g., metal-forming operations, tactile operations) and in situations where small motion errors could produce large unwanted forces (e.g., in parts assembly), direct control of motor torque would be necessary. For precise torque control, direct measurement of torque (e.g., strain gage sensors) would be required.

Schematic Representation of a DC Servo-System

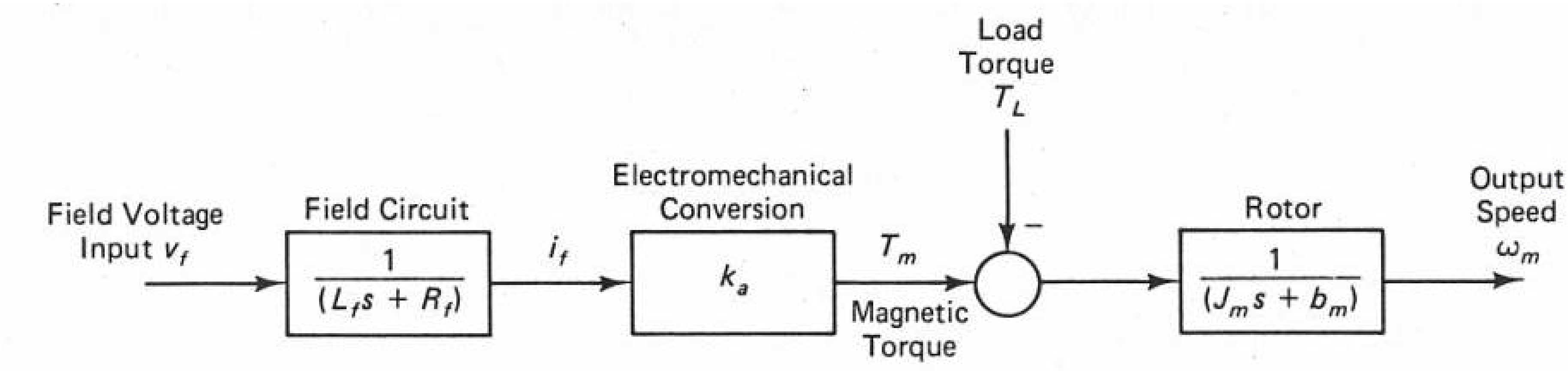


Note: power supplies are not shown

Open-Loop Block Diagram for an Armature-Controlled DC Motor

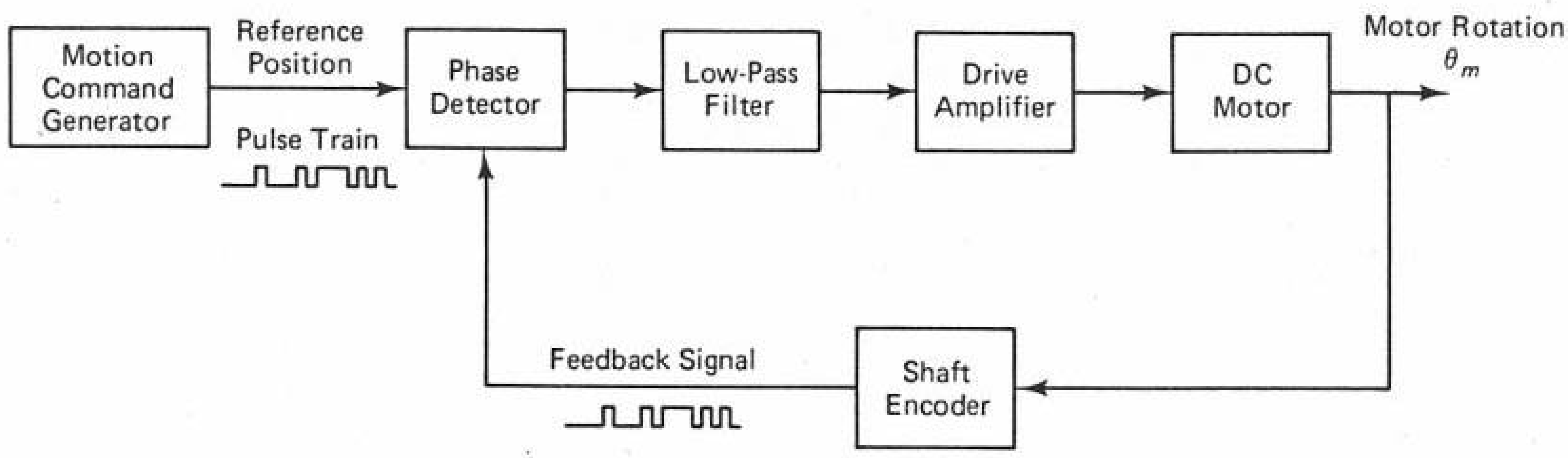


Open-Loop Block Diagram for a Field-Controlled DC Motor



- Phase-Locked Control
  - Phase-locked control is a modern approach to controlling DC motors.
  - This is a phase-control method. The objective is to maintain a fixed phase difference (ideally zero) between the reference signal and the position signal. Under these conditions, the two signals are phase-locked. Any deviation from the locked conditions will generate an error signal that will bring the motor motion back in phase with the reference command. In this manner, deviations due to external load changes on the motor are also corrected.
  - How do you determine the phase difference between two pulse signals? One method is by detecting the edge transitions. An alternative method is to take the product of the two signals and then low-pass filter the result.

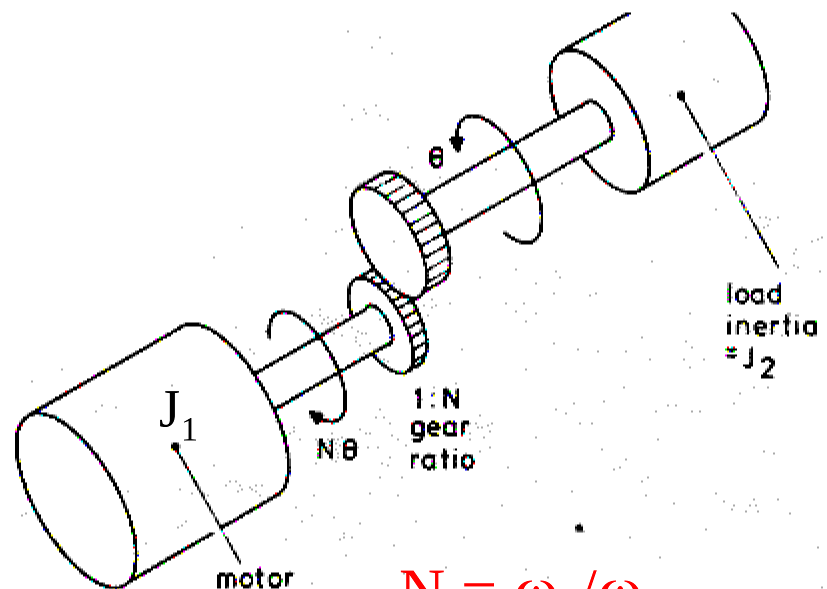
Schematic Diagram of a Phase-Locked Servo



Geared Systems, Optimum Gear Ratios, and Motor Selection

- Servomechanisms may be direct drive (motor coupled directly to load) or geared systems.
- For geared systems, choice of the motor also involves a choice of gear ratio.

Neglect backlash and elasticity in either gear teeth or shafts



$N = \omega_1/\omega_2$

- If we neglect frictional and other load effects, then the equation of motion for this system in terms of ω_2 is given by:

$$\left(J_2 + N^2 J_1\right)\frac{d\omega_2}{dt} = NT_m$$

- T_m is the electromagnetic torque from the motor.
- An important consequence of this result is that motor inertia is often the dominant inertia in a servo system. Consider the following numerical example:
  - Motor rotor inertia $J_1 = 1$, load inertia $J_2 = 100$, and gear ratio $N = 100$.
  - Physically the load appears 100 times "larger" than the motor, but because of the high (but not unusual) gear ratio, the motor's inertia effect is N^2J_1 or 10,000, i.e., 100 times larger than the load.
  - Thus measures to "lighten" the load inertia are misplaced; we should really be striving for a lower inertia motor.

- When frictional and other load effects are negligible and inertia is dominant, an optimum gear ratio that maximizes load shaft acceleration for a given input torque exists and may be found as follows:

$$\left(J_2 + N^2 J_1\right)\frac{d\omega_2}{dt} = NT_m$$

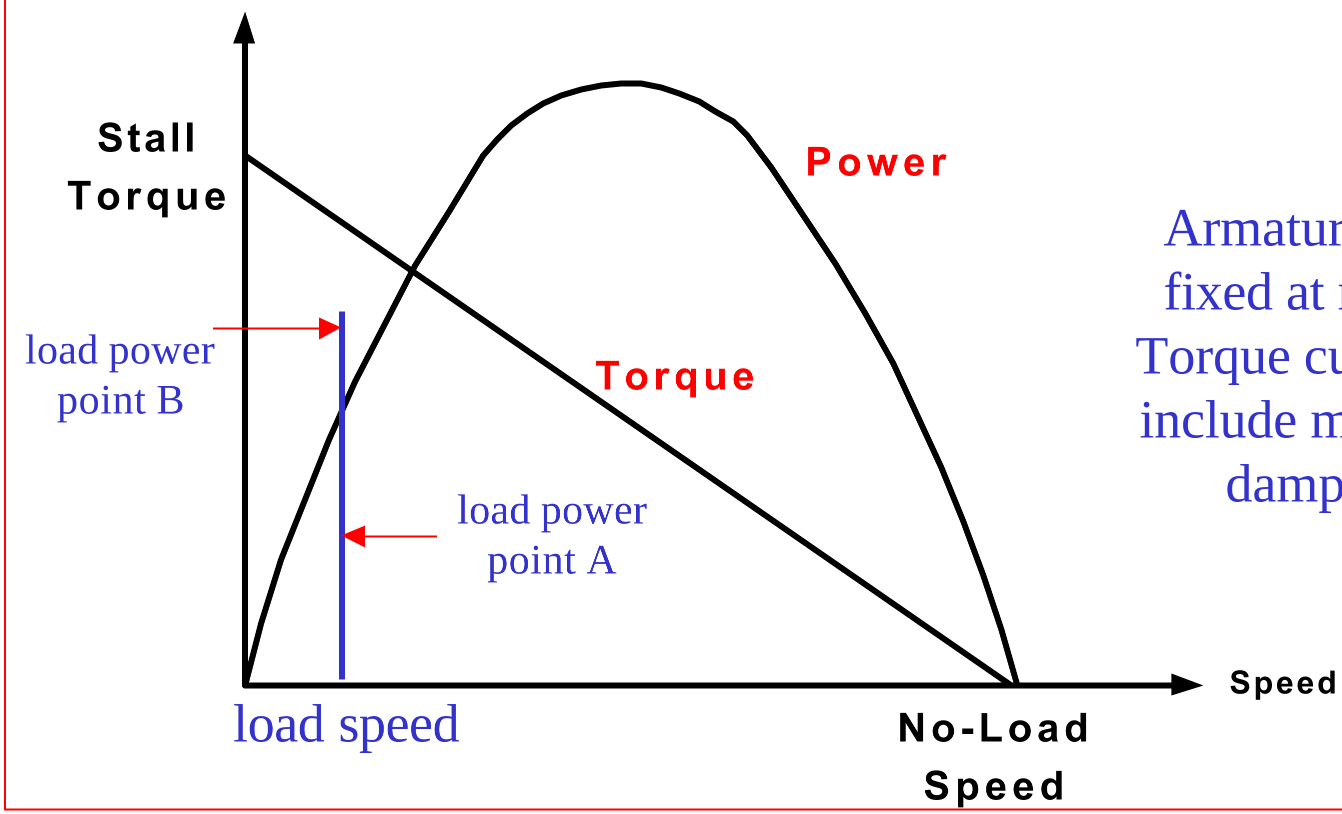
$$\frac{d\omega_2}{dt} = \frac{NT_m}{J_2 + N^2 J_1}$$

$$\frac{d}{dN}\left(\frac{d\omega_2}{dt}\right) = \frac{\left(J_2 + N^2 J_1\right)T_m - NT_m\left(2NJ_1\right)}{\left(J_2 + N^2 J_1\right)^2} = 0$$

$$N_{opt} = \sqrt{\frac{J_2}{J_1}}$$

Referred inertia of the motor rotor is equal to the actual load inertia

- When friction, load, and acceleration torques are all significant, gear ratio selection is less straightforward.
- Consider the linear speed/torque curve (typical of an armature-controlled dc motor with fixed field). Power is the product of speed and torque, hence the parabolic speed/power curve.



Armature voltage is fixed at rated value. Torque curve does not include motor viscous damping loss.

- One possible design criterion requires that we operate the motor at the maximum power point (which occurs at one half the no-load speed) when it is supplying the greatest demand of the load.
- One could compute the maximum power demand at maximum velocity; however, it is unlikely that this maximum speed will coincide with the peak power speed of the motor; thus a direct drive would not satisfy our requirement to operate at the peak power point.
- The figure shows two load power points at the same maximum load speed.
- If maximum load power corresponded to point A, a direct drive would be possible since the motor has more power than needed at this speed. The excess power simply means that more acceleration is available than is required.

- If instead our calculations had given point B, direct drive would not be possible since the load requires more power than the motor can supply at that speed. However, this load is less than the peak motor power, so suitable gearing can reconcile the supply/demand mismatch.
- Since one finds both direct-drive and geared systems in practical use, it is clear that the one scheme is not always preferable to the other.
- Direct drive is favored because of reduced backlash and compliance and longer motor life due to lower speed.
- The advantages of geared drives include smoother motor operation at higher speeds, possibly higher torsional natural frequency, and the lower cost of smaller motors. Of course, if a sufficiently large motor is simply not available, then a geared drive is a necessity.

Motor Selection Considerations

- Torque and speed are the two primary considerations in choosing a motor for a particular application. Motor manufacturers' data that are usually available to users include the following specifications:
- <u>Mechanical Specifications</u>
  - Mechanical time constant
  - Speed at rated load
  - Rated torque
  - Frictional torque
  - Dimensions and weight
  - No-load speed

- No-load acceleration
- Rated output power
- Damping constant
- Armature moment of inertia

- <u>Electrical Specifications</u>
  - Electrical time constant
  - Armature resistance and inductance
  - Compatible drive circuit specifications (voltage, current, etc.)
  - Input power
  - Field resistance and inductance

- <u>General Specifications</u>
  - Brush life and motor life

- Heat transfer characteristics
- Coupling methods
- Efficiency
- Mounting configuration
- Operating temperature and other environmental conditions

- It should be emphasized that there is no infallible guide to selecting the best motor. There are always several workable configurations. Constraints (e.g., space, positioning resolution) can often eliminate several designs.
- The engineer must make the motor drive system work both electrically and mechanically. Look at the motor-to-load interface before looking at the electrical drive-to-motor interface.
- In a typical motion control application the requirement will be to overcome some load frictional force and move a mass through a certain distance in a specified time.

- The designer should weigh the following:
  - Moment of Inertia
  - Torque
  - Power
  - Cost
- <u>Load Inertia</u>
  - For optimum system performance, the load moment of inertia should be similar to the motor inertia.
  - When gear reducers intervene between motor and load, the reflected load inertia is J_L/N^2, where N is the gear ratio.
  - If the motor inertia J_M is equal to the reflected load inertia, the fastest load acceleration will be achieved, or conversely, the torque to obtain a given acceleration will be minimized.

- Therefore, matched inertias are best for fast positioning.
- Peak power requirements are minimized by selecting the motor inertia so that the reflected load inertia is 2.5 times as large as the motor inertia. The torque will be increased but the maximum speed will be further reduced. A load inertia greater than 2.5 times the motor inertia is less than ideal, but it should not present any problems if the ratio is less than 5. A larger motor inertia implies that the same performance can be achieved at a lower cost by selecting a smaller motor.
- There is a wide range of motor inertias on the market today. Overlap is extensive. An engineer can virtually consider any type of motor including brushless and stepper, at the first stage of the design-inertia match.

- <u>Torque</u>
  - The motor must supply sufficient torque T_m to overcome the load friction and to accelerate the load over a distance (radians) *s* in time *t*.
  - Torque and acceleration at the motor are given by:

$$\left.\begin{aligned} \alpha_m &= N\alpha_L \\ T_L &= T_f + J_L\alpha_L \end{aligned}\right\} T_m = \frac{T_L}{N} + J_m\alpha_m \Rightarrow T_m = \frac{1}{N}\left[T_f + \alpha_L\left(J_L + N^2 J_m\right)\right]$$

  - For linear acceleration over distance *s* in time *t* :

$$\alpha_L = \frac{2s}{\tau^2}$$

  - For a damped ($\zeta = 0.7$) second-order response over distance *s* :

$$\left(\alpha_L\right)_{max} = \omega_n^2 s$$

- Allowances should always be made for variations in load and bearing behavior as well as motor production variations.
- An initial design should be planned without a gear reducer. In many cases direct drive is not possible because load torque requirements far exceed the torque delivered by a motor of reasonable size.
- Critical needs on space or weight can lead to gear reducers for otherwise perfectly matched motor/load systems.
- The problem with gear reducers is gear backlash. If gears mesh too tightly, there is severe sliding friction between teeth which can cause lockup. Thus, the teeth spacing, backlash, is a tradeoff between reducing the power loss within the gears (loose fit) or improving position accuracy (tight fit) of the load.

- Power
  - Besides maximum torque requirements, torque must be delivered over the load speed range. The product of torque and speed is power. Total power P is the sum of the power to overcome friction P_f and the power to accelerate the load P_a, the latter usually the dominant component:

$$P = P_f + P_a = T_f \omega + J\alpha\omega$$

  - Peak power required during acceleration depends upon the velocity profile. If the load is linearly accelerated over distance s in time t, the maximum power is:

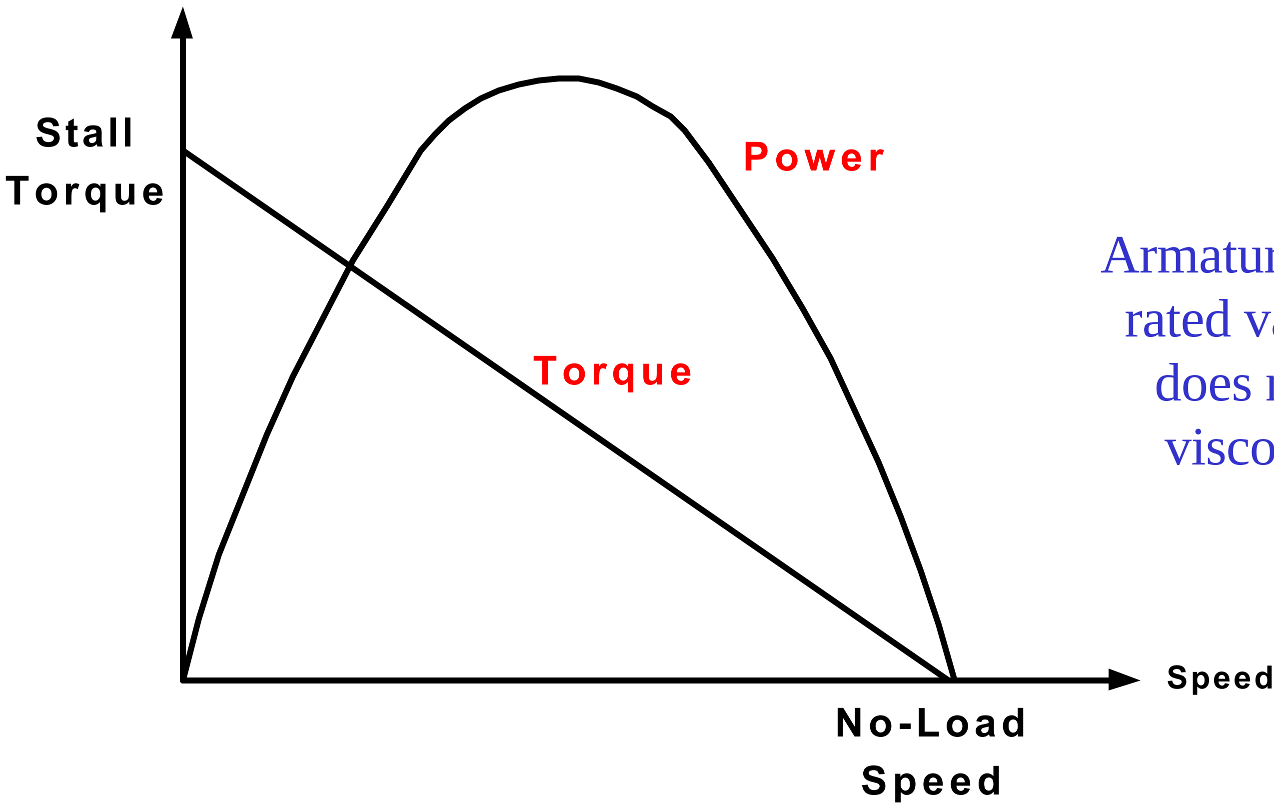
$$\left(P_a\right)_{max} = \frac{4Js^2}{\tau^3}$$

– If the load undergoes a damped ($\zeta = 0.7$) second-order response over distance *s*, the maximum power is:

$$\left(P_a\right)_{max} = \left(0.146\right) J\omega_n^3 s^2$$

– It is interesting to note that to accelerate a load in one half the time will require eight times the power.

– The torque-speed curve for permanent-magnet DC motors is a linear line from stall torque to no-load speed. Therefore, the maximum power produced by the motor is the curve midpoint or one-fourth the stall torque and maximum speed product.



Armature voltage is fixed at rated value. Torque curve does not include motor viscous damping loss.

- The maximum speed for a permanent-magnet DC motor is 10,000 RPM, while for a brushless DC motor it is > 20,000 RPM.
- As a starting point, choose a motor with double the calculated power requirement.

- Cost
  - Among several designs the single most important criterion is cost.
  - Although it may be more prudent to choose the first workable design when only several units are involved, high-volume applications demand careful study of the economic tradeoffs.

- For example, permanent-magnet DC motors operate closed loop and the cost of an encoder can equal if not exceed the motor cost. In addition, stepper and brushless motors have electronic expenses greater than brushed motor electronic expenses.