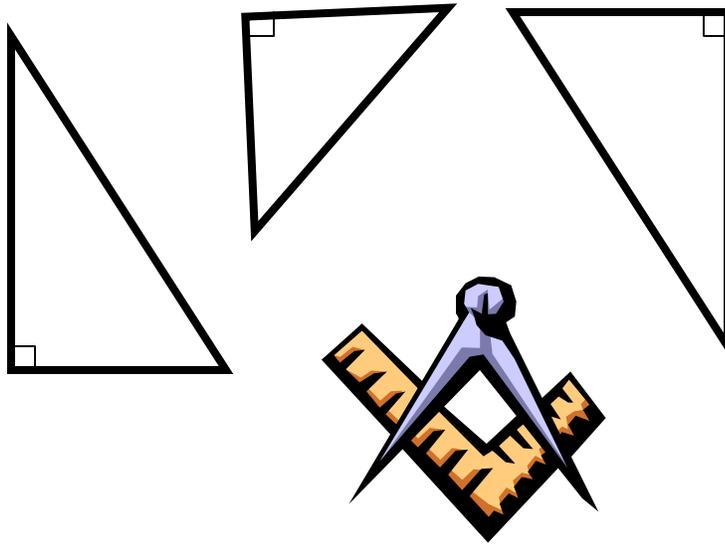


TRIGONOMETRY UNIT



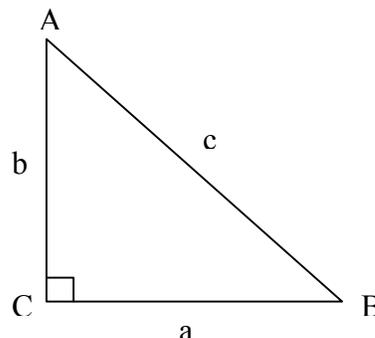
Originally TRIGONOMETRY was that branch of mathematics concerned with solving triangles using trigonometric ratios which were seen as properties of triangles rather than of angles. The word Trigonometry comes from the Greek words: Treis= three, Gonia=angle, and Metron=measure. Therefore, the word trigonometry means “measurement of triangles.” This unit will explore trigonometry as it relates to right triangles.

Math 99 Intermediate Algebra
Whatcom Community College
revised by H Ypma Winter 2003

Introduction to Trigonometry

Trigonometry is the branch of mathematics that studies the relationship between angles and sides of a triangle. Using “special trig functions”, you will be able to calculate the height of a tree from its shadow or the distance across a wide lake by walking and measuring along its shore. Let’s begin by working with a RIGHT TRIANGLE (one with a 90° angle in it) and using it to define those “special trig functions” we mentioned earlier.

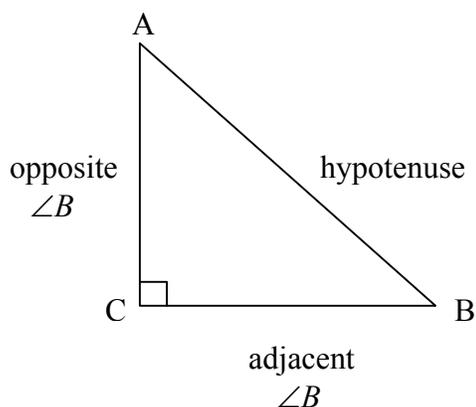
First of all, we will use **standard notation** when labeling our right triangles. Capital letters will represent the vertices or angles; A, B, and C will be our standard letters. C will be used to represent the right angle. We will use lower case letters to represent the sides; a, b, and c will be the standard choices here. The side opposite the angle labeled A will be side “a”, the side opposite angle B will be “b” and the side opposite angle C will be “c”.



In a Right Triangle, the right angle will always measure 90° and the two other angles will add up to 90° . You could have, for example, a right triangle with angles measuring 90° , 45° , and 45° , or you could have a right triangle with angles measuring 90° , 2° , and 88° . Notice that all three angles in a triangle (right triangle or not) will always add up to 180 degrees.

If you stood at the 90° angle, (the right angle) and pointed at the longest side of the triangle (which is opposite the 90° angle) you would be looking at the HYPOTENUSE. The hypotenuse always has the longest measurement of the three sides. Now let’s talk briefly about the other two angles.

In the drawing at the right, we also have angle A and angle B. If you stand at angle B, and point to the side directly in front of you, you would see the side OPPOSITE the angle B. Since you have found the hypotenuse side already, and you know the side opposite the angle, what do we call the remaining side of the triangle? It is called “the side ADJACENT to the angle B”. This side is always next to the angle.



Since we are working with a right triangle, the Pythagorean theorem applies:

$$a^2 + b^2 = c^2$$

OR $(\text{side adjacent})^2 + (\text{side opposite})^2 = (\text{hypotenuse})^2$

CHECK FOR UNDERSTANDING:

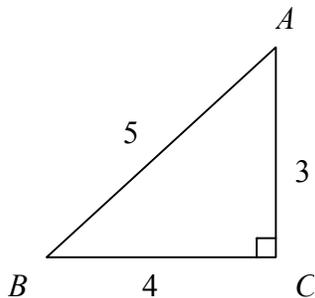
Draw a triangle and label the angles similar to the one illustrated in the example. Once you have drawn the triangle, write the given measurements next to the appropriate side. Find the measurements of the hypotenuse, the side opposite angle A, and the side adjacent to angle A. Do the same for angle B. (the hypotenuse will stay the same, but the other two sides change in their orientation to angle B) Verify that the Pythagorean theorem works. (one of the questions doesn't work)

EXAMPLE:

In triangle ABC: AB = 5, AC = 3, BC = 4

Find the measurements of the hypotenuse, the side opposite angle A, and the side adjacent to A. Do the same for angle B. Verify that this is a right triangle using the Pythagorean Theorem.

Solution:



| | | |
|------------------|---|--------------------|
| Hypotenuse: | 5 | |
| Side opposite A: | 4 | Side opposite B: 3 |
| Side adjacent A: | 3 | Side adjacent B: 4 |

$$\begin{aligned} (3)^2 + (4)^2 &= (5)^2 \\ 9 + 16 &= 25 \\ 25 &= 25 \end{aligned}$$

QUESTIONS:

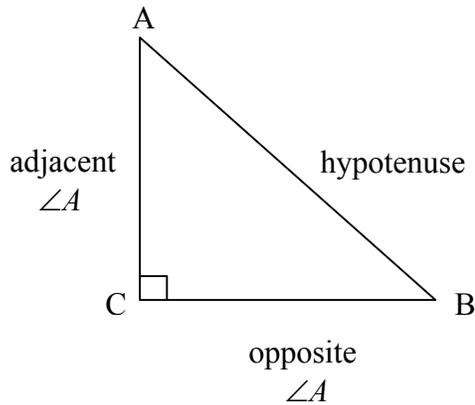
1. AB = 13, BC = 12, AC = 5
2. AB = 2, BC = 1, AC = $\sqrt{3}$
3. AB = $\sqrt{2}$, BC = 1, AC = 1
4. AB = 6, BC = 4, AC = 2

Right Triangle Trigonometry

The basic trigonometric functions are the ratios of the lengths of the sides of right triangles. The three fundamental trigonometric functions are:

sine which is abbreviated as **sin**
cosine which is abbreviated as **cos**
tangent which is abbreviated as **tan**

Let us once again consider the standard right triangle and look at the ratios that these basic trig functions represent in relationship to $\angle A$.

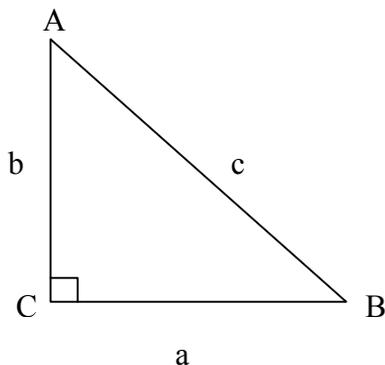


$$\sin A = \frac{\textit{opposite}}{\textit{hypotenuse}} = \frac{\textit{opp}}{\textit{hyp}}$$

$$\cos A = \frac{\textit{adjacent}}{\textit{hypotenuse}} = \frac{\textit{adj}}{\textit{hyp}}$$

$$\tan A = \frac{\textit{opposite}}{\textit{adjacent}} = \frac{\textit{opp}}{\textit{adj}}$$

Notice that in the previous diagram, the sides have been labeled in reference to angle A . We could also look at these three relationships in the triangle with relationship to angle B . In the example below, we will use the **standard notation** to identify both the sides and the angles of a right triangle. We can look at both angles A and B and the values of their trigonometric functions.



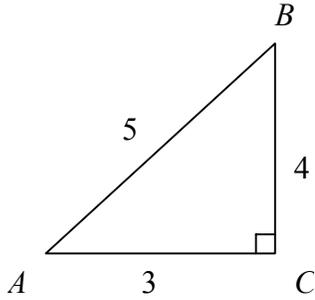
$$\frac{a}{c} = \sin A \leftarrow \boxed{\frac{\textit{opp}}{\textit{hyp}}} \rightarrow \sin B = \frac{b}{c}$$

$$\frac{b}{c} = \cos A \leftarrow \boxed{\frac{\textit{adj}}{\textit{hyp}}} \rightarrow \cos B = \frac{a}{c}$$

$$\frac{a}{b} = \tan A \leftarrow \boxed{\frac{\textit{opp}}{\textit{adj}}} \rightarrow \tan B = \frac{b}{a}$$

Once we understand the relationships given by this standard triangle and the three basic trig functions, we can then replace the variables with actual numbers.

Example 1; Given the triangle below, find



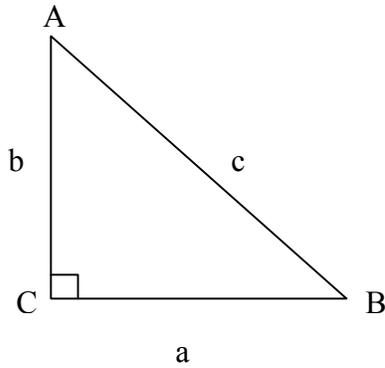
- (a) $\sin A$
- (b) $\sin B$
- (c) $\cos B$
- (d) $\tan B$

- Answers: a) $\frac{4}{5}$
 b) $\frac{3}{5}$
 c) $\frac{4}{5}$
 d) $\frac{3}{4}$

There are actually six trigonometric functions. The reciprocals of the ratios that define the sine, cosine, and tangent functions are used to define the remaining three trigonometric functions. These are:

- cosecant** which is abbreviated as **csc**
- secant** which is abbreviated as **sec**
- cotangent** which is abbreviated as **cot**

The relationships of these trigonometric functions in a standard right triangle are shown below.

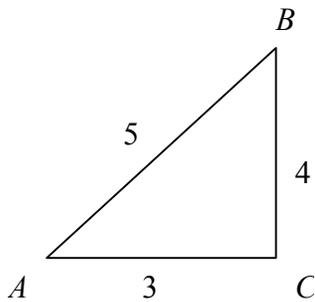


$$\csc A = \frac{c}{a} = \frac{\text{hyp}}{\text{opp}}$$

$$\sec A = \frac{c}{b} = \frac{\text{hyp}}{\text{adj}}$$

$$\cot A = \frac{b}{a} = \frac{\text{adj}}{\text{opp}}$$

Example 2: Given the triangle below, find

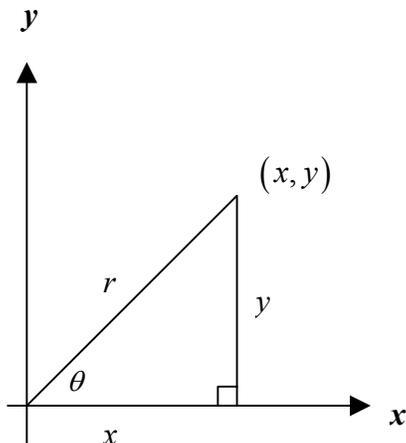


- (a) $\cot A$
- (b) $\csc B$
- (c) $\sec A$
- (d) $\csc A$

- Answers: a) $\frac{3}{4}$
 b) $\frac{5}{3}$
 c) $\frac{5}{3}$
 d) $\frac{5}{4}$

Another View of Right Triangle Trigonometry

When working with right triangles we do not always label our vertices A, B, and C. Sometimes we just use a symbol to indicate the angle of the triangle we wish to use. The most common labels used for angles in trig are the: θ pronounced “theta”, α pronounced “alpha”, and β pronounced “beta”. Often in trigonometry we view the triangle in the first quadrant of a rectangular coordinate system. In this way, the trigonometric functions can be viewed in terms of x and y .



$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

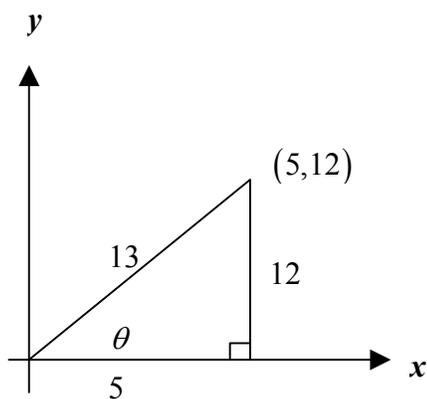
$$\tan \theta = \frac{y}{x}$$

Notice in this representation, the acute angle θ is between the x -axis and a line segment from the origin to a point (x, y) in the first quadrant. Notice that the *legs* (another word used to describe the sides forming the right angle) of the right triangle take on the value of the coordinates of the given point. The *hypotenuse* is renamed as r because this is actually the radius of a circle.

Let’s look at an example using the notation that we have just discussed.

Example 1: In this triangle find the six trigonometric functions of θ .

Answers: One leg = 5, Other leg = 12, $r = 13$



$$\sin \theta = \frac{12}{13} \qquad \csc \theta = \frac{13}{12}$$

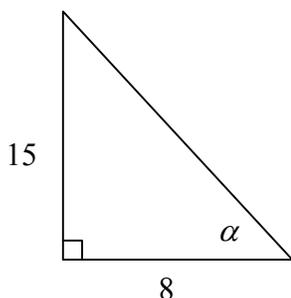
$$\cos \theta = \frac{5}{13} \qquad \sec \theta = \frac{13}{5}$$

$$\tan \theta = \frac{12}{5} \qquad \cot \theta = \frac{5}{12}$$

Evaluating Trigonometric Functions

Now that we have looked at the six different trigonometric functions and ways in which we can represent a right triangle, let's look at a couple more examples of working with these new functions.

Example 1: Find the values of the six trigonometric functions of angle α in the right triangle shown below.



Solution: Notice that you have been given only two sides of the right triangle. You first need to find the third side. By the Pythagorean Theorem, $(hyp)^2 = (opp)^2 + (adj)^2$, it follows that

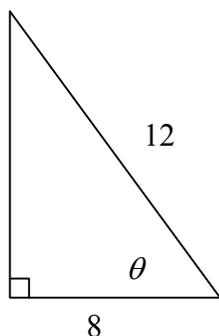
$$\begin{aligned} (hyp)^2 &= 15^2 + 8^2 \\ (hyp)^2 &= 225 + 64 \\ (hyp)^2 &= 289 \\ \sqrt{hyp^2} &= \sqrt{289} \\ hyp &= 17 \end{aligned}$$

We now have the following relationships: $adj \alpha = 8$, $opp \alpha = 15$, and $hyp = 17$. The six trigonometric relationships are:

$$\begin{array}{lll} \sin \alpha = \frac{15}{17} & \cos \alpha = \frac{8}{17} & \tan \alpha = \frac{15}{8} \\ \csc \alpha = \frac{17}{15} & \sec \alpha = \frac{17}{8} & \cot \alpha = \frac{8}{15} \end{array}$$

Example 2: Given the right triangle below, find the value of each trigonometric function.

- (a) $\sin \theta$ (b) $\cos \theta$ (c) $\tan \theta$



Solution: First, we apply the Pythagorean theorem to find the length of the leg opposite angle θ .

Letting a be the length of this leg, we find

$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + 8^2 &= 12^2 \\ a^2 &= 12^2 - 8^2 \\ a^2 &= 80 \\ a &= \sqrt{80} = 4\sqrt{5} \end{aligned}$$

Now using the trigonometric ratios, we obtain the values of the indicated trigonometric functions. (Notice that values are given in reduced radical form to preserve accuracy.)

$$\begin{array}{lll} \text{a) } \sin \theta = \frac{4\sqrt{5}}{12} = \frac{\sqrt{5}}{3} & \text{b) } \cos \theta = \frac{8}{12} = \frac{2}{3} & \text{c) } \tan \theta = \frac{4\sqrt{5}}{8} = \frac{\sqrt{5}}{2} \end{array}$$

Finding Angle Measures

In the following example we are confronted with a problem of the following type:

$$\sin A = 0.6$$

Up until now we've looked only at the ratio of a specific angle, now let's find the actual angle measure the ratio represents. First, we note that the problem can be written symbolically as either

$$A = \sin^{-1} 0.6 \quad \text{or} \quad A = \arcsin 0.6$$

Both of these expressions are read as " A is the angle whose sine is 0.6".

Notice on your calculator that you have the symbol " \sin^{-1} " located right above the "sin" button. Like many of the other functions on the calculator the function and its inverse can be found using the same button. You will see that above "cos" is " \cos^{-1} ", and above "tan" is " \tan^{-1} ". (Since these functions are located above the actual buttons, be sure to first press the 2nd key.) These function keys will take us from a trigonometric ratio back to the corresponding acute angle in degree measure when our calculator is set in degree mode.

Therefore to solve the previous problem, we have:

$$\sin A = 0.6$$

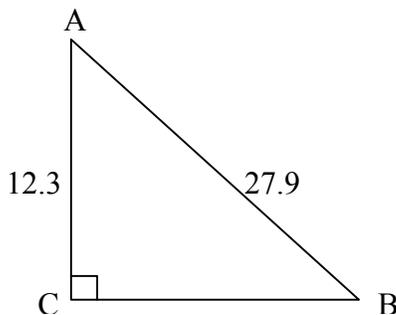
$$A = \sin^{-1} 0.6$$

$$A \approx 36.9^\circ$$

Caution: It is important to remember that $\sin^{-1} 0.6$ does not mean $\frac{1}{\sin 0.6}$

The superscript $^{-1}$ is part of a function symbol, and \sin^{-1} represents the **inverse sin function**.

Example: Find the measure of $\angle A$ and $\angle B$.



Solution: We know the side adjacent to A and also the hypotenuse. This suggests the use of the cosine ratio. We get

$$\cos A = \frac{12.3}{27.9}$$

$$A = \cos^{-1} \left(\frac{12.3}{27.9} \right)$$

$$A \approx 63.8^\circ$$

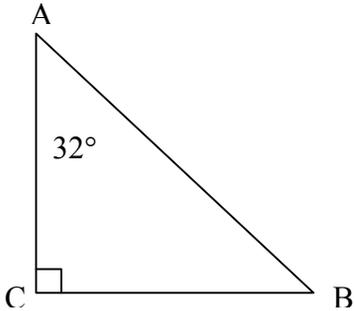
Now, since the measure of $\angle A$ and $\angle B$ must total 90° , we can find B by subtracting:

$$B = 90^\circ - 63.8^\circ = 26.2^\circ$$

Solving Right Triangles

To *solve* a right triangle means to find the lengths of all of its sides and the measures of all of its angles, provided that they are not already known. To solve a triangle, we use the trigonometric ratios, as needed to find the missing parts. We will look at some examples of different situations that involve finding missing information in a triangle.

Example 1: Find the $m\angle B$ (read as “the measure of angle B ”)



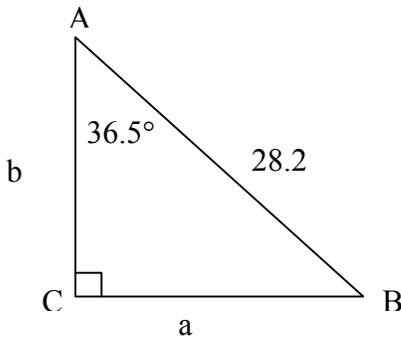
Solution: Since the angle measures of any triangle add up to 180° and the right angle measures 90° , the acute angles must add up to 90° :

$$m\angle A + m\angle B = 90^\circ$$

so,

$$m\angle b = 90^\circ - 32^\circ = 58^\circ$$

Example 2: In this triangle, find a and b .



Solution: To solve for a , we recognize that the information we have is the angle A , and the hypotenuse. The side we are looking for is the side opposite angle A . The trig function that deals with this relationship is the *sin* function.

$$\sin 36.5^\circ = \frac{a}{28.2}$$

Solving for a , we have

$$a = 28.2 \sin 36.5^\circ$$

Using our calculators*, we find

$$a \approx 16.77$$

***Note:** In order to use a calculator to solve these problems, make sure that your calculator is set in DEGREE mode.

We use a similar strategy to solve for b . When solving a right triangle, it is best to use the original information that was given whenever possible to avoid rounding errors in your answer. If we look at the original problem, b is the side adjacent to the angle given and we also know the hypotenuse. The trig function that deals with this relationship is the *cos* function.

$$\cos 36.5^\circ = \frac{b}{28.2}$$

Solving for b , we have

$$b = 28.2 \cos 36.5^\circ$$

$$b \approx 22.67$$

Notice we can also easily find the measure of the angle B : $90^\circ - 36.5^\circ = 53.5^\circ$.

We have now successfully solved this right triangle.

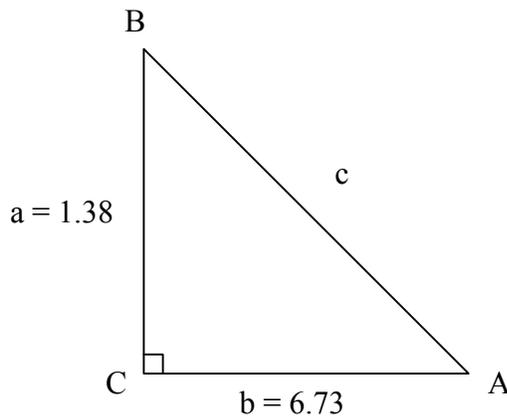
$$\angle A = 36.5^\circ \quad a = 16.77$$

$$\angle B = 53.5^\circ \quad b = 22.67$$

$$\angle C = 90^\circ \quad c = 28.2$$

Example 3: Solve the right triangle with $a = 1.38$ and $b = 6.73$.

Solution: In this problem we are not given a diagram so the first thing that needs to be done is to draw a triangle. We assume that standard notation is used.



Next, place the information given in the correct locations. It now is apparent that to solve this right triangle, we need to find side c , $\angle A$, and $\angle B$.

Let's start by finding c :

$$c^2 = a^2 + b^2$$

$$c^2 = (1.38)^2 + (6.73)^2$$

$$c^2 = 47.1973$$

$$\sqrt{c^2} = \sqrt{47.1973}$$

$$c \approx 6.87$$

Now let's find $\angle B$. To do so, use the original information given, to avoid error by rounding.

$$\tan B = \frac{6.73}{1.38}$$

$$B = \tan^{-1} \frac{6.73}{1.38}$$

$$B \approx 78.4^\circ$$

We can use this result to find $\angle A$: $90^\circ - 78.4^\circ = 11.6^\circ$.

We could have also found this angle by using a trigonometric relationship and the original information given.

$$\tan A = \frac{1.38}{6.73}$$

$$A = \tan^{-1} \frac{1.38}{6.73}$$

The triangle is now solved.

$$\angle A = 11.6^\circ \quad a = 1.38$$

$$\angle B = 78.4^\circ \quad b = 6.73$$

$$\angle C = 90^\circ \quad c = 6.87$$

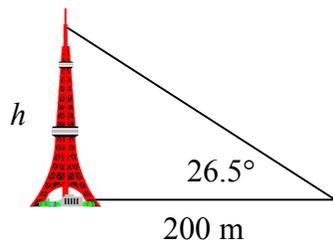
Applications of Right Triangles

Right triangles have many applications. To solve a problem, we draw a right triangle and for the part associated with the application question solve the triangle.

To solve a triangle problem:

- 1) Draw a sketch of the problem situation.
- 2) Look for triangles and sketch them in.
- 3) Mark the known and unknown sides and angles.
- 4) Express the desired side or angle in terms of known trigonometric ratios.
- 5) Solve.
- 6) Answer the question being asked. (use appropriate labels)

Example 1: An observer stands on level ground, 200 meters from the base of a TV tower, and looks up at an angle of 26.5° to see the top of the tower. How high is the tower above the observer's eye level?



Solution: We draw a diagram and see that a right triangle is formed. The information we have is an angle and the adjacent side to this angle. We need to find the opposite side. To relate these pieces of information, we can use the tangent function.

$$\tan 26.5^\circ = \frac{h}{200} \quad (\text{solve for } h)$$

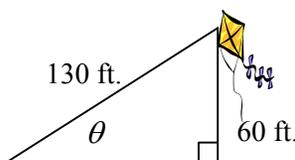
$$200 \cdot \tan 26.5^\circ = \frac{h}{200} \cdot 200$$

$$200 \cdot \tan 26.5^\circ = h$$

$$99.72 \approx h$$

The height of the tower is about 99.72 meters.

Example 2: A kite flies at a height of 60 feet when 130 feet of string is out. Assuming that the string is in a straight line, what is the angle that it makes with the ground?



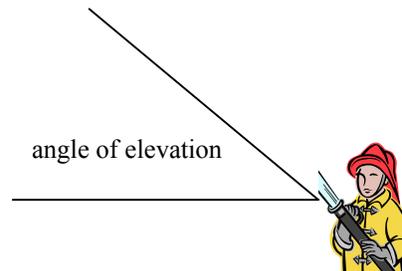
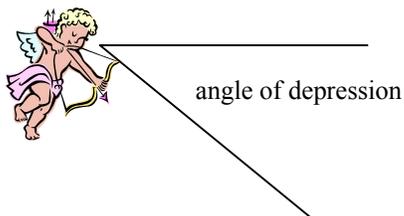
Solution: We draw a diagram of the situation presented. The information we have is the side opposite the angle we are looking for and the hypotenuse. To relate this information we use the sine function.

$$\sin \theta = \frac{60}{130}$$

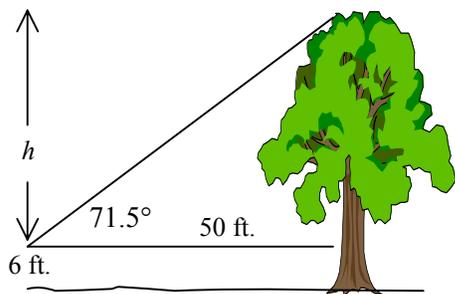
$$\theta = \sin^{-1}\left(\frac{60}{130}\right)$$

$$\theta \approx 27.5^\circ$$

The trigonometric functions are often used in calculating distances that are difficult to measure directly. In many of these instances, an angle is determined by a horizontal line and a line-of-sight. If a person is looking **up** at an object, the acute angle measured from the horizontal to a line-of-sight observation is called the **angle of elevation**. If a person is looking **down** at an object, the acute angle made by the line-of-sight observation of the object and the horizontal is called the **angle of depression**.



Example 1: A surveyor is standing 50 feet from the base of a large tree, as shown below. The surveyor measures the angle of elevation to the top of the tree as 71.5° . How tall is the tree if the surveyor is 6 feet tall?



Solution: The information given suggests the use of the tangent function.

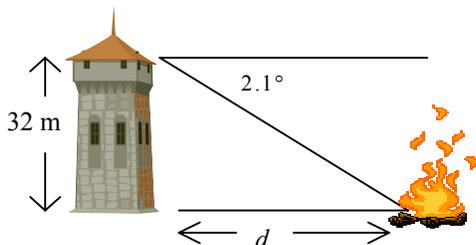
$$\tan 71.5^\circ = \frac{h}{50}$$

$$h = 50 \cdot \tan 71.5^\circ$$

$$h \approx 149.4 \text{ feet}$$

But when we find h we have to add in the surveyor's height, since we want the height of the entire tree. The height of the tree is about 155.4 feet.

Example 2: From the fire tower in Flatlands National Park, a forest ranger sighted a fire. To measure the angle of depression of the fire, the ranger used an instrument that was known to be 32 meters above the ground. The angle of depression from the tower to the fire was 2.1° . What was the distance between the fire and the base of the tower?



Solution: In this problem the angle given is not part of the right triangle we are going to solve. However, we can use the given angle to determine one of the acute angles of the triangle we need to solve since the angle of depression always uses a horizontal line. The angle inside the right triangle at the top of the tower will be: $90^\circ - 2.1^\circ = 87.9^\circ$

We now have an angle, the side adjacent to the angle and are looking for the side opposite the given angle. We use the tangent function to solve.

$$\tan 87.9^\circ = \frac{d}{32}$$

$$d = 32 \cdot \tan(87.9) \approx 872.7 \text{ meters}$$

Breakdown of Degrees

When working with degrees, we have used decimal representations to designate part of a degree. In the previous example, we looked at 2.1° , which represents an angle of degree measure 2 and one-tenth of a degree. A degree can also be divided further using minutes and seconds just as an hour is divided into minutes and seconds. Each degree is divided into 60 equal parts called **minutes**, and each minute is divided into 60 equal parts called **seconds**. Symbolically, minutes are represented by ' and seconds by ". Another way to think about it is that **one minute**, denoted by 1' is defined as $1/60$ degree. **One second**, denoted by 1" is defined as $1/60$ minute or, equivalently, $1/3600$ degree. Thus,

$$12^\circ 23' 14''$$

is a concise way of writing 12 degrees, 23 minutes, and 14 seconds.

Decimal degrees (DD) are useful in some instances and degrees-minutes-seconds (DMS) are useful in others. You should be able to go from one form to the other as demonstrated in the example below.

Example 1: Converting between Degrees, Minutes, Seconds, and Decimal Forms

- (a) Convert $50^\circ 6' 21''$ to a decimal in degrees.
- (b) Convert 21.256° to the $D^\circ M' S''$ form. Round off your answer to the nearest second.

Solution:

- (a) Because $1' = \frac{1}{60}^\circ$ and $1'' = \frac{1}{60}' = \left(\frac{1}{60} \cdot \frac{1}{60}\right)^\circ$, we convert as follows:

$$\begin{aligned} 50^\circ 6' 21'' &= \left(50 + 6 \cdot \frac{1}{60} + 21 \cdot \frac{1}{60} \cdot \frac{1}{60} \right)^\circ \\ &\approx (50 + 0.1 + 0.005833)^\circ \\ &= 50.105833^\circ \end{aligned}$$

- (b) We start with the decimal part of 21.256° , that is 0.256° :
 $0.256^\circ = (0.256)(1^\circ) = (0.256)(60') = 15.36'$ since $1^\circ = 60'$

Now we work with the decimal part of $15.36'$, that is, $0.36'$:

$$0.36' = (0.36)(1') = (0.36)(60'') = 21.6'' \approx 22''$$

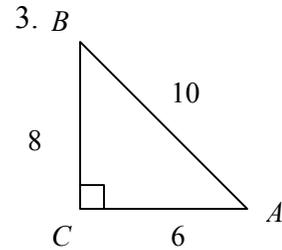
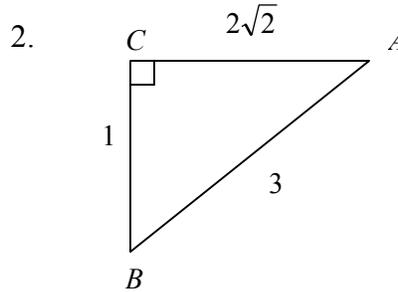
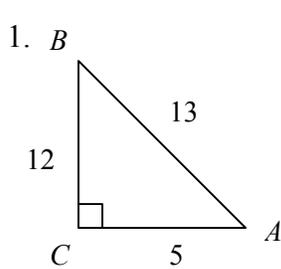
Thus,

$$\begin{aligned} 21.256^\circ &= 21^\circ + 0.256^\circ = 21^\circ + 15.36' = 21^\circ + 15' + 0.36' \\ &= 21^\circ + 15' + 21.6'' \approx 21^\circ 15' 22'' \end{aligned}$$

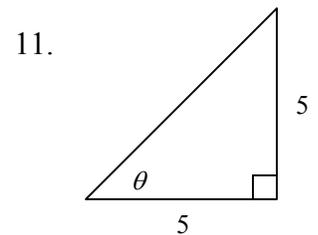
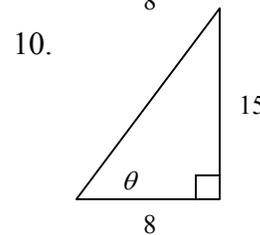
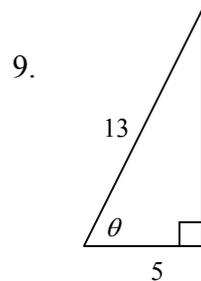
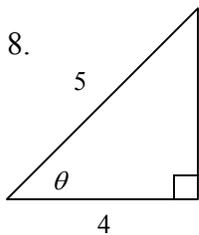
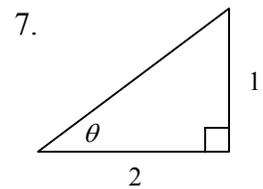
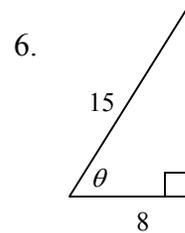
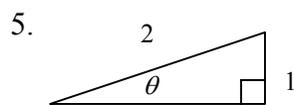
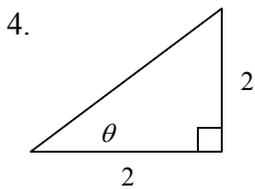
It is also possible to do these conversions on the calculator.

SET 1:**Evaluating Trigonometric Functions**

For each triangle, find (a) $\sin A$ (b) $\cos A$ (c) $\tan A$ (d) $\sin B$ (e) $\cos B$ (f) $\tan B$



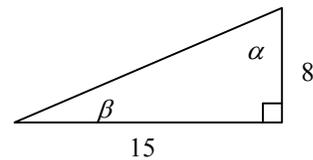
In exercises 4-11, use the Pythagorean Theorem to find the third side of the triangle. Then find the exact values of the six trigonometric functions of the angle θ . (Leave answers in exact simplified form.)



In exercises 12-14, use the right triangle shown in the sketch to find the values of the indicated trigonometric functions.

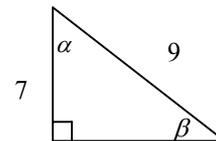
12. (a) $\sin \alpha$ (b) $\cos \alpha$ (c) $\cot \beta$

(d) $\tan \alpha$ (e) $\csc \beta$ (f) $\sec \beta$



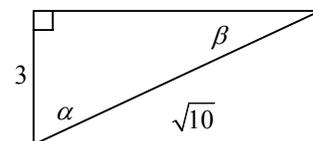
13. (a) $\tan \beta$ (b) $\csc \alpha$ (c) $\sec \alpha$

(d) $\sin \beta$ (e) $\cos \beta$ (f) $\cot \alpha$



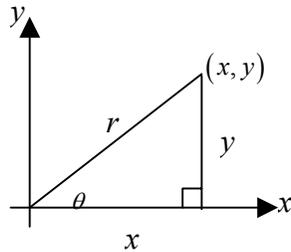
14. (a) $\sin \beta$ (b) $\cos \alpha$ (c) $\tan \beta$

(d) $\cot \alpha$ (e) $\sec \beta$ (f) $\csc \alpha$



SET 2:**Evaluating Trigonometric Functions (another view)**

Refer to the figure below to solve problems 1-12.



For each ratio in problems 1-6 identify the trigonometric function represented in relation to θ .

- | | | |
|------------------|------------------|------------------|
| 1. $\frac{y}{r}$ | 2. $\frac{r}{y}$ | 3. $\frac{x}{r}$ |
| 4. $\frac{x}{y}$ | 5. $\frac{y}{x}$ | 6. $\frac{r}{x}$ |

Write the ratios of sides corresponding to each trigonometric function in problems 7-12.

- | | |
|-------------------|-------------------|
| 7. $\sin \theta$ | 8. $\cot \theta$ |
| 9. $\csc \theta$ | 10. $\cos \theta$ |
| 11. $\sec \theta$ | 12. $\tan \theta$ |

Find each side or angle specified. (round answers to the nearest tenth)

- | | |
|--|--|
| 13. $r = 30, x = 24$, find y . | 14. $x = 8, y = 6$, find r . |
| 15. $y = 8, r = 10$, find x . | 16. $r = 23.2, y = 15.3$, find x . |
| 17. $\theta = 45^\circ, x = 17$, find y . | 18. $\theta = 60^\circ, y = 1.5$, find r . |
| 19. $\theta = 47^\circ, r = 23$, find x . | 20. $\theta = 51.7^\circ, y = 10$, find x . |
| 21. $r = 2, x = 1.2$, find θ . | 22. $r = 97, y = 72$, find θ . |
| 23. $x = 635, y = 446$, find θ . | 24. $x = 45, r = 68$, find θ . |

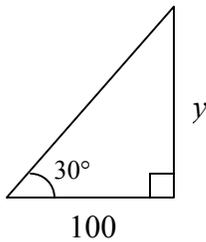
For problems 25-30 find each acute angle θ (in degrees) to two decimal places using your calculator.

- | | |
|--------------------------------|---------------------------------|
| 25. $\sin \theta = 0.9243$ | 26. $\cos \theta = 0.5277$ |
| 27. $\theta = \tan^{-1} 9.533$ | 28. $\theta = \sin^{-1} 0.0317$ |
| 29. $\theta = \arccos 0.7425$ | 30. $\theta = \arctan 4.296$ |

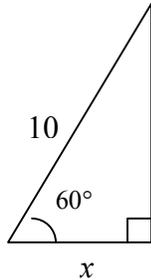
SET 3:

Applications Involving Right Triangles

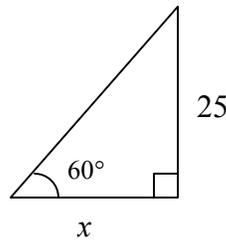
1. Solve for y .



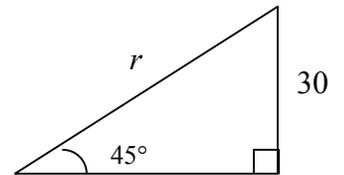
2. Solve for x .



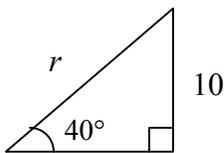
3. Solve for x .



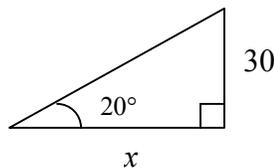
4. Solve for r .



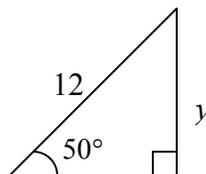
5. Solve for r .



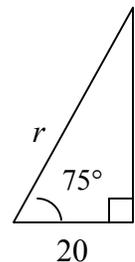
6. Solve for x .



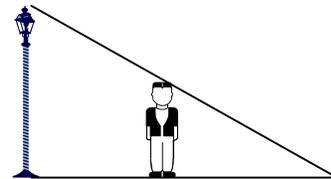
7. Solve for y .



8. Solve for r .



9. *Height* A 6-foot person standing 12 feet from a streetlight casts an 8-foot shadow (see figure). What is the height of the streetlight?

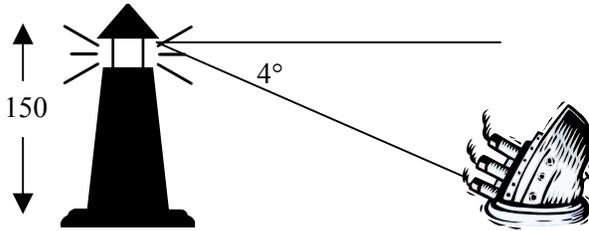
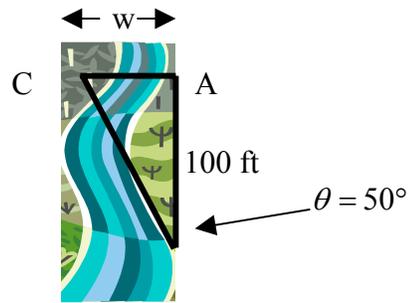


10. *Length* A 20-foot ladder leaning against the side of a house makes a 75° angle with the ground (see figure). How far up the side of the house does the ladder reach?

11. *Height* A 6-foot man walked from the base of a broadcasting tower directly toward the tip of the shadow cast by the tower. When he was 132 feet from the tower, his shadow started to appear beyond the tower's shadow. What is the height of the tower if the man's shadow is 3 feet long?

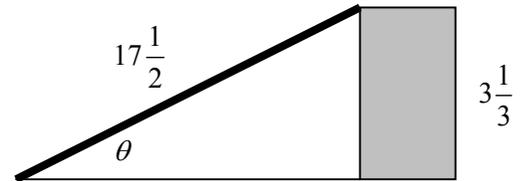
12. *Length* A 24-foot ladder is leaning against the side of an office building. It makes a 60° angle with the sidewalk. How far from the building is the base of the ladder?

13. *Width of a River* A biologist wants to know the width w of a river in order to properly set instruments for studying the pollutants in the water. From point A , the biologist walks downstream 100 feet and sights to point C . From this sighting, it is determined that $\theta = 50^\circ$ (see figure). How wide is the river?



14. *Distance* From a 150-foot observation tower on the coast, a Coast Guard officer sights a boat in difficulty. The angle of depression of the boat is 4° (see figure). How far is the boat from the shoreline?

15. *Angle of elevation* A ramp $17\frac{1}{2}$ feet in length rises to a loading platform that is $3\frac{1}{3}$ feet off the ground. (see figure). Find the angle θ



SET 4:

More Trig Applications

1. The hypotenuse of a right triangle is 3 feet. If one leg is 1 foot, find the degree measure of each angle.
2. A right triangle contains a 25° angle. If one leg is of length 5 inches, what is the length of the hypotenuse? (There are two answers)
3. Suppose you are headed toward a plateau 50 meters high. If the angle of elevation to the top of the plateau is 20° , how far are you from the base of the plateau?
4. A 22-foot extension ladder leaning against a building makes a 70° angle with the ground. How far up the building does the ladder touch?
5. At 10 am on April 26, 1998 a building 300 feet high cast a shadow 50 feet long. What was the angle of elevation of the sun?
6. To measure the height of Lincoln's caricature on Mt. Rushmore, two sightings 800 feet from the base of the mountain are taken. If the angle of elevation to the bottom of Lincoln's face (his chin) is 32° and the angle of elevation to the top (his forehead) is 35° , how long is Lincoln's face on the mountain?

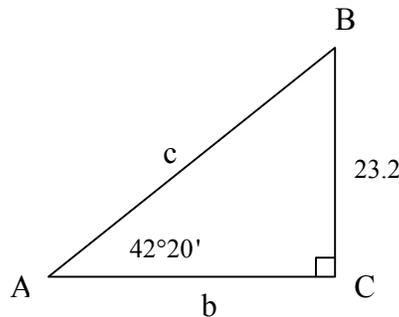
SET 5:**Using Degrees, Minutes, Seconds**

Solve these triangles. Standard lettering has been used.

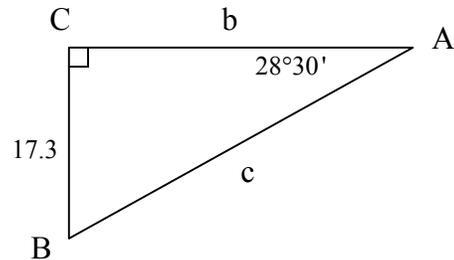
- | | |
|------------------------------------|----------------------------------|
| 1. $A = 36^\circ 10', a = 27.2$ | 2. $A = 87^\circ 40', a = 9.73$ |
| 3. $B = 12^\circ 40', b = 98.1$ | 4. $B = 69^\circ 50', b = 127$ |
| 5. $A = 17^\circ 20', b = 13.6$ | 6. $A = 78^\circ 40', b = 1340$ |
| 7. $A = 88^\circ 50', c = 3950$ | 8. $B = 82^\circ 20', c = 0.982$ |
| 9. $a = 12.0, b = 18.0$ | 10. $a = 16.0, c = 20.0$ |
| 11. $B = 56^\circ 30', c = 0.0447$ | 12. $b = 1.80, c = 4.00$ |

Solve these triangles.

13.

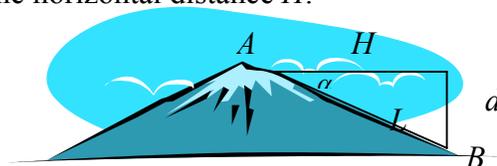


14.



Solve the following problems.

15. A device for measuring cloud height at night consists of a vertical beam of light, which makes a spot on the clouds. The spot is viewed from a point 135 feet away from the device. The angle of elevation is $67^\circ 40'$. Find the height of the clouds.
16. If a jet airliner climbs at an angle of $15^\circ 30'$ with a constant speed of 315 miles per hour, how long will it take (to the nearest minute) to reach an altitude of 8.00 miles? Assume there is no wind.
17. Horizontal distances must often be measured even though terrain is not level. One way of doing it is as follows. Distance down a slope is measured with a surveyor's tape, and the distance d is measured by making a level sighting from A to a pole held vertically at B , or the angle α is measured by an instrument placed at A . Suppose that a slope distance L is measured to be 121.3 feet and the angle α is measured to be $3^\circ 25'$. Find the horizontal distance H .



Answers

Set I

1. a) $\frac{12}{13}$ b) $\frac{5}{13}$ c) $\frac{12}{5}$ d) $\frac{5}{13}$ e) $\frac{12}{13}$ f) $\frac{5}{12}$
2. a) $\frac{1}{3}$ b) $\frac{2\sqrt{2}}{3}$ c) $\frac{\sqrt{2}}{4}$ d) $\frac{2\sqrt{2}}{3}$ e) $\frac{1}{3}$ f) $2\sqrt{2}$
3. a) $\frac{4}{5}$ b) $\frac{3}{5}$ c) $\frac{4}{3}$ d) $\frac{3}{5}$ e) $\frac{4}{5}$ f) $\frac{3}{4}$

| | $\sin \theta$ | $\cos \theta$ | $\tan \theta$ | $\csc \theta$ | $\sec \theta$ | $\cot \theta$ |
|-----|-------------------------|-----------------------|------------------------|----------------------------|-----------------------|---------------------------|
| 4. | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{2}}{2}$ | 1 | $\sqrt{2}$ | $\sqrt{2}$ | 1 |
| 5. | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{3}}{3}$ | 2 | $\frac{2\sqrt{3}}{3}$ | $\sqrt{3}$ |
| 6. | $\frac{\sqrt{161}}{15}$ | $\frac{8}{15}$ | $\frac{\sqrt{161}}{8}$ | $\frac{15\sqrt{161}}{161}$ | $\frac{15}{8}$ | $\frac{8\sqrt{161}}{161}$ |
| 7. | $\frac{\sqrt{5}}{5}$ | $\frac{2\sqrt{5}}{5}$ | $\frac{1}{2}$ | $\sqrt{5}$ | $\frac{\sqrt{5}}{2}$ | 2 |
| 8. | $\frac{3}{5}$ | $\frac{4}{5}$ | $\frac{3}{4}$ | $\frac{5}{3}$ | $\frac{5}{4}$ | $\frac{4}{3}$ |
| 9. | $\frac{12}{13}$ | $\frac{5}{13}$ | $\frac{12}{5}$ | $\frac{13}{12}$ | $\frac{13}{5}$ | $\frac{5}{12}$ |
| 10. | $\frac{15}{17}$ | $\frac{8}{17}$ | $\frac{15}{8}$ | $\frac{17}{15}$ | $\frac{17}{8}$ | $\frac{8}{15}$ |
| 11. | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{2}}{2}$ | 1 | $\sqrt{2}$ | $\sqrt{2}$ | 1 |

12. a) $\frac{15}{17}$ b) $\frac{8}{17}$ c) $\frac{15}{8}$ d) $\frac{15}{8}$ e) $\frac{17}{8}$ f) $\frac{17}{15}$
13. a) $\frac{7\sqrt{2}}{8}$ b) $\frac{9\sqrt{2}}{8}$ c) $\frac{9}{7}$ d) $\frac{7}{9}$ e) $\frac{4\sqrt{2}}{9}$ f) $\frac{7\sqrt{2}}{8}$
14. a) $\frac{3\sqrt{10}}{10}$ b) $\frac{3\sqrt{10}}{10}$ c) 3 d) 3 e) $\sqrt{10}$ f) $\sqrt{10}$

SET II

- | | | | | |
|------------------|-------------------|----------------------|---------------------------------|---------------------------------|
| 1. $\sin \theta$ | 7. $\frac{y}{r}$ | 13. $y = 18$ | 19. $x \approx 15.7$ | 25. $\theta \approx 67.6^\circ$ |
| 2. $\csc \theta$ | 8. $\frac{x}{y}$ | 14. $r = 10$ | 20. $x \approx 7.9$ | 26. $\theta \approx 58.1^\circ$ |
| 3. $\cos \theta$ | 9. $\frac{r}{y}$ | 15. $x \approx 6$ | 21. $\theta \approx 53.1^\circ$ | 27. $\theta \approx 84.0^\circ$ |
| 4. $\cot \theta$ | 10. $\frac{x}{r}$ | 16. $x \approx 17.4$ | 22. $\theta \approx 47.9^\circ$ | 28. $\theta \approx 1.8^\circ$ |
| 5. $\tan \theta$ | 11. $\frac{r}{x}$ | 17. $y = 17$ | 23. $\theta \approx 35.1^\circ$ | 29. $\theta \approx 42.1^\circ$ |
| 6. $\sec \theta$ | 12. $\frac{y}{x}$ | 18. $r \approx 1.7$ | 24. $\theta \approx 48.6^\circ$ | 30. $\theta \approx 76.9^\circ$ |

SET III

- | | | | | |
|---------------------|---------------------|---------------------|---------------|-------------------------------|
| 1. $y \approx 57.7$ | 4. $r \approx 42.4$ | 7. $y \approx 9.2$ | 10. 19.3 feet | 13. 119.2 feet |
| 2. $x = 5$ | 5. $r \approx 15.6$ | 8. $r \approx 77.3$ | 11. 270 feet | 14. 2145.1 feet |
| 3. $x \approx 14.4$ | 6. $x \approx 82.4$ | 9. 15 feet | 12. 12 feet | 15. $\theta \approx 11^\circ$ |

SET IV

1. $19.5^\circ, 70.5^\circ$ 2. 11.8 in, 5.5 in 3. 137.4 m 4. 20.7 ft 5. 80.5° 6. 60.3 ft

SET V

- | | |
|---|---|
| 1. $\angle B = 53^\circ 50'$ $b \approx 37.2$ $c \approx 46.1$ | 13. $\angle B = 47^\circ 40'$ $b \approx 25.5$ $c \approx 34.4$ |
| 2. $\angle B = 2^\circ 20'$ $b \approx 0.40$ $c \approx 9.7$ | 14. $\angle B = 61^\circ 30'$ $b \approx 31.9$ $c \approx 36.3$ |
| 3. $\angle A = 77^\circ 20'$ $a \approx 436.5$ $c \approx 447.4$ | 15. 328.6 feet |
| 4. $\angle A = 20^\circ 10'$ $a \approx 46.6$ $c \approx 135.3$ | 16. 6 minutes |
| 5. $\angle B = 72^\circ 40'$ $a \approx 4.2$ $c \approx 14.2$ | 17. 121.1 feet |
| 6. $\angle B = 11^\circ 20'$ $a \approx 6685.8$ $c \approx 6818.8$ | |
| 7. $\angle B = 1^\circ 10'$ $a \approx 3949.2$ $b \approx 80.4$ | |
| 8. $\angle A = 7^\circ 40'$ $a \approx 0.131$ $b \approx 0.973$ | |
| 9. $\angle A \approx 33.7^\circ$ $\angle B \approx 56.3^\circ$ $c \approx 21.6$ or $33^\circ 41'$ or $56^\circ 19'$ | |
| 10. $\angle A \approx 53.1^\circ$ $\angle B \approx 36.9^\circ$ $b \approx 12.0$ or $53^\circ 8'$ or $36^\circ 52'$ | |
| 11. $\angle A = 33^\circ 30'$ $a \approx 0.0247$ $b \approx 0.0373$ | |
| 12. $\angle A \approx 63.3^\circ$ $\angle B \approx 26.7^\circ$ $a \approx 3.57$ or $63^\circ 15'$ or $26^\circ 45'$ | |