Induction Machines

Construction

1. Stator - Three Phase Winding
2. Rotor
   2.1 Squirrel Cage
   2.2 Phase Wound (Slip Ring)

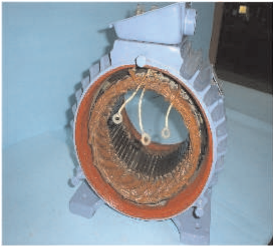
Both windings carry alternating currents. The alternating current is supplied to the stator winding and to the rotor winding by *induction*.

- The induction machine can operate both as a motor and as a generator.
- As a generator, its performance is not satisfactory. However it is used in wind turbines.
- It is extensively used as a motor.
- It is also called as an *asynchronous machine*.

Construction

Stator

- It is a distributed winding.
- The windings of each phase are distributed over several slots.
- When a poly phase current flows through it, it produces a *revolving mmf*.



Rotor

Squirrel cage rotor

- Aluminium or copper bars are inserted into the rotor slots.
- These bars are shorted by rings at both the ends.
- The rotor behaves like a short circuited winding.
- It is simple and robust.

Slip ring rotor (wound-rotor type)

- It has the same form as the stator winding. .
- The terminals of the rotor winding are connected to three slip rings.
- Using stationary brushes, they can be connected to an external circuit.

Three Phase Induction Machine

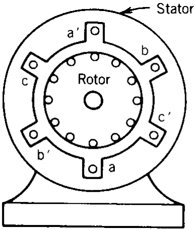
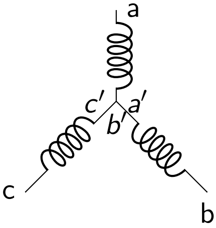
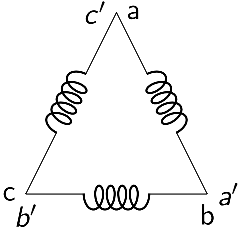


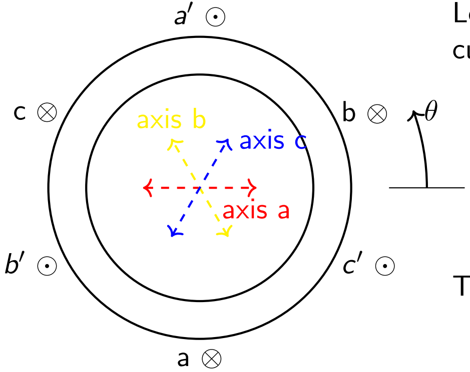
Figure: Three-phase Squirrel Cage Induction Machine

Stator Winding can be connected as





Rotating Magnetic Field



Let a balanced three pahse current flow through the winding.

$$i_a = I_m \sin \omega t$$

$$i_b = I_m \sin(\omega t - 120^\circ)$$

$$i_c = I_m \sin(\omega t - 240^\circ)$$

The total mmf along θ is

$$F(\theta) = F_a(\theta) + F_b(\theta) + F_c(\theta)$$

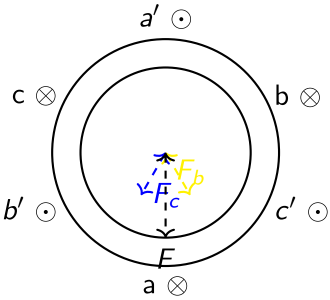
Graphical Method

At time instant $\omega t = 0$,

$$i_a = 0 \quad F_a = 0$$

$$i_b = -\frac{\sqrt{3}}{2} I_m \quad F_b = -\frac{\sqrt{3}}{2} F_m$$

$$i_c = +\frac{\sqrt{3}}{2} I_m \quad F_c = \frac{\sqrt{3}}{2} F_m$$

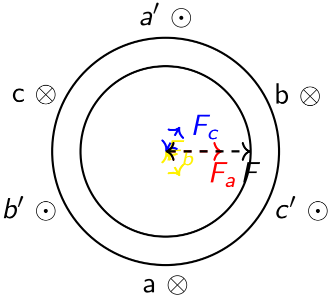


At time instant $\omega t = \frac{\pi}{2}$,

$$i_a = I_m \quad F_a = F_m$$

$$i_b = -\frac{1}{2} I_m \quad F_b = -\frac{1}{2} F_m$$

$$i_c = -\frac{1}{2} I_m \quad F_c = -\frac{1}{2} F_m$$

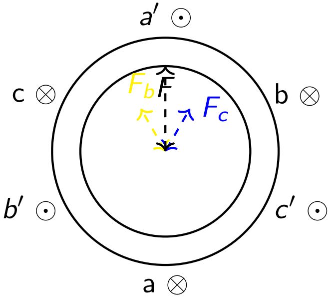


At time instant $\omega t = \pi$,

$$i_a = 0 \quad F_a = 0$$

$$i_b = \frac{\sqrt{3}}{2} I_m \quad F_b = \frac{\sqrt{3}}{2} F_m$$

$$i_c = -\frac{\sqrt{3}}{2} I_m \quad F_c = -\frac{\sqrt{3}}{2} F_m$$

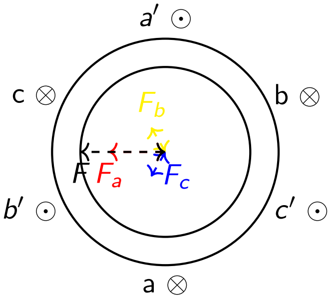


At time instant $\omega t = \frac{3\pi}{2}$,

$$i_a = -I_m \quad F_a = -F_m$$

$$i_b = \frac{1}{2} I_m \quad F_b = \frac{1}{2} F_m$$

$$i_c = \frac{1}{2} I_m \quad F_c = \frac{1}{2} F_m$$

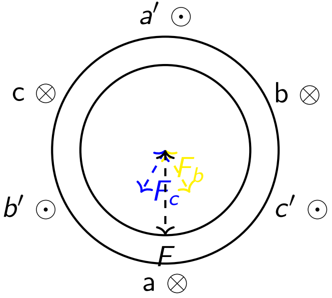


At time instant $\omega t = 2\pi$,

$$i_a = 0 \quad F_a = 0$$

$$i_b = -\frac{\sqrt{3}}{2} I_m \quad F_b = -\frac{\sqrt{3}}{2} F_m$$

$$i_c = +\frac{\sqrt{3}}{2} I_m \quad F_c = \frac{\sqrt{3}}{2} F_m$$



- The resultant mmf wave makes 1 revolution per cycle of the current in a two pole machine.
- In a P pole machine, it makes $\dfrac{2}{P}$ revolutions for a cycle.
- For a frquency of f cycles per second,

$$N = \frac{2f}{P} 60 = \frac{120f}{P} \text{ revolutions per minute.}$$

It is called as *synchronous speed*.

Analytical Method

$$F_a(\theta) = Ni_a \cos\theta$$
$$F_b(\theta) = Ni_b \cos(\theta - 120^\circ)$$
$$F_c(\theta) = Ni_c \cos(\theta - 240^\circ)$$

The net mmf

$$F(\theta) = Ni_a \cos\theta + Ni_b \cos(\theta - 120^\circ) + Ni_c \cos(\theta - 240^\circ)$$

Since the currents are functions of time,

$$\begin{aligned} F(\theta, t) = NI_m \cos\omega t \cos\theta + NI_m \cos(\omega t - 120^\circ) \cos(\theta - 120^\circ) \\ + NI_m \cos(\omega t - 240^\circ) \cos(\theta - 240^\circ) \end{aligned}$$

$$\begin{aligned} F(\theta, t) = & \frac{1}{2} NI_m \cos(\omega t - \theta) + \frac{1}{2} NI_m \cos(\omega t + \theta) \\ & \frac{1}{2} NI_m \cos(\omega t - \theta) + \frac{1}{2} NI_m \cos(\omega t + \theta - 240^\circ) \\ & \frac{1}{2} NI_m \cos(\omega t - \theta) + \frac{1}{2} NI_m \cos(\omega t + \theta + 240^\circ) \end{aligned}$$

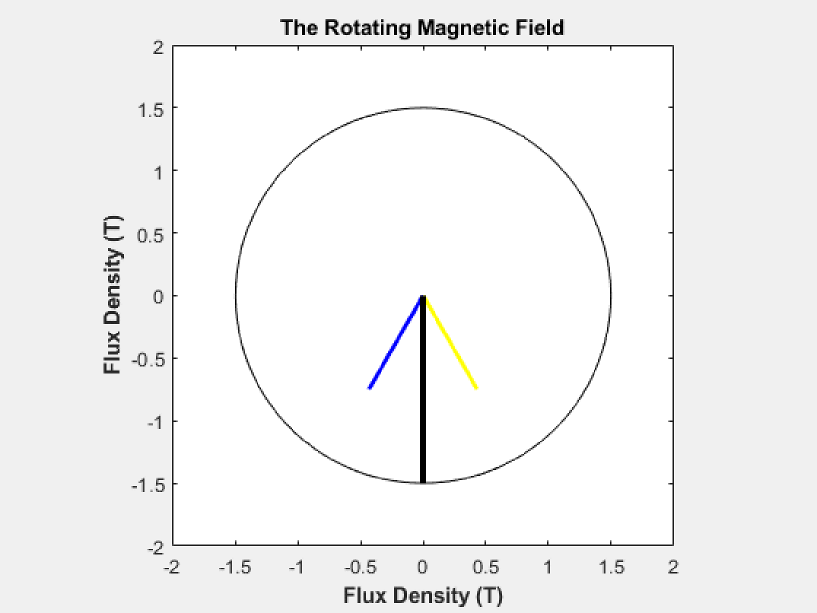
$$F(\theta, t) = \frac{3}{2} N I_m \cos(\omega t - \theta)$$

The angular velocity of the resultant mmf wave is $\omega = 2\pi f$ radians per second. It's rpm for a P pole machine is

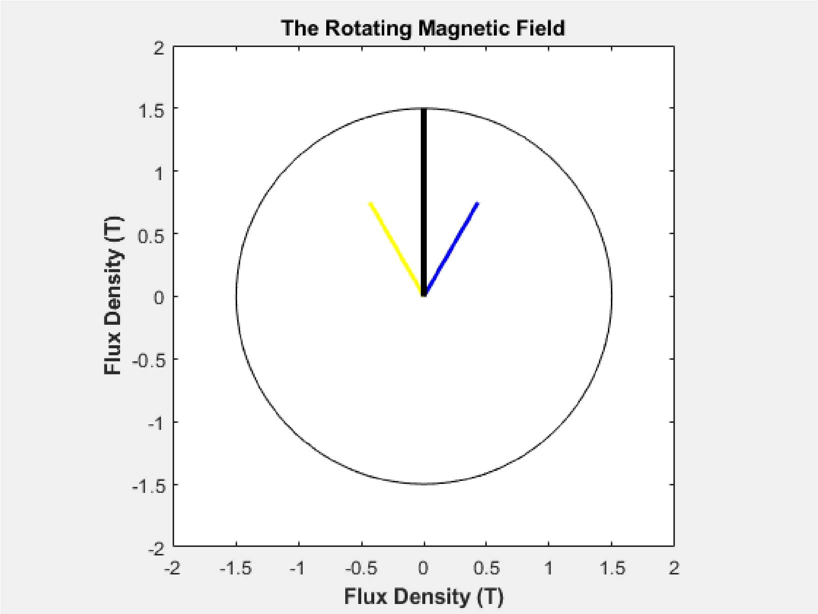
$$N_s = \frac{120f}{P}$$

- When a three phase balanced current flows through a three phase winding placed 120° (electrical degree) apart in space, a rotating magnetic field will be produced.
- The direction can be reversed by interchanging any two phases.
- In general, when a n phase balanced current flows through a n phase winding placed $\dfrac{360^\circ}{n}$ (electrical degree) apart in space, a rotating magnetic field will be produced.
- This is analogous to moving a field which is stationary with respect to space.

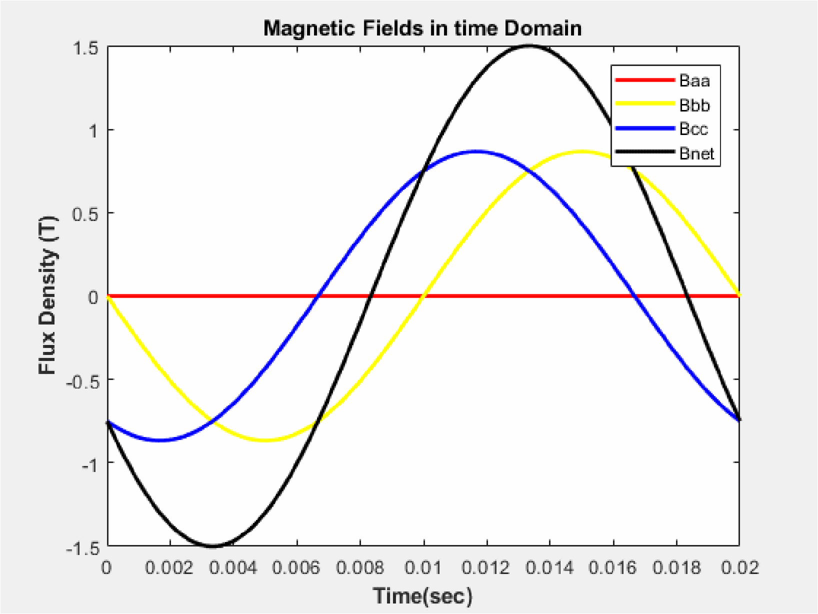
Rotating Magnetic Field (RMF) - Animation



RMF (Reversal) - Animation



RMF(Time Domain) - Animation



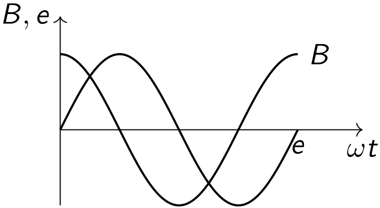
Induced Voltages

Since the flux pattern is varying sinusoidally in space as the flux wave form rotates, the flux linkage varies sinusoidally.
The flux linkage with stator coil *a* is

$$\lambda_a = N_{\mathrm{ph}}\phi_p \cos\omega t$$

By Faraday's law[1], the voltage induced in phase *a* is

$$e_a = -\frac{d\lambda_a}{dt} = \omega N_{\mathrm{ph}}\phi_p \sin\omega t$$



[1] In transformers, +ve induced emf circulates a current such that the field produced by it opposes the mutual flux.

The voltages induced in other phases are

$$e_b = \omega N_{\text{ph}} \phi_p \sin(\omega t - 120^\circ)$$

$$e_c = \omega N_{\text{ph}} \phi_p \sin(\omega t - 240^\circ)$$

The RMS voltage per phase is

$$E_{\text{rms}} = \sqrt{2} \pi f N_{\text{ph}} \phi_p$$

$$E_{\text{rms}} = 4.44 f N_{\text{ph}} \phi_p$$

The rotating field also induces a three phase voltage in the rotor winding.

Torque Production

- When a balanced three phase current flows through the stator winding, a rotating magnetic field is produced and rotates at synchronous speed in the air-gap.
- The field induces three phase balanced voltages in both stator and rotor windings.
- The closed rotor windings circulate a three phase balanced current. In slip ring induction motor, the rotor terminals have to be either short circuited or connected to an external circuit.
- The interaction of field due to rotor currents and the air-gap field produces torque.
- According to Lenz's law, the rotor rotates in the direction of rotating field such that the relative speed between the rotating field and the rotor decreases.
- The rotor will reach a steady speed N that is less than the synchronous speed N_s.
- At $N = N_s$, there will be no relative speed, induced voltage and current in the rotor and hence no torque.

The difference between the rotor speed N_r and the synchronous speed N_s is called the *slip* and is defined as

$$s = \frac{N_s - N_r}{N_s}$$

The relative speed between the rotating field and the rotor is the cause for induced voltages and currents in the rotor.

$$f_r = \frac{P(N_s - N_r)}{120} = \frac{sPN_s}{120} = sf_s$$

where, f_r is the frequency of rotor voltages and current and f_s is the supply (stator) frequency.
The voltage induced in the rotor at slip s is

$$E_{rs} = sE_r$$

where E_r is the voltage induced in the rotor at standstill.

Three phase balanced currents in the rotor will produce a revolving field.
Its speed with respect to the rotor is

$$N_{rf} = \frac{120 f_r}{P} = \frac{120 s f_s}{P} = s N_s$$

The speed of the rotor field with respect to the stator is

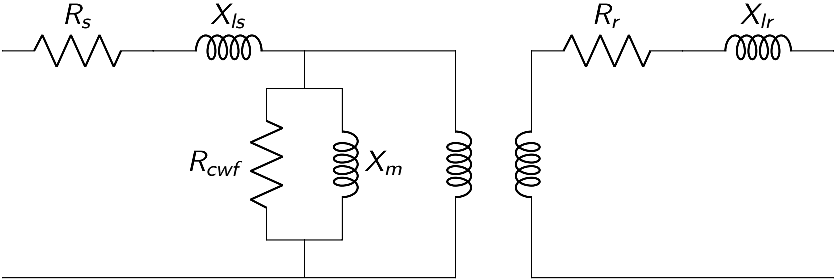
$$= N_r + N_{rf} = (1 - s) N_s + s N_s = N_s$$

The two fields are stationary with respect to each other.

Equivalent Circuit

Since the operation is balanced, per-phase equivalent circuit is sufficient.

When the rotor is open circuited, it behaves exactly like a transformer. The equivalent circuit under this condition is



R_s = per phase stator resistance

R_{cwf} = core loss plus friction and windage loss resistance

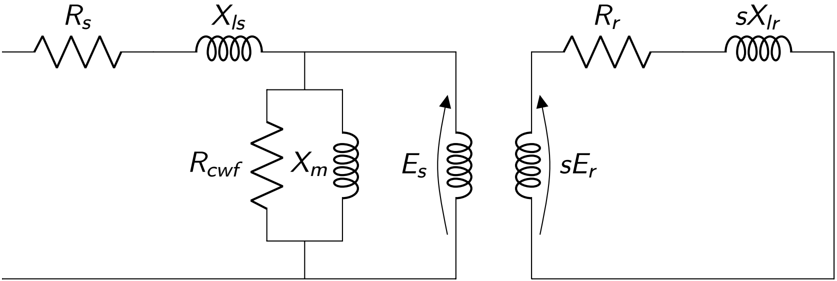
R_s = per phase rotor resistance

X_{ls} = per phase stator leakage reactance

X_m = magnetizing reactance

X_{ls} = per phase rotor leakage reactance

When the rotor is short circuited,

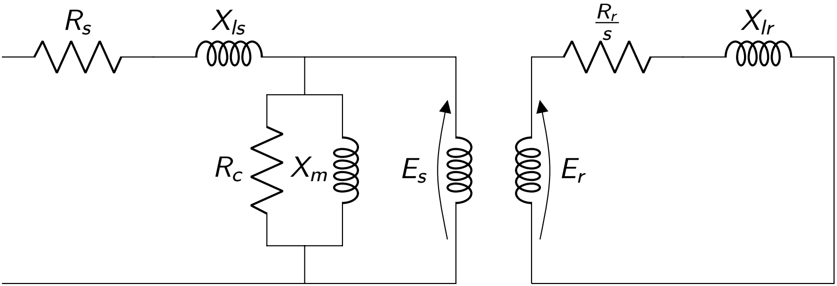


Stator and Rotor sides are at different frequencies.

$$I_r = \frac{sE_r}{R_r + \jmath s X_{lr}}$$

$$I_r = \frac{E_r}{\frac{R_r}{s} + \jmath X_{lr}}$$

This shows that the current in the rotor is at stator frequency.



Since the two sides are at the same frequency, the rotor side quantities can be referrred to the stator side.

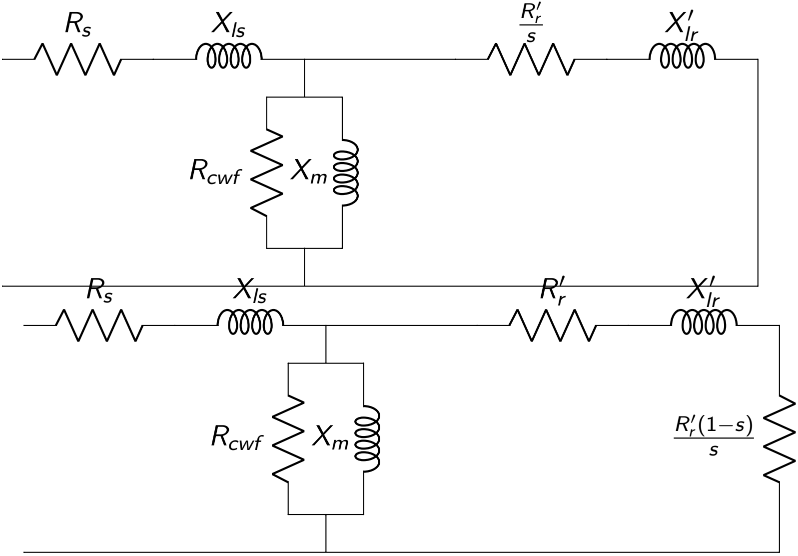


Figure: Exact Equivalent Circuit

$$\frac{R_r'}{s} = \text{air gap power component}$$
$$R_r' = \text{rotor copper loss component}$$
$$\frac{R_r'(1-s)}{s} = \text{mechanical power output component}$$

By moving the shunt path near the supply,

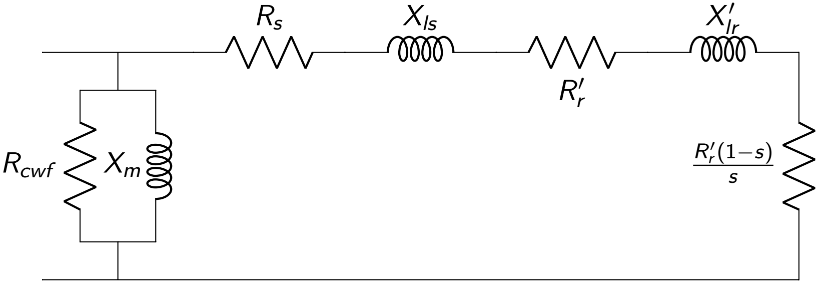


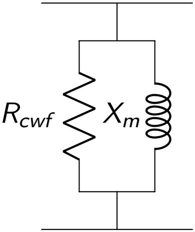
Figure: Approximate Equivalent Circuit

Determination of Circuit Parameters

For simplicity, let us the approximate equivalent circuit.
(Because of air-gap and high leakage reactance, this circuit gives less accurate results. Still we use it !)

No Load Test

At no load, $N_r \approx N_s$.Hence $s \approx 0$.



Let

$$V_{\text{NL}} = \text{stator voltage (Line - Line)}$$
$$I_{NL} = \text{no load stator line current}$$
$$P_{NL} = \text{no load input power (3 Phase)}$$

Unlike transformers, induction machines draw 30 to 50 % of full load current at no load (because of air-gap).
P_{NL} represents not only stator copper losses and stator core losses but also friction and windage losses. We have to subtract stator copper losses from it to get the rotational losses.
Assume the machine is star connected.

$$P_{\text{rot}} = P_{NL} - 3 \times I_{NL}^2 R_s$$

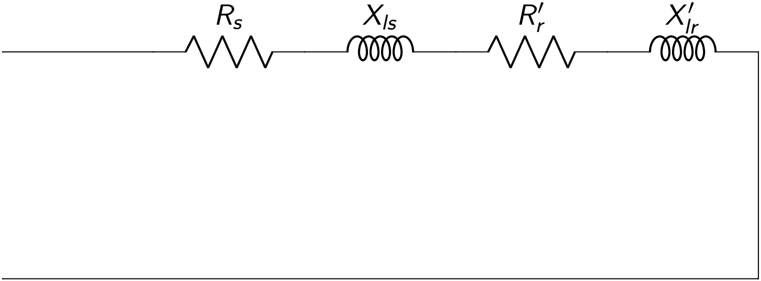
$$R_{cwf} = \frac{(V_{\text{NL}}/\sqrt{3})^2}{P_{\text{rot}}/3}$$

$$Z_{\text{NL}} = \frac{V_{\text{NL}}/\sqrt{3}}{I_{\text{NL}}}$$

$$X_m = \frac{1}{\sqrt{(\frac{1}{Z_{\text{NL}}})^2 - (\frac{1}{R_{cwf}})^2}}$$

Blocked Rotor Test

In this test, the rotor is prevented from rotation by mechanical means. $N_r = 0$ and $s = 1$.



Let

$$\begin{aligned} V_{\text{BR}} &= \text{applied stator voltage (Line - Line)} \\ I_{BR} &= \text{rated stator line current} \\ P_{BR} &= \text{input power during blocked rotor (3 Phase)} \end{aligned}$$

The voltage which is necessary to cause the rated current to flow in the stator is applied under this test.
Assume the machine is star connected.

$$(R_s + R_r') = \frac{P_{BR}/3}{I_{BR}^2}$$

If R_s is known, R_r' can be found.

$$(X_{ls} + X_{lr}') = \sqrt{\left(\frac{V_{BR}/\sqrt{3}}{I_{BR}}\right)^2 - (R_s + R_r')^2}$$

If the relationship between X_{ls} and X_{lr}' is given, they can be found.
Otherwise assume that $X_{ls} = X_{lr}'$.

Approximate Equivalent Circuit

Since the rotational loss consists of stator core losses and friction and windage losses, R_{cwf} can be removed from the approximate equivalent circuit.

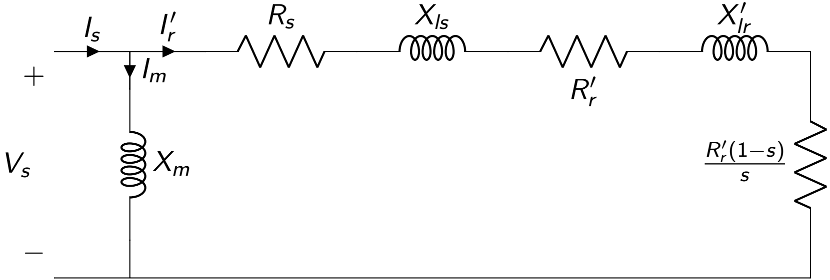


Figure: Approximate Equivalent Circuit

Example 1 (Kothari Example 9.4) :
A 400 V, 6-pole, 3-phase, 50 Hz, star connected induction motor gives the following measurements:

$$V_{NL} = 400; \mathrm{V} \quad I_{NL} = 7.5 \text{ A} \quad P_{NL} = 700 \text{ W}$$

$$V_{BR} = 150; \mathrm{V} \quad I_{BR} = 35 \text{ A} \quad P_{BR} = 4000 \text{ W}$$

R_s per phase is 0.55 Ω. $\dfrac{X_{ls}}{X'_{lr}} = 2.$ When the motor is operating at a slip of 4 %, calculate

1. stator current
2. power factor
3. net mechanical power output and torque
4. efficiency

From NL Test:

$$P_{\text{rot}} = 700 - 3 \times 7.5^2 \times 0.55 = 607 \text{ W}$$

$$R_{cwf} = \frac{(400/\sqrt{3})^2}{607/3} = 264\ \Omega$$

$$Z_{NL} = \frac{400/\sqrt{3}}{7.5} = 30.8\ \Omega$$

$$X_m = \frac{1}{\sqrt{(\frac{1}{30.8})^2 - (\frac{1}{264})^2}} = 31\ \Omega$$

From BR Test:

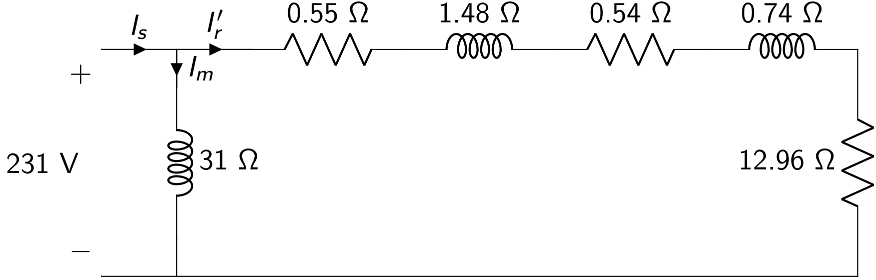
$$(R_s + R_r') = \frac{4000/3}{35^2} = 1.09\ \Omega$$

$$R_s = 0.55\ \Omega \quad R_r' = 0.54\ \Omega$$

$$(X_{ls} + X_{lr}') = \sqrt{(\frac{150/\sqrt{3}}{35})^2 - (1.09)^2} = 2.22\ \Omega$$

$$X_{ls} = 1.48\ \Omega \quad X'_{lr} = 0.74\ \Omega$$

For $s = 0.04$,



1. stator current

$$I_s = I'_r + I_m$$

$$I'_r = \frac{231}{14.05 + \jmath 2.22} = 16.24\angle -9^\circ \text{ A}$$

$$I_m = \frac{231}{\jmath 31} = 7.45\angle -90^\circ$$

$$I_s = 18.9\angle - 31.9^\circ \text{ A}$$

2. input power factor

$$pf = \cos 31.9^\circ = 0.85 \text{ lagging}$$

3. net mechanical power output

$$P_m(\text{gross}) = 3(I_r')^2 \frac{R_r'(1-s)}{s} = 3 \times 16.24^2 \times 12.96 = 10.25 \text{ kW}$$

$$P_m(\text{net}) = 10.25 - P_{\text{rot}} = 10.25 - 0.607 = 9.64 \text{ kW}$$

Since $N_r = (1-s)N_s$, $N_r = 960$ rpm.

$$T_m(\text{net}) = \frac{P_m(\text{net})}{\omega_m} = \frac{9.64 \times 10^3}{2 \times \pi 960/60} = 96 \text{ Nm}$$

4. efficiency

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100$$

$$P_{\text{in}} = \sqrt{3} V_s \times I_s \times pf = \sqrt{3} \times 400 \times 18.9 \times 0.85 = 11.13 \text{ kW}$$

$$\eta = \frac{9.64}{11.13} \times 100 = 86.61\%$$

The results are 10-12 % higher than the results obtained from the exact equivalent circuit. It is okay. But calculation is simple...

Torque - Slip Characteristics

From the approximate equivalent circuit,

$$I_r' = \frac{V_s}{\sqrt{(R_s + \frac{R_r'}{s})^2 + (X_{ls} + X_{lr}')^2}}$$

$$T = \frac{P_m}{\omega_m} = \frac{3(I_r')^2 \frac{R_r'(1-s)}{s}}{\omega_m}$$

Since $\omega_m = (1-s)\omega_s$,

$$T = \frac{3V_s^2(R_r'/s)}{\omega_s\left((R_s + \frac{R_r'}{s})^2 + (X_{ls} + X_{lr}')^2\right)}$$

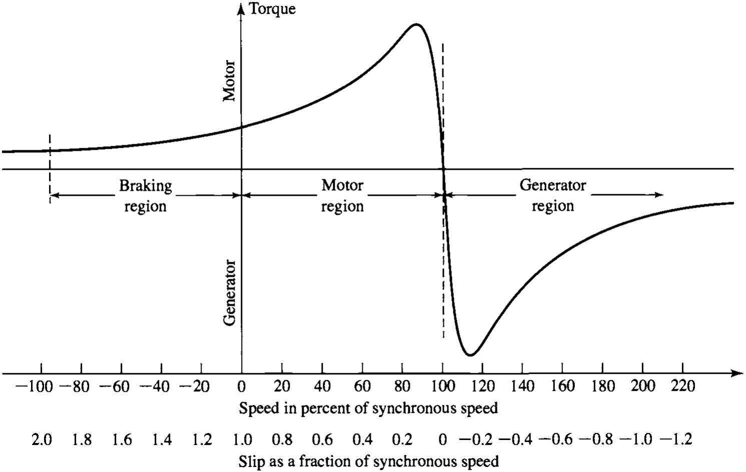
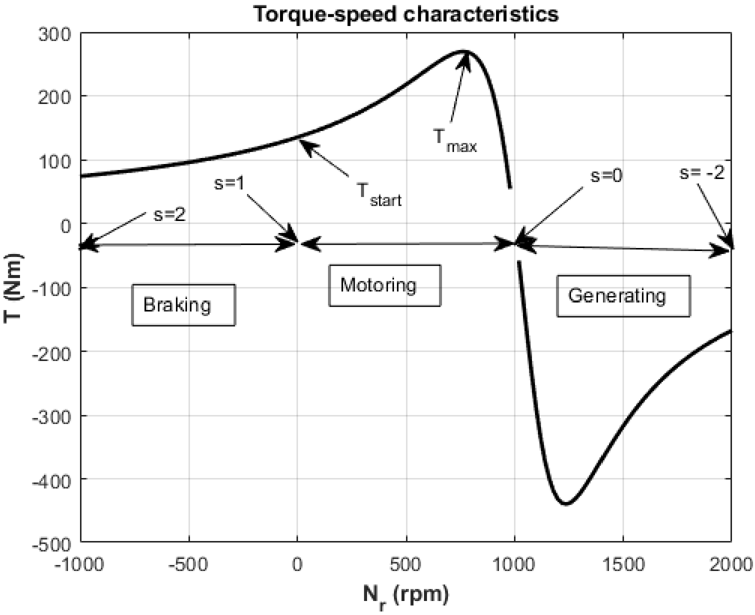


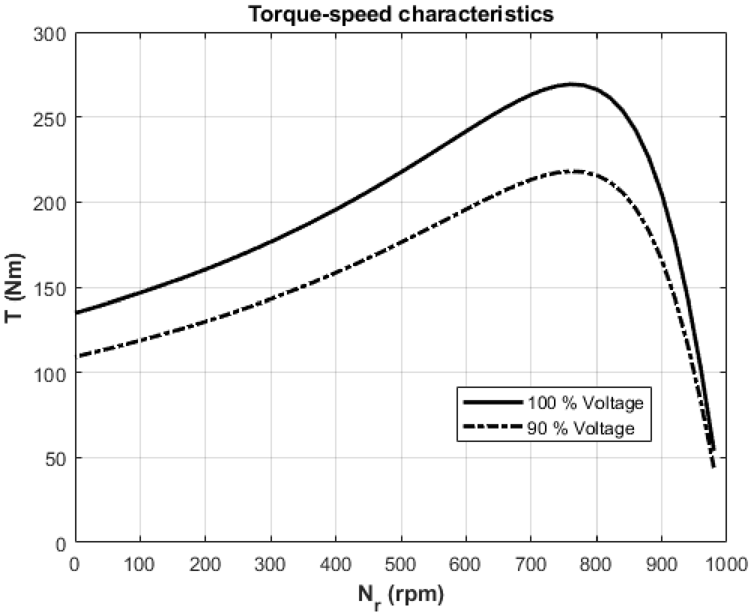
Figure: Complete Torque-slip Characteristics

Source : A. E. Fitzgerald et.al., "Electric Machinery"

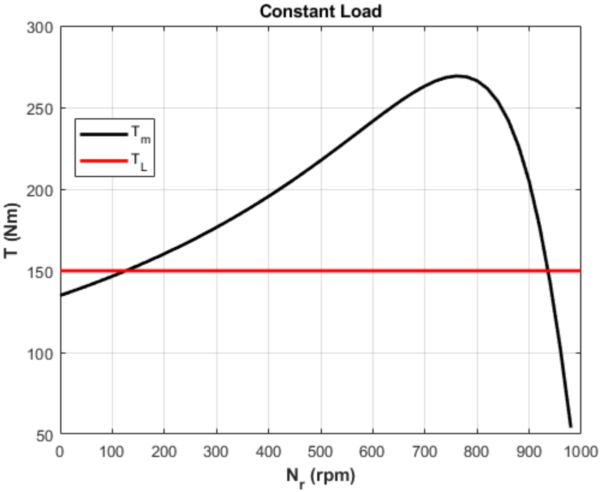
For the same example: $N_s = 1000$ rpm.



For the example 1 with two different input voltages:



Operating Point



The stable operating point is at 950 rpm.

To determine the maximum torque and the slip at which it occurs,

$$\frac{dT}{ds} = 0$$

$$s_{\text{max T}} = \frac{R_r'}{\sqrt{R_s^2 + (X_{ls} + X_{lr}')^2}}$$

Substituting this in the torque equation and simplifying it,

$$T_{\max} = \frac{3}{\omega_s}\left[\frac{0.5 V_s^2}{R_s + \sqrt{R_s^2 + (X_{ls} + X_{lr}')^2}}\right]$$

- The slip at which the maximum torque occurs depends on the rotor resistance.
- However, the maximum torque is independent of the rotor resistance.

Let us simplify by neglecting stator impedance.
$R_s = 0$ and $X_{ls} = 0$.

$$\boxed{T = \frac{3V_s^2(R_r'/s)}{\omega_s\left((\frac{R_r'}{s})^2 + (X_{lr}')^2\right)}}$$

$$s_{\text{max T}} = \frac{R_r'}{X_{lr}'} = \frac{R_r}{X_{lr}}$$

$$T_{\text{max}} = \frac{3V_s^2}{2\omega_s X_{lr}}$$

At starting, $s = 1$,

$$T_{\text{start}} = \frac{3V_s^2 R_r'}{\omega_s((R_r')^2 + (X_{lr}')^2)}$$

To get the maximum torque at starting.

$$s_{\text{max T}} = 1$$

$$R_r = X_{lr}$$

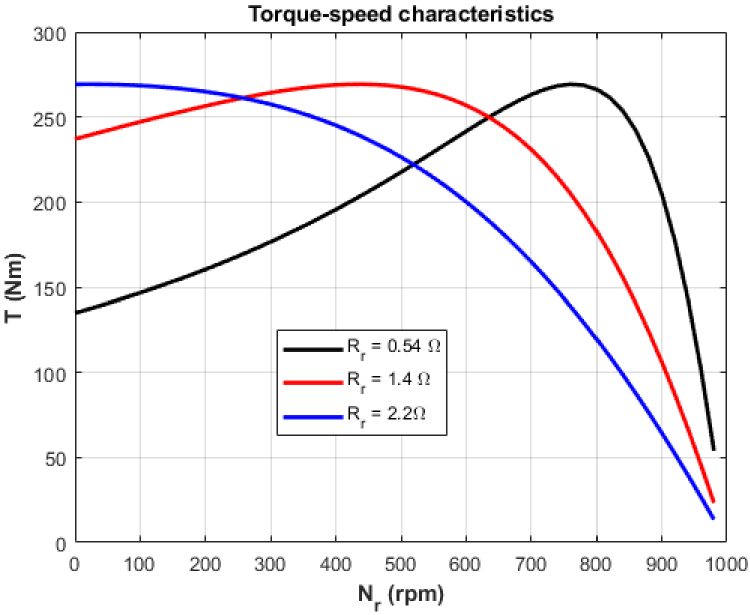
Slip Ring Induction Machine

In slip ring induction machines, by adding external resistance to the rotor circuit

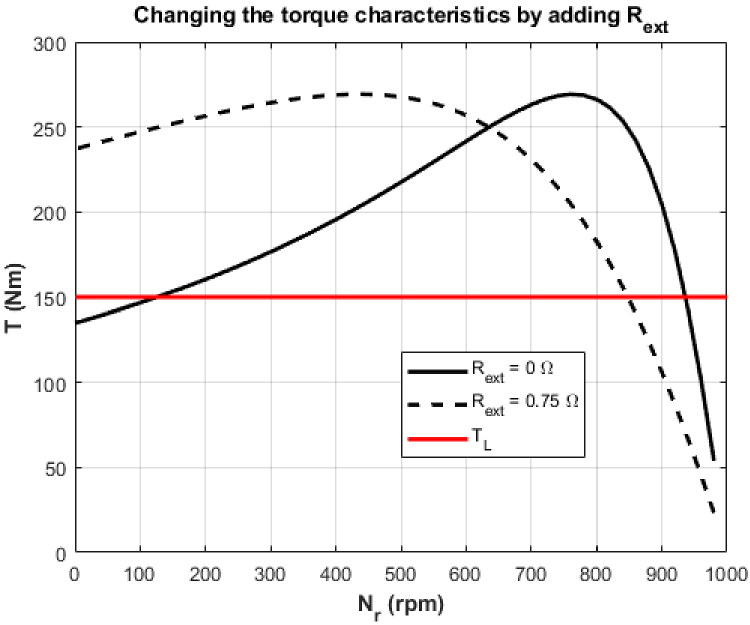
- the slip at which the maximum torque occurs can be varied.
- the starting torque can be varied.
- speed variations can be obtained.
- the starting current can also be reduced.

The maximum torque is independent of the rotor resistance

For the example 1 with different rotor resistance:



To make the starting torque higher than the load torque:



Example 2 :

A 400 V, 3-phase, 4 pole slip ring induction motor is supplied at rated voltage and frequency. The actual rotor resistance per phase is 3 Ω and the stand still rotor reactance per phase is 12 Ω. Neglect stator resistance,reactance and magnetizing reactance.

1. Determine the value of resistance to be added to the rotor circuit to get the maximum torque at starting.
2. Determine the value of resistance to be added to the rotor circuit to get 75 % of the maximum torque at starting.

1.

$$s_{\text{max T}} = 1$$

$$\frac{R_r + R_{\text{ext}}}{X_{lr}} = 1$$

$$\boxed{R_{\text{ext}} = 9\ \Omega}$$

2.

$$\frac{T_{\text{start}}}{T_{\text{max}}} = 0.75$$

From the known expressions (refer to previous slides),

$$\frac{T_{\text{start}}}{T_{\text{max}}} = \frac{2(R_r' + R_{\text{ext}})X_{lr}'}{(R_r' + R_{\text{ext}})^2 + (X_{lr}')^2)}$$

Let $R_r' + R_{\text{ext}} = A$,

$$0.75 = \frac{2 \times A \times 12}{A^2 + 144}$$

On solving this,

$$A = 26.6,\ 5.4$$

and

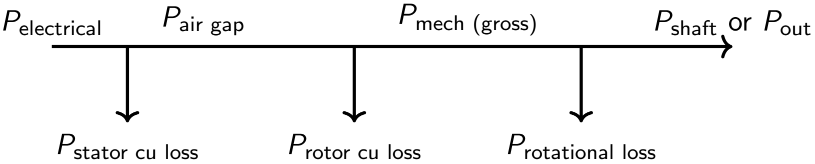
$$R_{\text{ext}} = 23.6\ \Omega,\ 2.4\ \Omega$$

We have to choose R_{ext} such that $s_{\text{max T}} < 1$.

$$\therefore \boxed{R_{\text{ext}} = 2.4\ \Omega}$$

Power Flow

Motoring Mode ($0 < s < 1$):



$$P_{\text{air gap}} = 3(I_r')^2 \frac{R_r'}{s}$$

$$P_{\text{rotor cu loss}} = sP_{\text{air gap}}$$

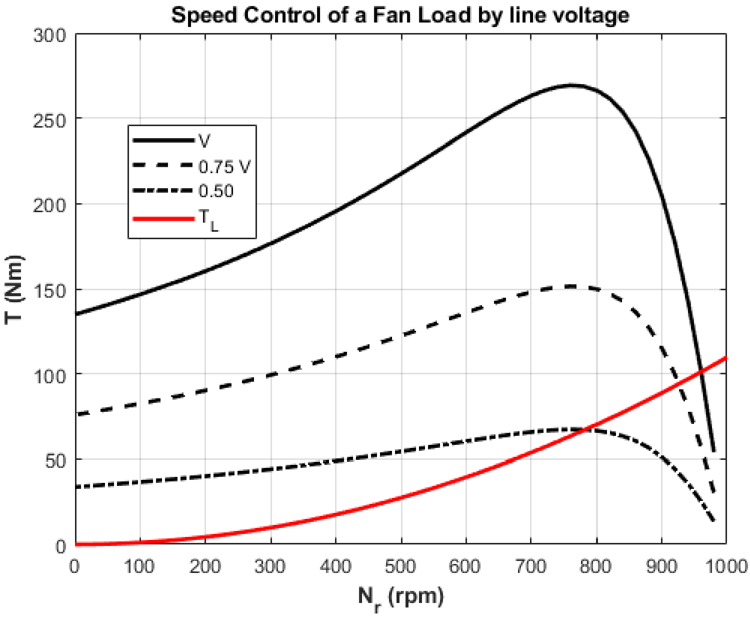
$$P_{\text{mech (gross)}} = (1 - s)P_{\text{air gap}}$$

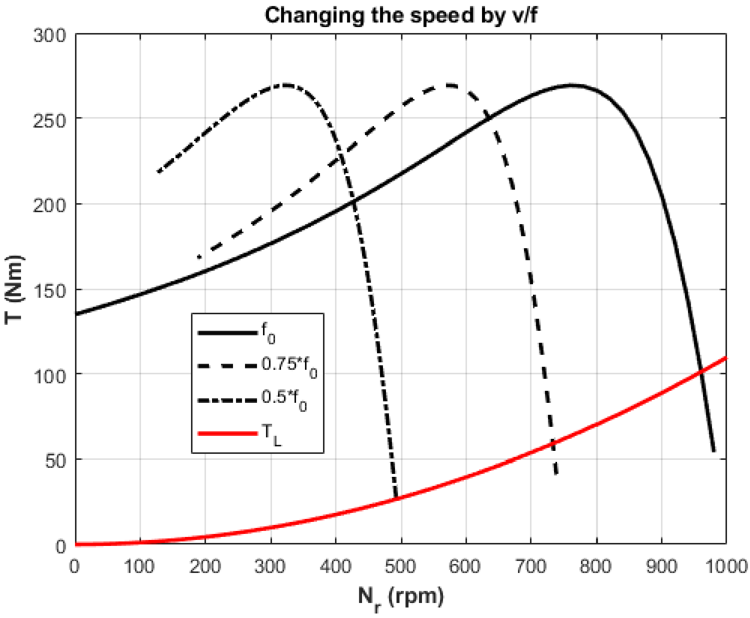
Speed Control

1. Pole changing
   Since the poles are changed in the ratio of 2 to 1, this method provides two synchronous speeds.
2. Line voltage control
3. Line frequency control

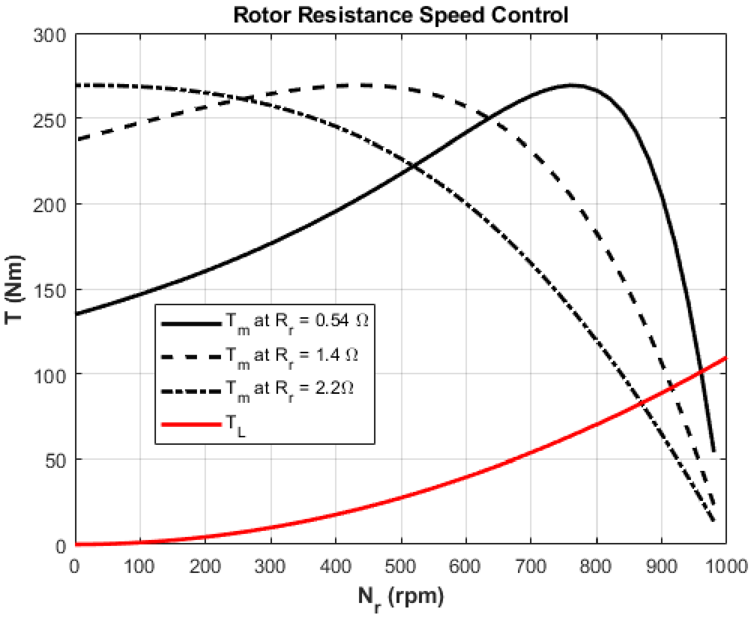
   $$\phi \propto \frac{V}{f}$$

   (To avoid saturation, the terminal voltage must also be varied in proportion to the frequency while reducing the frequency).
4. Rotor resistance control
   This is possible only in wound rotor (slip ring) induction machines.


Speed Control of a Fan Load by line voltage
T (Nm)
N_r (rpm)
V
0.75 V
0.50
T_L
0
50
100
150
200
250
300
0
100
200
300
400
500
600
700
800
900
1000



Changing the speed by v/f
T (Nm)
300
250
200
150
100
50
0
0
100
200
300
400
500
600
700
800
900
1000
N_r (rpm)
f_0
0.75*f_0
0.5*f_0
T_L


Rotor Resistance Speed Control
T (Nm)
N_r (rpm)
T_m at R_r = 0.54 Ω
T_m at R_r = 1.4 Ω
T_m at R_r = 2.2Ω
T_L
0
50
100
150
200
250
300
0
100
200
300
400
500
600
700
800
900
1000

Starting :

At Starting $s = 1$, $\dfrac{R_r'(1-s)}{s} = 0$.

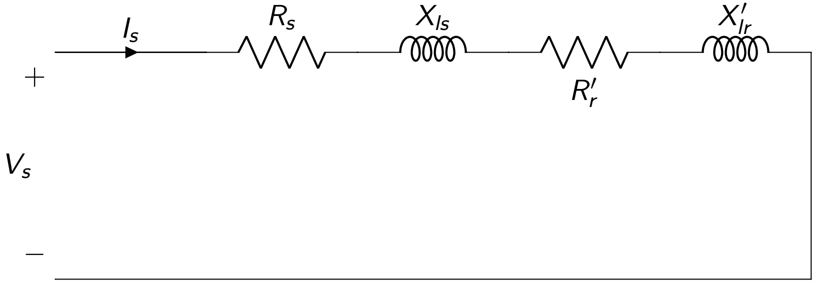


Figure: Approximate Equivalent Circuit at starting

$$I_s \approx I_r'$$

The starting torque

$$T_{\text{start}} = \frac{3I_s^2 R_r'}{\omega_s}$$

The torque at full load,

$$T_{fl} = \frac{3I_{fl}^2 R_r'/s_{fl}}{\omega_s}$$

$$\frac{T_{\text{start}}}{T_{fl}} = \left(\frac{I_s}{I_{fl}}\right)^2 s_{fl}$$

If induction motors are started by connecting them across the supply line, they may draw 5 to 8 times full load current.
Let $I_s = 5I_{fl}$ and s_{fl} =0.04.

$$\frac{T_{\text{start}}}{T_{fl}} = 5^2 \times 0.04 = 1$$

- With such a large starting current, the motor must accelerate and reach normal speed quickly. Otherwise, it may result in overheating and damage the motor.
- A large starting current causes appreciable voltage drop in the line and it may affect other drives connected to the line.

To avoid this, a reduced voltage starting must be used.

1. Auto Transformer starter
2. Star-delta starter
3. Rotor resistance control (only for slip ring motors)

However small rating motors may be started direct on line.

Example 3 (Kothari 9.18) : A 3-phase, wound rotor induction motor has a star connected rotor winding with a rotor resistance of 0.12 Ω/phase. With slip-rings shorted, the motor develops a rated torque at a slip of 0.04 and a line current of 100 A.

1. What external resistance must be inserted in each rotor phase to limit the starting current to 100 A?
2. What will be the per unit starting torque with the above rotor- resistance starting?

1.

$$s_{fl} = 0.04, \ I_{fl} = 100A$$

To have $I_s = I_{fl}$, R_{ext} is added to the rotor.

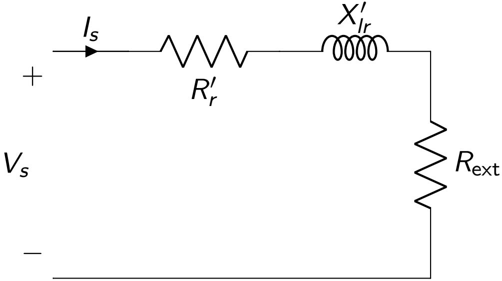


Figure: At starting

$$I_s^2 = \frac{V_s^2}{(R'_r + R_{\text{ext}})^2 + (X'_{lr})^2}$$

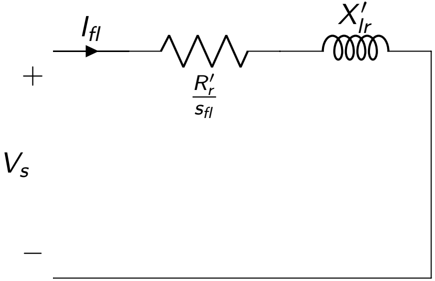


Figure: At full load

$$I_{fl}^2 = \frac{V_s^2}{(R_r'/s_{fl})^2 + (X_{lr}')^2}$$

Since $I_s = I_{fl}$,

$$(R_r'/s_{fl})^2 + (X_{lr}')^2 = (R_r' + R_{\text{ext}})^2 + (X_{lr}')^2$$

$$R_r' + R_{\text{ext}} = R_r'/s_{fl}$$

$$R_{\text{ext}} = \frac{0.12}{0.04} - 0.12 = 2.88\ \Omega$$

2.

$$T_{\text{start}} = \frac{3I_s^2(R_r' + R_{\text{ext}})}{\omega_s}$$

$$T_{fl} = \frac{3I_{fl}^2 R_r'/s_{fl}}{\omega_s}$$

$$\frac{T_{\text{start}}}{T_{fl}} = \frac{(R_r' + R_{\text{ext}})}{R_r'/s_{fl}} = 1$$

$$T_{\text{start}} = 1 \text{ p.u.}$$

Example 4: A three phase squirrel cage induction motor has a starting current of seven times the full load current and full load slip of 5 %. Suppose the motor is started using a star-delta starter. Determine

1. the per unit starting current. (It is the ratio of the starting current to the full load current.)
2. the per unit starting torque. (It is the ratio of starting torque to the full load torque.)

1. When it is started as a Y connected machine,

$$I_{L,Y} = I_p = \frac{V_L}{\sqrt{3}Z}$$

When it is started as a Δ connected machine,

$$I_{L,\Delta} = \sqrt{3}I_p = \frac{\sqrt{3}V_L}{Z}$$

Since $I_{L,Y} = \frac{I_{L,\Delta}}{3}$,

$$I_{st,Y} = \frac{1}{3} I_{st,\Delta}$$

$$\frac{I_{st,Y}}{I_{fl}} = \frac{7}{3} = 2.333 \text{ p.u.}$$

2. When it is started in Y, the phase current is

$$I_{s,Y} = \frac{1}{\sqrt{3}} I_{s,\Delta}$$

We know that

$$\frac{T_{\text{start}}}{T_{fl}} = \left(\frac{I_s}{I_{fl}}\right)^2 s_{fl}$$

Therefore,

$$\frac{T_{\text{start, Y}}}{T_{fl}} = \frac{1}{3} \left(\frac{I_s}{I_{fl}}\right)^2 s_{fl} = \frac{1}{3} \times 49 \times 0.05 = 0.816 \text{ p.u.}$$