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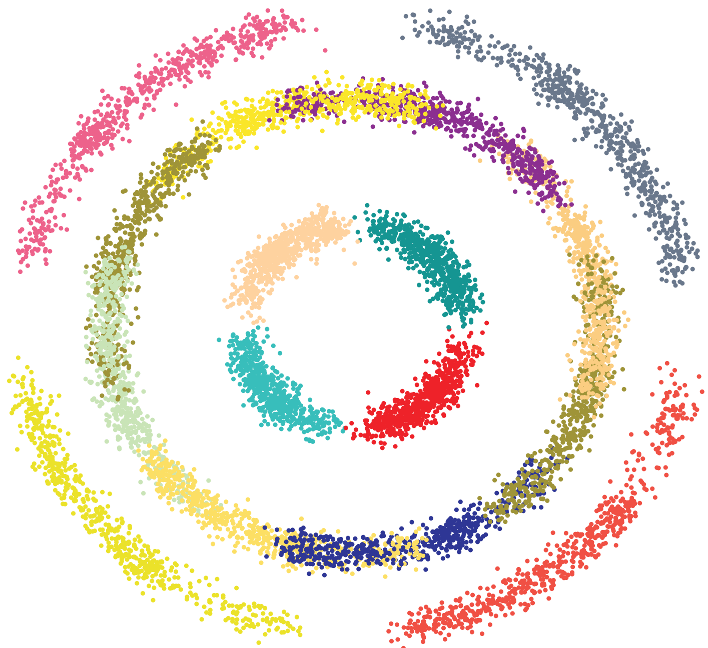

Phase Noise in Communication Systems
Modeling, Compensation, and Performance Analysis

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THESIS FOR DEGREE OF DOCTOR OF PHILOSOPHY

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Göteborg, November 2015

Khanzadi, M. Reza
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Doktorsavhandlingar vid Chalmers tekniska högskola
ISBN 978-91-7597-277-0
Serial Number 3958
ISSN 0346-718X

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This thesis has been prepared using LaTeX.

Front Cover: The figure on the front cover is the distorted received signal constellation of a 16-QAM communication system due to transmitter and receiver oscillator phase noise.

Printed by Chalmers Reproservice,
Göteborg, Sweden, October 2015.

To those who have dedicated their lives to make the world a better place,
to my family,
to my beloved Fatemeh.

ABSTRACT

The continuous increase in demand for higher data rates due to applications with massive number of users motivates the design of faster and more spectrum efficient communication systems. In theory, the current communication systems must be able to operate close to Shannon capacity bounds. However, real systems perform below capacity limits, mainly because of considering too idealistic or simplistic assumptions on the imperfections such as channel estimation errors or hardware impairments.

Oscillator phase noise is one of the hardware impairments that is becoming a limiting factor in high data rate digital communication systems. Phase noise severely limits the performance of systems that employ dense constellations. Moreover, the level of phase noise (at a given off-set frequency) increases with carrier frequency, which means that the problem of phase noise may be worse in systems with high carrier frequencies.

The focus of this thesis is on: i) finding accurate statistical models of phase noise, ii) designing efficient algorithms to mitigate the effect of this phenomenon, iii) analyzing the Shannon capacity of the single and multiple-antenna communication systems affected by phase noise.

First, a new statistical model of phase noise valid for free-running and phase-locked-loop-stabilized oscillators is provided. The new model incorporates white and colored noise sources inside the oscillator circuitry. The new model is used in order to connect the performance of phase-noise affected communication systems, in terms of error-vector-magnitude, with oscillator phase-noise measurements. The results can be used by hardware and frequency generator designers to better understand the impairing effects of phase noise on the system performance and optimize their design criteria respectively.

Second, the proposed phase-noise model is employed for estimation of phase noise generated from white and colored noise sources. A soft-input maximum a posteriori phase noise estimator and a modified soft-input extended Kalman smoother are proposed. The performance of the proposed algorithms is compared against that of those studied in the literature, in terms of mean square error of phase noise estimation, and symbol error rate of the considered communication system. The comparisons show that considerable performance gains can be achieved by designing estimators that employ correct knowledge of the phase-noise statistics. The performance improvement is more significant in low-SNR or low-pilot density scenarios.

Finally, the capacity of single and multiple antenna communication systems affected by phase noise is investigated. For the SISO Wiener phase-noise channel, upper and lower bounds on the capacity are obtained, which are tight for a wide range of SNR values. In addition, a family of input distributions, which result in a tight lower bound are introduced. The high-SNR capacity of single-input multiple-output (SIMO) and multiple-output single-input (MISO) phase-noise channels for two different oscillator configurations is investigated. The provided analysis shows that driving antennas at the base station by separate (independent) local oscillators is beneficial for the SIMO channel compared to driving all the antennas with a common oscillator. In contrast, larger gains are achieved for the MISO channel when a common oscillator is employed.

Keywords: Oscillator phase noise, free-running oscillator, phase-locked loop, colored phase noise, phase noise model, Bayesian Cramér-Rao bound, maximum a posteriori estimator, extended Kalman filter/smoother, mean square error, channel capacity, Wiener process, capacity achieving distribution, multiple antennas, distributed oscillators.

LIST OF PUBLICATIONS

Appended Publications

Paper A
M. Reza Khanzadi, Dan Kuylenstierna, Ashkan Panahi, Thomas Eriksson, and Herbert Zirath, "Calculation of the Performance of Communication Systems from Measured Oscillator Phase Noise," *IEEE Transactions on Circuits and Systems I,* May. 2014.

Paper B
M. Reza Khanzadi, Rajet Krishnan, and Thomas Eriksson, "Estimation of Phase Noise in Oscillators with Colored Noise Sources," *IEEE Communications Letters,* Nov. 2013.

Paper C
M. Reza Khanzadi, Rajet Krishnan, Johan Söder, and Thomas Eriksson, "On the Capacity of the Wiener Phase-Noise Channel: Bounds and Capacity Achieving Distributions," *IEEE Transactions on Communications,* Jul. 2015.

Paper D
M. Reza Khanzadi, Giuseppe Durisi, and Thomas Eriksson, "Capacity of SIMO and MISO Phase-Noise Channels with Common/Separate Oscillators," *IEEE Transactions on Communications,* Sep. 2015.

Other Publications

Below are other contributions by the author which are not included in this thesis, either due to contents overlapping with that of the appended publications, or due to contents not related to this thesis.

Patents

[P1] **M. Reza Khanzadi**, Mats Rydström, "Phase Noise Suppression For Large Antenna Array Communication Systems," *Patent Application,* Jul. 2015.

Journal Papers

[J1] Rajet Krishnan, **M. Reza Khanzadi**, Narayanan Krishnan, Yongpeng Wu, Alexandre Graell i Amat, Thomas Eriksson, and Robert Schober, "Linear Massive MIMO Precoders in the Presence of Phase Noise: A Large-Scale Analysis," *IEEE Transactions on Vehicular Communications,* Jan. 2015.

[J2] Rajet Krishnan, **M. Reza Khanzadi**, Thomas Eriksson, and Tommy Svensson, "Soft metrics and their Performance Analysis for Optimal Data Detection in the Presence of Strong Oscillator Phase Noise," *IEEE Transactions on Communications,* Jun. 2013.

[J3] Cristian B. Czegledi, **M. Reza Khanzadi**, and Erik Agrell, "Bandlimited Power-efficient Signaling and Pulse Design for Intensity Modulation," *IEEE Transactions on Communications,* Aug. 2014.

[J4] Rajet Krishnan, **M. Reza Khanzadi**, Narayanan Krishnan, Alexandre Graell i Amat, Thomas Eriksson, Nicolò Mazzali, and Giulio Colavolpe, "On the impact of oscillator phase noise on the uplink performance in a massive MIMO-OFDM system," *Submitted to IEEE Signal Processing Letters,* 2015.

[J5] Hani Mehrpouyan, **M. Reza Khanzadi**, Michail Matthaiou, Robert Schober, and Yingbo Hua, "Improving Bandwidth Efficiency in E-band Communication Systems," *IEEE Communications Magazine,* Sep. 2012.

Conference Papers

[C1] **M. Reza Khanzadi**, Rajet Krishnan, and Thomas Eriksson, "Receiver Algorithm based on Differential Signaling for SIMO Phase Noise Channels with Common and Separate Oscillator Configurations," *IEEE Global Communications Conference (GLOBECOM),* Dec. 2015.

[C2] **M. Reza Khanzadi**, Giuseppe Durisi, and Thomas Eriksson, "High-SNR Capacity of Multiple-Antenna Phase-Noise Channels with Common/Separate RF Oscillators," *IEEE International Conference on Communications (ICC),* Jun. 2015.

[C3] Rajet Krishnan, **M. Reza Khanzadi**, Narayanan Krishnan, Yongpeng Wu, Alexandre Graell i Amat, Thomas Eriksson, and Robert Schober, "Large Scale Analysis of Linear Massive MIMO Precoders in the Presence of Phase Noise," *IEEE International Conference on Communications (ICC),* Jun. 2015.

[C4] **M. Reza Khanzadi**, Rajet Krishnan, Dan Kuylenstierna, and Thomas Eriksson, "Oscillator Phase Noise and Small-Scale Channel Fading in Higher Frequency Bands," *IEEE Global Communications Conference (GLOBECOM),* Dec. 2014.

[C5] Cristian B. Czegledi, **M. Reza Khanzadi**, and Erik Agrell, "Bandlimited Power-Efficient Signaling for Intensity Modulation," *IEEE European Conference on Optical Communications (ECOC),* Sep. 2014.

[C6] **M. Reza Khanzadi**, Rajet Krishnan, and Thomas Eriksson, "Effect of Synchronizing Coordinated Base Stations on Phase Noise Estimation," *IEEE Conference on Acoustic, Speech, and Signal Processing (ICASSP),* May. 2013.

[C7] **M. Reza Khanzadi**, Ashkan Panahi, Dan Kuylenstierna, and Thomas Eriksson, "A model-based analysis of phase jitter in RF oscillators," *IEEE International Frequency Control Symposium (IFCS),* May. 2012.

[C8] Rajet Krishnan, **M. Reza Khanzadi**, Lennart Svensson, Thomas Eriksson, and Tommy Svensson, "Variational Bayesian Framework for Receiver Design in the Presence of Phase Noise in MIMO Systems," *IEEE Wireless Communications and Networking Conference (WCNC),* Apr. 2012.

[C9] **M. Reza Khanzadi**, Hani Mehrpouyan, Erik Alpman, Tommy Svensson, Dan Kuylenstierna, and Thomas Eriksson,"On models, bounds, and estimation algorithms for time-varying phase noise,"*IEEE International Conference on Signal Processing and Communication Systems (ICSPCS),* Dec. 2012.

[C10] **M. Reza Khanzadi**, Kasra Haghighi, and Thomas Eriksson, "Optimal Modulation for Cognitive Transmission over AWGN and Fading Channels,"*IEEE European Wireless Conference,* Apr. 2011.

[C11] **M. Reza Khanzadi**, Kasra Haghighi, Ashkan Panahi, and Thomas Eriksson, "A Novel Cognitive Modulation Method Considering the Performance of Primary User," *IEEE Wireless Advanced Conference (WiAD)* Jun. 2010.

[C12] Arash Toyserkani, **M. Reza Khanzadi**, Erik Ström, and Arne Svensson, "A Complexity Adjustable Scheduling Algorithm for Throughput Maximization in Clusterized TDMA Networks,"*IEEE Vehicular Technology Conference (VTC),* May. 2010.

ACKNOWLEDGEMENTS

I am getting close to the end of a five-year-long battle against *phase noise*. An adventurous journey, with many ups and downs, with countless happy and sad moments. Wondering if I defeated the beast, or we have just decided to coexist… Regardless of how it ended, the journey could have never been possible without the help of my fellow companions.

Thomas, this journey was impossible without your help, support, and guidance. You once said: "The most important product of the PhD education is nothing but you; you and how you develop as a PhD will be the fruit of this process". Now, looking back and thinking about my professional and personal developments, I can clearly see the effect of these five years on my life. Thank you for everything, for your kindness, for level of trust and belief in me. You know what I will miss the most after my graduation? Our Friday afternoons' pointless conversations! Expect me to come back to you for a new series of such discussions!

Herbert, thank you for recruiting me and giving me the freedom to choose my research path. I am really grateful for your generous support. Dan, I would also like to thank you for your constructive comments and warm encouragement. I really enjoyed our futuristic discussions on horizons of technology. Yes, humanity is on a tipping point of something big, a knowledge singularity that will change every dimension of our lives. I am looking forward for more of those discussions in the future.

Erik A., it has been an honor to be involved in a part of your research in optical communications; a great and rewarding experience. Giuseppe, thank you for your guidance and for helping me to develop my view in the field of information theory. Tommy, I always enjoyed our discussions including our annual appraisal talks, where I could clearly monitor my improvements over time. My deep appreciation also goes to Erik S. for giving me the opportunity to be a part of the ComSys family. ComSyS and S2 are my home ground and my safe beach. Thanks for hosting me during the last five years. Thanks to Agneta, Lars, Natasha, Malin, Madeleine, and Karin; Commsys environment would not have been this pleasant if not for all of you.

Now, time for my "partner in crime"– Rajet! Dr. Krishnan! HaHaHa! who never understand what does "silence" means and can never practice it! If you do some googling, Rajet is the strongest antonym for silence! Rajet, you should know what you mean to me. You were not only a colleague or a friend, but the most influential person in my personal and professional life during these years. I always admire your high level of positive energy, your generosity in sharing ideas and opportunities, and for understanding the value of collaboration. I am looking forward for the rest of this journey towards our mutual goals.

To all my friends at Chalmers, former and current colleagues at S2 and MC2, thanks for your friendship, support, and all the never-forgetting memories. Thank you to my friends outside Chalmers. You guys are amazing!

To my parents, thank you for raising me to be the person I am today. I wish you were not gone and could have seen this. How happy you would be ...

Shirin, you built the foundation of who I am, where I am ..., and your shocking loss reformed my view of life ... I promise to use my energy and knowledge to improve the health, safety, and wellbeing of people worldwide, to make their life more enjoyable.

Shohreh, and Giti, your energy and kindness is what move me forward. Thank you for being always there for me.

Last but never ever least, it is time to say thank you to the most important person in my life, my main source of energy and inspiration behind the scene, to my best companion through all the tears and laughs of the PhD life, to my beloved Fatemeh! Thank you for tolerating and guiding me during the down times, with your endless love, with your immeasurable kindness. I know that you are tired of hearing about *the battle of Reza vs. Phase Noise*, but yet you still are here for me. Thank you for your support, and patience. Believe it or not, my journey, my success and what I am now, could have never been possible without you, the only "ESHGHOOL" in the world! I wish you the best of bests; I wish all your dreams come true; And I promise to be there for you to achieve all your goals.

And finally, a thank you to you the reader, whoever you are. You have just finished my acknowledgment and have 170 more pages to go ...

M. Reza Khanzadi
Göteborg Oct. 2015

CONTENTS

ACRONYMS

ADC:	Analog to Digital Converter
AR:	Autoregressive
AWGN:	Additive White Gaussian Noise
BCRB:	Bayesian Cramér-Rao Bound
BER:	Bit Error Rate
BIM:	Bayesian Information Matrix
CLO:	Common Local Oscillator
CPE:	Common Phase Error
CRB:	Cramér-Rao Bound
EKS:	Extended Kalman Smoother
EVM:	Error-Vector Magnitude
FIM:	Fisher Information Matrix
GaN:	Gallium Nitride
HEMT:	High-Electron-Mobility Transistor
ICI:	Intercarrier Interference
I/Q:	Inphase / Quadrature Phase
LNA:	Low-Noise Amplifier
LoS:	Line-of-Sight
MAP:	Maximum a Posteriori
MBCRB:	Modified Bayesian Cramér-Rao Bound
MIMO:	Multiple-Input Multiple-Output
MISO:	Multiple-Input Single-Output

ML: Maximum Likelihood

MMIC: Monolithic Microwave Integrated Circuit

MMSE: Minimum Mean Square Error

MSE: Mean Squared Error

OFDM: Orthogonal Frequency Division Multiplexing

PA: Power Amplifier

PDF: Probability Density Function

PLL: Phase-Locked Loop

PN: Phase Noise

PSD: Power Spectral Density

QAM: Quadrature Amplitude Modulation

RF: Radio Frequency

SER: Symbol Error Rate

SIMO: Single-Input Multiple-Output

SLO: Separate Local Oscillators

SNR: Signal-to-Noise Ratio

SSB: Single-Side-Band

Part I

Introduction

CHAPTER 1

OVERVIEW

1 Introduction

Digital communication has become an indispensable part of our personal and professional lives. Data transmission rates over fixed and wireless networks have been increasing rapidly due to applications with massive number of users such as smartphones, tablets, wired and wireless broadband Internet connections, cloud computing, the machine-to-machine communication services, etc. The figures show that demands for higher data rates continue to increase at about 60% per year [1].

This increasing demand motivates the need for design of faster and more spectrum and power-efficient communication systems. During the past decades, substantial research has been done in order to design communication systems that operate close to the Shannon capacity limits. However, several of those studies have been based on some idealized assumptions such as perfect channel state information, perfect synchronization, ideal hardware, etc. Hence, despite the progress in theoretical aspects of such systems, the analog front end still fails to deliver what is promised mainly due shortcomings such as hardware impairments. Hardware impairments in radio-frequency (RF) and optical communications, such as amplifier nonlinearity, I/Q imbalance, optical channel nonlinearities, and RF oscillator or laser phase noise, can be seen as the bottlenecks of the performance [2].

The remedy to this problem is to include the effect of such impairments in design and analysis of communication systems. A key step in this avenue is to find mathematical models that can accurately represent the physical impairments. Motivated by Moore's law, there is an increasing interest in handling those problems by means of *digital* signal processing algorithms. This further indicates the need for accurate modeling of the hardware impairments. In many prior studies, simple models have been employed for this purpose. For instance, abstracting all unwanted effects such as hardware impairments in an additive white Gaussian noise term has been a normal practice. However, to be able to come up with efficient algorithms, better understanding and modeling of the system imperfections is needed.

Oscillator phase noise is one of the hardware imperfections that is becoming a limiting factor in high data rate digital communication systems. Phase noise severely limits the performance of systems that employ dense constellations. The negative effects of phase noise are more pronounced in high-carrier-frequency systems, e.g., E-band (60-80 GHz), mainly due to the high level of phase noise in oscillators designed for such frequencies [3–6]. Moreover, phase noise affects the performance of multiple-antenna communication systems [7–10], and also destroys the orthogonality of the subcarriers in orthogonal frequency division multiplexing (OFDM) systems and degrades the performance by producing intercarrier interference [11–13].

In many prior studies, phase noise has been modeled as a discrete random walk with uncorrelated (white) Gaussian increments between each time instant (i.e., the discrete Wiener process). This model is acceptable for free-running oscillators with white noise sources [14]. However, numerous studies show the presence of colored noise sources inside oscillators [14–19].

The Shannon capacity of a communication system can be used as a benchmark for performance evaluation. Unfortunately, a closed-form expression for the capacity of communication systems impaired by phase noise is not available. Asymptotic capacity characterizations for large signal-to-noise ratio (SNR), and nonasymptotic capacity bounds are available in the literature. However, several of those studies have considered non-realistic mathematical models for phase noise. Moreover, the literature is particularly lacking studies on the capacity of multiple-antenna communication systems affected by phase-noise.

2 Aim and Scope of the Thesis

In summary, accurate phase-noise modeling, design of efficient algorithms to mitigate the effect of phase noise, and derivation of the channel capacity of single and multiple-antenna communication systems, affected by phase noise, form the core of this thesis. In this thesis,

- An accurate statistical model of phase noise, inspired by phase-noise measurements of practical oscillators, is provided. This model can further be used for accurate simulations of phase-noise-affected communication systems, as well as design of algorithms for mitigation of phase-noise effects.

- The performance of a phase-noise-affected communication system for given phase-noise measurements has been investigated. Such results can have impacts on design of hardware for frequency generation.

- Realistic phase-noise models are employed in order to design and implement more efficient estimation and mitigation algorithms. More specifically, in presence of phase noise with colored-noise sources, considerable performance improvements can be achieved by using the proposed algorithms, especially in low signal-to-noise ratio (SNR) and low-pilot-density scenarios.

- Capacity of the single and multiple-antenna channels, impaired by phase-noise, is studied. Tight upper and lower bounds on capacity of the discrete Wiener phase-noise channel is found. High-SNR capacity of multiple-antenna systems, considering various oscillator configurations, is investigated.

In this thesis, the concentration is on oscillator phase noise. The other determining effects such as nonlinearities, system architectural problems, or noise from other components are modeled as an additive white Gaussian noise term. In addition, perfect time synchronization is considered in the system models.

The main focus of the thesis is on line-of-sight (LoS) backhauling applications, where the channel can be assumed to be relatively stable over long time periods. Therefore, a deterministic, time-invariant channel model, where the channel coefficients are known to the transmitter and receiver has been considered. Moreover, the channel is frequency-flat due to the LoS properties. Hence, single-carrier communication systems are the main focus of the thesis. Design and implementation of digital signal processing algorithms in order to resolve the phase-noise problem has been the main interest. Therefore, analog (including RF) approaches for phase-noise mitigation are not investigated in this thesis.

3 Thesis Outline

The thesis is organized as follows: In Chapter 2 we review different sources of hardware impairments in practical communication systems. Then we focus on phase noise and its effects on the performance of communication systems. In Chapter 3, we focus on structure of oscillators and their role in communication systems, then we investigate the source of phase noise in oscillators and study the phase-noise generation mechanism. In Chapter 4, we discuss the system models that capture the effect of phase noise in single and multiple-antenna systems. In Chapter 5, an overview of phase-noise models available in the literature is given. Further, a new statistical model for the phase noise of real oscillators is presented. Chapter 6 deals with the design of algorithms for phase noise compensation. Non-Bayesian and Bayesian estimators are reviewed, and the chapter is concluded by introducing methods for evaluation of the performance of Bayesian estimators. Capacity of the phase-noise channel and information-theoretic tools, used for calculation of the capacity, is presented in Chapter 7. Finally, a summary of the contributions of the appended publications is provided in Chapter 8.

CHAPTER 2

PRACTICAL COMMUNICATION SYSTEMS

Communication systems have become an essential part of our everyday lives. Growing demands for faster communication links and more power-efficient communication systems has increased the requirements for efficient design and implementation of such systems. This chapter deals with hardware impairments in practical communication systems. We particularly focus on oscillator phase noise which is the main topic of this thesis.

1 Hardware Impairments

Any practical communication system consists of several components that play a specific role in the transmission of data stream from one point to another. Fig. 2.1 illustrates a simplified hardware block diagram of a typical wireless communication system.

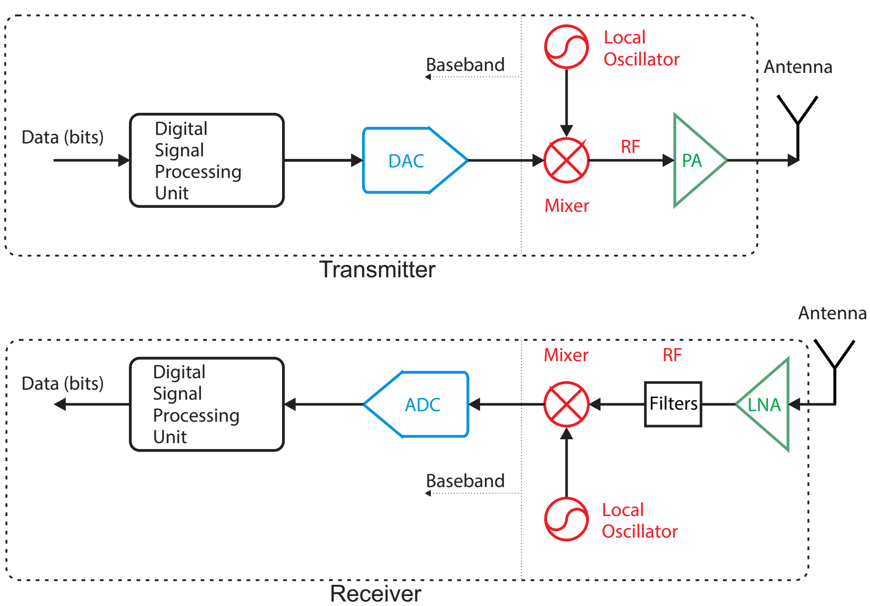


Figure 2.1: Hardware block diagram of the transmitter and receiver in a typical communication system.

All the system components are made of electronic devices such as resistors, inductors, capacitors, diodes, and transistors, which are composed of conductor and semiconductor materials. These electronic devices are not perfect and can behave differently from what is predicted for many reasons such as temperature variation, aging, pressure, mismatch between the components, etc. Hardware impairments are an inevitable consequence of using non-ideal components in communication systems. Amplifier nonlinearities [20], I/Q imbalance [21], quantization noise [22], jitter in digital sampling clock [23], and RF oscillator phase noise [24] are examples of such impairments.

In this thesis, the focus is on RF oscillator phase noise, one of the major hardware impairments affecting the performance of wireless communication systems. Oscillators are one of the main building blocks in a communication system. Their role is to create stable reference signals for frequency and timing synchronization. RF oscillators are used to upconvert/downconvert the baseband signal to/from RF signal (or intermediate frequency signal). Unfortunately, any real oscillator suffers from hardware imperfections that introduce phase noise to the communication system. The fundamental source of the phase noise is the inherent noise of the passive and active components (e.g., thermal noise) inside the oscillator circuitry [15, 24, 25].

2 Effect of Phase Noise on Communication Systems

Phase noise from transmitter and receiver RF oscillators destroys the transmitted signals. Random rotation of the received signal constellation [26–29], amplitude variation of received signal in multi-antenna systems [30], and spectrum regrowth [11, 12, 31–34] are the main effects of phase noise.

Random rotation of the received signal constellation due to phase noise may lead to symbol detection errors in phase-modulated transmissions [26–29]. Fig. 2.2a illustrates the received signal constellation of a single-antenna single-carrier system employing 16-QAM modulation format over an AWGN channel. Fig. 2.2b, on the other hand shows the effect of phase noise from a free-running oscillator on the received signal constellation. The constellation rotation is due to the random variation of the phase of the received signal. The dashed lines show the Voronoi regions of a coherent detector, designed for the AWGN channel. Phase noise cause decision errors by moving the received signals to wrong decision regions.

Fig. 2.3 shows the effect of phase noise before compensation on symbol error rate (SER) of the system, where it is compared against the SER of pure AWGN channel.

In multiple-antenna systems, phase noise may result in random amplitude variation of the received signal [29]. This can happen mainly due to noncoherent (out-of-phase) combining of received signals at the receiver [35]. Fig. 2.4 shows the effect of phase noise on the receive constellation of a 2×1 multiple-input single-output (MISO) channel, when transmit antennas are fed by independent oscillators. The amplitude variation effect can be observed more clearly by comparing the received signal constellation Fig. 2.4 with that of SISO channel, shown in Fig. 2.2b.

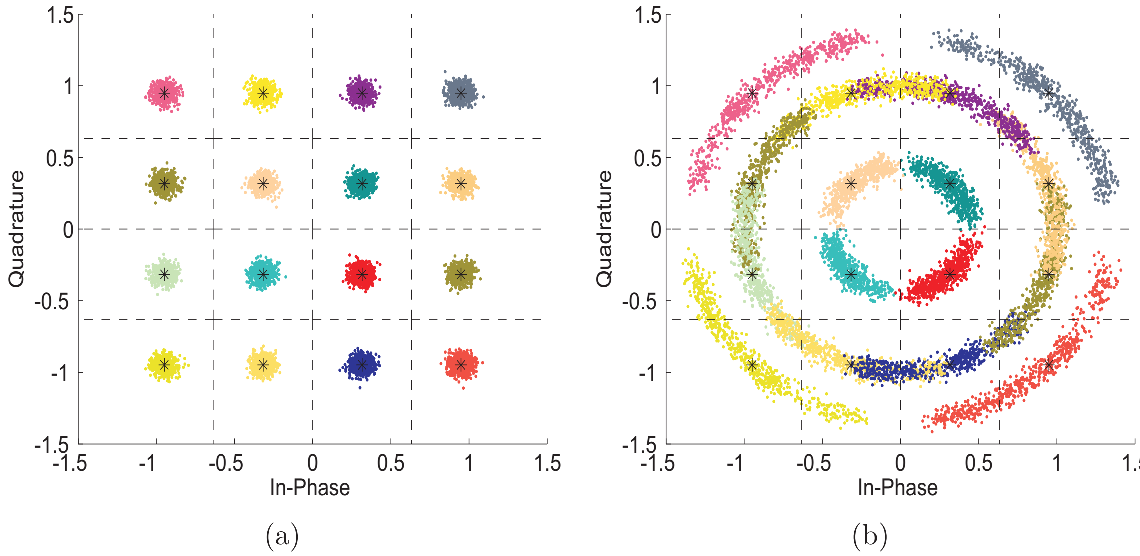


Figure 2.2: Received signal constellation of a 16-QAM system. (a) Pure AWGN channel with SNR=25 dB. (b) AWGN channel affected by phase noise, SNR=25 dB and phase noise increment variance of 10^{-4} rad^2/symbol duration.

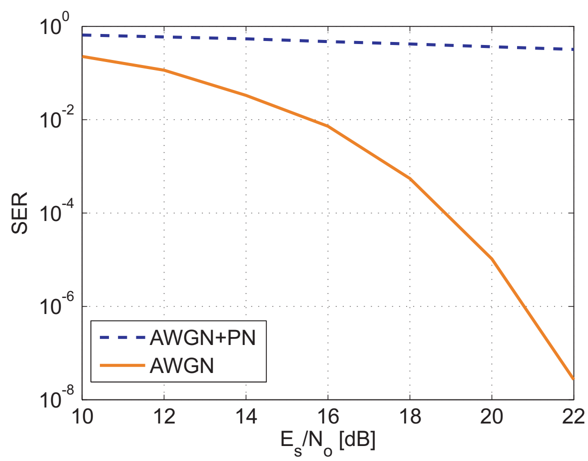


Figure 2.3: Effect of phase noise before compensation on the SER of a 16-QAM system.

Phase noise may result in spectral regrowth, which can further cause adjacent channel interference in frequency division multiplexing systems [36, 37]. Fig. 2.5a shows two systems with the bandwidth of 22 MHz, and central frequencies of 2.412 and 2.437 GHz. Both systems use low-phase-noise oscillators that results in no overlapping between the bands. On the other hand, Fig. 2.5b illustrates a case where the first system employs a noisy oscillator that cause power leakage to the other band, resulting in inter channel interference.

Phase noise has two detrimental effects on the performance of OFDM systems. First, phase noise rotates all the subcarriers in the same OFDM symbol with a common phase, which is called common phase error (CPE). More importantly, it destroys the orthog-

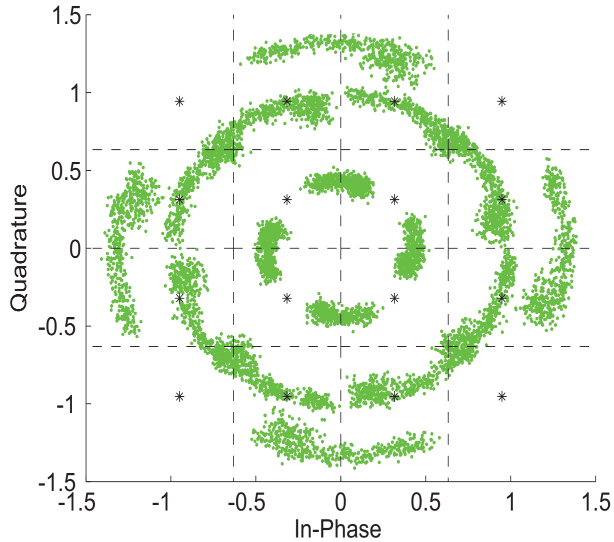
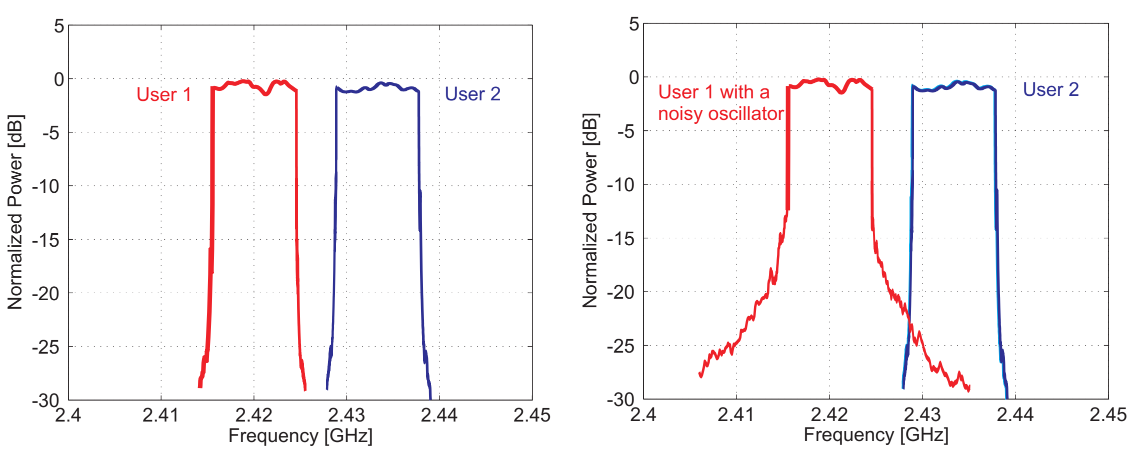


Figure 2.4: Received signal constellation of a 2×1 MISO system using 16-QAM modulation format. Here, SNR=25 dB and phase noise increment variance is 10^{-4} rad^2/symbol duration, and channel gains between the receive and the transmit antennas are equal to unity.

onality of the subcarriers by spreading the power from one subcarrier to the adjacent subcarriers. This effect is called inetrcarrier interference (ICI) [11, 12, 38]. In Fig. 2.6 the effect of phase noise on the main lobs of the subcarriers are illustrated. Fig. 2.7 illustrates the effect of phase noise on the received constellation diagram of a 16-QAM OFDM system. The constellation rotation is due to CPE and the AWGN-like cloud is the effect of ICI.



(a) (b)

Figure 2.5: Effect of using ideal or noisy oscillators on the spectrum. (a) Two systems employing low phase noise oscillators. (b) Two systems, one using a noisy oscillator, which interferes with the other channel by spectrum regrowth.

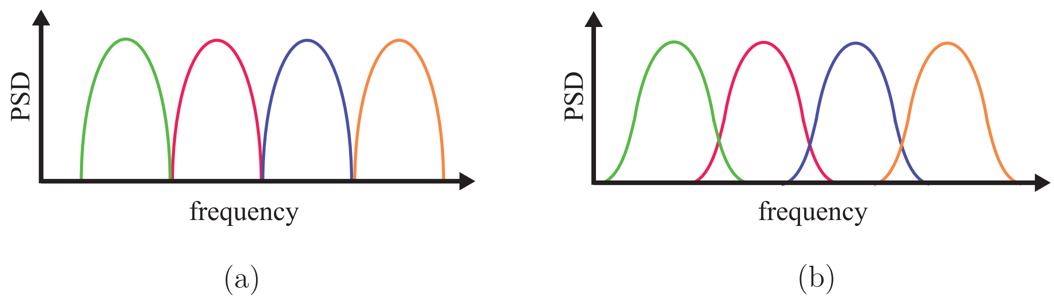


Figure 2.6: In (a), PSD of an OFDM signal with 4 subcarriers is shown. Note that only the main lobs of each subcarrier is shown. In (b), effect of phase noise on the main lobs of subcarriers are illustrated.

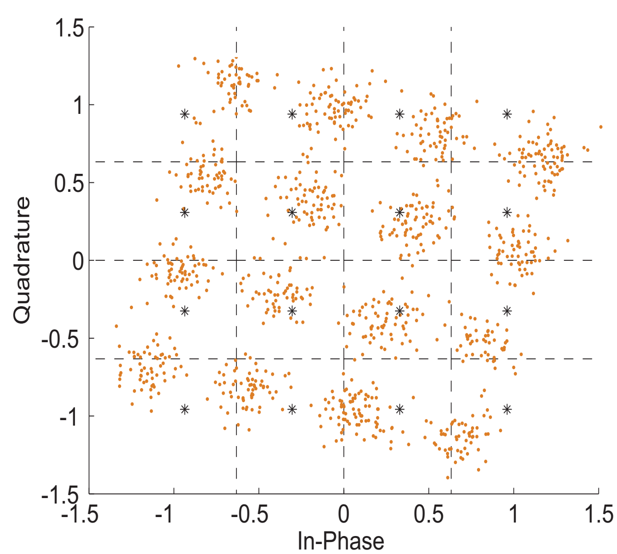


Figure 2.7: Received signal constellation of a 16-QAM OFDM system, affected by phase noise. Here, a received OFDM symbol with 1024 subcarriers is plotted.

CHAPTER 3

PHASE NOISE IN OSCILLATORS

1 Oscillators

Oscillators are autonomous systems, which provide a reference signal for frequency and timing synchronization. Harmonic oscillators with sinusoidal output signals are used for up-conversion of the baseband signal to an intermediate/radio frequency signal at the transmitters, and down-conversion from radio frequencies to baseband at the receivers (see Fig. 2.1).

Fig. 3.1 illustrates the block diagram of a feedback oscillator consisting of an amplifier and a feedback network. Roughly speaking, the oscillation mechanism is based on positive feedback of a portion of the output signal to the input of the amplifier through the feedback network [25, 39].

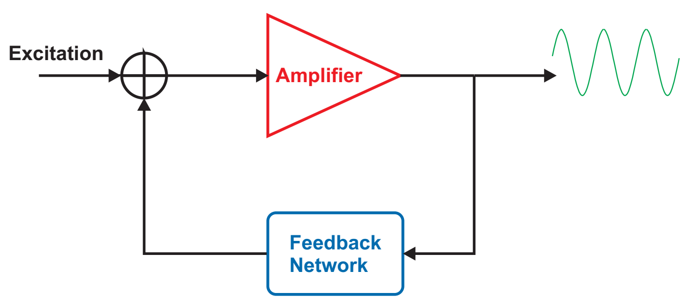


Figure 3.1: Block diagram of a typical feedback oscillator.

The feedback network is a resonator circuit, e.g., an LC network, and the amplifier is normally composed of diodes or transistors (see Fig. 3.2). One of the main sources of phase noise in the output oscillatory signal is the noise of such internal electronic devices.

In an ideal oscillator, the phase transition over a given time interval is constant and the output signal is perfectly periodic. However in practical oscillators, the amount of phase increment is a random variable. This phase variation is called phase noise increment or phase jitter, and the instantaneous deviation of the phase from the ideal value is called

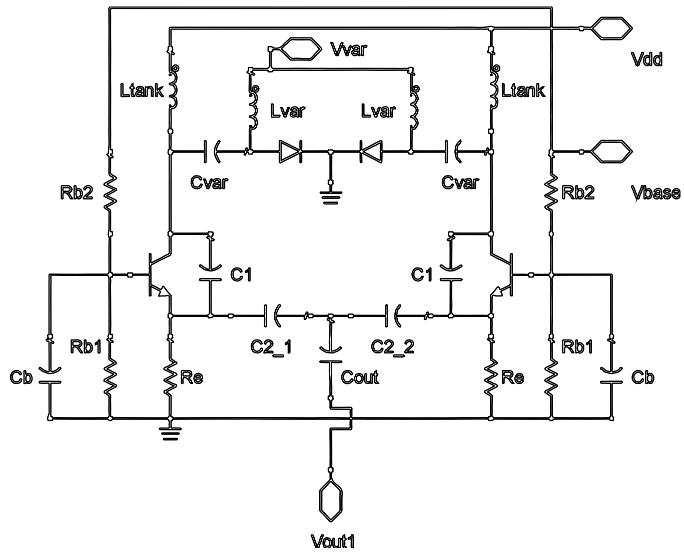


Figure 3.2: Schematic view of an InGaP/GaAs HBT voltage controlled oscillator [40].

phase noise [15, 24, 36].

In time domain, the output of a harmonic oscillator with normalized amplitude can be expressed as

$$v(t) = (1 + a(t)) \cos (2\pi f_0 t + \phi(t)) \, , \tag{3.1}$$

where f_0 is the oscillator's central frequency, and $a(t)$ and $\phi(t)$ denote the amplitude noise and phase noise processes, respectively [15]. The amplitude noise and phase noise are modeled as two independent random processes. According to [15, 41], the amplitude noise has insignificant effect on the output signal of the oscillator. Thus hereinafter, the effect of amplitude noise is neglected and the focus is on the study of the phase noise process.

Fig. 3.3 compares the output signal of an ideal oscillator with a noisy one. It can be seen that in the output of the noisy oscillator, the zero-crossing time changes randomly due to phase noise .

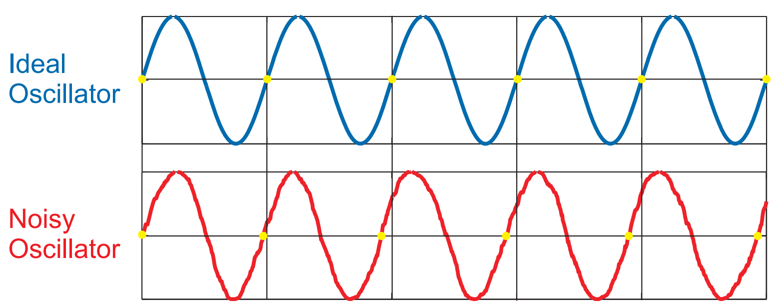


Figure 3.3: Effect of phase noise on the output oscillatory signal.

For an ideal oscillator where the whole power is concentrated at the central frequency f_0, the power spectral density would be a Dirac delta function, while in reality, phase

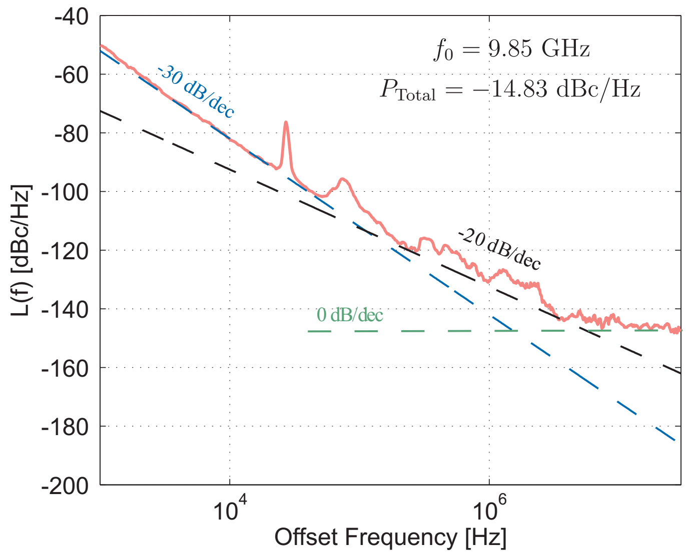


Figure 3.5: The SSB phase noise spectrum from a GaN HEMT MMIC oscillator. Drain voltage $V_{\mathrm{dd}} = 6$ V and drain current $I_{\mathrm{d}} = 30$ mA.

noise results in spreading the power over frequencies around f_0 (Fig. 3.4).

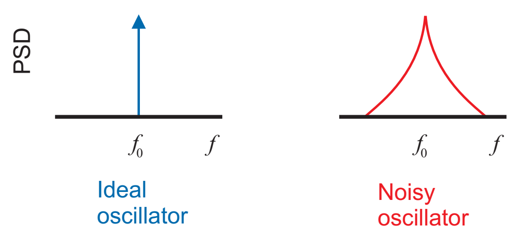


Figure 3.4: Effect of phase noise on the oscillator's power spectral density.

In frequency domain, phase noise is most often characterized in terms of single-side-band (SSB) phase noise spectrum [14, 15], defined as

$$\mathcal{L}(f) = \frac{P_{\mathrm{SSB}}(f_0 + f)}{P_{\mathrm{Total}}}, \tag{3.2}$$

where $P_{\mathrm{SSB}}(f_0 + f)$ is the single-side-band power of oscillator within 1 Hz bandwidth around the offset frequency f from the central frequency f_0, and P_{Total} is the total power of the oscillator. By plotting the experimental measurements of free-running oscillators on a logarithmic scale, it is possible to see that $\mathcal{L}(f)$ usually follows slopes of -30 dB/decade and -20 dB/decade, until a flat noise floor is reached at higher frequency offsets. Fig. 3.5 shows the measured SSB phase noise spectrum from a GaN HEMT MMIC oscillator.

2 Phase Noise Generation

Oscillator phase noise originates from the noise inside the circuitry. The internal noise sources can be categorized as white (uncorrelated) and colored (correlated) noise sources [14,

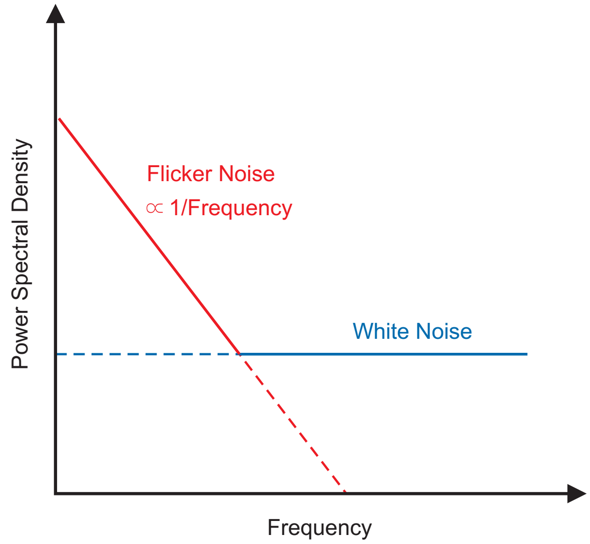


Figure 3.6: Power spectral density of noise inside the oscillator circuitry, plotted on a logarithmic scale.

15, 42]. A white noise process has a flat power spectral density (PSD), while this is not the case for colored noise sources. Noise sources such as thermal noise and shot noise inside the devices are modeled as white, while substrate and supply-noise sources, as well as low-frequency noise, are modeled as colored sources. Thermal noise is caused by Brownian motion of free electrons [43]. The shot noise represents the small current fluctuations due to finite charge of an electron and the randomness of the electron emission [44]. A significant part of the colored noise sources inside the circuit can be modeled as flicker noise [15]. Fig. 3.6, shows a typical PSD of the noise sources inside the oscillator circuitry, plotted on a logarithmic scale.

The phase noise generation mechanism in the oscillator is usually explained as the integration of the white and colored noise sources [14, 42]. However, this cannot describe the entire frequency properties of the practical oscillators. Hence, we consider a more general model in our analysis, where phase noise is partially generated from the integration of the circuit noise, and the rest is a result of amplification/attenuation of the noise inside the circuitry. For example, phase noise with −30 and −20 dB/decade slopes originate from integration of flicker noise (colored noise) and white noise, respectively

$$\phi(t) = \int_0^t \Phi(\tau)\mathrm{d}\tau. \tag{3.3}$$

Here, $\Phi(\tau)$ denotes the superposition of white and colored noise sources inside the oscillator circuitry. Due to the integration, $\phi(t)$ has a *cumulative* nature [14, 45]; phase noise accumulation over the time delay T

$$\Delta(t, T) = \phi(t) - \phi(t-T) = \int_{t-T}^t \Phi(\tau)\mathrm{d}\tau, \tag{3.4}$$

is usually called the phase noise increment process [19], self-referenced phase noise [46], the differential phase noise process [47], or the phase noise innovation process [16, 19, 48].

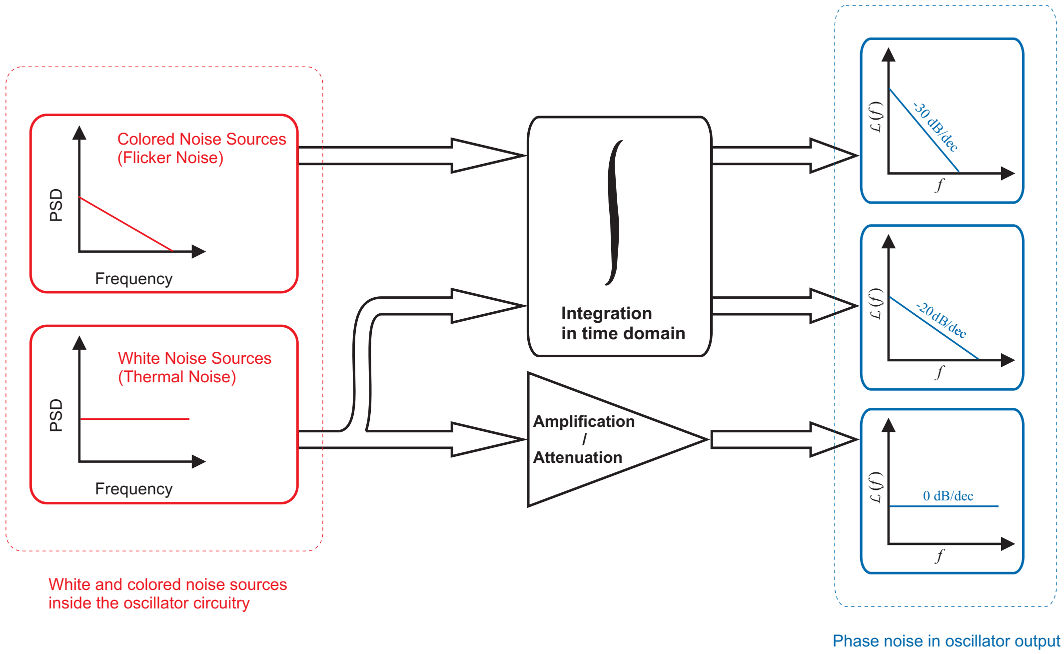


Figure 3.7: Phase noise generation mechanism.

The flat noise floor in the PSD, also known as white phase noise, originates directly from the thermal noise and the shot noise. Fig. 3.7 illustrates the phase noise generation mechanism.

In our analysis we are interested in the PSD of the phase noise process denoted as $S_\phi(f)$. It is possible to show that at high offset frequencies, $S_\phi(f)$ is well approximated with SSB phase-noise spectrum $\mathcal{L}(f)$ [14, 45, 49, 50]

$$S_\phi(f) \approx \mathcal{L}(f) \text{ for large } f. \tag{3.5}$$

The offset frequency range where this approximation is valid depends on the phase noise performance of the studied oscillator [51]. For wideband systems, the final system performance is not sensitive to close-in phase noise (phase noise at low frequency offsets) due to a large channel width [19]. Thus for low frequency offsets, $S_\phi(f)$ can be modeled in such a way that it follows the same slope as that of higher frequencies.

3 Phase-Locked Loop (PLL)

In many practical systems, the free running oscillator is stabilized inside a phase-locked loop (PLL) [14,52–54]. A PLL is a feedback loop that compares the phase of a free running oscillator (usually a voltage-controlled oscillator) with that of a reference oscillator in order to minimize the deference.[1] It basically locks the phase and frequency of a free-running oscillator to that of a reference signal. A PLL architecture that is widely used in modern communication systems is shown in Fig. 3.8. The output phase of the free-running oscillator is scaled down by a frequency divider, then compared with phase of a reference

[1]The PLL, which is discussed here, should not be mistaken for trackers used at the receiver to lock to the received carrier phase and frequency. Such trackers will be discussed in detail in Chapter 6.

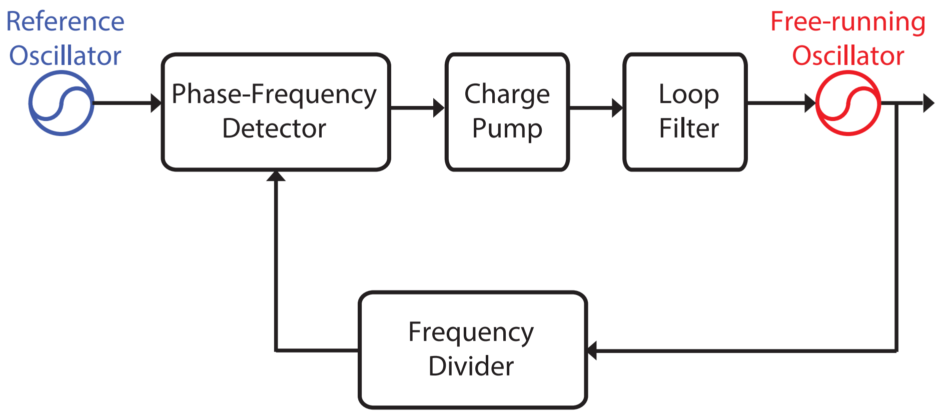


Figure 3.8: Schematic view of a charge-pump PLL.

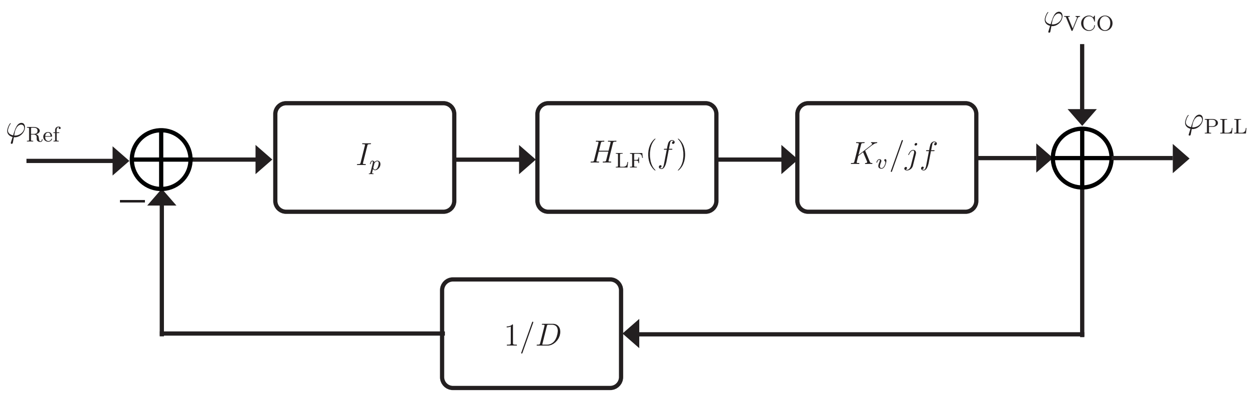


Figure 3.9: LTI phase-domain model for the charge-pump PLL.

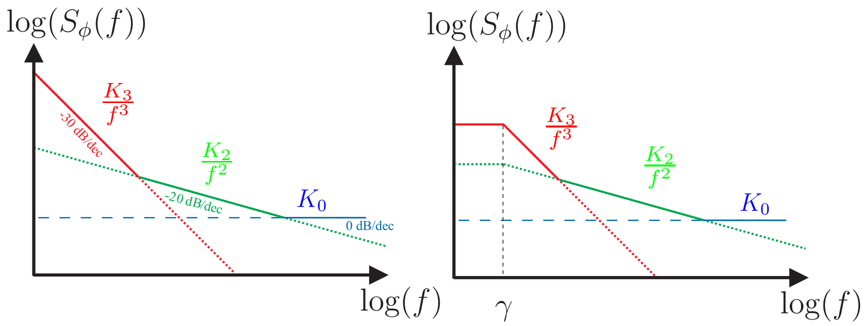
oscillator by means of a phase-frequency detector and an error signal is generated. The charge pump converts the error signal to an error current, then it is low-pass filtered to generate a control signal for the free-running oscillator [14, 46, 55–58].

Noise sources inside any of the components in the PLL loop may contribute to the output phase noise of the PLL. A simplified LTI phase-domain model for the charge-pump PLL is shown in Fig. 3.9. The phase variable φ_{PLL} is phase noise in the PLL output, φ_{Ref} represents the phase noise process of the reference oscillator, and φ_{VCO} is the phase noise of the free-running oscillator (a VCO here). The charge pump component is models as a gain block with gain I_p, the loop filter transfer function, which usually has a low-pass characteristics, is denoted as $H_{\mathrm{LF}}(f)$, and the VCO is modeled as an integrator with transfer function K_v/jf. The transfer functions from φ_{Ref} and φ_{VCO} to the output φ_{PLL} are calculated as

$$H_{\mathrm{Ref}}(f) = \frac{DI_pK_vH_{\mathrm{LF}}(f)}{j2\pi fD + I_pK_vH_{\mathrm{LF}}(f)} \tag{3.6}$$

$$H_{\mathrm{VCO}}(f) = \frac{j2\pi fD}{j2\pi fD + I_pK_vH_{\mathrm{LF}}(f)}, \tag{3.7}$$

respectively. The reference noise is amplified by the frequency divider and is low-pass filtered in the loop. On the other hand, the PLL behaves as a high-pass filter for the noise from the free-running oscillator [50, 57]. Thus in general, above a certain frequency, PSD of phase noise at the output of the PLL is identical to that of the free-running oscillator, while it follows the reference below that frequency. In [57, 58], comprehensive models for the PSD of phase noise in PLL-stabilized oscillators are proposed and analyzed. The provided models consider the effect of various noise sources inside the oscillator circuitry as well as the noise from other components of the PLL. However, here, only the noise sources from the free-running oscillator are considered, and the output PSD is modeled as constant below a certain frequency [14, 46, 55, 56]. The provided model can easily be generalized to include the noise from other components. Fig. 3.10 shows the spectrum model for a free and a PLL-locked oscillator, ignoring the effect of reference noise.



(a) (b)

Figure 3.10: Phase noise PSD of a typical oscillator. (a) shows the PSD of a free running oscillator. (b) is a model for the PSD of a locked oscillator, where γ is the PLL loop's bandwidth. It is considered that the phase noise of the reference oscillator is negligible compared to the phase noise of the free running oscillator in the PLL.

In general for both free-running and PLL-stabilized oscillators, we model the PSD of $\phi_3(t)$, $\phi_2(t)$, and $\phi_0(t)$ as modified power-law spectrums [45]:

$$S_{\phi_3}(f) = \frac{K_3}{\gamma^3 + f^3}, \quad S_{\phi_2}(f) = \frac{K_2}{\gamma^2 + f^2}, \quad S_{\phi_0}(f) = K_0, \tag{3.8}$$

where K_3, K_2 and K_0 are the PSD levels, and γ is a low cut-off frequency. Assuming independent noise sources, the total PSD of phase noise is given by

$$S_\phi(f) = S_{\phi_3}(f) + S_{\phi_2}(f) + S_{\phi_0}(f). \tag{3.9}$$

CHAPTER 4

COMMUNICATION SYSTEM MODELS IN PRESENCE OF PHASE NOISE

1 SISO Channel

Consider a single-carrier single-antenna communication system. The transmitted signal $x(t)$ is

$$x(t) = \sum_{n=1}^{N} s[n]p(t - nT), \tag{4.1}$$

where $s[n]$ denotes the modulated symbol from constellation $\mathcal{C}$ with an average symbol energy of E_s, n is the transmitted symbol index, $p(t)$ is a bandlimited square-root Nyquist shaping pulse function with unit-energy, and T is the symbol duration [59]. The continuous-time complex-valued baseband received signal after down-conversion, affected by the transmitter and receiver oscillator phase noise, can be written as

$$r(t) = x(t)e^{j\phi(t)} + \tilde{w}(t), \tag{4.2}$$

where $\phi(t)$ denotes the phase noise process and models the phase noise from transmitter and receiver local oscillators, and $\tilde{w}(t)$ is zero-mean circularly symmetric complex-valued additive white Gaussian noise (AWGN), that models the effect of noise from other components of the system. Note that in (4.2), the channel gain between the transmit and receive antennas is assumed deterministic, constant, and without loss of generality it is here set equal to unity.

The received signal (4.2) is passed through a matched filter $p^*(-t)$ and the output is

$$\begin{aligned} y(t) = &\int_{-\infty}^{\infty} \sum_{n=1}^{N} s[n]p(t - nT - \tau)p^*(-\tau)e^{j\phi(t-\tau)}\mathrm{d}\tau \\ &+ \int_{-\infty}^{\infty} \tilde{w}(t-\tau)p^*(-\tau)\mathrm{d}\tau. \end{aligned} \tag{4.3}$$

Assuming phase noise does not change over the symbol duration, but changes from one symbol to another so that no intersymbol interference arises, sampling the matched filter output (4.3) at nT time instants results in

$$y(nT) = s[n]e^{j\phi(nT)} + w(nT). \tag{4.4}$$

We change the notation as

$$y[n] = s[n]e^{j\phi[n]} + w[n], \tag{4.5}$$

where $\phi[n]$ represents the phase noise of the nth received symbol in the digital domain that is bandlimited after the matched filter, and $w[n]$ is the filtered (bandlimited) and sampled version of $\tilde{w}(t)$ that is a zero-mean circularly symmetric complex-valued AWGN with variance σ_w^2. Note that in this thesis, the focus is on oscillator phase synchronization and, hence, similar to several former studies (e.g., [60–64] and references therein), time synchronization is assumed to be perfect.

The slow-varying phase noise assumption and the introduced discrete-time model (4.5), are well accepted in related literature and several practical scenarios (see e.g., [9, 11, 12, 26–28, 30, 32, 33, 60–69]). However, the reader is referred to the recent studies of this model, where the phase noise variations over the symbol period has also been taken into consideration, and the possible loss due to the slow-varying phase noise approximation has been investigated [70–74]. Nevertheless, according to our studies and also results of [72, 73], the loss is negligible for standard oscillators.

2 Multiple-Antenna Channels

In multiple-antenna systems, phase noise acts differently depending on whether the RF circuitries connected to each antenna are driven by separate (independent) local oscillators (SLO) or by a common local oscillator (CLO) [35, 75–78] (see Fig. 4.1). The CLO configuration is intuitively more appealing because it results in a less-complex less-expensive hardware and also a single phase-noise process to be tracked. However, the SLO configuration is unavoidable when the large spacing between antennas is needed in order to exploit the available spatial degrees of freedom, and, hence, achieve multiplexing or diversity gains [79, 80].

Now consider an $1 \times M$ single-input multiple-output (SIMO) system, affected by phase noise. By assuming root-Nyquist pulses for transmission, match filtering followed by sampling at the receiver, the received signal model is given by

$$\mathbf{y}[n] = \mathbf{h}\boldsymbol{\Theta}[n]s[n] + \mathbf{w}[n]. \tag{4.6}$$

Here, the phase rotation matrix is defined as $\boldsymbol{\Theta}[n] = \mathrm{diag}\{e^{j\phi_{\mathrm{r}1}}[n], \ldots, e^{j\phi_{\mathrm{r}M}[n]}\} \times e^{j\phi_{\mathrm{t}}[n]}\mathbf{I}$, where $\phi_{\mathrm{t}}[n]$ denotes the phase noise sample from the transmitter oscillator and $\phi_{\mathrm{r}m}[n],\ m \in \{1, \ldots, M\}$, is the phase noise sample from the oscillator connected to the mth receive antenna, and $\mathbf{I}$ denotes the $M \times M$ identity matrix. Note that for the CLO configuration, the phase rotation matrix simplifies to a scalar $\boldsymbol{\Theta}[n] = e^{j(\phi_{\mathrm{r}}[n]+\phi_{\mathrm{t}}[n])}$. The vector $\mathbf{h} = [h_1, \ldots, h_M]$ contains path-loss coefficients between the transmit and receive antennas, which in this thesis, are assumed to be deterministic, time-invariant, and known to the transmitter and the receiver. Finally, the vector $\mathbf{w}[n] = [w_1[n], \ldots, w_M[n]]$ contains the AWGN samples.

For a $M \times 1$ multiple-input single-output (MISO) phase-noise channel, the received signal is written as

$$y[n] = \mathbf{h}^T \mathbf{\Theta}[n]\mathbf{s}[n] + w[n]. \tag{4.7}$$

Here, the phase rotation matrix is defined as $\mathbf{\Theta}[n] = \mathrm{diag}\{e^{j\phi_{\mathrm{t}1}[n]}, \ldots, e^{j\phi_{\mathrm{t}M}[n]}\} \times e^{j\phi_{\mathrm{r}}[n]}\mathbf{I}$, where $\phi_{\mathrm{t}m}[n], \in \{1, \ldots, M\}$, denotes the phase noise sample from the oscillator connected to the mth transmit antenna, and $\phi_{\mathrm{r}}[n]$ is the phase noise of oscillator at the receiver. The path-loss vector $\mathbf{h}$ is defined similar to that of the SIMO case.

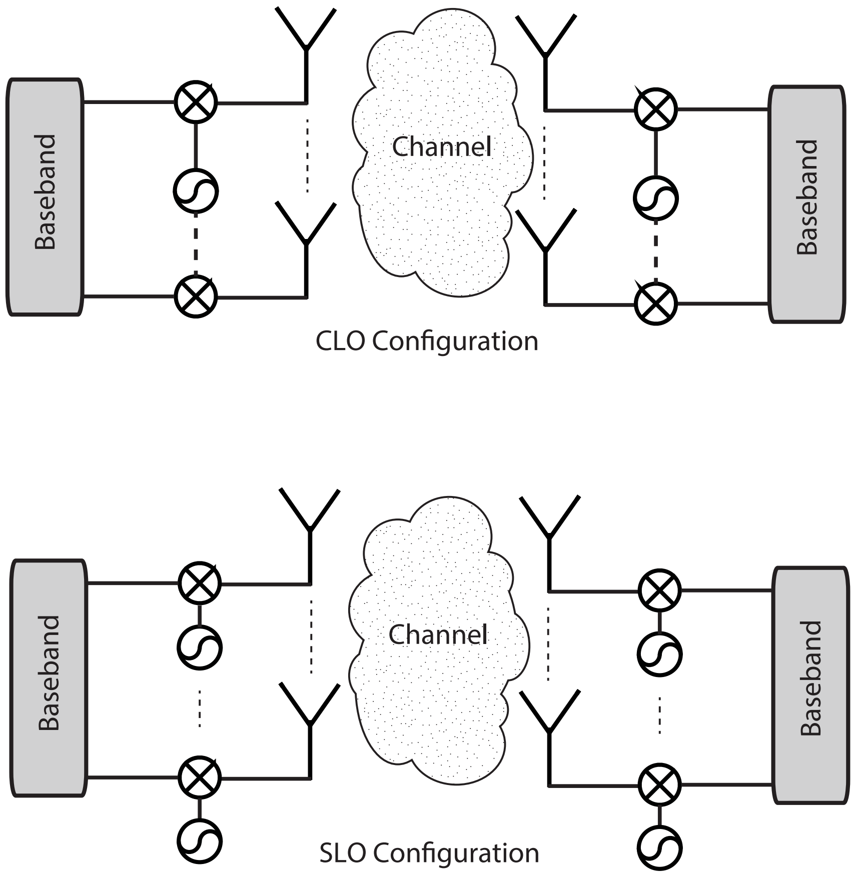


Figure 4.1: The CLO vs. the SLO configuration.

CHAPTER 5

DISCRETE-TIME PHASE NOISE MODELS

Accurate mathematical models of phase noise are required in order to analyze the performance of phase-noise-affected communication systems, as well as for design of algorithms to mitigate the effect of phase noise.

1 Available Models in the Literature

In this section, an overview of the mathematical models used for modeling of phase noise in digital communication systems is provided. The models range from simple to more comprehensive and accurate models.

Noncoherent Systems: Consider a case where phase noise is modeled as a stationary memoryless process with marginal distribution over $[0, 2\pi)$. This scenario models a noncoherent communication system, where the phase of the transmitted signal cannot be used to transmit information [81]. This model is often used in order to simplify the information theoretic analysis. However, it is oversimplified and does not capture the characteristics of phase noise in real oscillators. Hence, uniform phase noise model cannot be used for designing algorithms to compensate phase noise in communication systems.

Partially Coherent Systems: When a phase tracker/estimator (e.g., a PLL that is locked to the received carrier) is employed at the receiver, the output signal after phase tracking is impaired only by residual phase errors. Systems employing phase trackers are sometimes referred to as partially coherent [81]. It is often assumed that the residual phase-error process, denoted as $\theta[n]$, is stationary and memoryless. This is a fairly valid assumption when an ideal phase tracker is used that can minimize the memory of the residual phase noise process. The statistic of residual phase errors is modeled by a Tikhonov distribution (von Mises distribution)

$$f_\theta(\theta) = \frac{e^{\lambda \cos\theta}}{2\pi I_0(\lambda)}, \tag{5.1}$$

where $1/\lambda$ is the variance of θ, which depends on oscillator phase noise quality and also on the parameters of the tracker [82]. It is worth mentioning that this distribution is closely approximated by the wrapped Gaussian distribution.

Phase Noise with Memory: The case of the phase-noise process with memory is relevant when a free running oscillator is used or when the phase tracker/PLL is not able to completely remove the memory of the phase-noise process [19, 24]. In this case, the phase noise samples are typically modeled using a random-walk process [14, 24, 46, 83], according to which

$$\phi[n+1] = (\phi[n] + \Delta[n]) \bmod (2\pi), \tag{5.2}$$

where $\Delta[n]$ are zero-mean Gaussian random samples.

Fig. 5.1 compares the histograms of two time-separated frames of phase noise increment samples from a GaN HEMT MMIC oscillator. This observation shows that the phase noise increments can be reasonably well modeled with a Gaussian distribution. It can also be seen that the mean and variance of the samples within the frames are almost equal and do not change over time.

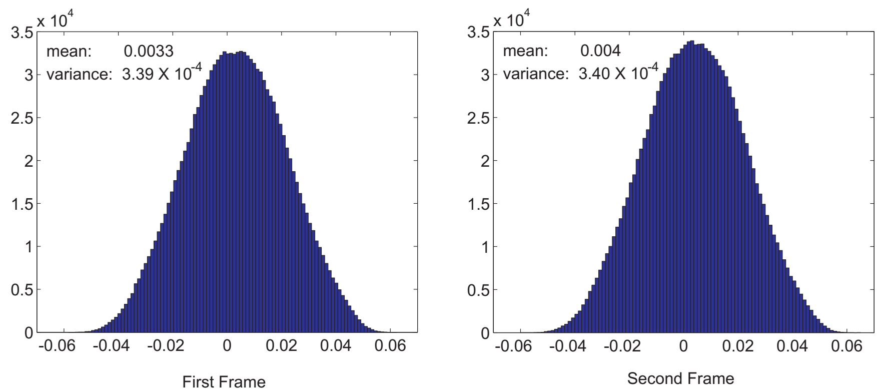


Figure 5.1: Histograms of two time-separated frames of phase noise increment samples from a GaN HEMT MMIC oscillator. The mean corresponds to a frequency offset.

The Wiener process is extensively used in the literature for modeling phase noise. This model considers independent samples of $\Delta[n]$, drawn from a $\mathcal{N}(0, \sigma_\Delta^2)$ distribution. Hence, the sequence $\{\theta[n]\}$ is a Markov process, i.e.,

$$f_{\phi[n]|\phi[n-1],\ldots,\phi[0]} = f_{\phi[n]|\phi[n-1]} = f_\Delta, \tag{5.3}$$

where the wrapped Gaussian distribution

$$f_\Delta(\delta) = \sum_{k=1}^{\infty} \frac{1}{\sqrt{2\pi\sigma_\Delta^2}} \exp\left(-\frac{(\delta - 2\pi l)^2}{2\sigma_\Delta^2}\right), \quad \delta \in [0, 2\pi) \tag{5.4}$$

is the pdf of the innovation Δ modulus 2π. Note that the probability of wrapping increases when the innovation variance σ_Δ^2 is high. For the oscillators commonly used in wireless transceivers, σ_Δ^2 is in a range that Δ can be well approximated by a an unwrapped Gaussian random variable [48, Fig. 2].

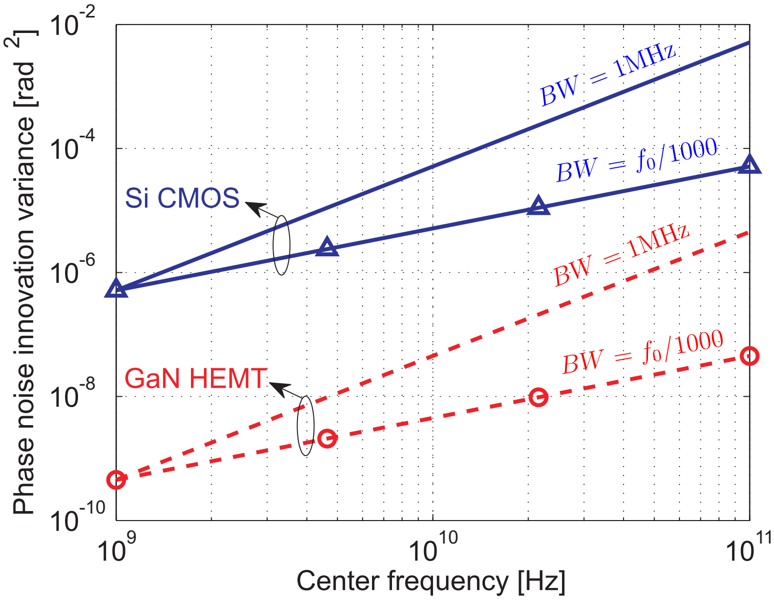


Figure 5.2: Phase noise innovation variance σ_Δ^2 for Si CMOS and GaN HEMT technologies versus the center frequency f_0 of the oscillator.

The Wiener process is an easy model to use in analysis and simulations. However, it is only accurate when free-running oscillators are considered and only white noise sources inside oscillator circuitry contribute to phase noise [14]. This corresponds to the case, where the phase noise PSD follows a -20 dB/decade slope for all offset frequencies. The innovation variance in this case is equal to [48]

$$\sigma_\Delta^2 = c f_0^2 T, \tag{5.5}$$

where f_0 is the operating center frequency, T is the sampling interval, and c is a constant that depends on design parameters of the oscillator such as the quality factor of the resonator, operating currents and voltages, etc. As observed from (5.5), the phase noise innovation variance grows quadratically with f_0 and decreases linearly with bandwidth $BW = 1/T$. In Fig. 5.2, the lowest possible σ_Δ^2 that can be achieved for free-running oscillators in Si CMOS [84] and GaN HEMT [85] technologies is illustrated against f_0. Two scenarios are considered; in the first case a fixed bandwidth is used, $BW = 1$MHz. In the second case BW is linearly increased with f_0, to study the case of a constant relative bandwith. When BW is fixed, the innovation variance grows quadratically with f_0, while it scales linearly when $BW = 0.001 f_0$.

The Winer process can also be used as an approximate model for phase noise in presence of colored noise sources or for PLL stabilized oscillators (for any low-pass phase noise process). However, σ_Δ^2 should be adjusted such that the approximation error of the model is minimized.

2 The Proposed Phase Noise Model

In order to study the effect of phase noise and to design algorithms for mitigation of its effects, models that can accurately capture the characteristics of this phenomenon are required. As mentioned, in most prior studies, the effects of phase noise are studied using simple models, e.g, the Wiener process [14, 16, 28, 30, 62, 68, 71–73]. The Wiener process does not take into account colored noise sources [42], and, hence, cannot describe frequency and time-domain properties of phase noise properly [16, 17, 19, 86]. This motivates the use of more realistic phase noise models in study and design of communication systems.

In **Paper A**, a time-domain model of phase noise is proposed, which captures the effect of both white and colored noise sources inside the oscillator. Assuming independent noise sources, phase noise is modeled as a superposition of three independent processes

$$\phi[n] = \phi_3[n] + \phi_2[n] + \phi_0[n], \tag{5.6}$$

where $\phi_3[n]$ and $\phi_2[n]$ are random-walk processes with Gaussian increments

$$\phi_3[n+1] = \phi_3[n] + \Delta_3[n] \qquad \phi_2[n+1] = \phi_2[n] + \Delta_2[n], \tag{5.7}$$

and model phase noise that originates from integration of flicker noise (colored noise) and white noise sources, respectively. Further, $\phi_0[n]$ is a zero-mean white Gaussian process and models the direct effect of white noise sources. The performance of a phase-noise-affected communication systems depends on the statistics of phase noise. In **Paper A**, the phase-noise statistics for the proposed phase noise spectrum model (3.9) are derived. Some of the preliminaries and results are presented in the following.

The variance of the white Gaussian noise process $\phi_0[n]$ is given by

$$\sigma^2_{\phi_0} = \frac{K_0}{T}, \tag{5.8}$$

where K_0 is the level of the noise floor that can be found from the measured PSD. For the integrated noise, major focus of former studies has been on mathematical calculation of the variance of the phase-noise increments. Based on our analysis, phase-noise increments can be correlated over time in presence colored noise sources, or when a PLL-stabilized oscillator is considered. Hence, both the variance and correlation properties of the phase-noise increments are required for an accurate modeling.

The autocorrelation function of $\Delta_2[m]$ is computed as

$$R_{\Delta_2}[m] = \frac{K_2\pi}{\gamma}\left(2e^{-2\gamma\pi T|m|} - e^{-2\gamma\pi T|m-1|} - e^{-2\gamma\pi T|m+1|}\right), \tag{5.9}$$

where K_2 is the PSD level and can be found from the measured spectrum. For a PLL-stabilized oscillator γ can be set to the PLL bandwidth, and for a free-running oscillator, the autocorrelation function can be found by taking the limit of (5.9) as γ approaches 0 that results in

$$R_{\Delta_2}[m] = \begin{cases} 4K_2\pi^2 T & \text{if } m = 0 \\ 0 & \text{otherwise} \end{cases}. \tag{5.10}$$

Results in (5.9) and (5.10) show that for a PLL-stabilized oscillator samples of $\Delta_2[n]$ process are correlated over time, while they are uncorrelated for a free-running oscillator. Fig. 5.3 compares $R_{\Delta_2}[m]$ of a free-running and a PLL-stabilized oscillator. The phase noise PSD level is $K_2 = 25$ rad^2 Hz, sampling time $T = 10^{-6}$ sec, and the PLL bandwidth $\gamma = 1$ MHz. Observe that for a free-running oscillator $R_{\Delta_2}[m \neq 0] = 0$, while it is not the case for a PLL-locked oscillator.

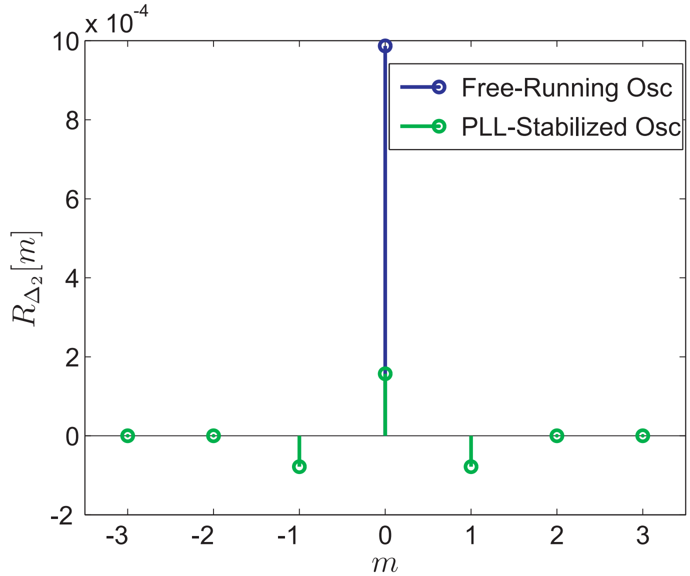


Figure 5.3: The autocorrelation function of $\Delta_2[m]$ for a free-running and a PLL-stabilized oscillator.

The autocorrelation function of $\Delta_3[m]$ is given by

$$R_{\Delta_3}[m] \approx \begin{cases} -8K_3\pi^2T^2\left(\Lambda + \log(2\pi\gamma T)\right), & \text{if } m = 0 \\ -8K_3\pi^2T^2(\Lambda + \log(8\pi\gamma T)), & \text{if } m = \pm 1 \\ -8K_3\pi^2T^2\Big[-m^2(\Lambda + \log(2\pi\gamma T|m|)) & \\ \qquad +\dfrac{(m+1)^2}{2}(\Lambda + \log(2\pi\gamma T|m+1|)) & \text{otherwise} \\ \qquad +\dfrac{(m-1)^2}{2}(\Lambda + \log(2\pi\gamma T|m-1|))\Big], & \end{cases} \tag{5.11}$$

where $\Lambda \triangleq \Gamma - 3/2$, and $\Gamma \approx 0.5772$ is the Euler-Mascheroni constant [87], K_3 can be found from the measurements. The cut-off frequency γ must be set to a small value for a free running oscillator, while for a PLL-stabilized oscillator it can be set to the PLL bandwidth. Note that in (5.11), a quadratic approximation of the autocorrelation function is provided. This approximation is accurate when $T|m|$ is small. Fig. 5.4 illustrates how the relative error of the approximation scales by $T|m|$. The exact value of $R_{\Delta_3}[m]$ is found by numerical integration. Fig. 5.5 shows the evaluated $R_{\Delta_3}[m]$ for a free-running oscillator. It can be observed from the figure that the samples of $\Delta_3[m]$ are highly correlated.

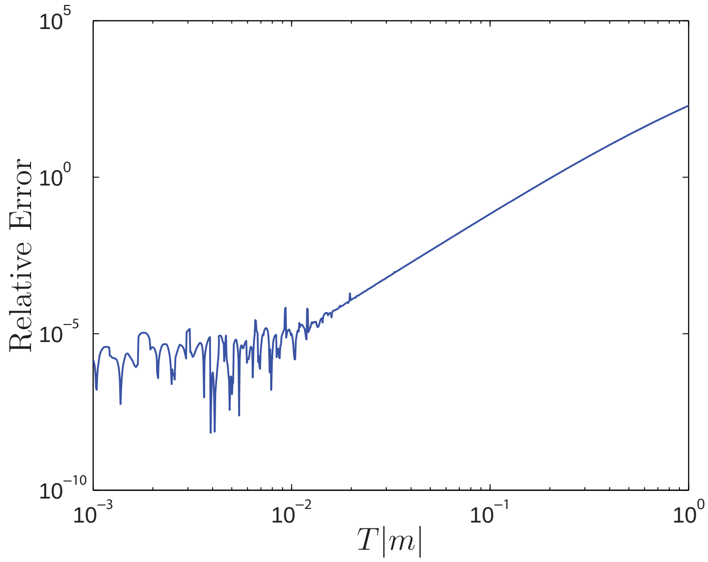


Figure 5.4: Relative error in quadratic approximation of $R_{\Delta_3}[m]$ vs. $T|m|$. Here, $\gamma = 1$ Hz.

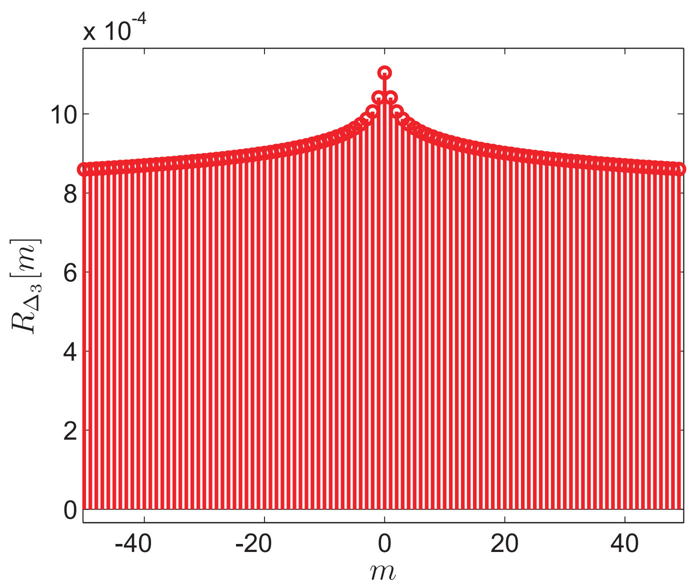


Figure 5.5: Autocorrelation of $\Delta_3[m]$, showing that the samples are highly correlated.

CHAPTER 6

PHASE NOISE COMPENSATION

1 Overview

Design of communication systems in presence of phase noise has been an active field of research during the last decades. The ultimate goal of such studies is to achieve a performance close to that of the coherent systems. Various techniques are proposed to mitigate the impairing effects of phase noise. The commonly used methods can be categorized into transmitter-side and receiver-side techniques.

In transmitter-side techniques, the transmitted signal is manipulated before transmission to make it more immune to distortions from phase noise. Constellation design [29], and differential modulation [88] fit within this category.

The receiver-side techniques are mainly based on tracking/estimation of phase noise. Tracking methods (e.g., phase tracking loops [89]) reduce the phase variation of the received signal. Estimators provide an estimated value of phase noise or its statistics, which can further be used for symbol detection [28]. For example, estimation of the phase noise and cleaning the received signal, followed by a coherent detector or a phase-noise-optimized detector [28] is proposed in [67]. Joint phase noise estimation-symbol detection, as well as iterative estimation-detection methods are other receiver-side methods [28, 30, 62, 66, 90]. In iterative methods, usually pilot symbols are used to generate an initial estimate of phase noise. The initial estimates (or their statistics) are employed by a detector for a preliminary detection. The detected symbols along with pilots will be used for the next round of phase-noise estimation. Iteration continues till a convergence criterion is satisfied [18,90]. Fig. 6.1 shows a simplified sketch of a communication receiver employing an iterative phase noise estimation-data detection algorithm.

Phase noise estimation and tracking algorithms can be categorized based on how they treat the phase-noise parameter. The so called Bayesian methods consider phase noise as a random variable with a given prior distribution, while in the non-Bayesian approaches phase noise is considered as a deterministic parameter [91,92]. The non-Bayesian methods use only the received signal for tracking/estimation, while the Bayesian methods employ a priori statistical information of the phase noise process as well. In the following, examples of non-Bayesian and Bayesian phase-noise tracker/estimators are presented.

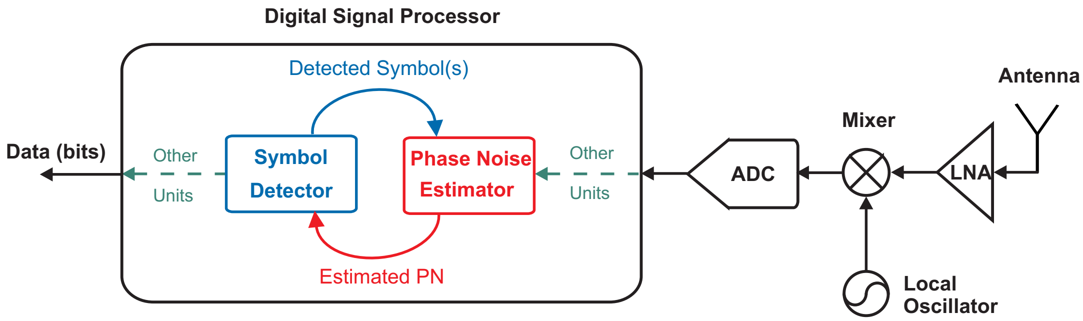


Figure 6.1: Receiver of a communication system designed for the phase-noise channel.

2 Non-Bayesian Methods

As mentioned before, the non-Bayesian methods use only the received signal that is a noisy observation of the phase-noise parameter. A simple non-Bayesian method of phase noise estimation is block averaging (time averaging) [26]. The phase-noise estimate is given by the average of the phase of the received signal. However, such method is only applicable in slow-varying phase-noise scenarios, where phase noise stays almost constant over a block of transmitted symbols.

Phase tracking loops are another type of non-Bayesian methods [89]. Fig. 6.2 shows the sketch of a first-order tracking loop. Similar to the block averaging method, it is assumed that the data symbols are known, which is motivated by use of pilot symbols or a priori detected symbols.

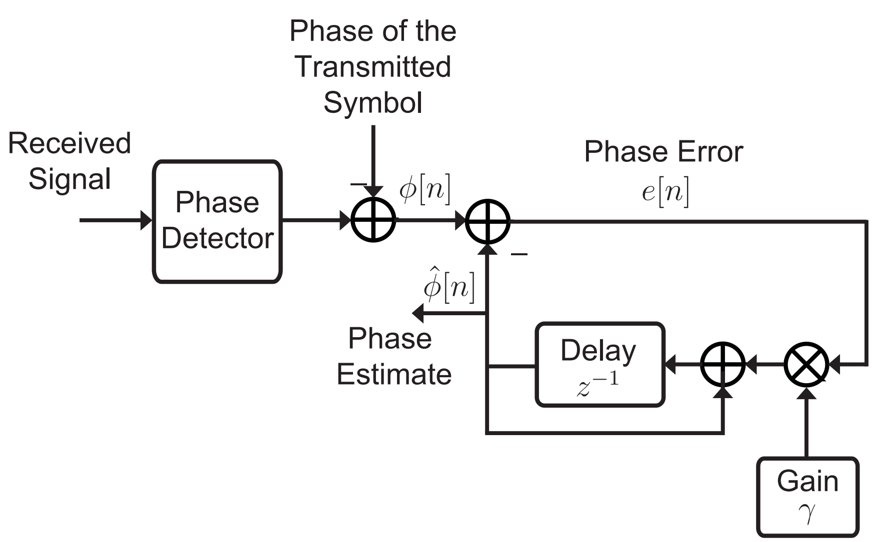


Figure 6.2: First-order phase tracking loop.

The first-order tracking loop, first generates a phase error signal, which is the difference between the input phase and the current estimate

$$e[n] = \phi[n] - \hat{\phi}[n]. \tag{6.1}$$

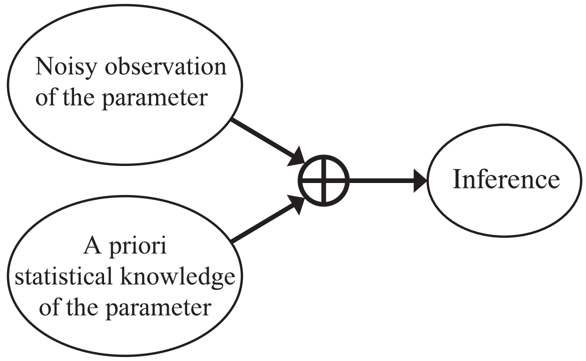


Figure 6.3: Bayesian inference.

Note that the phase distortion due to AWGN is neglected. The phase estimate for the next time instant is given by

$$\hat{\phi}[n+1] = \hat{\phi}[n] + \gamma e[n], \tag{6.2}$$

where γ is a damping gain (a step size). The reader is referred to [89] and references therein for more detailed discussions on phase tracking loops.

In [67], another non-Bayesian phase noise estimator is proposed, which is based on approximation of the phase-noise trajectory over a block by means of discrete cosine transform (DCT) basis functions. Pilot symbols are used to estimate the DCT coefficients. The approximated trajectory is later used to estimate phase noise of the data symbols. The algorithm is used for synchronization of OFDM systems in [93], and it is extended to an iterative scheme in [90].

3 Bayesian Methods

In the Bayesian framework, unknown parameters are considered as random. The Bayesian approach perfectly fits the phase noise estimation problem, because phase noise is a random phenomenon by its physical nature. In Bayesian analysis, the knowledge about the random parameter of interest is mathematically summarized in a prior distribution. In phase noise inference problems, the prior distribution can be chosen subjectively by studying the physical characteristics of this phenomenon. This further motivates the accurate statistical modeling of phase noise. The ultimate goal in Bayesian estimation is to find an estimate that minimizes a Bayesian risk function [91]. The Bayesian risk functions are usually defined as the expectation of a specific cost functions. In this thesis, we are mainly interested in quadratic and uniform (hit-or-miss) cost functions [91, 92]. The minimizer of the quadratic risk is called the Bayesian minimum mean square error (MMSE) estimator, and it is possible to show that this estimator is equal to the mean of the posteriori distribution. Kalman filter is an example of MMSE estimators [92]. Further, the minimizer of the uniform risk is equal to the mode of the posteriori distribution and it is called maximum a posteriori (MAP) estimator.

In particular, when the linear Gaussian model is adopted, the mean and the mode of the posterior distribution are identical and the MAP estimator performs similar to

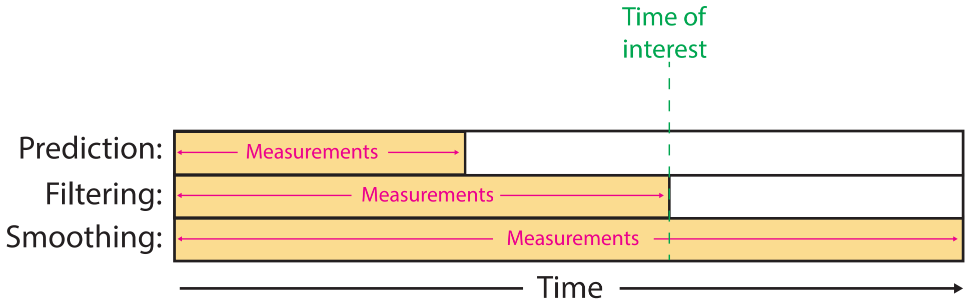


Figure 6.4: Prediction, filtering, and smoothing.

the MMSE estimator. There are other special scenarios where the MAP estimator also minimizes the quadratic risk function and it is optimal in the MMSE sense (e.g., when mean and mode of the posteriori are equal).

Three general design possibilities for phase-noise estimators are as follows: prediction, filtering, and smoothing (see Fig. 6.4). In prediction, current and past observations (measurements) are used to predict the phase noise parameter in the future, while they are used in filtering for estimation of current phase-noise sample. Smoothing algorithms use a block of observations to find an estimate of phase-noise samples over the block. Prediction and filtering can be done in real time, while to perform the smoothing a complete block of measurements is needed. Smoothing algorithms are usually more complex compared to filtering and prediction. However, as more measurement samples are available, performance of smoothers generally outperforms that of prediction and filtering.

Kalman filter is a Bayesian filtering algorithm that employs observations along with the statistics of the parameter of the interest [92]. The algorithm consists of two steps: a prediction step, where the parameter's statistic (state equation) is used to make an initial estimate of the parameter and its statistic. In the next step, this estimate will be updated by using the observations. Kalman filter combines the information from observations and the parameter's statistics in a smart way by means of an adaptive gain. More specifically, the filter relies more on observations when they are less noisy, otherwise it follows the parameters statistics.

The observation and state equations in the phase-noise problem are given by

$$y[n] = s[n]e^{j\phi[n]} + w[n] \tag{6.3}$$

$$\phi[n] = \phi[n-1] + \Delta[n-1]. \tag{6.4}$$

For linear Gaussian models, Kalman filter is an optimal MMSE estimator. However as we see in (6.3), the observation is a nonlinear function of the phase-noise parameter $\phi[n]$. To resolve the nonlinearity problem, an extended Kalman filter (EKF) is proposed, which basically linearizes the observation function around the predicted value of the phase noise parameter at each time instant.

Another suggested method for handling the nonlinearity of the phase noise problem in the Kalman filtering is to track the phasor instead of the phase noise parameter. This will resolve the nonlinearity in the observation. However, a state equation that models the evolution of the phasor must be found. For this purpose, the phasor $g[n] \triangleq e^{j\phi[n]}$ is approximated as a 1st order autoregressive (AR) process

$$g[n] = \rho g[n-1] + v[n-1], \tag{6.5}$$

where $v \sim \mathcal{CN}(0, \sigma_v^2)$. The variance of the process noise σ_v^2 is minimized by setting $\rho = e^{-\sigma_\Delta^2/2}$.

Note that both methods mentioned for handling the non-linearity are based on approximations. Such approximations are accurate when the phase-noise innovation variance is small.

When a block of received symbols are available, it is possible to employ a smoothing algorithm. The Rauch–Tung–Striebel (RTS) smoother [94] is an example of such an algorithm. It consists of a forward step, which is identical to the Kalman filter, as well as a backward smoothing step.

One of the main assumptions in derivation of the Kalman filter is whiteness of the process noise (consecutive samples are uncorrelated). However, as mentioned in Chapter 5, phase noise with colored noise sources is modeled by a random walk with colored increments. Hence, the Kalman filter must be designed to include the memory. In **Paper B**, we study the design of algorithms for estimation of phase noise with colored noise sources. A modified Kalman smoother (RTS smoother [94]) and a MAP estimator are proposed. To modify the Kalman algorithm, phase-noise process is approximated by a pth-order AR process. Then the state equation is modified by employing this AR model. The MAP estimator is a Bayesian estimator and uses both observations and the a priori distribution of the phase-noise process to perform the estimation.

4 Performance of Bayesian Estimators

In order to assess the estimation performance, Cramér-Rao bounds (CRBs) can be utilized, which provide a lower bound on the mean square error (MSE) of estimation [92]. In the case of random parameter estimation, e.g., phase noise estimation, the Bayesian Cramér-Rao bound (BCRB) gives a tight lower bound on the MSE [63, 91].

Consider the phase-noise estimation of a block symbols with length N. Denote the phase-noise vector, and its estimate as $\boldsymbol{\varphi}$, and $\hat{\boldsymbol{\varphi}}$, respectively. The BCRB bounds the covariance matrix of the estimation error by the following inequality

$$\mathbb{E}\left[(\hat{\boldsymbol{\varphi}} - \boldsymbol{\varphi})(\hat{\boldsymbol{\varphi}} - \boldsymbol{\varphi})^T\right] \geq \mathbf{BCRB}, \tag{6.6}$$

$$\mathbf{BCRB} = \mathbf{B}^{-1}, \tag{6.7}$$

where $\mathbf{B}$ is called Bayesian information matrix (BIM). The BIM is calculated in two parts, $\mathbf{B} = \boldsymbol{\Xi} + \boldsymbol{\Psi}$, where $\boldsymbol{\Xi}$ is derived from the likelihood function and $\boldsymbol{\Psi}$ from the prior distribution function of $\boldsymbol{\varphi}$. The reader is referred to [19, 63, 91] for detailed steps involved in derivation of BRCB.

In **Paper A**, we have derived the BCRB for estimation of phase noise with colored noise sources. The calculated BCRB directly depends on the statistics of phase noise, through the prior distribution of phase noise vector. As mentioned before, the phase-noise statistics are calculated from the PSD measurements of phase noise.

CHAPTER 7

CHANNEL CAPACITY

1 Overview

A fundamental way to assess the impact of phase noise on the throughput of communication links is to determine the corresponding Shannon capacity. Unfortunately, a closed-form expression for the capacity of wireless channels impaired by phase noise is not available. Nevertheless, asymptotic capacity characterizations for large signal-to-noise ratio (SNR) and as well as nonasymptotic capacity bounds are available in the literature. Specifically, Lapidoth [95] characterized the first two terms in the high-SNR expansion of the capacity of a SISO stationary phase-noise channel. Focusing on memoryless phase-noise channels, Katz and Shamai [81] provided upper and lower bounds on the capacity that are tight at high SNR. They showed that the capacity achieving distribution of the noncoherent memoryless channel is discrete with an infinite number of mass points. The results in [81, 95] have been generalized to block-memoryless phase-noise channels in [96, 97]. There has been limited number of studies on characterizing the capacity of channels affected by phase noise with memory. The Wiener phase noise channel belongs to this family of channels. Lapidoth results can be used to characterize the capacity of the Wiener phase noise channel at high SNR. Capacity results of [95] have been recently extended to multi-antenna systems in [9, 10, 77].

2 Definition of the Channel Capacity

Consider a communication link, where a source conveys a message M, randomly chosen from a set of finite messages, over a channel to a destination. An encoder at the source maps the message m to a length-n sequence of channel input symbols $(x_1, \dots, x_n) \in \mathcal{X}$, where $\mathcal{X}$ denotes the input alphabets. The output sequence $(Y_1, \dots, Y_n) \in \mathcal{Y}$ (were $\mathcal{Y}$ denotes the output alphabets) will be generated according to the channel probability law

$$P_{Y^n|X^n}(y_1, \dots, y_n|x_1, \dots, x_n). \tag{7.1}$$

For a memoryless channel, (7.1) can be rewritten as

$$P_{Y^n|X^n}(y_1,\ldots,y_n|x_1,\ldots,x_n) = \prod_{k=1}^{n} P_{Y|X}(y_k|x_k). \tag{7.2}$$

In many practical scenarios of interest, the choice of channel input sequence is limited to certain conditions. In this thesis we only consider the average-power constraint, where

$$\frac{1}{n}\sum_{k=1}^{n}\mathbb{E}\left[|x_k|^2\right] \leq 2\rho. \tag{7.3}$$

For memoryless channels with finite discrete input and output alphabets, Shannon showed that capacity is [98]

$$C = \max_{P_X} I(Y;X), \tag{7.4}$$
$$\tag{7.5}$$

where maximum is over all probability distributions on X that satisfy

$$\mathbb{E}_{P_X}\left[|x_k|^2\right] \leq E_\mathrm{s}. \tag{7.6}$$

Shannon's result has been generalized by Dobrušin [99] to channels with memory and continuous alphabets

$$C = \lim_{n\to\infty}\frac{1}{n}\sup_{P_{X^n}\in\mathcal{P}(\mathcal{X}^n)} I(X^n;Y^n), \tag{7.7}$$

where $\mathcal{P}(\mathcal{X}^n)$ is the set of all probability distributions P_{X^n} on X^n that satisfy the average-power constraint

$$\frac{1}{n}\sum_{k=1}^{n}\mathbb{E}_{P_{X^n}}\left[|X_k|^2\right] \leq E_\mathrm{s}. \tag{7.8}$$

2.1 Definition of Mutual Information

For notational convenience, consider the memoryless case with continuous alphabets. The mutual information term in (7.7) can be defined in terms of differential entropies as

$$\begin{aligned} I(X;Y) &= h(X) - h(X|Y) && (7.9)\\ &= h(Y) - h(Y|X). && (7.10) \end{aligned}$$

Mutual information can also be defined in terms of input-output probability measures as [100, Eq. 8.49]

$$I(X;Y) = \mathbb{E}_{P_X}\left[\mathcal{D}\Big(P_{Y|X}||P_Y\Big)\right] \tag{7.11}$$

where $\mathcal{D}(\cdot||\cdot)$ denotes the relative entropy (or Kullback-Leibler divergence) between two probability distributions [100, Eq. 8.46].

3 Bounds on the capacity

Solving the optimization in (7.7) is mathematically intractable for most interesting channel scenarios. Even numerical evaluation of such optimization is generally infeasible. Therefore, an alternative practice to identify the capacity is to obtain upper and lower bounds for it.

3.1 Lower Bound

Any choice of input probability distribution $P_X \in \mathcal{P}(\mathcal{X})$ results in a lower bound on the capacity. However, in order to obtain a tight lower bound, P_X should be chosen as close as possible to the capacity-achieving input distribution of the channel. Moreover, in order to keep the derivation steps tractable, an input distribution that results in a mathematically tractable expression of $I(X;Y)$ should be chosen.

3.2 Duality-Based Upper Bounds

Finding capacity upper bounds might be tougher, mainly because of difficulties involved in performing the maximization over all possible input distributions. Duality approach [101] is a technique that results in more tractable expressions needed for calculation of upper bounds. Mutual information (7.11) can be upper bounded by using duality as follows [101, Th. 5.1]:

$$I(X;Y) = \mathbb{E}_{P_X}\left[\mathcal{D}\Big(P_{Y|X}||P_Y\Big)\right] \tag{7.12}$$

$$= \mathbb{E}_{P_X}\left[\mathcal{D}\Big(P_{Y|X}||Q_Y\Big)\right] - \mathcal{D}\Big(P_Y||Q_Y\Big) \tag{7.13}$$

$$\leq \mathbb{E}_{P_X}\left[\mathcal{D}\Big(P_{Y|X}||Q_Y\Big)\right] \tag{7.14}$$

Here, Q_Y denotes an arbitrary distribution on the output. In (7.13), Topsøe's identity [102] is used, and (7.14) follows because of nonnegativity of relative entropy [100, Thm. 8.6.1]. Accordingly, a capacity upper bound is given by [103]

$$C = \sup_{P_X} \mathbb{E}_{P_X}\left[\mathcal{D}\Big(P_{Y|X}||P_Y\Big)\right] \tag{7.15}$$

$$\leq \sup_{P_X} \mathbb{E}_{P_X}\left[\mathcal{D}\Big(P_{Y|X}||Q_Y\Big)\right], \tag{7.16}$$

where in (7.15), we set (7.11) into (7.7), and the inequality follows from (7.14). From (7.16) we observe that any choice of Q_Y on the output results in an upper bound on channel capacity. Replacing P_Y with an axillary distribution Q_Y that does not depend on P_X makes derivation of the bound simpler. Choosing a proper Q_Y on the output helps to establish a tight upper bound on capacity (e.g., see [10], **Paper C**, and **Paper D**).

4 Capacity of the Phase Noise Channel

As already mentioned, a closed-form expression for the capacity of the phase noise channel, other than Lapidoth high-SNR result [95], is not available. Lapidoth proved the following asymptotic characterization of the capacity of SISO phase noise channel

$$C(\mathsf{SNR}) = \eta \ln(\mathsf{SNR}) + \chi + o(1), \quad \mathsf{SNR} \to \infty \tag{7.17}$$

where $\eta = 1/2$ and

$$\chi = \ln(2\pi) - h\left(\{\phi[k]\}\right). \tag{7.18}$$

Here in (7.17), $o(1)$ denotes a function that vanishes as SNR grows large, and $h\left(\{\phi[k]\}\right)$ is the differential entropy rate of the phase noise process.

The factor $\eta = 1/2$ in (7.17) is the so-called capacity *prelog*, defined as the asymptotic ratio between capacity and the logarithm of SNR as SNR grows to infinity: $\eta = \lim_{\mathsf{SNR}\to\infty} C(\mathsf{SNR})/\ln(\mathsf{SNR})$. The capacity prelog can be interpreted as the fraction of complex dimensions available for communications in the limiting regime of high signal power, or equivalently vanishing noise variance [104]. For the phase-noise channel (4.5), only the amplitude of the transmitted signal, $|s[n]|$, can be perfectly recovered in the absence of additive noise, whereas the phase, $\angle s[n]$, is lost. Hence, the fraction of complex dimensions available for communication is $\eta = 1/2$.

We have called the second term in the high-SNR expansion of the capacity (7.17), the *phase-noise number* (see **Paper D**)

$$\chi = \lim_{\mathsf{SNR}\to\infty} \left\{C(\mathsf{SNR}) - \eta \ln(\mathsf{SNR})\right\}. \tag{7.19}$$

We can observe from (7.18) that the phase-noise number depends only on the statistics of the phase-noise process.

In **Paper C**, we improved the Lapidoth results and provided upper and lower bounds on the capacity of the SISO phase-noise channel, which are tight for a wide range of SNR values.

In **Paper D**, we built on Lapidoth results and calculated the capacity prelog and the phase-noise number of the SIMO and MISO channels for the SLO and CLO configurations.

CHAPTER 8

CONTRIBUTIONS, AND FUTURE WORK

In this thesis, various aspects of phase-noise problem in communication systems is investigated. First, we derive statistics of phase noise in real oscillators. Then, we employ those statistics for design of phase noise estimation algorithms, as well as calculation of the performance of phase noise-affected communication systems. Capacity of the single and multiple-antenna systems, affected by phase noise, is also investigated.

1 Contribution

The contribution of the appended papers are shortly described in the following.

Paper A: Calculation of the Performance of Communication Systems from Measured Oscillator Phase Noise

In this paper, we investigate the relation between real oscillator phase noise measurements and the performance of communication systems. To this end, we first derive statistics of phase noise with white and colored noise sources in free-running and PLL-stabilized oscillators. Based on the calculated statistics, analytical BCRB for estimation of phase noise with white and colored sources is derived. Finally, the system performance in terms of error vector magnitude of the received constellation is computed from the calculated BCRB.

According to our analysis, the influence from different noise regions strongly depends on the communication bandwidth, i.e., the symbol rate. For example, in high symbol rate communication systems, cumulative phase noise that appears near carrier is of relatively low importance compared to the white phase noise far from carrier.

The paper's findings can be used by hardware and frequency generator designers to better understand the effect of phase noise with different sources on the system performance and optimize their design criteria respectively. Moreover, the computed phase noise statistics can further be used in design of phase noise estimation algorithms.

Paper B: Estimation of Phase Noise in Oscillators with Colored Noise Sources

In this work design of algorithms for estimation of phase noise with colored noise sources is studied. A soft-input maximum a posteriori phase noise estimator and a modified soft-input extended Kalman smoother are proposed.

We show that deriving the soft-input MAP estimator is a concave optimization problem at moderate and high SNRs. To be able to implement the Kalman algorithm for filtering/smoothing of phase noise with colored increments, the state equation is modified by means of an autoregressive modeling.

The performance of the proposed algorithms is compared against those studied in the literature, in terms of mean square error of phase noise estimation, and symbol error rate of the considered communication system. The comparisons show that considerable performance gains can be achieved by designing estimators that employ correct knowledge of the phase noise statistics. The gain is more significant in low-SNR or low-pilot density scenarios.

Paper C: On the Capacity of the Wiener Phase-Noise Channel: Bounds and Capacity Achieving Distributions

Quantifying the capacity of the SISO phase-noise channel is the main focus of this paper. Phase noise is modeled by discrete-time Wiener process. Tight upper and lower bounds on the capacity of this channel are developed.

The upper bound is obtained by using the duality approach, and considering a specific distribution over the output of the channel. The candidate output distribution is formed as a mixture of distributions that resulted in tight upper bounds for two extreme cases of high and low SNRs. At low SNR, AWGN dominates and the channel behaves similar to the AWGN channel, while at high SNR, phase noise is dominating.

In order to lower-bound the capacity, first a high-SNR approximation of the channel is considered. A functional optimization of the mutual information of the approximate channel is solved and a family of capacity-achieving input distributions is obtained. Then, lower bounds on the capacity of the original channel are obtained by drawing samples from the proposed distributions and through Monte-Carlo simulations.

The proposed input distributions are circularly symmetric, non-Gaussian, and the input amplitudes are correlated over time. They can be used in order to design new modulation formats or error correction codes for the phase-noise channels.

The evaluated capacity bounds are tight for a wide range of signal-to-noise-ratio (SNR) values, and thus they can be used to quantify the capacity. Specifically, the bounds follow the well-known AWGN capacity curve at low SNR, while at high SNR, they coincide with the high-SNR capacity result available in the literature for the phase-noise channel.

Paper D: Capacity of SIMO and MISO Phase-Noise Channels with Common/Separate Oscillators

In this paper, capacity of the multiple antenna systems affected by phase noise is investigated. More specifically, high-SNR expressions of the capacity for SIMO and MISO channels, considering the CLO and the SLO configurations are derived. The results show that the prelog is the same for all scenarios (SIMO/MISO and SLO/CLO), and equal to 1/2. On the contrary, the phase-noise number, turns out to be scenario-dependent.

For the SIMO case, the SLO configuration provides a diversity gain, resulting in a larger phase-noise number than for the CLO configuration. For the case of Wiener phase noise, a diversity gain of at least $0.5\ln(M)$ can be achieved, where M is the number of receive antennas. This gain, implies that to achieve the same throughput in the high-SNR regime, the oscillator used in the CLO configuration must be at least M times better than any of the oscillators used in the SLO configuration.

For the MISO, the CLO configuration yields a higher phase-noise number than the SLO configuration. This is because with the CLO configuration one can obtain a coherent-combining gain through maximum ratio transmission (a.k.a. conjugate beamforming). This gain is unattainable with the SLO configuration. The capacity achieving-strategy for the SLO configuration turns out to be antenna selection, i.e., activating only the transmit antenna that yields the largest SISO high-SNR capacity, and switching off all other antennas.

2 Future Work

Some possible topics for future research are described in the following:

- Find a joint power consumption-phase noise model for oscillators.
- Evaluate the complexity, cost, and power consumption of the digital signal processing algorithms proposed for compensation of phase noise, and compare it against analog solutions.
- Develop low-complexity phase-noise estimation/compensation algorithms for massive MIMO systems.
- Design error correcting codes that incorporate the impairing effects of phase noise.
- Design new modulation formats for the Wiener phase-noise channel, based on the capacity-achieving distributions proposed in **Paper C**.
- Extend our work in **Paper D** to analyze the prelog and phase-noise number of $N \times M$ MIMO systems, for both SLO and CLO configurations.
- Design differential space-time coding and detection methods for MIMO systems in the presence of phase noise.

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