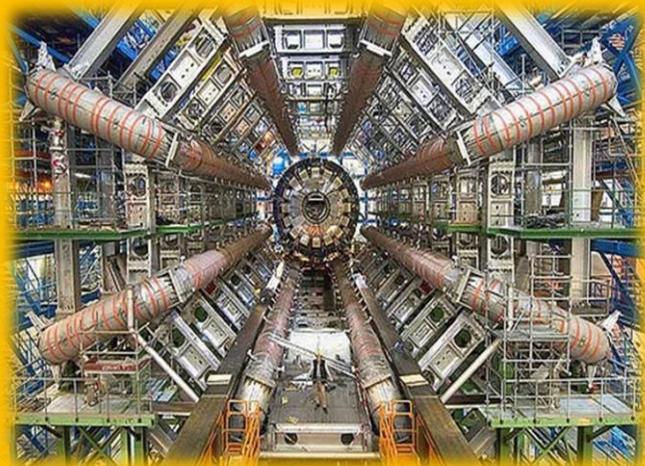
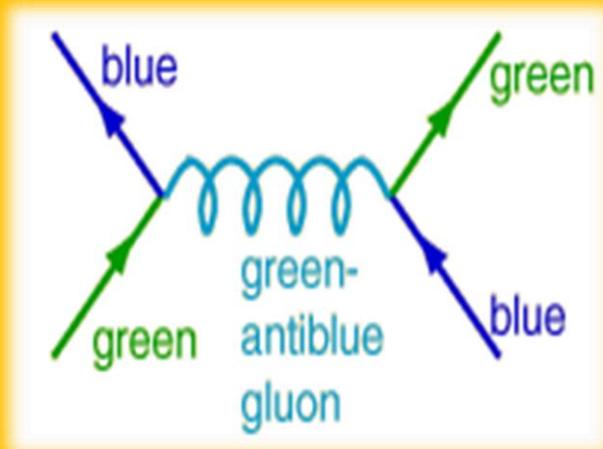
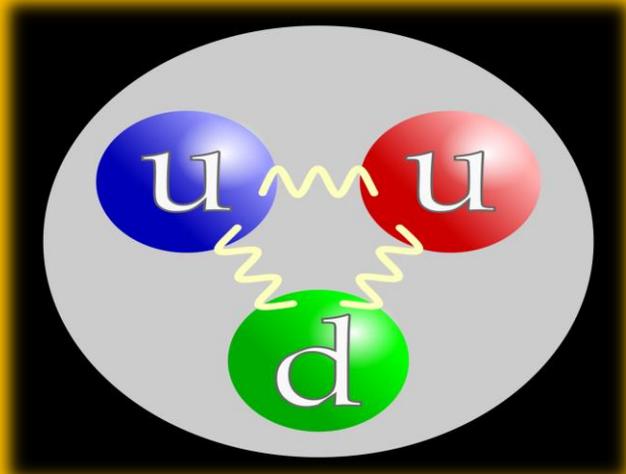
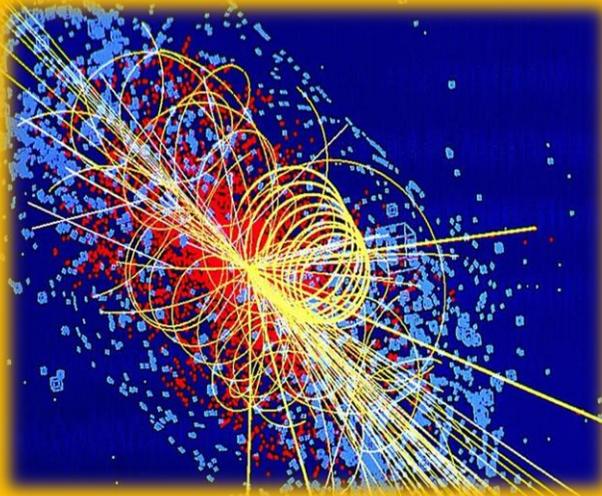




**MSCPH 558**

**M. Sc. IV SEMESTER**

# **PARTICLE PHYSICS**



**DEPARTMENT OF PHYSICS**

**SCHOOL OF SCIENCE**

**UTTARAKHAND OPEN UNIVERSITY, HALDWANI**

# PARTICLE PHYSICS

**MSCPH 558**



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**UNIT 1****ELEMENTARY PARTICLES**

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**Structure of the Unit**

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  - 1.8.4 Parity Violation
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- 1.10 References
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- 1.12 Terminal Questions

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## 1.1 INTRODUCTION

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Ever since the evolution of scientific thought, one of the most fascinating areas of physical researches has been the study of the ultimate structure of matter. At the start of the 19th century, Dalton put forward his atomic hypothesis in order to explain the course of chemical reaction processes and the structure of molecules. He contended that an atom is the ultimate constituent of matter but it soon began to be realized that the atom also possess a rich structure. By the end of 19th century (1897), J.J. Thomson firmly established the existence of a particle, the electron which even till now is classified as a fundamental particle and that it is an essential constituent of all atoms. The studies and discoveries of what are called the elementary or fundamental particles, started after the completion of the structural picture of the atom in which Rutherford's experiments and Bohr's ideas in early 20th century, established that the atom consisted of positively charged nucleus and the electrons revolve around it. Earlier experiments on radioactivity conducted by curies (Mrs. and Mr.) and Rutherford indicated that the nucleus could be subdivided. In 1919, Lord Rutherford was successful in carrying out the first ever artificial nuclear transmutation (that of nitrogen nucleus by means of  $\alpha$ -particles), the identification of the products of which led to the discovery of another fundamental particle- the proton. With the discovery of another fundamental particle- the neutron in 1932, it was firmly established that the nucleus consisted only of two fundamental particles the protons and the neutrons. Thus, the atomic picture became quite clear in early 1930's and the atomic scene was found to have only four characters, viz. electron, proton, neutron and photon. The photon is the quantum unit of radiation i.e. it is the building block of the electromagnetic field. It is uncharged and has zero rest mass but it is the carrier of electromagnetic energy.

In 1932 itself (the year of the discovery of the neutron) another particle called positron, was discovered by Anderson in cosmic rays showers. This particle is an antiparticle of the electron and has had the status of being the first antiparticle to be discovered. Dirac had speculated theoretically the existence of the particle in 1930 in connection with his relativistic theory of the electron but it could be discovered only in 1932.

The next necessary addition to the list of elementary particles arose out of the behavior of neutron. We know, in the bound state (inside the nucleus), the neutron can survive indefinitely but in the unbound state (outside the nucleus or in free state), it is unstable with a lifetime of 18 minutes and ejects a  $\beta$ -particle converting itself into a proton. The proton and

electron together are about 1.5 electron mass lighter than the neutron. So this much of mass appears to be lost in the decay process which from the relation  $E=mc^2$  amounts to 0.78MeV energy. Thus, this much of energy must show up as the kinetic energy of decay products viz. Proton and electron. When the energy of the proton and the electron was experimentally measured, it was found that they rarely have so much of energy. Is it violation of the well-established energy conservation law? No, definitely it is not. Then where does the energy deficit go? To answer this question and thus to account for this energy discrepancy, Pauli suggested the existence of another particle having no charge and having zero rest mass, in the decay process. The particle was later named neutrino by Fermi.

In 1935, Yukawa, (a Japanese scientist) in a bid to explain the origin of the strong nuclear force which holds the nucleons together in such a small volume as that of the nucleus, even in the wake of strong repulsive and thus disrupting force, predicted the existence of another fundamental particle the  $\pi$ - meson or pion. He was tempted to put forward his so-called meson field theory of nuclear forces by the success with which electromagnetic forces could be explained in terms of the field Quantum of the electromagnetic field- the photon. He assumes that the nuclear forces are the result of the exchange of a field quantum of nuclear forces called the meson, between the nucleons. He deduced some characteristics of this particle from the known properties of the nuclear forces. Since nuclear forces are extremely short-range forces, he deduced that mesons unlike the photons must have a finite rest mass (about two to three hundred times the electron mass) and also that the particle should exist in both, the charged and uncharged states.

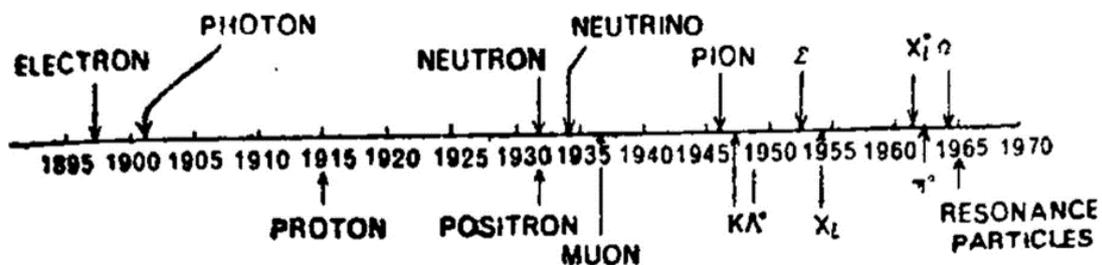
Soon after Yukawa's theoretical prediction i.e. 1936, Anderson and Neddermeyer detected a particle- the  $\mu$ -meson(muon) which had a mass  $207m_e$  and spin  $1/2$  and existed in two states, positively charged and negatively charged. This particle was found to have a lifetime of  $10^{-6}$  second. and decays into an electron, a neutrino and an antineutrino. For quite some time, this new particle( $\mu$ -meson) was mistaken for the mesons predicted by Yukawa. However, for that role its properties were all wrong as it does not interact strongly with nucleons. This resulted in a lot of confusion that prevailed for about 10 years and could be resolved only in 1947 when Yukawa's predicted particle was experimentally discovered by Lattes, Occhialini and Powell. The particle was named  $\pi$ -meson or pion and was found to exist in all the three charged states: positively charged negatively charged and neutral. Its mass was estimated to be  $270m_e$  as expected and it interacted strongly with nucleons. Thus, the pion had all the

requisite properties to behave as the Yukawa particle. The pion decays into a muon with a mean lifetime time  $10^{-8}$  sec.

Around 1950, elementary particle physics entered an era of great confusion when a whole procession of new particles was discovered in cosmic ray showers, which occur when a high energy particle hits a lead plate in a cloud chamber. During the same period the 2BeV proton synchrotron at Brookhaven and 6BeV proton synchrotron at Berkeley were commissioned and number of transient particles were found to be created in the high energy beams of from these accelerations. Among the cosmic ray shower tracks in a cloud chamber, some V-shaped tracks were found. This event was taken to be due to the decay of neutral particle into two charged particles. These particles, were first call V- particles. Through an analysis of V-particle tracks, it was concluded that there were at least two new neutral particles, one of which decays into a proton and a negative pion was named Lambda particle which was later found to be a member of the whole family of particles called hyperons.

The other which decays into a positive and negative pion was called the K-meson or Kaon. The Kaons were heavier than the pions and had masses around  $1000m_e$ . The hyperons were found to have masses in excess of the neutron-mass but less than the mass of deuteron. Soon after, to the family of hyperons, two more new particles viz. the sigma-hyperons which exist in charge as well as uncharged states, and the Xi particles which exists in negatively charged and neutral states, were added. Also, the Kaons were found to exist in positive and negative charged states. Then the  $\Omega$ -hyperons and a large number of resonance particles were also discovered.

Following table shows the discovery scheme of the elementary particles.



**Fig1.** Time table of Discovery of elementary particles.

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## ***1.2 OBJECTIVES***

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After studying this unit, you should be able to explain about -

- Historical developments of elementary particles.
- Classification of the elementary particles.
- Types of the Fundamental interactions involved between elementary particles.
- Various properties associated with elementary particles e.g. Lepton and baryon number, Isospin, Strangeness, Hypercharge etc.
- Various symmetry and conservation laws involved in the elementary particles.

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## ***1.3 CLASSIFICATION OF ELEMENTARY PARTICLES***

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Having given an account of the discovery time-table of elementary particles, we can now proceed to group the particles in accordance with their common properties. So, after the discovery of the fundamental particles, a great deal of effort had gone into exploring the properties like masses, spins, parity, lifetimes and decay modes of these particles.

The elementary particle can be grouped into two broad categories differentiated from each other by a property called 'spin' which is intimately connected to the kind of statistics the particles are governed by.

We know that almost all the elementary particles spin about their axes and if they are charged, spin makes them tiny magnets. In units of  $\hbar$ , the electron, proton and neutron all have  $1/2$  spin while 2 photon has a spin of 1. An important property of the elementary particles related to spin; is the kind of statistics the particles follow. It has been found that half odd integer spin particles (in units of  $\hbar$ ) are governed by Pauli exclusion principle and obey Fermi-Dirac statistics and so are called Fermions. Proton, neutron and electron, all fall in this category of fermions. The integral spin particles such as photons and pions, Bose Einstein statistics and as such are called Bosons. One of the most important differences between these classes, viz. Fermions and Bosons, is that although there is no conservation law, governing the total number of bosons in the universe but the total number of Fermions in the universe is strictly conserved in all transformations.

Next, these two general groups of elementary, particles, viz. the Fermions and the Bosons can be further classified in accordance with the rest masses of the particles.

The fermions and the bosons, can be further classified in accordance with the rest masses of the particles.

### 1.3.1 Leptons (Weakly Interacting Fermions):

This type of the particles consists of Fermions of a spin  $\frac{1}{2}$  which have a mass lesser than that of nucleons. These particles are subjected to electromagnetic and weak (Fermi) interaction only. Members of this group are:

Muons ( $\mu^+$ ,  $\mu^-$ ), electron( $e^-$ ), positron( $e^+$ ) neutrino muon( $\nu_\mu$ ), Antineutrino muon( $\bar{\nu}_\mu$ ), neutrino electron( $\nu_e$ ) and antineutrino electron ( $\bar{\nu}_e$ ).

Leptons carry an additively conserved internal quantum number, called the Lepton number L, which has a value  $L = +1$  for leptons ( $e^-, \mu^-, \nu_e, \nu_\mu$ ) and  $L = -1$  for anti-leptons ( $e^+, \mu^+, \bar{\nu}_e, \bar{\nu}_\mu$ ) and zero for all other particles

### 1.3.2 Baryons (Strongly Interacting Fermions):

These particles comes in the category of fermions of half-integral spin and have masses equal to or in excess of nucleon mass. They are subject to all the three types of interactions, viz. strong weak and electromagnetic particles of this category which are heavier than nucleons are collectively known as hyperons. Members of this group are :-

Omega hyperon ( $\Omega^-$ ), Cascade hyperons ( $\Xi^0, \Xi^-$ ), Sigma hyperons ( $\Sigma^+, \Sigma^0, \Sigma^-$ ), Lambda hyperon( $\Lambda^0$ ), and the nucleons (proton and neutron).

Baryons carry an additively conserved internal quantum number, called the Baryon number B which has a value  $B = +1$  for the baryons and  $B = -1$  for antibaryons and zero for all other particles.

Similarly, Bosons also fall into two lighter and heavier categories.

(a) **Massless Bosons** (applicable to electromagnetic interaction only):- This group consists of integral spin Bosons i.e. with spins 1,2 or so and rest mass equal to zero. This group contains a sole member- the photon. The other conjectured member the graviton, has eluded detection thus far. The photon is subject to electromagnetic induction only.

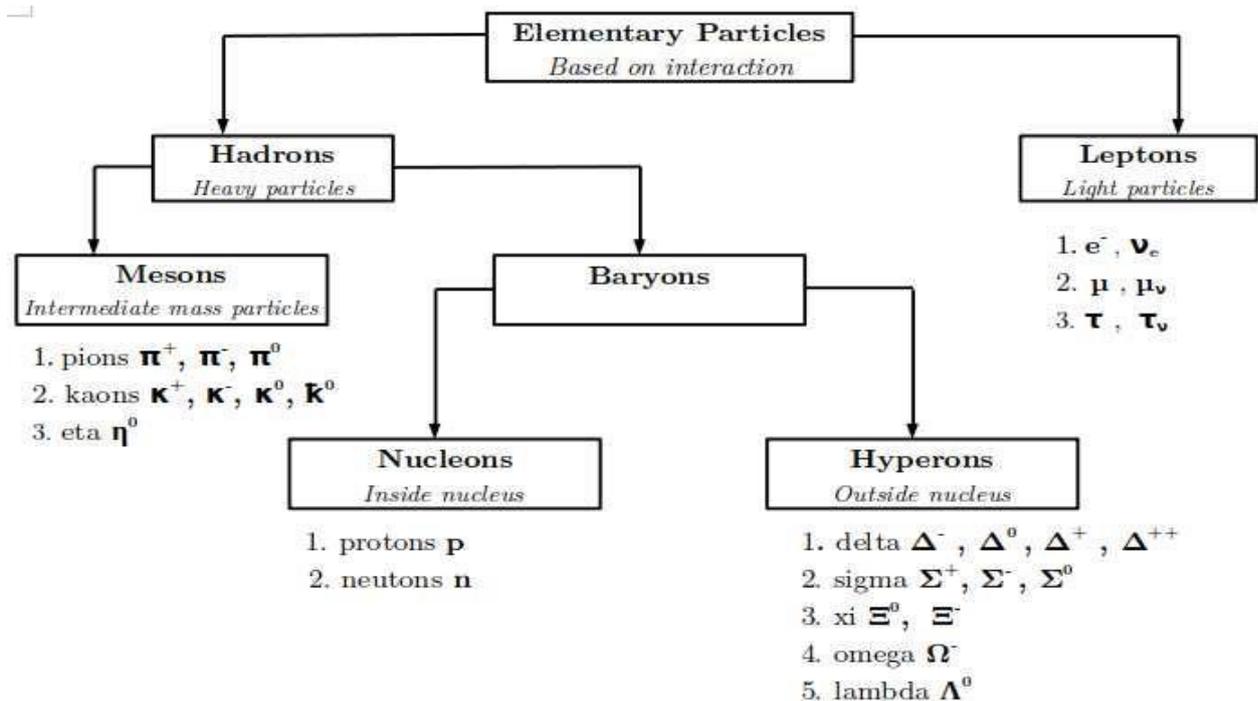
(b) **Mesons** (Strongly Interacting Bosons):- This group consists of four Bosons of spin 0 and having masses intermediate between the leptons and the nucleons and are subjected to all the three types of interaction, viz. strong weak and electromagnetic. Members of this group are the following:

$\eta$  – meson ( $\eta^0$ ), K-mesons ( $K^+, K^-, K^0, \bar{K}^0$ ) and  $\pi$ -mesons ( $\pi^+, \pi^-, \pi^0$ )

Look to the irony of fate- the first particle to be called a meson viz., the  $\mu$ -meson, does not fall into the category of mesons at all. Baryons and Mesons together are also termed as hadrons as they are strongly interacting particles.

By definition, hadrons are those particles which take part in strong interactions, but hadrons participate into other type of interactions also.

**TABLE1: CLASSIFICATION SCHEME OF ELEMENTARY PARTICLES**



## 1.4 FUNDAMENTAL INTERACTIONS

In order to study the decay modes of the elementary particles and their other properties, it is essential to have a knowledge of the various fundamental interactions to which they are subjected.

All the phenomena in high energy physics experiments can be explained in terms of the behavior of a few classes of elementary particles and those particles are governed by only four types of fundamental interactions i.e., there are only four different types of physical forces that are known although we speak of many more such as chemical, electrical, muscular, molecular etc. but they all can be classified as examples of these four fundamental interactions.

In order to explore the nature of any physical force, we proceed on the following lines:

- i. What kind of objects (Particles) participate in the interaction? Or what is the change that acts as a source?
- ii. How does the interaction depend upon the distance between the 'charges'?
- iii. What is the direction of the force?
- iv. Does the interaction depend upon the relative velocity and orientation of the participants?
- v. What is the strength of the force relative to the other three?
- vi. How is the interaction propagated through space? Is there any messenger? Does the effect take time or propagate instantaneously?

The four fundamental interactions are:

**(D)Gravitational Interaction-** The gravitational interaction is always attractive and does not depend upon the color size or any other parameter except inertia. Here 'charge' is the "mass".

The force is  $1/r^2$  type and acts along the line joining the masses or there spins. Its magnitude is relatively small and is conjectured to be propagated by means of particle called 'graviton' which has eluded observation so far. The force is characterized by a dimensionless coupling constant

$$\frac{GM_N^2}{\hbar c} = \frac{(6.67 \times 10^{-11})(1.67 \times 10^{-27})^2}{(1.6 \times 10^{-3})(3 \times 10^8)} \cong 5 \times 10^{-39} \quad \dots\dots\dots(1)$$

Where  $M_N$  is nucleon mass and G, gravitational constant. Its characteristic time is  $10^{16}$  sec.

**(II)Electromagnetic Interaction-** The electromagnetic interactions are charge -dependent and are attractive as well as repulsive. The force is of comparatively long-range character in as much as it is  $1/r^2$  type.

Electromagnetic interaction depends upon the velocity and magnitude of the charge and is not directed towards the charge but perpendicular to that direction and also to the direction of motion. The quantum of the electromagnetic field is the photon. The force is characterized by a dimensionless coupling constant,

$$\frac{e^2}{\epsilon_0 \hbar c} \frac{(1.6 \times 10^{-19})^2}{(8.854 \times 10^{-12}) \times (1.06 \times 10^{-34}) \times (3 \times 10^8)} \cong \frac{1}{137} \dots \dots \dots (2)$$

called the fine structure constant and has a characteristic time of  $10^{-20}$  sec. its magnitude is much greater than that of the gravitational interaction.

**(III)Strong Interaction-** We know that in order to explain the existence of stable nuclei, a force of a nuclear origin, strong enough to overcome the large repulsive forces between the density packed protons in the nucleus, is needed. The force cannot be of gravitational origin as it is too weak to supply the necessary binding energy.

The strong nuclear force is charge independent i.e. the p-p, n-n and n-p forces are the same. The force cannot be easily described in terms of strength-distance relationship. It certainly is not  $1/r^2$  type but is a very short-range force. It does not depend upon the relative orientation of the nucleons. The quantum of the field is the  $\pi$ -meson or Kaon. It is characterized by a coupling constant  $g^2/\hbar c = 1$  and has a characteristic time of  $10^{-23}$  sec.

**(IV)Weak Interactions-** Most of the elementary particles have short lives. Hyperons decay down into nucleons and mesons, in times of the order of  $10^{-10}$  sec. The pions decay into muons in about  $10^{-8}$  sec and muons collapse to electrons in about  $10^{-6}$  sec. These times are small on human scale but they are very large on nuclear time scale, a suitable unit of which is the time taken by a photon to cross the nuclear diameter, which comes out to be of the order of  $10^{-23}$  sec.

All the strong interactions take place in times of the order of  $10^{-23}$  sec. In the case of  $\beta$ - decay of radioactive nuclei, it is found that the process is very slow and does not take effect until a time  $10^{13}$  times greater than that involved in strong interactions. The strong nuclear and electromagnetic interactions cannot account for such a long stability. So, either the particles

are not subject to these forces or else some new prohibition forbids the decay. Since eventually the decay takes place, there must be a fourth type of interaction. The existence of such an interaction was proposed by Fermi in 1930's to explain  $\beta$ -decay. The force is comparatively very weak, and is characterized by a dimensionless coupling constant  $g_w^2/(\hbar c)^2 \approx 5 \times 10^{-14}$ , where  $g_w$  is the characteristic time of weak interactions is of the order of  $10^{-10}$  sec.

Calculation in natural units have shown that the relative strengths (by a parameter  $\alpha$ ) of the four fundamental interactions are in the order

$$\alpha_{\text{Strong}} : \alpha_{\text{EM}} : \alpha_{\text{Weak}} : \alpha_{\text{Grav}} = 1 : 10^{-2} : 10^{-7} : 10^{-38}$$

The essential characteristics of the four fundamental interactions are tabulated below.

**TABLE-2: COMPARISON OF FOUR FUNDAMENTAL INTERACTIONS**

<b>Interaction</b>	<b>Relative Magnitde</b>	<b>Carrier Particle</b>	<b>Characteristic time</b>	<b>Range</b>
<b>Gravitational Interaction</b>	$10^{-39}$	Graviton	$10^{16}$ sec	$\infty$
<b>Electromagnetic Interaction</b>	$10^{-2}$	Photon	$10^{-21}$ sec	$\infty$
<b>Strong Interaction</b>	1	Pion, Kaon	$10^{-23}$ sec	$10^{-15}$ m
<b>Weak interaction</b>	$10^{-14}$	Intermediate bosons	$10^{-10}$ sec	Almost zero

---

## 1.5 PARAMETERS OF ELEMENTARY PARTICLES

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For a systematic study of the properties and characteristics of the various elementary particles, the knowledge of their certain parameters is essential. these parameters are as follows:

**(a)Mass** - Elementary particles are distinguished from each other by the values of their mass which is expressed either in terms of energy needed to produce them or in gms. The electron mass in terms of energy is about 0.5 me and that of proton and neutron is about 1 BeV.

**(b)Particles, Anti – particles-** Until 1932, only three elementary particles viz, the electron, the proton and the photon, were known to physicist. But the year 1932 added two more particles to the list. These were the neutron- a neutral particle and the positron which has had the status of being the first anti-particle (that of electron) to have been discovered. The positron has a mass just equal to that of an electron and a charge just equal to the electronic charge but with opposite (positive) sign. Positron is as stable as an electron. The existence of positron was predicted theoretically by Dirac in his relativistic theory of the free electron.

**TABLE 3 :RELATIONSHIP BETWEEN PARTICLE AND ANTI-PARTICLES**

Mass	Same
Spin	Same
Charge	Of opposite sign but of same magnitude
Magnetic moment	Of opposite sign but of same magnitude
Mean life in free decay	Same
Annihilation	In pairs
Creation	In pairs
Total Isotopic spin	Same
Third component of isotopic spin	Of same magnitude but of opposite sign
Intrinsic parity	Same for Bosons Opposite for Fermions
Strangeness quantum number	Of opposite sign but of same magnitude

---

## 1.6 ELEMENTARY PARTICLE QUANTUM NUMBERS

---

In classifying the various elementary particles, several discrete quantum numbers are used. We are already familiar with two such quantum numbers, namely those that describe a particle's *charge* and *spin*. These quantum numbers specify measurable physical properties and are always conserved. We know that all elementary charges are 0 or  $\pm 1$ . The charge is conserved in all processes and no exceptions are known. The spin quantum number  $J$  is either an integer or a half odd integer for the particles so far detected. Particles with integer spin obey the Bose-Einstein statistics and are called bosons. Particles with half odd integer spins obey the Fermi-Dirac statistics and are called fermions. The other quantum numbers are more abstract and it is not always clear precisely to what aspect of physical reality they refer.

### 1.6.1: Baryon number

Each baryon is given a baryon number  $B=1$ , each corresponding antibaryon is given a baryon number  $B=-1$ . All other particles have  $B=0$ . The law of conservation of baryons states that the sum of the baryon numbers of all the particles after a reaction or decay must be the same as their sum before. This rule ensures that a proton cannot change into an electron, even though a neutron can change into a proton. Baryon conservation ensures the stability of the proton against decaying into a particle of smaller mass.

### 1.6.2: Lepton number

Leptons are supposed to possess a property called *Lepton number* ( $L$ ). Since the neutrinos associated with electrons and with muons are recognised as different, we introduce two lepton numbers  $L_e$  and  $L_\mu$ , both of which must be conserved separately in particle reactions and decays. The number  $L_e = 1$  is assigned to the electron and the  $e$ -neutrino and  $L_e = -1$  to their antiparticles. All other particles have  $L_e = 0$ . Also, the number  $L_\mu = 1$  is assigned to the muon and the  $\mu$ -neutrino and  $L_\mu = -1$  to their antiparticles. All other particles have  $L_\mu = 0$ .

**Example1:** In the decay of the neutron, in which  $B=1$  and  $L_e = 0$  before and after to show the particle number conservation.

$$n \rightarrow p + e^- + \bar{\nu}_e \quad \dots\dots\dots(3)$$

$$\text{Baryon number } (B): +1 \rightarrow (+1) + 0 + 0$$

$$\text{Lepton number } (L_e): 0 \rightarrow 0 + (+1) + (-1)$$

The above shown example is the only way in which the neutron can decay and still conserve both energy and baryon number.

**Example2: Pion decay**

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu \quad \dots\dots\dots(4)$$

$$\text{Conservation of } L_\mu: 0 \rightarrow (+1) + (-1)$$

**1.6.3: Isospin and Isospin Quantum Number, I and I<sub>3</sub>**

Isospin is a concept in particle physics that emerged from the study of hadrons, particularly nucleons (protons and neutrons), and their interactions. It is a quantum number associated with the symmetry between particles with similar properties but different electric charges.

Isospin is similar to the concept of spin, which is a quantum property that characterizes the intrinsic angular momentum of particles. However, isospin is not related to the physical rotation of particles but rather to a mathematical symmetry.

The isospin quantum number (I) can take on integer or half-integer values, similar to spin. It is denoted by I and can have values of 0, 1/2, 1, 3/2, and so on. Isospin is a symmetry that arises from the fact that protons and neutrons have similar properties and interact through the strong nuclear force, despite their different electric charges.

The concept of isospin is rooted in the charge independence of the strong interaction,  $nn \approx ne \approx ee$ . This doublet of neutrons and protons is said to have isospin 1/2 with the projection  $I_3 = +1/2$  for the proton and  $I_3 = -1/2$  for the neutron. The three pions ( $\pi^+, \pi^-, \pi^0$ ) compose a triplet, suggesting isospin  $I = 1$ . The projections  $I_3 = +1$  (for  $\pi^+$ ), 0 (for  $\pi^0$ ) and  $-1$  (for  $\pi^-$ ). At the quark level, an isospin doublet ( $I = 1/2$ ) is formed by the up and down quarks with the projection  $I_3 = +1/2$  assigned to up quark and  $I_3 = -1/2$  to down quark. The strange quark is in a class by itself and has isospin  $I = 0$ .

I is the additive quantum number and Clebsch -Gordon coefficients are used for the addition of isospins. The charge multiplicity is given by  $2I + 1$ . The antiparticle has the same I as the particle but the opposite  $I_3$ . Total isospin (I) is conserved in the strong interaction but breaks

down in the electromagnetic and the weak interactions. The third component  $I_3$  of the system of hadrons is conserved in the strong and the EM interactions but is violated in the weak interactions ( $\Delta I_3 = \pm 1/2$ ).

For example, the process

$$\Sigma^+ \rightarrow p + \eta^0 \quad \dots \dots \dots (5)$$

hasn't been observed even though it conserves the charge, angular momentum and the baryon number. It is forbidden by the fact that it doesn't conserve the isospin,  $I = 1 \neq 1/2 + 0$ , as required by the strong interaction.

Isospin is related to other quantum numbers for the particles by Gell-Mann formula,

$$Q = e \left( I_3 + \frac{S+B}{2} \right) \quad \dots \dots \dots (6)$$

Where S is the strangeness and B is the baryon number.

**1.6.4 STRANGENESS AND STRANGE PARTICLES**

Strangeness is a property of subatomic particles that was introduced to explain certain phenomena observed in particle interactions. It is a quantum number that characterizes the relative stability and decay properties of particles containing strange quarks. The concept of strangeness was first proposed by Murray Gell-Mann and Kazuhiko Nishijima in the 1950s as a way to understand the behavior of particles produced in high-energy collisions. It was observed that certain particles, such as the kaon (K) and the lambda baryon ( $\Lambda$ ), had longer lifetimes and unique decay patterns compared to other particles.

Strangeness is a quantum number that is conserved in strong and electromagnetic interactions but can change in weak interactions. It is denoted by the letter "S" and takes the value of +1 for particles containing strange quarks and -1 for their antiparticles. Particles without strange quarks have zero strangeness.

One of the key features of strange particles is their relatively long lifetime compared to other particles. This is due to the fact that strong interactions, which govern the stability of

particles, do not readily change the strangeness quantum number. Therefore, strange particles tend to survive longer before undergoing weak decays.

The conservation of strangeness in strong and electromagnetic interactions led to the discovery of the "principle of strangeness" or "strangeness conservation." This principle states that strong and electromagnetic interactions do not produce or destroy strange particles, only transform them into other strange particles or their antiparticles.

However, strangeness is not conserved in weak interactions. Weak decays can change the strangeness quantum number by transforming a strange quark into an up or down quark, or vice versa. This allows strange particles to decay into lighter particles, resulting in the eventual disappearance of strangeness. The study of strangeness has contributed to the understanding of the quark structure of matter and the behavior of elementary particles. It played a crucial role in the development of the quark model, which describes particles in terms of their constituent quarks and their associated quantum numbers.

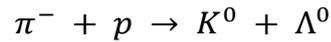
Table 4: Isospin (I) and the strangeness (S) for various hadron multiplets.

I/S	-3	-2	-1	0	1	2	3
0	$\Omega^-$		$\Lambda$		$\bar{\Lambda}$		$\bar{\Omega}^+$
				p			
		$\Xi^0$	$K^-$	n	$K^+$	$\bar{\Sigma}^-$	
1/2		$\Xi^-$	$K^0$	$\bar{p}$	$\bar{K}^0$	$\bar{\Sigma}^+$	
				$\bar{n}$			
			$\Sigma^+$	$\pi^+$	$\bar{\Sigma}^-$		
1			$\Sigma^0$	$\pi^0$	$\bar{\Sigma}^0$		
			$\Sigma^-$	$\pi^-$	$\bar{\Sigma}^+$		

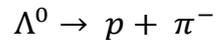
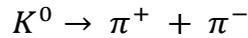
You can understand by the following example given below.

**Example:** Heavy unstable particles such as the  $\Lambda$  and  $\Sigma$  baryons, and the  $K$  mesons are produced at a rapid rate in the high energy collisions but decay slowly, i.e. they have exceptionally long lifetimes.

Their production is typical of strong interaction such as



and their decay is through weak interaction.



with lifetimes  $\tau_K \sim 0.89 \times 10^{-10}$  s and  $\tau_\Lambda \sim 2.63 \times 10^{-10}$  s which is larger compared with the strong interaction time scale of  $10^{-23}$  s.

They are named strange particles. A new quantum number  $S$ , strangeness is introduced to distinguish them from other particles. Gell-Mann formula relates  $S$  with other quantum numbers. Strangeness  $S$  is an additive quantum number like the electric charge, and is thus, conserved in the production process. Strangeness  $S$  is conserved in the strong and the electromagnetic interactions, i.e.  $\Delta S = 0$ , but breaks down in the weak interactions, i.e.  $\Delta S = \pm 1$ .

In summary, strangeness is a quantum number that characterizes the relative stability and decay properties of particles containing strange quarks. It is conserved in strong and electromagnetic interactions but can change in weak interactions. The study of strangeness has deepened our understanding of particle interactions and has been instrumental in the development of the quark model and our knowledge of the fundamental constituents of matter.

### 1.6.5 HYPERCHARGE

Hypercharge is a quantum number that represents the overall electric charge of particles in the context of the electroweak interaction, a unified theory combining the electromagnetic and weak interactions. Hypercharge is in general equal to the sum of strangeness and baryon numbers of the particle families. It is denoted by the symbol  $Y$  ( $Y = S + B$ ). Hypercharge is related to the electric charge ( $Q$ ) and the weak isospin ( $I_3$ ) of a particle through the equation:

$$Y = 2(Q - I_3) \quad \dots \dots \dots (7)$$

where  $Q$  represents the electric charge and  $T_3$  represents the third component of weak isospin. The factor of 2 in the equation is a convention to match the observed electric charges of particles.

The concept of hypercharge arises from the symmetry group associated with the electroweak interaction, called  $U(1)$ , which is related to the conservation of electric charge. The hypercharge quantum number allows us to assign specific values to particles based on their electric charge and weak isospin, providing a consistent framework for understanding the electric charge properties of particles within the context of the electroweak theory.

Hypercharge plays a crucial role in determining the behavior of particles under the electroweak interaction. It influences their interactions with the electroweak gauge bosons ( $W^+$ ,  $W^-$ , and  $Z^0$ ) and their coupling strengths to these bosons. By knowing the hypercharge of a particle, one can understand its electric charge properties and its role in electroweak interactions.

In the Standard Model of particle physics, particles are classified into different representations of the electroweak symmetry group based on their hypercharge and weak isospin. This classification allows for the description of particle interactions and the formulation of gauge theories that incorporate the electroweak interaction.

The concept of hypercharge has been crucial in the development of the electroweak theory and our understanding of the unified nature of the electromagnetic and weak interactions. It provides a framework for consistent and unified descriptions of particle properties and interactions in the realm of particle physics.

In summary, hypercharge is a quantum number that represents the overall electric charge of particles in the context of the electroweak interaction. It is related to the electric charge and weak isospin of a particle and allows for a consistent description of particle properties and interactions within the framework of the electroweak theory. Hypercharge plays a crucial role in our understanding of the unified nature of the electromagnetic and weak interactions in the Standard Model of particle physics.

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## 1.7 GELL-MANN NISHIJIMA RELATION

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The Gell-Mann-Nishijima relation, also known as the charge quantization rule, is a fundamental relation in particle physics that connects the electric charge of elementary particles to certain properties of their associated quantum numbers. It was formulated independently by Murray Gell-Mann and Kazuhiko Nishijima in the 1950s.

The Gell-Mann-Nishijima formula relates the baryon number  $B$ , the strangeness  $S$ , the isospin  $I_3$  of quarks and hadrons to the electric charge  $Q$ . The original form of the Gell-Mann-Nishijima formula is:

$$Q = I_3 + \frac{B + S}{2} \quad \dots \dots \dots (8)$$

Or it can be written as

$$Q = I_3 + \frac{Y}{2} \quad \dots \dots \dots (9)$$

where  $Y$  is the hypercharge of the particle. Note that  $Q$ ,  $I_3$ ,  $B$ , and  $S$  are all additive quantum numbers, and Gell-Mann Nishijima relation is a linear equation.

Isospin is a quantum number that describes the behaviour of particles under the strong nuclear force, similar to how spin describes their behaviour under rotations. Hypercharge, on the other hand, is a quantum number associated with the symmetry group called  $U(1)$  and plays a role in the electroweak interactions.

The Gell-Mann-Nishijima relation provides a systematic way to assign electric charges to particles based on their isospin and hypercharge quantum numbers. It helps establish a consistent framework for understanding the electromagnetic interactions of elementary particles, such as quarks and leptons. By determining the values of isospin and hypercharge for a particle, one can calculate its electric charge using the relation.

The Gell-Mann-Nishijima relation played a crucial role in the development of the quark model and the understanding of the underlying symmetries in particle physics. It provided a way to explain the observed patterns in particle charges and facilitated the discovery of new particles with specific combinations of isospin and hypercharge.

In summary, the Gell-Mann-Nishijima relation is an important relation in particle physics that connects the electric charge of elementary particles to their isospin and hypercharge quantum numbers. It has been instrumental in establishing a consistent framework for understanding the electromagnetic interactions of particles and has contributed to the development of the quark model and our understanding of particle symmetries.

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## ***1.8 SYMMETRY AND CONSERVATION LAWS***

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Conservation laws are critical to an understanding of particle physics. Strong evidence exists that energy, momentum, and angular momentum are all conserved in all particle interactions. The annihilation of an electron and positron at rest, for example, cannot produce just one photon because this violates the conservation of linear momentum. The special theory of relativity modifies definitions of momentum, energy, and other familiar quantities. In particular relativistic momentum of a particle differs from its classical momentum by a factor  $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$  that varies from 1 to  $\infty$  depending on the speed of the particle. But whatever is the quantity that is getting conserved is always leads to some symmetry. Closely related to conservation laws are three symmetry principles that apply to changing the total circumstances of an event rather than changing a particular quantity. The three symmetry operations associated with these principles are: charge conjugation (C), which is equivalent to exchanging particles and antiparticles; parity (P), which is a kind of mirror-image symmetry involving the exchange of left and right; and time-reversal (T), which reverses the order in which events occur. According to the symmetry principles (or invariance principles), performing one of these symmetry operations on a possible particle reaction should result in a second reaction that is also possible. However, it was found in 1956 that parity is not conserved in the weak interactions, i.e., there are some possible particle decays whose mirror-image counterparts do not occur. Although not conserved individually, the combination of all three operations performed successively is conserved; this law is known as the CPT theorem. There is an insightful connection between the conservation laws obeyed by a physical system and the symmetries. This connection is contained in the Noether's theorem.

### 1.8.1 Noether's Theorem:

Noether theorem states that every symmetry in nature is related to a conservation law and *vice versa*. The following table gives the idea about Noether's theorems.

Invariance under	Leads to
Translations in time	conservation of energy
Translations in space	conservation of momentum.
Rotation in space	conservation of angular momentum
Gauge transformation	conservation of charge

### 1.8.2 Conservation Laws for Elementary Particles

All the interactions are accompanied by certain conservation laws. Conservation laws for nuclear reactions has already been discussed in the SLM of nuclear physics in the previous semester. However, there exist a few conservation laws, which are valid for some interactions and not for all other interactions. For example, parity, isospin, and other flavour quantum numbers are not conserved in the weak interactions. The stronger an interaction the more symmetrical it is happens to exist a rule for conservation laws.

The familiar conservation laws of classical physics, i.e.

- ❖ Conservation of energy
- ❖ Conservation of charge
- ❖ Conservation of linear momentum
- ❖ Conservation of angular momentum

also hold good for particle physics, provided we have made relativistic formula for to mass momentum, etc.

**1.8.2.1 Conservation of Electric Charge**

The electric charge is one of the quantum numbers, which is exactly conserved. This conservation is associated with the masslessness of the photon. It is due to the conservation of charge that the electron can't decay, for instance, via

$$e^- \rightarrow \nu_e + \nu_e + \bar{\nu}_e$$

$$e^- \rightarrow \nu_e + \gamma \quad \dots\dots\dots(10)$$

**1.8.2.2 Conservation of Baryon Number**

No known interaction or decay process in nature alters the net baryon number. The neutron along with all the heavier baryons decays directly to the proton or eventually forms proton, since the proton is the least massive baryon. This indicates that the proton can't decay further without violating the conservation of baryon number, which means that if the conservation of baryon number holds exactly, the proton should be completely stable against any decay. One prediction of the grand unification of forces is that the proton can also decay. Until now, such a possibility hasn't been experimentally verified

Conservation of baryon number forbids a decay of the type:

$$p + n \rightarrow p + \mu^+ + \mu^-$$

$$B = 1 + 1 \neq 1 + 0 + 0 \quad \dots\dots\dots(11)$$

Although, with adequate energy permits pair production in the reaction

$$p + n \rightarrow p + n + p + \bar{p} \quad \dots\dots\dots(12)$$

$$B = 1 + 1 = 1 + 1 + 1 - 1$$

**1.8.2.3 Conservation of Lepton Number**

Each of the three sets of leptons ( $L_e, L_\mu, L_c$ ) are to be conserved separately. For example, decay processes

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

$$\mu^- \rightarrow e^- + \nu_e^- + \nu_\mu \quad \dots\dots\dots(13)$$

Since, a well-defined muon energy is observed from the decay, the first reaction (decay of  $\pi^-$  is known to be a two-body decay. However, the decay of muon ( $\mu^-$ ) into electron produces a distribution of electron energies, indicating that it is at least a three-body decay. In order for both  $L_e$  and  $L_\mu$  to be conserved, the other particles must be  $\nu_e^-$  and  $\nu_\mu$ .

**Example :** Which of the following processes are forbidden by the laws of lepton number ?

- (i)  $n \rightarrow p + e^- + \nu_e$
- (ii)  $\pi^+ \rightarrow \mu^+ + e^- + e^+$
- (iii)  $\pi^- \rightarrow \mu^- + \nu_\mu$
- (iv)  $p + e^- \rightarrow n + \nu_e$
- (v)  $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$
- (vi)  $K^- \rightarrow \mu^- + \bar{\nu}_\mu$

**Solution :**

(i) Forbidden as there are 2 more leptons  $e^-$ ,  $\nu_e$  on the right hand side as compared to zero on the left.

(ii) Forbidden because this corresponds to a change of lepton number by zero on the left and -1 on the right.

(iii) forbidden because  $\mu^-, \nu_\mu$  are both leptons, which means  $\Delta L = 2$ .

(iv) Allowed.

(v) Allowed.

(vi) Allowed.

### 1.8.3 Additional Conservation Laws: -

Other additional conservation laws are obeyed by some of the interactions not by all interactions. So, it's important to discuss these conservation laws like Conservation of Parity(P), Charge Conjugation (C), Isospin(I), Strangeness(S) and Hypercharge (Y). All above conservation laws are obeyed only in the strong interactions.

TABLE5 : Conservation laws for three type of fundamental interactions

Quantity	Strong	Electromagnetic	Weak
Charge (Q)	Yes	Yes	Yes
Baryon Number(B)	Yes	Yes	Yes
Angular Momentum(J)	Yes	Yes	Yes
Mass and Energy	Yes	Yes	Yes
Linear Momentum	Yes	Yes	Yes
Isospin	Yes	No	No
Third component of Isospin	Yes	Yes	No $\Delta I_z = \pm 1/2$
Strangeness	Yes	Yes	No $\Delta S = \pm 1$
Parity	Yes	Yes	No
Charge conjugation	Yes	Yes	No
Lepton Number	Yes	Yes	Yes
Charge and Parity(CP)	Yes	Yes	No
Time Reversal(T)	Yes	Yes	No
CPT	Yes	Yes	Yes

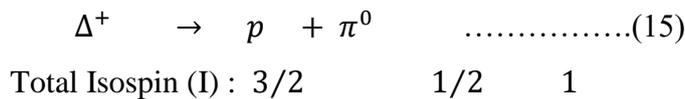
### 1.8.3.1 Conservation of Isospin (I)

According to law of conservation of isospin, the total isospin for particles subjected to strong interactions is always same before and after the particle reaction/decay. Remember that like angular momentum addition, isospin adds vectorially

$$\vec{I} = \vec{I}_1 + \vec{I}_2 \quad \dots\dots\dots(14)$$

that means I can have discrete values starting from  $|I_1 - I_2|$  to  $|I_1 + I_2|$  differing by unity. For example, isospin 1/2 when added to isospin 1, would give 1/2 and 3/2 as total isospin.

Consider the following decay:



in which total isospin of the final state ( $1/2 + 1 = 3/2$ ) match with that of the parent particles, so this decay is allowed to occur through the strong interactions.

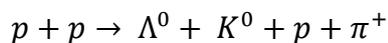
Now consider the decay of



Before the decay isospin is 1 and after the decay, it is zero. Hence, this decay is not allowed by strong interaction. However, this decay is possible via the electromagnetic interactions, as total isospin may not be conserved in these interactions.

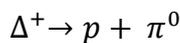
**1.8.3.2 Conservation of Component of Isospin (I<sub>3</sub>)**

According to this conservation law, for strong and electromagnetic interactions I<sub>3</sub>, the third component of isospin before and after particle reaction/decay always remains the same. Consider the following reaction:



Total I<sub>3</sub> component before the reaction is 1/2+1/2 = 1, After the reaction total I<sub>3</sub>, component is 0-1/2 + 1/2 +1=1. The component I<sub>3</sub> is same before and after the reaction, and hence this reaction is possible.

Similarly, for the following decay:



Third Component of Isospin ( $I_3$ ) :       $1/2 \quad 1/2 \quad 0 \quad \dots\dots\dots(18)$

But for the decay of

$$\Lambda \rightarrow p + \pi^-$$

Third Component of Isospin ( $I_3$ ):       $0 \quad 1/2 \quad -1 \quad \dots\dots\dots(19)$

Component  $I_3$  before the decay is zero and after the decay  $1/2 - 1 = -1/2$ , therefore,  $I_3$  is not conserved in this reaction as shown by equation(19) . Hence, this reaction cannot proceed via strong or electromagnetic interactions but can take place through weak interactions.

**1.8.3.3 Conservation of Hypercharge (Y)**

According to this conservation law, for strong and electromagnetic interactions, the total hypercharge remains the same before and after the particle reaction/decay. Consider the reaction

$$p + p \rightarrow \Lambda^0 + K^0 + p + \pi^+ \quad \dots\dots\dots(20)$$

Hypercharge before the reaction is  $1+1=2$  and after the reaction, it is  $0+1+1+0=2$ . Hypercharge before and after the reaction is same ( $=2$ ), hence this reaction is possible via strong or electromagnetic interactions. However, weak interactions can violate hypercharge conservation like  $\Lambda \rightarrow p + \pi^-$ .

**1.8.3.4 Conservation of Parity (P)**

Parity is a fundamental concept in physics that refers to the symmetry of physical systems under spatial reflections. It is associated with the transformation that reverses all spatial coordinates, effectively flipping a system across a mirror plane. The parity transformation, often denoted as P, changes the sign of each spatial coordinate, while leaving time unchanged. In other words, if a point in space has coordinates (x, y, z), under the parity transformation, it is transformed to (-x, -y, -z). Parity transformation essentially reflects a system across a mirror plane, interchanging left and right.

The particle's parity is a characteristic of the wave function representing its quantum mechanical state. Positive parity refers to a wave function that represents a single particle that

does not undergo a sign change upon reflection through the origin, whereas negative parity refers to a wave function that undergoes a sign change. So, we have

$$\Psi(x, y, z) = \Psi(-x, -y, -z) = \Psi(x, y, z), \text{ for positive parity .....(21)}$$

$$\Psi(x, y, z) = \Psi(-x, -y, -z) = -\Psi(x, y, z), \text{ for negative parity .....(22)}$$

A wave function that describes multiple particles can be expressed as the sum of the wave functions of each individual particle or as a composite of those wave functions.

Since the parities of the individual particle wave functions are a product, the parity of the entire system is evidently determined by this.

The concept of parity plays a significant role in particle physics and quantum mechanics. For a physical system to possess parity symmetry, it must exhibit the same behavior under a parity transformation. This means that the laws of physics governing the system should remain unchanged if all spatial coordinates are reversed. In certain physical processes, such as electromagnetic interactions, the laws of physics are indeed invariant under parity transformations. This property is known as "parity conservation" or "parity symmetry." For example, in classical electromagnetism, the equations governing the behavior of electric and magnetic fields are symmetric under parity transformations.

However, in the mid-20th century, experimental observations in weak nuclear interactions, particularly in the decay of certain particles, revealed a violation of parity symmetry. This discovery, known as the "parity violation," was a significant milestone in physics and led to a deeper understanding of the fundamental forces and the nature of elementary particles.

It was experimentally demonstrated that weak nuclear interactions violate parity symmetry, meaning that the laws governing these interactions do not remain the same under a parity transformation. This discovery paved the way for the development of the theory of weak interactions and contributed to the formulation of the electroweak theory, which unifies the electromagnetic and weak interactions.

It plays a crucial role in understanding the behavior of physical systems and the fundamental forces governing them. Indeed, parity conservation is found to hold true only in the strong and electromagnetic interactions, while weak nuclear interactions such as beta decays violate parity symmetry, leading to significant advancements in our understanding of particle physics.

Nucleons and electrons are assigned *positive* or even intrinsic parity. Then pions which are involved in strong interactions with nucleons should have *negative or odd* intrinsic parity (since parity is conserved in strong interactions) K-mesons and  $\eta^0$  mesons are allotted *negative* parity.  $\Lambda^0$  parity is taken to be *positive* and so is  $\Sigma$ -hyperon parity.  $\Xi$ -hyperons are also assigned positive parity. Although the parity of  $\Omega$ -hyperons has not been established conclusively but there are strong reasons to believe that it has positive parity.

### 1.8.3.5 Conservation of Charge Conjugation Parity(C):

Charge conjugation is a fundamental concept in particle physics that involves changing the sign of the electric charge of particles while keeping other properties unchanged. It is denoted by the symbol C and represents a theoretical operation that reverses the charge of particles.

$$\textit{Particle} \rightarrow \textit{Antiparticle}$$

Or

$$\textit{Antiparticle} \rightarrow \textit{Particle}$$

Under charge conjugation, particles with positive electric charge are transformed into corresponding antiparticles with negative electric charge, and vice versa. For example, an electron (-e) would be transformed into a positron (+e) under charge conjugation.

Charge conjugation also affects other properties associated with particles, such as baryon number, lepton number, and weak isospin. When applying charge conjugation, these properties are reversed as well. The baryon number of a proton (+1) is transformed into the baryon number of an antiproton (-1) under charge conjugation. For example, if in a hydrogen atom, the proton is replaced by an antiproton and the electron is replaced by a positron, then this antimatter atom will behave exactly like an ordinary atom, if observed by people also made of antimatter. In fact, C is not conserved in the weak interaction.

It is important to note that not all particles are affected by charge conjugation in the same way. In the Standard Model of particle physics, charge conjugation is a valid symmetry operation for electrically charged particles (fermions) but not for neutral particles or particles with zero electric charge (such as photons or neutral mesons). For these neutral particles, the concept of charge conjugation does not apply.

In the context of quantum field theory, charge conjugation is often combined with other symmetry operations, such as parity (P) and time reversal (T), to form the combined operation known as CPT. The CPT symmetry is a fundamental property of relativistic quantum field theories and is believed to be preserved in all known physical interactions.

Charge conjugation plays a significant role in understanding the properties of particles and the behavior of physical systems. It helps establish the relationship between particles and antiparticles, as well as the conservation laws associated with various quantum numbers. By studying the effects of charge conjugation, physicists gain insights into the fundamental symmetries and interactions governing the subatomic world.

### 1.8.3.6 Time Reversal Symmetry (T):

Time reversal is a fundamental concept in physics that involves reversing the direction of time in the evolution of physical systems. It is denoted by the symbol T and represents a theoretical operation that reverses the flow of time, effectively running physical processes backward. The idea of time reversal arises from the fundamental symmetry of the laws of physics under time reversal transformations. If a physical process obeys time reversal symmetry, it means that the laws governing that process remain unchanged if time is reversed. In other words, the physics of the system should be the same whether time progresses forward or backward.

Under a time reversal transformation, the positions and momenta of all particles in a system are reversed. For example, if a particle is moving forward in time with a certain momentum, under time reversal, it would be moving backward with the opposite momentum. Essentially, the roles of the past and future are interchanged. Prior to 1964, time parity T was considered to be conserved in every interaction. It was discovered in 1964 that one form of the  $K^0$  kaon can decay into  $\pi^+ + \pi^-$ , which violates the conservation of T. The concept of time reversal has profound implications in various areas of physics. In classical mechanics, if the laws of motion are invariant under time reversal, then the trajectories of particles should be reversible. However, in practice, the irreversibility of certain processes, such as the dissipation of energy or the increase of entropy, indicates that time reversal symmetry is not universally valid.

In quantum mechanics, time reversal symmetry is more subtle due to the presence of quantum superposition and measurement. While the fundamental laws of quantum mechanics are time reversal invariant, the process of measurement can introduce a preferred direction of time.

This phenomenon is known as the "arrow of time," where certain quantum processes are irreversible, and the observed world appears to have a preferred temporal direction.

Experimental studies have also revealed instances of time reversal violation in certain physical phenomena, particularly in weak interactions. For example, the discovery of CP-violation (combined charge conjugation and parity violation) in particle physics demonstrated a violation of time reversal symmetry.

### 1.8.3.7 CPT Theorem

As per the advanced quantum mechanics (or quantum field theory), all interactions should be invariant under the combination of C, P and T, i.e., CPT operation.

$$\psi_{particle}(\vec{x}, t) \rightarrow \psi_{antiparticle}(-\vec{x}, -t) \quad \dots\dots\dots(24)$$

This is also called as CPT theorem. In the experiments, pions and muons were found to come with both particle and antiparticle species. In 1957, CPT (C = charge conjugation, P= parity, T = time reversal) theorem was formulated which enunciates that *"for every particle that exists in nature there is a corresponding antiparticle"*. The theorem also predicts that the antiparticle has the same mass as its particle, the same lifetime (if the particle is not stable), the opposite charge and the opposite of all other internal quantum numbers. Certain particles act as their own antiparticle, like photon and  $\pi^0$ .

### 1.8.4 Parity Violation

Parity violation is a phenomenon in physics where certain physical processes do not exhibit symmetry under spatial reflections, known as parity transformations. It refers to situations where the laws of physics governing a system are not the same when left and right directions are interchanged. The discovery of parity violation was a significant milestone in the development of particle physics. In 1956, Chien-Shiung Wu and her collaborators conducted an experiment known as the Wu experiment, which demonstrated a violation of parity symmetry in weak nuclear interactions. The experiment involved the beta decay of cobalt-60 nuclei and showed that the emitted electrons were preferentially emitted in one direction, indicating an asymmetry under parity transformations.

The violation of parity symmetry in weak interactions had profound implications for our understanding of the fundamental forces and the nature of elementary particles. Prior to this

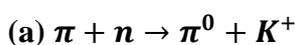
discovery, it was widely believed that all known interactions, including electromagnetic and strong nuclear interactions, exhibited parity symmetry. However, the Wu experiment revealed that the weak nuclear force violates this symmetry.

The violation of parity symmetry in weak interactions led to the formulation of the V-A (vector minus axial vector) theory by Richard Feynman and Murray Gell-Mann. This theory described the behavior of weak interactions by introducing the concept of left-handed and right-handed particles, and it successfully accounted for the experimental observations of parity violation.

The discovery of parity violation had a profound impact on our understanding of the fundamental forces and the structure of matter. It demonstrated that the laws of physics are not always symmetric under spatial reflections and highlighted the importance of studying the underlying symmetries and their violations.

Since the discovery of parity violation, further experimental investigations have revealed violations of other symmetries, such as CP-violation (combined charge conjugation and parity violation) and time reversal symmetry. These discoveries have deepened our understanding of the fundamental interactions and the behavior of particles, contributing to the development of the Standard Model of particle physics.

**Example1:** Let us take a few examples by taking into the consideration the application of these conservation laws, to decide which reactions are allowed and which are forbidden.

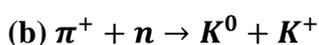


The particles taking part in the reaction are a pion and a neutron and hence the reaction shall proceed by way of strong interaction.

Applying conservation laws for the validity strong interactions.

(i) Charge number Q :	$+1 + 0$	$= 0 + 1$ ,	Q is conserved
(ii) Baryon number B:	$0 + 1$	$= 1 + 0$	B is conserved
(iii) Strangeness S	$0 + 0$	$= -1 + 1$ ,	S is conserved
(iv) I3	$+1 - \frac{1}{2}$	$= 0 + \frac{1}{2}$ ,	I3 is conserved

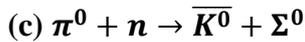
Hence the reaction is allowed.



Applying the conservation laws

- (i) Charge number  $Q$   $+1 + 0 = 0 + 1$ ,  $Q$  is conserved  
 (ii) Baryon number  $B$  :  $0 + 1 = 0 + 0$   $B$  is not conserved  
 (iii) Strangeness  $S$  :  $0 + 0 = +1 + 1$   $S$  is not conserved  
 (iv)  $I_3$ ;  $+1 - \frac{1}{2} = -\frac{1}{2} + \frac{1}{2}$ ,  $I_3$  is not conserved

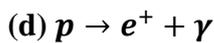
So the reaction is not allowed as  $B, S$  and  $I_3$  are not conserved.



Applying the conservation laws appropriate to strong interactions.

- (i) Charge number  $Q$   $0 + 0 = 0 + 0$   $Q$  is conserved  
 (ii) Baryon number  $B$   $0 + 1 = 0 + 1$   $B$  is conserved  
 (iii) Strangeness  $S$   $0 + 0 = -1 - 1$ ,  $S$  is not conserved  
 (iv)  $I_3$ :  $0 - \frac{1}{2} = +\frac{1}{2} + 0$   $I_3$  is not conserved.

The reaction is not allowed as  $S$  and  $I_3$  are not conserved.

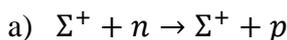


Applying conservation laws,

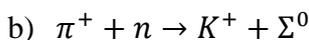
- (i) Charge number  $Q$ ;  $+1 = +1 + 0$ ,  $Q$  is conserved  
 (ii) Baryon number  $B$ ;  $+1 = 0 + 0$ ,  $B$  is not conserved  
 (iii) Lepton number  $L$ ;  $0 = -1 + 0$   $L$  is not conserved.

So the decay is not permitted.

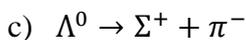
**Example 2:** Which of the following reactions are allowed and forbidden through the strong interactions ?



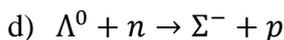
Forbidden as charge is not conserved.



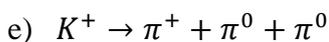
Allowed by strong interaction as strangeness is conserved.



Forbidden as energy is not conserved.



Allowed, as strangeness is conserved.



Forbidden as strangeness is not conserved.

**Example3.** Classify the following processes as they are either strong, electromagnetic, weak or totally forbidden by the conservation of parity (P), strangeness(S), isospin(I) and third component of isospin( $I_3$ ).

a)  $\pi^- + p \rightarrow \Lambda^0 + K^0$

Strong - Parity, isospin, strangeness and  $I_3$  are conserved.

b)  $\pi^- + p \rightarrow n + \pi^0$

Strong - Parity, isospin, strangeness and  $I_3$  are conserved

c)  $p + \gamma \rightarrow p + \pi^0$

Electromagnetic - Parity, strangeness,  $I_3$  are conserved but isospin is not conserved.

(d)  $\Sigma^0 \rightarrow \Lambda^0 + \gamma$

Electromagnetic-Parity, strangeness,  $I_3$  are conserved but isospin not conserved.

(e)  $\Lambda^0 \rightarrow p + \pi^-$

Weak - Parity, strangeness, isospin and  $I_3$  not conserved.

(f)  $\Xi^- \rightarrow \Lambda^0 + \pi^-$

Weak - Parity, strangeness, isospin and  $I_3$  not conserved.

(g)  $K^0 \rightarrow \pi^+ + \pi^-$

Weak - Parity, strangeness, isospin and  $I_3$  not conserved.

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## 1.9 SUMMARY

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After studying the unit students will be able to understand

- Historical developments of elementary particles.
- Classification of the elementary particles.
- Types of the Fundamental interactions involved between elementary particles.
- Various properties associated with elementary particles e.g. Lepton and baryon number, Isospin, Strangeness, Hypercharge etc.
- Various symmetry and conservation laws involved in the elementary particles

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### ***1.11 SUGGESTED READINGS***

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3. Hughes, J.S., Elementary Physics, Cambridge University Press, Cambridge, 1991
4. B.R.Martin, Nuclear and Particle Physics, Wiley, New York, 2006.
5. Gell-Mann, M., What are the Building Blocks of Matter-The Nature of the Physical Universe. Wiley, New York, 1979.
6. Cheng, D.C. and O'Neill, G.K., Elementary Particle Physics, Addison-Wesley, Reading (MAL 1979).
7. Perkins, D.H., Introduction to High Energy Physics, Addison-Wesley, Reading (MA), 1982.

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### ***1.12 TERMINAL QUESTIONS***

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1. What are baryon and lepton conservation laws?
2. Explain the concept of charge conjugation.
3. Explain the concept of isospin.
4. What is strangeness?
5. How do you differentiate between leptons and baryons?

6. Explain the concept of strangeness and associated production. Discuss the Gell-Mann-Nishijima relation.
7. Explain the terms isotopic spin and strangeness. In what respect are they important in the classification of elementary particles?
8. Discuss the quantum numbers associated with elementary particles. Give corresponding conservation laws. Give at least one example in support of each conservation.
9. Discuss the various types of interactions between elementary particles giving the characteristic coupling constant and lifetimes. What are the interaction carriers electromagnetic and strong Interactions between nuclei?
10. Discuss the following intrinsic quantum numbers associated with elementary particles
  - a) Charge number
  - b) Lepton number
  - c) Hypercharge
  - d) Isospin quantum number
  - e) Strangeness
  - f) Decay process of mesons
11. Define parity, charge conjugation and time reversal, State CPT theorem.
12. Discuss in detail the strange particles.
13. Give the Gell-Mann-Nishijima classification scheme of strange particles.
14. What is parity? Describe an experiment illustrating violation of parity conservation.
15. Discuss whether the following particle reactions are allowed or forbidden under conservation of charge  $Q$ , Baryon number  $B$  and strangeness  $S$  ?
  - (i)  $\pi^+ + n \rightarrow \Lambda^0 + K^+$
  - (ii)  $\pi^+ + p \rightarrow \Lambda^0 + \pi^0$
16. Classify the following particle reactions in terms of the type of interaction using conservation of strangeness, isospin  $I$  and third component  $I_3$ .
  - (i)  $\pi^- + p \rightarrow n + \pi^0$
  - (ii)  $p + \gamma \rightarrow p + \pi^0$
  - (iii)  $\Lambda^0 \rightarrow p + \pi^0$

## UNIT 2

# QUARK MODELS

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### Structure of the Unit

- 2.1 Introduction
- 2.2 Objectives
- 2.3 CP Violation in Mesons
- 2.4 CPT Invariance
- 2.5 Mesons and Yukawa's Hypothesis: Pions
- 2.6 Basic Constituents of Particles
- 2.7 The Eightfold Way
- 2.8 The Quark Model
  - 2.8.1 Need of Color in Quarks
  - 2.8.2 Gluons
  - 2.8.3 Charm, Bottom and Top
  - 2.8.4 Three Generation of Quarks
- 2.9 Summary
- 2.10 References
- 2.11 Suggested Readings
- 2.12 Terminal Questions

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## 2.1 INTRODUCTION

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In the previous unit we have obtained the basic information about elementary particles and its classification on the basis of spin statistics followed by these particles. In this consequence we have discussed the basic quantum number possessed by them and also studied the various conservation laws exhibited by them. Now, in this unit we shall study about the CP violations in mesons and CPT invariance. In this unit we will study about the Quark Model theory according to it baryons and mesons are made up of quarks and antiquarks.

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## 2.2 OBJECTIVES

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After studying the unit, the learners will be able to understand

- CP violation in mesons
- CPT Invariance
- Mesons and Yukawa's Hypothesis: Pions
- The Eightfold Way
- The Quark Model
- Need of Colour in Quarks
- Gluons

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## 2.3 CP VIOLATION IN MESONS

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The term "CP violation in mesons" refers to the phenomena of combined parity (P) and charge conjugation (C) violation seen in some mesons' decay processes. The CP violation in the decays of mesons, which are composite particles made of a quark and an antiquark, was crucial in helping to understand how fundamental symmetries in particle physics were broken.

James Cronin and Val Fitch, who shared the 1964 Nobel Prize in Physics for their discovery, made the first observation of CP violation in the decays of neutral kaons ( $K^0$  mesons).  $K^0$  and its antiparticle,  $\bar{K}^0$ , are the two different states that make up the neutral kaon system. These two states are capable of oscillating back and forth between one another.

The interference between the direct decay process and the decay via  $K^0$ - $\bar{K}^0$  oscillation in the neutral kaon system is how the CP violation is seen. The combined CP symmetry is broken because the rates of decay of  $K^0$  and  $\bar{K}^0$  distinct end states are not the same.

Epsilon ( $\epsilon$ ), a quantity that expresses the asymmetry between the rates of  $K^0$  and  $\overline{K}^0$  decays, is used to describe CP violation in neutral kaon decays. The discovery of CP violation through the measurement of  $\epsilon$  opened up new research directions for understanding the fundamental interactions and symmetries of particle physics.

A significant understanding of the nature of matter and antimatter asymmetry in the universe was revealed by the finding of CP violation in mesons. It improved our knowledge of the Sakharov requirements, which are required to produce the apparent matter-antimatter asymmetry in the early cosmos. In order to explain why the cosmos is mostly made up of matter rather than being symmetric with equal proportions of matter and antimatter, CP violation is a key component.

To better understand the underlying physical mechanisms, further study on CP violation in mesons has been done. Other meson systems, such as B mesons, have also generated a lot of interest in the investigation of CP violation. Significant improvements in theoretical models and experimental methods have resulted from research on CP violation in meson decays.

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## ***2.4 CPT INVARIANCE***

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The most significant symmetry characteristic of relativistic quantum field theory is CPT invariance. It claims that the principles of physics remain unaffected by the combined operation of charge conjugation (C), parity inversion (P), and time reversal (T). In other words, if a physical process abides by CPT invariance, it is anticipated to behave in the same way when charge, parity, and time are all reversed at the same time.

The discovery that these three transformations combine to create a complete symmetry operation that upholds the fundamental principles of physics gives rise to the idea of CPT invariance. The formulation of the Standard Model of particle physics, which describes the fundamental particles and their interactions, depends critically on CPT invariance.

Particles undergo charge conjugation (C), which reverses their electric charge to create their antiparticles. The system is essentially reflected across a mirror plane via parity inversion (P), which flips the spatial coordinates. Physical processes are carried out in the reverse order than they are in the original process. Time reversal (T) reverses the direction of time.

Under these coupled transformations, symmetry of the physical laws is guaranteed by CPT invariance. The simultaneous reversal of the charge, the spatial coordinates, and the time implies

that physical processes have the same characteristics. All presently understood physical interactions, including electromagnetic, strong, and weak interactions, are thought to preserve this symmetry.

The consequences for particle characteristics and interactions are what give CPT invariance its relevance. It results in the equality of the masses, lifetimes, and decay rates of particle-antiparticle pairs. Additionally, it imposes restrictions on how particles behave, such as the conservation of a number of quantum constants, including as the electric charge, baryon number, lepton number, and strangeness.

In numerous high-precision investigations, including measurements of particle-antiparticle mass differences and lifetime ratios, the CPT invariance has been put to the test. The data support the theoretical foundation of relativistic quantum field theory and CPT invariance.

These are listed below:

- (i) The fact that every particle has an antiparticle.
- (ii) The masses and lifetimes of particles and antiparticles are the same.
- (iii) Every antiparticle's internal quantum number is the reverse of that of the particles.

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## ***2.5 MESONS AND YUKAWA'S HYPOTHESIS: PIONS***

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Mesons are composite particles which come under the category of hadrons, together with protons and neutrons. They are created when a quark and an antiquark are bound together and held together by the strong nuclear interaction via gluons. Mesons are categorised as bosons because they have integral spins of 0, 1, or 2.

Hideki Yukawa put up the Yukawa hypothesis in 1935, and it was crucial in advancing knowledge of the nuclear force and the presence of mesons. Yukawa proposed that the exchange of a particle served as the medium via which the strong nuclear force, which holds atomic nuclei together, operated. The "meson of intermediate mass" was the name given to this exchange particle, which was eventually determined to be a meson. The idea of exchange particles, or virtual particles, served as the foundation for Yukawa's theory. A particle's interaction with another particle can be thought of as the exchange of virtual particles, according to the quantum field theory, which treats particles as quanta of their respective fields.

These imaginary particles, which only briefly exist during the encounter, moderate the forces between the physical particles. Yukawa postulated that the interaction of mesons between protons and neutrons served as a medium for the strong nuclear force between them.

Theoretically, these mesons would be between the lightest particles, like electrons, and the heaviest particles, like protons and neutrons.

Meson exchange between nucleons gave a mechanism for the nuclear force's attractiveness and provided an explanation for the short-range nature of the attraction.

Observations made during experiments subsequently confirmed Yukawa's theory. In cosmic ray studies conducted in the 1940s and 1950s, the first mesons, including the pion (meson) were found. The presence of the intermediate-mass mesons that Yukawa predicted as well as the idea of exchange particles mediating fundamental forces were both substantiated by these observations.

The discovery of the pion and other pioneering mesons did not have the essential characteristics to adequately explain the strong nuclear force, it was later revealed.

The true mediators of the strong force, the gluons, were found in the 1970s, and the quantum chromodynamics (QCD) hypothesis gave a more thorough understanding of the strong interaction.

However, the Yukawa theory and the early knowledge of mesons set the path for the growth of QCD and our comprehension of the fundamental forces and particles in the universe. Mesons, which are composite particles made up of quark-antiquark pairs, are still of considerable interest to researchers in particle physics because they shed light on the strong interaction and the behaviour of quarks inside hadrons.

### **BASIC PROPERTIES OF PIONS**

Pions, which are mesons with zero spin, are composed of first-generation quarks. In the quark model, an up quark and an anti-down quark make up a  $\pi^+$  whereas a down quark and an anti-up quark make up the  $\pi^-$  and these are the antiparticles of one another. The neutral pion  $\pi^0$  is a combination of an up quark with an anti-up quark or a down quark with an anti-down quark. The two combinations have identical quantum numbers, and hence they are only found in superpositions. The lowest-energy superposition of these is the  $\pi^0$ , which is its own antiparticle. Together, the pions form a triplet of isospin. Each pion has isospin ( $I = 1$ ) and third-component isospin equal to its charge ( $I_z = +1, 0$  or  $-1$ ).

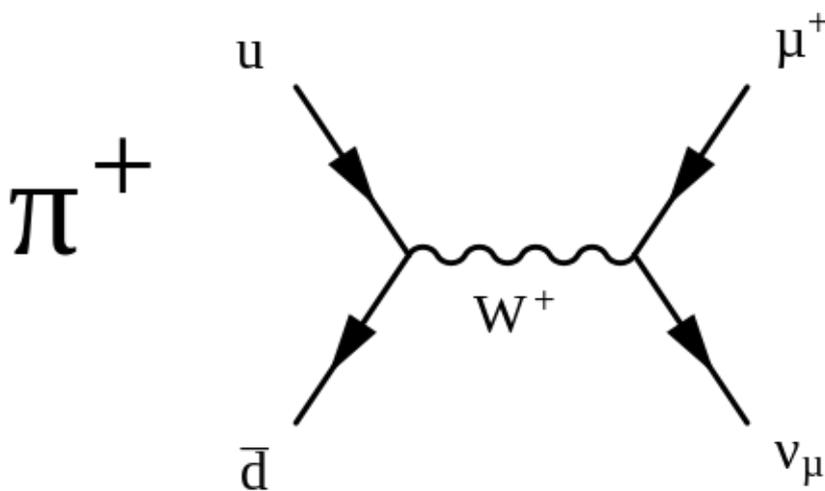
**Charged pion decays**

The  $\pi^{0\pm}$  mesons have a mass of  $139.6 \text{ MeV}/c^2$  and a mean lifetime of  $2.6033 \times 10^{-8} \text{ s}$ . They decay due to the weak interaction. The primary decay mode of a pion, with a branching fraction of  $0.999877$ , is a leptonic decay into a muon and a muon neutrino:

$$\begin{aligned} \pi^+ &\rightarrow \mu^+ + \nu_\mu \\ \pi^- &\rightarrow \mu^- + \bar{\nu}_\mu \end{aligned} \quad \dots\dots\dots(1)$$

The second most common decay mode of a pion, with a branching fraction of  $0.000123$ , is also a leptonic decay into an electron and the corresponding electron antineutrino. This "electronic mode" was discovered at CERN in 1958.

$$\begin{aligned} \pi^+ &\rightarrow e^+ + \nu_e \\ \pi^- &\rightarrow e^- + \bar{\nu}_e \end{aligned} \quad \dots\dots\dots(2)$$



**Fig.1:** Feynman diagram of the dominant leptonic pion decay.

**Neutral pion decays**

The  $\pi^0$  meson has a mass of  $135.0 \text{ MeV}/c^2$  and a mean lifetime of  $8.5 \times 10^{-17} \text{ s}$ . It decays via the electromagnetic force, which explains why its mean lifetime is much smaller than that of the charged pion (which can only decay via the weak force).

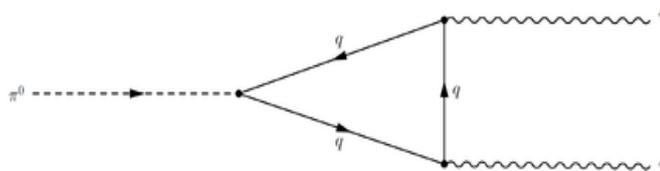
The dominant  $\pi^0$  decay mode is into two photons:

$$\pi^0 \rightarrow 2\gamma \quad \dots\dots\dots(3)$$

The decay

$$\pi^0 \rightarrow 3\gamma \quad \dots\dots\dots(4)$$

(as well as decays into any odd number of photons) is forbidden by the C-symmetry of the electromagnetic interaction: The intrinsic C-parity of the  $\pi^0$  is +1, while the C-parity of a system of n photons is  $(-1)^n$ .



**Fig.2:** Anomaly-induced neutral pion decay.

## 2.6 BASIC CONSTITUENT OF PARTICLES

Leptons and quarks are the fundamental components of matter, according to these findings and theoretical research. In other words, they are now recognised as the "elementary particles."

Quarks and leptons can be categorised into four-member families (the quark and lepton quartet) as a result of recent breakthroughs. Surprisingly, the first-generation constituents, which include "up" and "down" quarks, the electron, and the electron-type neutrino, may describe all the elements of our regular everyday world. Two further generations of quarks and leptons enter the picture at higher energy, which can be seen in some extra terrestrial phenomena or can be induced intentionally using particle accelerators.

## 2.7 THE EIGHTFOLD WAY

There have been numerous attempts to find a system of organisation that grouped elementary particles into more significant groupings of identification. The Eightfold Way, which is adapted from Buddhism, is the only one that has enjoyed significant success. The Eightfold Way organises baryons and mesons into geometric patterns with the same baryon number, spin, and parity. It was independently proposed in 1961 by both Gell-Mann and Yuval Ne'eman. The baryon octet, which is made up of the eight lightest baryons, is a visualisation of one of these shapes.

**Meson Octet:** The eightfold way organizes eight of the lowest spin-0 mesons into an octet. Diametrically opposite particles in the diagram are anti-particles of one-another while particles in the centre are their own anti-particle.

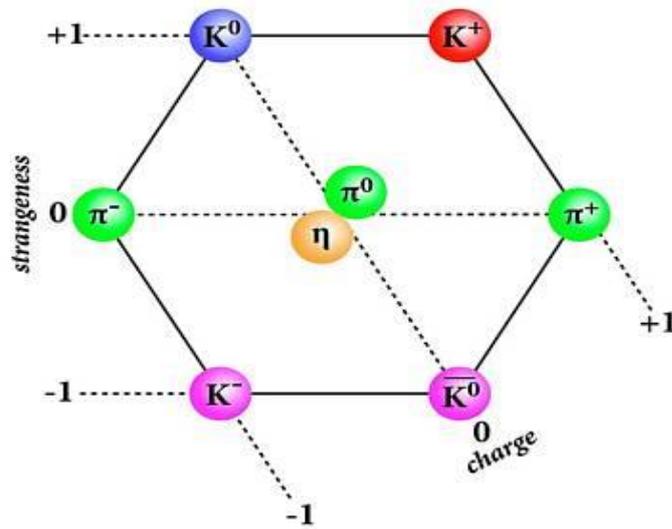


Fig 3: Meson Octet

**Baryon Octet:** The eightfold way organizes the spin- 2 baryons into an octet. Diametrically opposite particles in the diagram are anti-particles of one-another while particles in the centre are their own anti-particle.

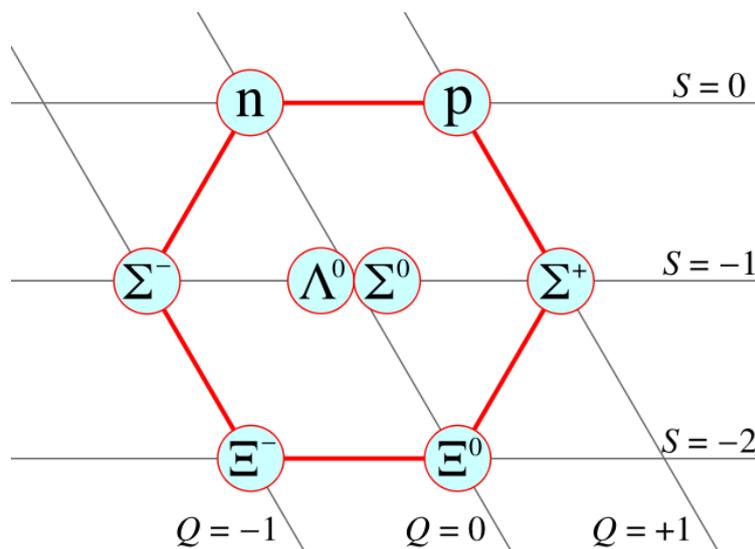


Fig 4: Baryon Octet

**Baryon decuplet:** The principles of the eightfold way also applied to the spin-3/2 baryons, forming a decuplet. However, one of the particles of this decuplet had never been previously observed when the eightfold way was proposed. Gell-Mann called this particle the  $\Omega^-$  and predicted in 1962 that it would have a strangeness -3, electric charge -1 and a mass near 1680 MeV/c<sup>2</sup>. In 1964, a particle closely matching these predictions was discovered by a particle accelerator group at Brookhaven. Gell-Mann received the 1969 Nobel Prize in Physics for his work on the theory of elementary particles.

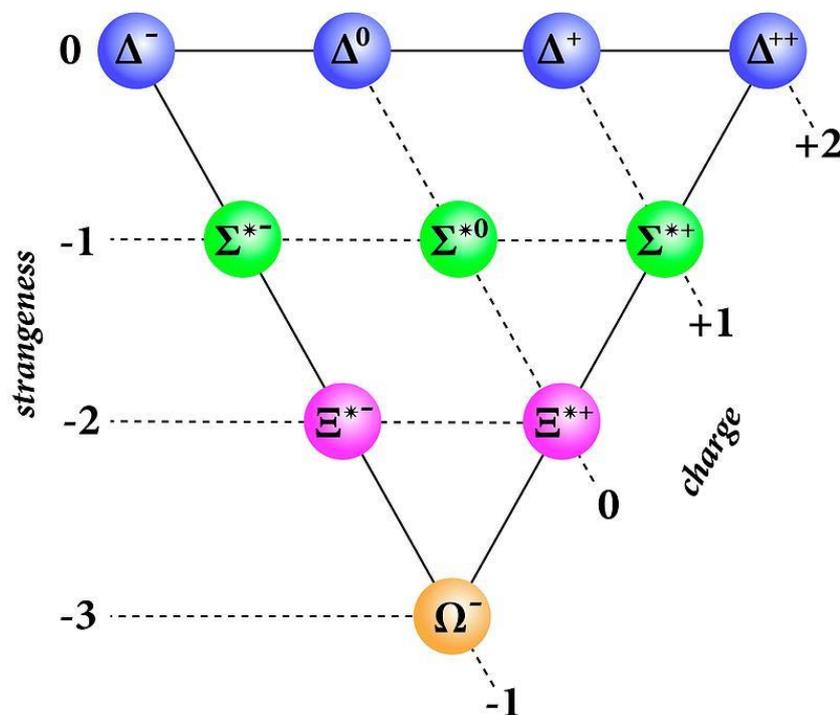


Fig. 5: Baryon decuplet

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## 2.8 THE QUARK MODEL

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Murray Gell-Mann and G. Zweig proposed the quark model in 1964. This theory is based on the idea that the hadrons are built up from a limited number of "fundamental" units, which

have acquired the name quarks. Originally three quarks were labeled by up(u), down(d) and strange (s).

$u$  quark has electric charge  $+\frac{2}{3}$  and strangeness 0 .

$d$  quark has electric charge  $-\frac{1}{3}e$  and strangeness 0 .

$s$  quark has electric charge  $-\frac{1}{3}e$  and strangeness -1 .

Each quark has a baryon number of  $B = 1/3$ .

Each quark has an antiquark associated with it  $\bar{u}, \bar{d},$  and  $\bar{s}$ . The magnitude of each of the quantum numbers for the antiquarks has the same magnitude as those for the quarks, but the sign is changed. As shown in the following table 1.

**Table1:** Representation of Quark and Antiquark

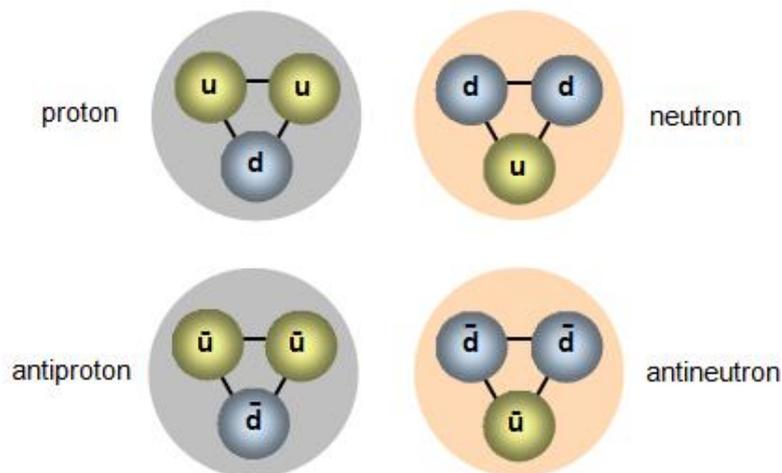
Quarks					
Name	Symbol	Spin	Charge	Baryon number	Strangeness
Up	$u$	$\frac{1}{2}$	$+\frac{2}{3}e$	$\frac{1}{3}$	0
Down	$d$	$\frac{1}{2}$	$-\frac{1}{3}e$	$\frac{1}{3}$	0
Strange	$s$	$\frac{1}{2}$	$-\frac{1}{3}e$	$\frac{1}{3}$	-1
Antiquarks					
Antiup	$\bar{u}$	$\frac{1}{2}$	$-\frac{2}{3}e$	$-\frac{1}{3}$	0
Antidown	$\bar{d}$	$\frac{1}{2}$	$+\frac{1}{3}e$	$-\frac{1}{3}$	0
Anti-strange	$\bar{s}$	$\frac{1}{2}$	$+\frac{1}{3}e$	$-\frac{1}{3}$	+1

The assumption of the model is that:

Hadrons are not ‘fundamental’, but they are built from ‘valence quarks’, i.e. quarks and antiquarks, which give the quantum numbers of the hadrons

$$\text{Baryons} = q q q \text{ and Mesons} = q \bar{q}$$

Hadrons as we have already studied that is composed of baryons or mesons. A baryon is made up of three quarks. For example, the proton is made up of two  $u$  quarks and a  $d$  quark ( $uud$ ). For these quarks, the electric charges are  $+2/3$ ,  $+2/3$ , and  $-1/3$  for a total value of  $+1$ . The baryon numbers are  $+1/3$ ,  $+1/3$  and  $+1/3$ , for a total of  $+1$ . The strangeness numbers are  $0$ ,  $0$  and  $0$  for a total strangeness of  $0$ . All are in agreement with the quantum numbers for the proton. Fig.6 shows quark models of the proton, antiproton, neutron, and antineutron.



**Fig 6:** Quark model for proton, antiproton, neutron, and antineutron

A meson is made up of one quark and one antiquark. For example, the  $\pi^+$  meson is the combination of a  $u$  quark and a  $d$  antiquark ( $u\bar{d}$ ). Electric charges of these quarks are  $+2/3$  and  $+1/3$  for a total of  $+1$ . The baryon numbers are  $+1/3$  and  $-1/3$  for a total baryon number of  $0$ . The strangeness numbers are  $0$  and  $0$  for a total of  $0$ . All of these are in agreement with the quantum numbers for the pi-meson. Quarks all have spins of  $1/2$ , which accounts for the observed hadrons. All known hadrons can be explained in terms of the various quarks and their antiquarks. Table 2 shows the quark contents of hadrons.

**Table 2:** Composition of Hadrons According to Quark Model

Mesons		Baryons	
Particle	Quark composition	Particle	Quark composition
$\pi^0$	$u\bar{u}$	$p$	$uud$
$\pi^+$	$u\bar{d}$	$n$	$udd$
$\pi^-$	$d\bar{u}$	$\Lambda^0$	$uds$
$\eta^0$	$d\bar{d}$	$\Sigma^+$	$uus$
$K^+$	$u\bar{s}$	$\Sigma^0$	$uds$
$K^0$	$d\bar{s}$	$\Sigma^-$	$dds$
$K^-$	$s\bar{u}$	$\Xi^0$	$uss$
$\bar{K}^0$	$s\bar{d}$	$\Xi^-$	$dss$
		$\Omega^-$	$sss$

### 2.8.1 Need of Colour in Quarks

There were problems with the quark model, one of them being  $\Omega^-$  hyperon. It was believed to contain three identical s quarks. (sss). This violates the Pauli exclusion principle, that prohibits two or more fermions from occupying identical quantum states. The proton, neutron, and others with two identical quarks would violate this principle also. We can resolve this difficulty by assigning a new property to the quarks. We can regard this new property as an additional quantum number that can be used to label the three otherwise identical quarks in the  $\Omega^-$ . If this additional quantum number can take any one of three possible values, we can restore the Pauli principle by giving each quark a different value of this new quantum number, which is known as colour. The three colours are labelled red ( $R$ ), blue ( $B$ ), and green ( $G$ ). The  $\Omega^-$  for example, would then  $s_R s_B s_G$ . The antiquark colours are antired ( $R$ ) antiblue ( $B$ ) and antigreen ( $G$ ). There were problems with the quark model, one of them being  $\Omega^-$  hyperon. It was believed to contain three identical s quarks. (sss). This violates the Pauli exclusion principle, that prohibits two or more fermions from occupying identical quantum states. The proton, neutron, and others with two identical quarks would violate this principle also. We can resolve this difficulty by assigning a new property to the quarks. We can regard this new property as an additional quantum number that can be used to label the three otherwise identical quarks in the  $\Omega^-$ . If this additional quantum number can take any one of three

possible values, we can restore the Pauli principle by giving each quark a different value of this new quantum number, which is known as colour. The three colours are labelled red ( $R$ ), blue ( $B$ ), and green ( $G$ ). The  $\Omega^-$  for example, would then  $s_R s_B s_G$ . The antiquark colours are antired ( $\bar{R}$ ) antiblue ( $\bar{B}$ ) and antigreen ( $\bar{G}$ ).

An essential component of the quark model with colour is that all observed meson and baryon states are "colourless", i.e., either colour anticoulour combinations in the case of mesons, or equal mixtures of  $R, B$  and  $G$  in the case of baryons.

Since hadrons seem to be composed of quarks, the strong interaction between hadrons should ultimately traceable to an interaction between quarks. The force between quarks can be modeled as an exchange force, mediated by the exchange of massless spin-1 particles called gluons. Eight gluons have been postulated. The field that binds the quarks is a colour field. Colour is to the strong interaction between quarks as electric charge is to the electromagnetic interaction between electrons. It is the fundamental strong "charge" and is carried by the gluons. The gluons must therefore be represented as combinations of a colour and a possibly different anticoulour. The gluons are massless and carry their colour-anticoulour properties just as other particles may carry electric charge.

### 2.8.2 Gluons

Gluons are the exchange particles for the color force between quarks, analogous to the exchange of photons in the electromagnetic force between two charged particles. The gluon is considered to be a massless vector boson with spin 1. The gluon can be considered to be the fundamental exchange particle underlying the strong interaction between protons and neutrons in a nucleus. That short-range nucleon-nucleon interaction can be considered to be a residual color force extending outside the boundary of the proton or neutron. That strong interaction was modeled by Yukawa as involving an exchange of pions, and indeed the pion range calculation was helpful in developing our understanding of the strong force.

Gluon interactions are often represented by a Feynman diagram. Note that the gluon generates a color change for the quarks. The gluons are in fact considered to be bi-colored, carrying a unit of color and a unit of anti-color as suggested in the diagram at right. The gluon exchange picture there converts a blue quark to a green one and vice versa. The range of the strong force is limited by the fact that the gluons interact with each other as well as with quarks in the context of quark confinement. These properties contrast them with photons, which are

massless and of infinite range. The photon does not carry electric charge with it, while the gluons do carry the "color charge".

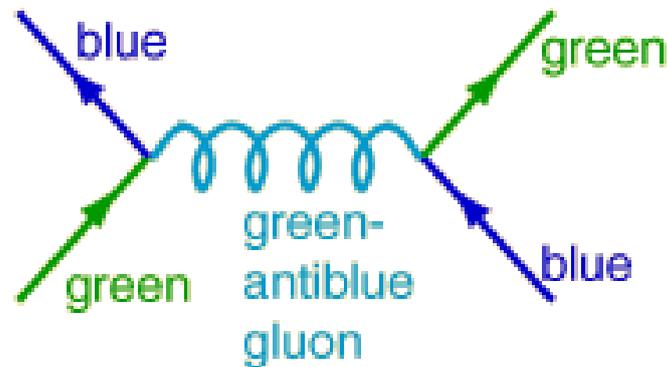


Fig 7: Feynman diagram for an interaction between quarks generated by a gluon

Within their range of about a fermi, the gluons can interact with each other, and can produce virtual quark-antiquark pairs. The property of interaction with each other is very different from the other exchange particles, and raises the possibility of gluon collections referred to as "glueballs". The internal state of a hadron is viewed as composed of a fixed net number of quarks, but with a dynamic cloud of gluons and quark-antiquark pairs in equilibrium.

### 2.8.3 Charm, Bottom, and Top.

In 1970, Glashow, Iliopoulos, and Maiani proposed the existence of fourth quark, called *c* or charmed quark. The charmed quark was suggested to explain the suppression of certain decay processes that are not observed. With only three quarks, the processes would proceed at measurable rates and should have been observed. The charm quark has a charge of  $\frac{2}{3}e$ , strangeness 0 and a charm quantum number of +1. Other quarks have 0 charm.

In 1977, a new particle was discovered at Fermi Lab that provided evidence for yet another quark. This particle, called the *upsilon*-meson, was thought to be made up of the new quark called *b* (for bottom or beauty) along with the associated antiquark  $\bar{b}$ .  $\bar{b}$  quark has electric charge  $-\frac{1}{3}e$ .

Because quarks seem to come in pairs, it is expected that there is a partner to the b quark, called t (for top, if b = bottom, or truth, if b = beauty). It has a charge of  $+\frac{2}{3}e$ . The following table explains the properties of six quark flavours.

**Table3:** Quark Flavours and its Properties

Property	d	u	s	c	b	t
Q-Electric charge	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$
I- isospin	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
$I_3$ -isospin 3 <sup>rd</sup> component	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
S-strangeness	0	0	-1	0	0	0
C-charm	0	0	0	+1	0	0
B-bottomness	0	0	0	0	-1	0
T-topness	0	0	0	0	0	+1

Subsequent discovery of new particles, like D, D<sub>s</sub>, B and Λ<sub>c</sub>, hadrons, introduced new quantum numbers, called charm (C), bottom (b), and top (t). Including these quantum numbers, hypercharge can be expressed as the following form

$$Y = B + S + C + b + t \quad \dots\dots\dots(5)$$

thus extending the Gell-Mann-Nishijima formula to the following form

$$Q = I_3 + \frac{B+S+C+b+t}{2} \quad \dots\dots\dots(6)$$

This permits us to give an alternative definition of the hypercharge, as twice the difference between the electric (Q) charge and isospin component ( $I_3$ ) of the particle, i.e.

$$Y = 2(Q - I_3) \quad \dots\dots\dots(7)$$

A particle and its antiparticle have opposite value of both and  $I_3$ , hence the hypercharge will be numerically equal in magnitude but opposite in sign, or zero.

### 2.8.4 Three generations of quarks and leptons

Both leptons and quarks appear to come in three generations of doublets, with all particles having spin 1/2. Table3 shows the properties of the three generations of quarks and leptons. The first generation contains two leptons, the electron and the electron neutrino, and two quarks, up and down. All the properties of ordinary matter can be understood on the basis of these particles. The second generation includes the muon and muon-neutrino and the charm and strange quarks. These particles are responsible for most of the unstable particles and resonances created in high energy collisions. The third generation includes the tau and the tau-neutrino and the top and bottom quarks.

**Table4:** Properties of three generation of Quarks and Leptons

Generation	Quarks		Leptons		Anti-Quarks		Anti-Leptons	
	Charge +2/3	Charge -1/3			Charge -2/3	Charge +1/3		
1 <sup>st</sup>	up	down	electron	electron neutrino	Anti-up	Anti-down	positron	Anti-electron neutrino
	u	d	e	$\nu$	$\bar{u}$	$\bar{d}$	$e^+$	$\bar{\nu}$
2 <sup>nd</sup>	charm	strange	muon	muon neutrino	Anti-charm	Anti-strange	Anti-muon	Anti-muon neutrino
	c	s	$\mu$	$\nu_\mu$	$\bar{c}$	$\bar{s}$	$\bar{\mu}$	$\bar{\nu}_\mu$
3 <sup>rd</sup>	top	bottom	tau	tau neutrino	Anti-top	Anti-bottom	Anti-tau	Anti-tau neutrino
	t	b	$\tau$	$\nu_\tau$	$\bar{t}$	$\bar{b}$	$\bar{\tau}$	$\bar{\nu}_\tau$

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## 2.9 SUMMARY

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After studying the unit students will be able to understand

- CP violation in mesons
- CPT Invariance
- Mesons and Yukawa's Hypothesis: Pions
- The Eightfold Way
- The Quark Model
- Need of Colour in Quarks
- Gluons

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### ***2.13 TERMINAL QUESTIONS***

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1. 1.Name different types of quarks.
2. What is CP violation?
3. Outline the basic assumptions and properties of quarks
4. Construct quark structure of a nucleon and a pion.
5. Explain in detail the quark model and explain the various types of quarks (flavours) along with their properties.
6. What are quarks? Give the elementary theory of structure of a few hadrons on the basis of quark model.
7. What are quarks? Give the qualitative description of quark model.
8. Give the quark model of
  - (i) proton and antiproton
  - (ii) neutron and antineutron
9. What are quarks? Explain the quark model. Also give two factors which do not support the existence of quarks.
10. What is meant by colour of a quark? Give the colours associated with quarks.
11. Discuss the quark model and explain how mesons and baryons are formed using quarks.
12. What is quark model? Is there any evidence in its support.

**UNIT 3****UNITARY SYMMETRIES AND APPLICATION IN THE  
PHYSICS OF ELEMENTARY PARTICLES**

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**Structure of the Unit**

- 3.1 Introduction
- 3.2 Objectives
- 3.3 Particle Symmetries
- 3.4 Basics of Unitary Groups
  - 3.4.1 Properties
- 3.5 Fundamental Representation of SU(2)
- 3.6 Fundamental Representation of SU(3)
- 3.7 Generators of SU(2), SU(3) and U(2)
  - 3.7.1 Generators of SU(2)
  - 3.7.2 Generators of SU(3)
  - 3.7.3 Generators of U(2)
- 3.8 Weight Diagram of Fundamental Representation of SU(2), SU(3) and U(2)
  - 3.8.1 Weights
  - 3.8.2 Weights in SU(2)
    - 3.8.2.1 Weight for Meson states
    - 3.8.2.2 Weight for Baryon states
  - 3.8.3 Weights for SU(3)
  - 3.8.4 Weight Diagram for the fundamental representation of U(2)
- 3.9 Weight for first fundamental representation of SU(3)
- 3.10 Shift Operators I, U and V spins
- 3.11 GellMann Okubo Mass Formula
- 3.12 Summary
- 3.13 References
- 3.14 Suggested Readings
- 3.15 Terminal Questions

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### ***3.1 INTRODUCTION***

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The concept of symmetry has acquired great and fundamental significance in classical as well as in modern physics, more particularly in particle physics. The term symmetry means that a system, state or quantity remains unchanged (or invariant) as a result of a particular transformation, such between different physical states, which can be connected through the corresponding as a change in space coordinates, or change in time, etc. Symmetries also predict degeneracy between different physical states, which can be connected through the corresponding transformations. Every conservation law is related to a particular invariance (or symmetry) principle. For example, conservation of total energy is due to the invariance of a system under shift in time, called time translation. Conservation of linear or angular momentum is due to invariance under displacement in space (space translation) or rotation in space respectively. These transformations can be abstract also, i.e. they may have no relation to actual space, time, etc. Examples of these are like conservation of charge, lepton number, baryon number, isospin etc. Besides these continuous symmetries, there also exist discrete symmetries like parity and C-parity.

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### ***3.2 OBJECTIVES***

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After studying the unit learners will be able to explain

- Particle Symmetries
- Basics of Unitary Groups
- Fundamental representation of  $SU(2)$ ,  $SU(3)$  and  $U(2)$
- Generators of  $SU(2)$ ,  $SU(3)$  and  $U(2)$
- Weight diagram of  $SU(2)$ ,  $SU(3)$  and  $U(2)$
- Shift Operators I, U and V spins
- Gell Mann Okubo Mass Formula

### 3.3 PARTICLE SYMMETRIES

In this section, we will describe briefly some aspects of special symmetries in particle physics, which are SU(2) symmetry, SU(3) symmetry, and higher symmetries

#### ❖ SU(2) SYMMETRY

Besides having same spin. proton and neutron have many similarities. Particularly, it is well known that nuclear forces (strong forces) are charge-independent. That means strong binding force between -p p-n, and n-n is essentially the same. The small difference between the attributed to the electromagnetic interactions, which arise due to difference in charges of proton and neutron. These facts suggested the existence of new kind of nuclear spin, called isospin, originally proposed by Heisenberg in 1934. Stating differently, in the world of strong interactions, we can think of proton and neutron as the two orthogonal states of the same particle, called nucleon. As has been mentioned before, a general isospin state is described by two quantum numbers, I and I<sub>3</sub>, where I<sub>3</sub> can have values ranging from -I to I separated by unity. Thus, for isospin I, there are 2I+1 degenerate states available. Isospin conservation is consequence of the invariance of the strong interaction Hamiltonian under the rotations in isospin space. Isospin symmetry also reflects in the physical states belonging to an isospin multiplet, which have the same mass. Consequently, the nucleon doublet. i.e. proton and neutron, have the same mass due to the strong isospin SU(2) symmetry". SU(2) denotes special (S) unitary (U) group in two complex dimensions, that means it is a group of 2 x 2 unitary matrices U with unity determinant. A unitary matrix is that whose inverse is given by complex conjugate of its transpose. Such a matrix, can be written as

$$U = \exp\left(i \sum_{j=1}^3 \sigma_j \theta_j\right) \dots\dots\dots(1)$$

where  $\sigma_j$ , are the well-known Pauli matrices,

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \dots\dots\dots(2)$$

and  $\theta_j$ , denotes the angles of rotation in isospin space. SU(2) allows all integral (2I+ 1) multiplets; 1, 2, 3, 4, 5...respectively for I= 0, 1/2, 1, 3/2, 2,... states. The fundamental particle-multiplet of the SU(2) is two-dimensional which is given by two-component column matrices. For example, we can imagine proton and neutron to be described by the following isospin states of the isospin symmetry:

$$p = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, n = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \dots\dots(3)$$

on which the Pauli matrices can operate.

Strong interactions respect the isospin symmetry leading to conservation of isospin in strong decays or reactions. This is quite similar to the conservation of angular momentum due to the invariance of the Hamiltonian under the rotations in ordinary space (more precisely in spin space). Isospin symmetry is broken by the electromagnetic and weak interactions. However, the electromagnetic interactions conserve the third component of the isospin, Breaking of isospin symmetry permits us to differentiate between a proton and a neutron.

### ❖ SU(3) Symmetry

The number of observed hadrons is more than hundred and more of them are yet to be seen. All the observed hadrons are extended objects having size at the Fermi scale. Obviously, all of the hadrons could not be treated as elementary. A certain pattern, similar to the Mendeleev periodic table of atoms, among them was found independently by Gell-Mann and Y. Ne'eman. Consider a system of coordinates in which x-axis and y-axis are used to represent third component of isospin ( $I_3$ ) and Hypercharge ( $Y = B+S$ ) respectively. If we draw mesons of same spin and parity. Say  $J^P=0$ , they create geometric symmetric octet patterns, called weight diagrams. Similar pattern is also created by  $J = 1/2$  baryons Gell-Mann called this scheme" Eightfold Way as term octet was occurring again and again. In fact, all the observed baryons, known till then, seemed to appear in the grouping of 1 (singlet), 8 (octet) and 10 (decuplet), whereas the mesons fall into groups of 1 and 8. As observed patterns, SU(3) symmetry, is a mathematical generalization of the SU(2) isospin symmetry. SU(3) is the

group of 3 x 3 unitary matrices with unit determinant, and its fundamental multiplet is obviously three-dimensional. In contrast to SU(2), SU(3) permits selective multiplets, say

$$1, 3, 3, 6, 6^*, 8, 10, 10^*, 15, \dots$$

The SU(3) could explain several regularities observed in the hadron world. In fact, the SU(3) symmetry scheme achieved wonderful confirmation with the discovery of  $\Omega$ -hyperon, which was predicted by Gell-Mann, as a missing partner of the already observed nine spin 3/2 baryons filling the decuplet. However, this symmetry is not exact and is broken even by the strong interactions, though in a particular way. Assuming that the SU(3) breaking also follows eightfold way, several properties of the hadrons could be explained. For instance, Gell-Mann-Okubo obtained a well-satisfied mass formula:

$$2N + 2\Xi = 3\Lambda + \Sigma$$

$$1129 \text{ MeV} \quad 1135 \text{ MeV} \dots\dots\dots(4)$$

where particle symbol denotes the average mass of the particles in that isospin multiplet.

❖ **Higher Symmetries**

Later higher symmetries, described by SU(4), SU(5), SU(6) and higher groups, have also been employed to study the hadronic behaviour. For instance, SU(6), which combines the SU(3) symmetry with the SU(2) spin symmetry, could explain reasonably the ratio of nucleon magnetic moments  $\mu_p/\mu_n = -\frac{3}{2}$

$$\text{Experimental value}-1.46$$

SU(4) and SU(S) are used to describe spectra of hadrons carrying charm and bottom quantum numbers.

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**3.4 BASICS OF UNITARY GROUPS**

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Unitary groups are fundamental mathematical objects in the field of linear algebra and group theory. They play a crucial role in various areas of mathematics and physics, particularly in quantum mechanics. The unitary group is a collection of matrices that preserve the inner product of vectors.

Let's define the unitary group and explore its basic properties:

Definition: The unitary group of degree  $n$ , denoted as  $U(n)$ , is the group of  $n \times n$  unitary matrices. A unitary matrix  $U$  satisfies the condition  $UU^\dagger = I$ , where  $U^\dagger$  denotes the conjugate transpose (or Hermitian adjoint) of  $U$ , and  $I$  is the identity matrix.

### 3.4.1 Properties:

1. **Unitarity:** A matrix  $U$  is unitary if and only if its columns (or rows) form an orthonormal set of vectors. This means that the inner product of any two distinct columns (or rows) is zero, and the inner product of a column (or row) with itself is one.
2. **Group Structure:** The unitary group  $U(n)$  forms a group under matrix multiplication. The group operation is associative, and the identity element of  $U(n)$  is the  $n \times n$  identity matrix, denoted as  $I$ .
3. **Inverses:** Every element  $U$  in  $U(n)$  has an inverse, denoted as  $U^{-1}$ , which is also a unitary matrix. The inverse of a unitary matrix is simply its conjugate transpose, i.e.,  $U^{-1} = U^\dagger$ .
4. **Subgroup:** The special unitary group  $SU(n)$  is a subgroup of  $U(n)$  consisting of matrices with determinant 1. In other words,  $SU(n)$  contains those unitary matrices that also have a determinant of magnitude 1.
5. **Lie Group:** The unitary group  $U(n)$  is a Lie group, which means it is a smooth manifold and has a group structure that is compatible with the smooth structure. This makes it amenable to analysis and differential geometry.
6. **Unitary Transformations:** Unitary matrices are often used to represent unitary transformations in linear algebra and quantum mechanics. A unitary transformation preserves the norms of vectors and the inner product between them. Therefore, unitary matrices play a vital role in studying quantum systems and quantum operations.

These are some of the basic properties and concepts related to unitary groups. Unitary matrices have numerous applications in areas such as quantum mechanics, signal processing, coding theory, and more. They provide a powerful mathematical framework for understanding and manipulating complex systems.

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### 3.5 FUNDAMENTAL REPRESENTATION OF SU(2)

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The fundamental representation of SU(2) refers to the smallest-dimensional complex representation of the special unitary group SU(2). This representation plays a crucial role in various areas of physics, particularly in the study of angular momentum and the behavior of spin-1/2 particles. Let's explore the fundamental representation of SU(2):

**Definition:** The special unitary group SU(2) is the group of 2x2 complex unitary matrices with determinant 1. Mathematically, it is defined as:

$$SU(2) = \{ U \in C^{(2 \times 2)} \mid U^*U = I \text{ and } \det(U) = 1 \}$$

where  $U^*$  denotes the conjugate transpose of  $U$ , and  $I$  is the 2x2 identity matrix.

**Fundamental Representation of SU(2):** The fundamental representation of SU(2) is a 2-dimensional complex representation of the group. It is obtained by associating each group element in SU(2) with a 2x2 complex matrix that acts on a 2-dimensional complex vector space. Let  $\sigma_i$  be the Pauli matrices: The matrix representation is defined as follows:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \dots\dots(5)$$

where “I” represents the imaginary unit.

Then, for any  $U$  in SU(2), the fundamental representation is given by:

$$\rho(U) = \cos(\theta/2) I + I \sin(\theta/2) (n \cdot \sigma),$$

where:

- $\theta$  is the rotation angle around the axis  $n$ .
- $I$  is the 2x2 identity matrix.
- “ $\cdot$ ” denotes the dot product.
- “ $n$ ” is a unit vector ( $n \cdot n = 1$ ) representing the axis of rotation.

In other words, the matrix  $\rho(U)$  is a linear combination of the identity matrix and the Pauli matrices, where the coefficients are determined by the rotation angle  $\theta$  and the axis of rotation “ $n$ .”

The fundamental representation of  $SU(2)$  is of great importance in theoretical physics, especially in quantum mechanics. It provides a mathematical framework for describing the behavior of spin-1/2 particles, such as electrons, neutrinos, and quarks. In quantum mechanics, the spin operators are related to the Pauli matrices, and the fundamental representation of  $SU(2)$  plays a crucial role in the theory of angular momentum and its quantization.

In summary, the fundamental representation of  $SU(2)$  is a 2-dimensional complex representation of the special unitary group  $SU(2)$ . It is an essential concept in quantum mechanics and is used to describe the behavior of spin-1/2 particles and angular momentum.

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### ***3.6 FUNDAMENTAL REPRESENTATION OF $SU(3)$***

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The fundamental representation of  $SU(3)$  refers to the smallest-dimensional complex representation of the special unitary group  $SU(3)$ .  $SU(3)$  is a group of  $3 \times 3$  complex unitary matrices with determinant 1. The fundamental representation plays a significant role in the study of quantum chromodynamics (QCD), which describes the strong interaction between quarks and gluons.

Let's delve into the fundamental representation of  $SU(3)$ :

Definition: The special unitary group  $SU(3)$  is defined as:

$$SU(3) = \{ U \in C^{(3 \times 3)} \mid U^*U = I \text{ and } \det(U) = 1 \}$$

where  $U^*$  denotes the conjugate transpose of  $U$ , and  $I$  is the  $3 \times 3$  identity matrix.

Fundamental Representation of  $SU(3)$ : The fundamental representation of  $SU(3)$  is a 3-dimensional complex representation. It associates each group element in  $SU(3)$  with a  $3 \times 3$  complex matrix that acts on a 3-dimensional complex vector space.

To define the matrix representation, we introduce the Gell-Mann matrices, which are a set of eight linearly independent  $3 \times 3$  Hermitian matrices:

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

**Fig.01:** Gell Mann matrices

These matrices span the space of traceless Hermitian 3x3 matrices, forming a basis for the Lie algebra  $su(3)$  associated with  $SU(3)$ .

For any  $U$  in  $SU(3)$ , the fundamental representation is given by:

$$\rho(U) = \exp(i\theta a, \lambda a),$$

where:

- $\theta a$  are the parameters associated with the group element  $U$ .
- $\lambda a$  are the Gell-Mann matrices.

The exponential term represents a matrix exponentiation, and it allows the representation to take into account the group structure and composition of  $SU(3)$ .

The fundamental representation of  $SU(3)$  is widely used in theoretical physics, particularly in the study of quantum chromodynamics (QCD), the theory of strong interactions. In QCD, quarks and gluons, the elementary particles that interact via the strong force, are described by fields transforming under the fundamental representation of  $SU(3)$ . The Gell-Mann matrices provide a basis for the color charge associated with the strong force.

Furthermore, the fundamental representation of  $SU(3)$  plays a crucial role in the classification of hadrons, which are composite particles made up of quarks. Hadrons are classified into different multiplets based on their transformation properties under  $SU(3)$  symmetry, and the fundamental representation helps to organize and understand their properties.

In summary, the fundamental representation of SU(3) is a 3-dimensional complex representation of the special unitary group SU(3). It is essential in the study of quantum chromodynamics and provides a mathematical framework for describing the behavior of quarks and the strong interaction between them. The Gell-Mann matrices form a basis for the representation and allow for the incorporation of group structure and composition.

**3.7 GENERATORS OF SU (2), SU (3) AND U (2)**

Two significant special unitary groups are SU(2) and SU(3) in the context of group theory and the study of Lie algebras. In the representation theory of each of these groups, there are associated weights and generators that are essential components.

**3.7.1 Generators of SU(2):**

SU(2) is the special unitary group of 2x2 complex unitary matrices with determinant 1. It is commonly associated with the description of spin in quantum mechanics and has important applications in various areas of physics.

SU(2) has three generators, usually denoted by the Pauli matrices:  $\sigma_1$ ,  $\sigma_2$ , and  $\sigma_3$ . These matrices are defined as follows:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \dots\dots\dots(6)$$

These generators correspond to the x, y, and z components of the total angular momentum or spin. They are Hermitian matrices.

These generators satisfy the SU(2) commutation relations:

$$[\sigma_i, \sigma_j] = 2i\epsilon_{ij}^k \sigma_k \quad \dots\dots\dots(7)$$

where  $\epsilon_{ij}^k$  is the Levi-Civita symbol.

**3.7.2 Generators of SU(3):**

SU (3) is the special unitary group of 3x3 complex unitary matrices with determinant 1. It plays a crucial role in the description of quantum chromodynamics (QCD), which is a fundamental force governing the strong interactions between quarks and gluons.

SU (3) has eight generators, which are usually represented by the Gell-Mann matrices. The Gell-Mann matrices are defined as follows (normalized by 1/2 for convenience):

The Gell-Mann matrices can be written as:

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

**Fig.02:** Gell Mann matrices

These generators satisfy the SU(3) commutation relations.

**3.7.3 Generators of U (2):** In U(2), the generators are the matrices that span the associated Lie algebra u(2). The Lie algebra u(2) is a 4-dimensional vector space over the field of complex numbers. The generators of U(2) can be represented as linear combinations of the Pauli matrices and the 2x2 identity matrix I.

The generators of U (2) are given by:

$$\begin{aligned} T_1 &= \frac{1}{2} (J_x + I J_y), \\ T_2 &= \frac{1}{2} (J_x - I J_y) \\ T_3 &= \frac{1}{2} J_z \quad \dots\dots\dots(8) \\ T_4 &= \frac{1}{2} I \end{aligned}$$

These generators are Hermitian and form a basis for the Lie algebra  $u(2)$ . The commutation relations of the Lie algebra  $u(2)$  are as follows:

$$[T_a, T_b] = i \epsilon_{abc} T_c \quad \dots\dots\dots(9)$$

where  $\epsilon_{abc}$  is the Levi-Civita symbol.

The generators of  $U(2)$  provide a basis for understanding the algebraic structure and transformations of  $U(2)$  and its representations. They play a fundamental role in various areas, including quantum mechanics, quantum information theory, and the study of symmetries in physical systems.

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### ***3.8 WEIGHT DIAGRAM OF FUNDAMENTAL REPRESENTATION OF SU (2), SU (3) and U (3)***

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#### **3.8.1 Weights**

Since they correspond to the independent, concurrent observables, we are looking for the set of commuting Hermitian operators for a particular system. The Cartan subalgebra is the group of commuting generators. The weights are the eigenvalues connected to them. One weight vector is created for each base vector using the eigenvalues of all the commuting generators. (The weights of the adjoint representation are referred to as the roots; they are connected to the raising and lowering operators.)

The generators in  $SU(2)$  are most simply represented by the  $2^2-1 = 3$   $2 \times 2$  Pauli matrices. Since the generators do not commute together, the set of “commuting generators” is simple and only consists of one generator.

The generator  $J$ , is chosen, giving rise to 2 weight vectors, ( $m=-1/2$ ) and ( $m=+1/2$ ) – these are just points on a line. In  $SU(3)$ , the simplest representation is in terms of the  $3 \times 3$  & matrices, of which there are  $(3^2-1)=8$ . There are 2 commuting generators,  $\lambda_3$  and  $\lambda_8$ . As, giving rise to 3 weight vectors, of the form  $(m, Y)$ - these are points in a plane. The weights are useful for classifying states and combinations of states.



To the triplet and singlet already identified from combining two states, we add the weight diagram corresponding to the third state, as illustrated in Fig.05



**Fig.05:** Weight diagram for the combination of three particles in SU (2)

The result is a quadruplet and two doublets.

The weight diagrams help us to identify the quantum numbers of the resultant states – in what follows, we will use  $I$  to identify explicitly the isospin. To find the quark description of the states built from three particles, we proceed as with the mesons:

1. Construct the maximally aligned state:  $uuu$  corresponding to  $I = 3/2, I_3 = 3/2$ .
2. Use the lowering operator:  $I_- = I^a + I^b + I^c$  to create states of lower  $I_3$ .
3. Construct other states of lower  $I$  using orthogonality and use the raising or lowering operators to complete the multiplets

The weight diagram is a graphical representation of the weights associated with the fundamental representation of a Lie group. It provides a visual depiction of the irreducible representations and their corresponding weights.

### 3.8.3 Weights for SU(3)

The commuting generators in SU(3) (Cartan subalgebra earlier discussed) are:

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \quad \dots\dots\dots(10)$$

This implies there are two simultaneously observable quantum numbers, along with  $I^2$ . With an eye to the description of hadrons, rather than QCD, we define:

$$\text{Isospin } I_3 = \frac{1}{2} \lambda_3$$

$$\text{Hypercharge } Y = \frac{1}{\sqrt{3}}\lambda_8$$

The diagonal matrices allow for easy identification of the weights:

$$I_3 = +\frac{1}{2}, -\frac{1}{2}, 0 \quad \text{and} \quad Y = \frac{1}{3}, \frac{1}{3}, -\frac{2}{3}$$

In the same way that raising and lowering operators were defined for SU(2), which involved moving between distinct weight vectors represented by points on a line, raising and lowering operators for SU(3) will involve moving between different weight vectors represented by points in the plane:

$$\begin{aligned} I_{\pm} &= \frac{1}{2}(\lambda_1 \pm \lambda_2) \quad , \\ U_{\pm} &= \frac{1}{2}(\lambda_6 \pm i\lambda_7) \quad , \\ V_{\pm} &= \frac{1}{2}(\lambda_4 \pm i\lambda_5) \quad \dots\dots\dots(11) \end{aligned}$$

In SU (3) representation theory, the generators act on a vector space, and the eigenvectors of these generators are called weights. The weights of SU (3) representations are often denoted as  $(\lambda_1, \lambda_2)$ , where  $\lambda_1$  and  $\lambda_2$  are integers or half-integers that satisfy certain conditions arising from the representations of SU (3).

The weights characterize different irreducible representations of SU (3) and are crucial in the study of hadrons and the classification of quarks

Both SU (2) and SU (3) have rich representation theories with various applications in particle physics, condensed matter physics, and other areas of theoretical physics. The concept of weights and generators is essential for understanding the properties and behavior of these groups in different physical systems.

In both SU (2) and SU (3), the diagonal generators and their corresponding weights provide a basis for understanding the group's structure and representation theory. They play a central role in describing the behaviour of particles, the composition of irreducible representations, and the symmetries of physical systems.

#### 4.8.4 Weight Diagram of the Fundamental Representation of $U(3)$ :

The weight diagram of the fundamental representation of the unitary group  $U(3)$  corresponds to the irreducible representation of the Lie algebra associated with this group. In this representation, the group  $U(3)$  acts on a three-dimensional vector space.

To construct the weight diagram, we consider the action of the Cartan subalgebra generators, which are represented by diagonal matrices. The weights are the eigenvalues of these diagonal matrices. Since we are dealing with the fundamental representation, the highest weight state will be the one with all the eigenvalues as large as possible.

To visualize the weight diagram, we often use a graph where the weights are represented as points in a three-dimensional space. The weights are typically arranged in a triangular pattern, reflecting the symmetry of the  $U(3)$  group.

The general structure of the weight diagram for the fundamental representation of  $U(3)$ .

1. The highest weight state is located at the top of the diagram, represented as a point with coordinates  $(\lambda_1, \lambda_2, \lambda_3)$ . Here,  $\lambda_1 \geq \lambda_2 \geq \lambda_3$  are the eigenvalues of the Cartan generators.
2. The other weight states are obtained by subtracting simple roots from the highest weight. Simple roots are vectors that form the basis for the Cartan subalgebra. They correspond to the roots of the Lie algebra of  $U(3)$ .
3. The weight diagram will have a triangular shape, reflecting the fact that  $U(3)$  is a rank-2 group.
4. Each weight state corresponds to an irreducible representation of  $U(3)$ , and the dimension of the representation is related to the number of boxes in the corresponding Young tableau.

Keep in mind that the specific values of  $\lambda_1$ ,  $\lambda_2$ , and  $\lambda_3$  depend on the conventions and normalization chosen for the Cartan generators. The above description provides a general understanding of the weight diagram for the fundamental representation of  $U(3)$ . To obtain specific values, you would need to refer to a group theory or representation theory textbook or software.

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### ***3.9 WEIGHT OF FIRST FUNDAMENTAL REPRESENTATION OF SU(3)***

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The first fundamental representation of SU(3), often denoted as the fundamental representation (3), is a three-dimensional representation of the special unitary group SU(3). The weight of this representation refers to the eigenvalues associated with the diagonal generators  $\lambda_3$  and  $\lambda_8$  (or hypercharge).

In the case of the first fundamental representation of SU(3), the weights are given by: (1, 0): This corresponds to the eigenvalues (1, 0) for  $\lambda_3$  and  $\lambda_8$ , respectively. (0, 1): This corresponds to the eigenvalues (0, 1) for  $\lambda_3$  and  $\lambda_8$ , respectively. (-1, -1): This corresponds to the eigenvalues (-1, -1) for  $\lambda_3$  and  $\lambda_8$ , respectively.

These weights represent the different eigenvalue combinations that label the basis states of the first fundamental representation. The weights play a crucial role in characterizing the transformation properties and behaviour of particles and fields under the symmetry of SU(3).

It is worth noting that the weights are often represented using the Dynkin labels or Young diagrams, which provide a compact and systematic way to describe the irreducible representations of SU(3). The first fundamental representation is labeled as (1, 0), which indicates the number of boxes in each row of the Young diagram.

Overall, the weights of the first fundamental representation of SU(3) are (1, 0), (0, 1), and (-1, -1), corresponding to the eigenvalue combinations for the diagonal generators  $\lambda_3$  and  $\lambda_8$ . These weights characterize the basis states and transformation properties of the first fundamental representation.

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### ***3.10 SHIFT OPERATORS I, U, V SPINS***

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In the context of quantum mechanics, the shift operators are operators that act on states and change the value of a quantum number, such as spin. The specific operators I, U, and V can be used to manipulate spin states. Let's explore these operators in the context of spin.

1. **Identity Operator (I):** The identity operator, denoted by I, is an operator that leaves a state unchanged when it acts on it. In the context of spin, the identity operator does not change the spin state of a particle. It is represented as a 2x2 identity matrix:

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \dots\dots\dots(12)$$

When the identity operator acts on a spin state, it simply returns the same spin state unchanged.

2. **Pauli Spin Operators (U, V):** The Pauli spin operators U and V are used to manipulate and transform spin states. They are represented by the Pauli matrices:

$$U = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$V = \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \dots\dots\dots(13)$$

These operators are Hermitian and act on spin states to rotate or transform them. They change the direction of the spin vector and can mix different spin components.

For example, applying the U operator to a spin-up state ( $|\uparrow\rangle$ ) would result in a superposition of spin-up and spin-down states:

$$U |\uparrow\rangle = |\downarrow\rangle$$

Similarly, applying the V operator to a spin-up state would result in a superposition of spin-up and spin-down states, but with a phase difference:

$$V |\uparrow\rangle = i |\downarrow\rangle$$

The U and V operators, together with the identity operator, form a basis for the 2x2 matrix space of operators that act on spin states.

In summary, the identity operator I leaves spin states unchanged, while the Pauli spin operators U and V are used to manipulate and transform spin states by rotating the spin vector and creating superpositions of spin-up and spin-down states.

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### 3.11 GELL MANN OKUBO MASS FORMULA

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The Gell-Mann-Okubo mass formula, also known as the Gell-Mann-Okubo relation, is an empirical formula that provides a relationship between the masses of certain hadrons within the framework of SU(3) flavor symmetry. It was proposed independently by Murray Gell-Mann and Kazuhiko Okubo in the 1960s.

In other words, the Gell-Mann-Okubo mass formula is derived from the assumption that the hadrons can be organized into SU(3) multiplets, which are representations of the SU(3) symmetry group. The formula relates the masses of the octet and decuplet baryons, which are composed of three quarks.

The Gell-Mann-Okubo mass formula, also known as the Gell-Mann-Okubo relation, is a mathematical formula that relates the masses of certain baryons, which are a type of subatomic particle. It was proposed independently by Murray Gell-Mann and Kazuhiko Nishijima in 1956, and later extended by Shoichi Sakata. This formula played a crucial role in the development of the quark model and our understanding of the underlying symmetries in particle physics.

The formula is based on the concept of approximate SU(3) symmetry of the strong nuclear force, also known as flavor symmetry, which was observed in the behavior of certain baryons. The SU(3) symmetry describes the interactions among three types of quarks: up (u), down (d), and strange (s) quarks, which are the elementary particles that constitute protons, neutrons, and other hadrons.

The Gell-Mann-Okubo mass formula is given by:

$$M = aI_3 + b(I_3)^2 + cY, \quad \dots\dots\dots(14)$$

where:

- $M$  is the mass of the baryon.
- $I_3$  is the third component of the isospin (a quantum number related to the weak force) of the baryon, which takes values  $-1/2$  and  $+1/2$  for up-type and down-type quarks, respectively.
- $Y$  is the hypercharge of the baryon, a quantum number related to the electromagnetic force and the third component of the baryon's isospin. It is defined as the sum of the

number of strange quarks ( $n_s$ ) and the difference between the number of up quarks ( $n_u$ ) and down quarks ( $n_d$ ):

$$Y = n_s + \frac{1}{2}(n_u - n_d) \quad \dots\dots\dots(15)$$

The coefficients  $a$ ,  $b$ , and  $c$  are constants that depend on the specific baryon octet under consideration. The baryon octet includes eight baryons that exhibit SU(3) symmetry: the proton and neutron, which are the lightest baryons, along with six other more massive baryons.

By using this formula, Gell-Mann and Nishijima successfully predicted the masses of certain baryons, providing strong evidence for the existence of quarks and the underlying SU(3) symmetry. The quark model eventually became a central component of the Standard Model of particle physics, providing a framework for understanding the subatomic world and its interactions.

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### ***3.12 SUMMARY***

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After going through this unit, learners have been able to understand

- Particle Symmetries
- Basics of Unitary Groups
- Fundamental representation of SU(2), SU(3) and U(3)
- Generators of SU(2), SU(3) and U(3)
- Weight diagram of SU(2), SU(3) and U(3)
- Shift Operators I, U and V spins
- Gell Mann Okubo Mass Formula

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### 3.14 SUGGESTED READINGS

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2. Quarks and Patrons by FE Close:
3. T.P.Cheng and G.LF Li by Gauge Field Theory.
4. Nuclear Physics by Irving Kaplan, Narosa Publishing House
5. Elementary Nuclear Theory, 2<sup>nd</sup> ed. by Bethe and Morrison, Wiley: New York.
6. Introduction to High Energy Physics by D. H. Perkins:

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### 3.15 TERMINAL QUESTIONS

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1. Define a symmetry group. Demonstrate through an example that symmetry group obeys the properties of closure, identity, inverse and associativity.
  2. Define the basic properties of a symmetry group.
  3. Define a Lie group and its generators. Show that the generators of U(2) group are the Pauli matrices. Compute the number of generators of O(n)
  4. Define SU(n) groups. Show that the generators of SU(n) group are group. (n-1) traceless hermitian matrices. Mentioning the generators of SU(3)
  5. Define an SU(2) group. Show that this group can describe both spin and isospin.
  6. What is meant by fundamental and adjoint representations of SU(2) groups?
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7. Define fundamental representation of  $SU(2)$  group.
8. Define an  $SU(3)$  group and show that the matrices form the fundamental representation of  $SU(3)$  group. Show also that the diagonal generators of  $SU(3)$  group obey the commutation relation.
9. What are the shift operators of  $SU(3)$ ? Deduce the commutation relations between them and explain their properties. Show that the I-y plot of  $SU(3)$  represents the octet representation of  $SU(3)$ .
10. Define the octet representation of  $SU(3)$  group and show that the particle states of the representation can be changed into one another by shift operators.
11. Deduce Gell-Mann-Okubo mass formula. How far do the baryon and meson masses agree with their masses predicted by this formula?
12. What are the applications of  $SU(3)$  group?

## UNIT 4

# METHOD OF YOUNG TABLEAUX AND ITS APPLICATIONS

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### Structure of the Unit

4.1 Introduction

4.2 Objectives

4.3 Young Tableaux

4.3.1 Rules to Construct Irreducible representations of the group SU (N)

4.4 Unitary Symmetry

4.4.1 Application of Young's tableaux for unitary symmetry

4.5 Standard arrangements of Young Tableaux

4.6 Dimensionality of the representations of SU(N)

4.7 Multiplets of SU(N-1)

4.8 Subgroup of SU(N)

4.9 Baryon multiplets in different representations

4.10 General rule and its application for reducing Kronecker product of two representations

4.11 Kronecker product of three particle state vectors

4.12 Summary

4.13 References

4.14 Suggested Readings

4.15 Terminal Questions

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## ***4.1 INTRODUCTION***

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In the previous unit we have discussed various symmetries involved in particle physics and also its applications. Now, in this unit we shall study about Young tableaux which are combinatorial objects used in the representation theory of symmetric groups and other related algebraic structures. It provides a convenient way to describe and analyse the irreducible representations of these groups. On the other hand, Unitary symmetry, refers to a type of symmetry exhibited by physical systems, particularly in quantum mechanics, where the Hamiltonian (energy operator) of the system commutes with a unitary symmetry operator.

Young tableaux and unitary symmetry are not directly related concepts, they can be connected through their applications in different areas of mathematics and physics.

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## ***4.2 OBJECTIVES***

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After studying the unit learners will be able to explain

- Young Tableaux
- Unitary Symmetry
- Applications of Young Tableaux
- Standard arrangements of Young Tableaux
- Dimensionality of the representation of  $SU(N)$
- Multiplets of  $SU(N-1)$
- Baryon multiplets in different representation
- Kronecker product of the three particle state vectors

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## ***4.3 YOUNG TABLEAUX***

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Young tableaux are arrangements of boxes in rows and columns, typically denoted by diagrams, which satisfy certain conditions. They are used to characterize the irreducible representations of the symmetric group, which is a group of permutations of a finite set. Each irreducible representation of the symmetric group corresponds to a unique Young tableau, and vice versa.

The symmetry properties of the Young tableaux reflect the underlying symmetry of the symmetric group. They can be used to determine the decomposition of a tensor product of irreducible representations into a direct sum of irreducible representations, known as the

branching rules. Young tableaux also find applications in algebraic combinatorics, algebraic geometry, and other areas of mathematics.

In this process of diagonalizing the Hamiltonian of systems with many identical systems with many identical particles it will be important to find subspaces of Hilbert space which carry at the same time irreducible representations of quantum mechanics rotation group  $SU(2)$  as well as the permutation group  $Y(N)$ . It turns out the representation which are simultaneously irreducible with respect to both groups come about rather naturally as well as we shall now see. In other words the irreducible representations of the unitary group are Young tableaux technique. In this technique the fundamental representation is denoted by a box and the conjugate representation (for example  $\bar{2}$  is the complex conjugate of 2) is denoted by a column of boxes whose number is one less than the dimension of the group.

In  $SU(2)$  we will denote,

$$2 = \begin{array}{|c|} \hline \square \\ \hline \end{array} \text{ (Row)}, \quad \bar{2} = \begin{array}{|c|} \hline \square \\ \hline \end{array} \text{ (column)}$$

In constructing  $2 \otimes 2$  we write,

$$\begin{array}{|c|} \hline a \\ \hline \end{array} \otimes \begin{array}{|c|} \hline b \\ \hline \end{array} = \begin{array}{|c|c|} \hline a & b \\ \hline \end{array} \oplus \begin{array}{|c|} \hline a \\ \hline b \\ \hline \end{array}$$

Where RHS is obtained by seeing that in how many ways the box b can be attached to a box a. Now we have to calculate the dimensions in the RHS, LHS is obviously  $2 \otimes 2$ . In the RHS, the dimension  $\begin{array}{|c|c|} \hline \square & \square \\ \hline \end{array}$  and  $\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array}$  are calculated by finding the numerator and

denominator (N) and the denominator (D) of each tableau. The dimension of each tableau would be equal to  $N/D$ . Since we are dealing with  $SU(2)$  representation, we have to note 2 of  $SU(2)$ , so for N, of the first tableau write,

$$\begin{array}{|c|c|} \hline \square & \square \\ \hline \end{array} = \begin{array}{|c|c|} \hline 2 & 2+1 \\ \hline \end{array}$$

Implying that  $N = 2(2+1) = 6$ . For D, draw lines 1 and 2 and notice

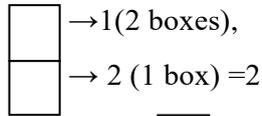
$$\begin{array}{|c|c|} \hline \square & \square \\ \hline \end{array} \rightarrow \begin{array}{|c|c|} \hline \square & \square \\ \hline \end{array} \begin{array}{c} \downarrow \quad \downarrow \\ 1 \quad 2 \end{array}$$

that the line 1 crosses through (2) boxes and line 2 through 1 box. So,  $D = 2 \times 1 = 2$ , the difference between line 1 and 2 is that they are passing through different columns, therefore the dimensions of  $SU(2)$   $\begin{array}{|c|c|} \hline \square & \square \\ \hline \end{array} = 6/2 = 3$ , Likewise, N for

$$\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} = \begin{array}{|c|} \hline 2 \\ \hline 1 \\ \hline \end{array} = 2 \times 1 = 2$$

Where it has to be noted that the number of decreases while going down a column.

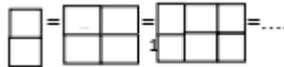
The D for



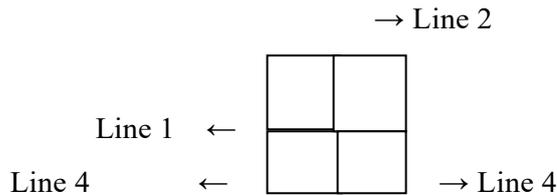
Therefore, the dimension of SU (2) =  $\frac{2}{2}=1$ . Hence in SU (2) the representation implies

$$2 \otimes 2 = 3 \oplus 1 \dots\dots\dots(1)$$

It can be noted that in SU (2)



All have dimension 1, because for example N for  =  $2 \times 3 \times 1 \times 2 = 12$  and for the D for these four boxes we have



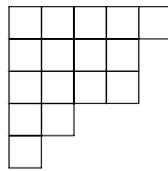
The lines 1,2,3 and 4 cross respectively 3,2,2 and 1. Hence  $D = 3 \times 2 \times 2 \times 1 = 12$  and the dimension for these four boxes  $N/D=12/12=1$ . Young's Tableau (YT) is more useful to know the irreducible representations at higher dimensions.

**4.3.1 RULES TO CONSTRUCT IRREDUCIBLE REPRESENTATIONS OF THE GROUP SU (N)**

The group SU (N) i.e. the group of  $N \times N$  complex unitary matrices ( $UU^\dagger = 1$ ) with unit determinant ( $\det (U) = 1$ ).The complex multiplet  $\psi_i$  ( $i = 1,2 \dots, N$ ) which belong to the fundamental representation of SU (N) i.e. the lower dimension non trivial representation,  $\psi_i \rightarrow U_{ij}\psi_j$  is represented by a box

$$\psi_i \equiv \square \equiv N$$

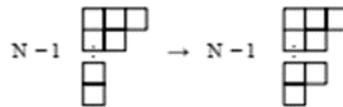
A Young tableau is a diagram of left-justified rows of boxes where any row is no longer than the row on top of it, e.g.



- Any column cannot contain more than N boxes.
- Any column with exactly N boxes can be crossed out since it corresponds to the trivialrepresentation (the singlet),

$$N \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \vdots \\ \hline \square \\ \hline \end{array} \equiv 1, \quad N \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \square & \square & \\ \hline \square & & \\ \hline \square & & \\ \hline \end{array} \equiv \begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \\ \hline \end{array}$$

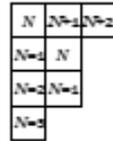
- Any Young tableau which respects the rules above corresponds to an irrep of SU (N)and vice-versa.
- The complex conjugate of a given irrep is represented by a tableaux obtained byswitching any column of k boxes with a column of (N -k) boxes , e.g.



The dimension d of a Young tableau i.e. the dimension of the associated irrep) can be obtained by the following ratio:  $d = \text{numerator}/\text{denominator}$ .

**Numerator:** Start writing the number N in the top left box of the Young tableau. Moving to the right, write the number increased by a unit at each step. Moving to the bottom, write the number decreased by a unit at each step. The numerator is obtained by the product of the entries in each box. E.g

e.g.



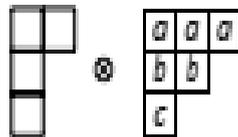
, numerator= $N(N+1)(N+2)(N-1)N(N-2)(N-1)(N-3)$  .

**Denominator:** Write in each box the number of boxes being to its right plus the number of boxes being below it plus a unit (the hook length). The denominator is obtained by the product of the entries in each box. e.g



, denominator= $6 \times 4 \times 4 \times 2 \times 3$  .

- Any irrep of SU (N) can be constructed starting from the fundamental irrep. The direct product of irreps can be decomposed in a direct sum of irreps with the following rules.
- Write the two tableaux which correspond to the direct product of irreps and label successive rows of the second tableau with indices a, b, c, . . . , e.g.



- Attach the boxes from the second to the first tableau, one a time following the order a, b, c. . . in all the possible way. The resulting diagrams should be valid Young tableaux with no two (or more) a in the same column neither b nor c or. . .).
- Two generated tableaux with the same shape but labels distributed differently have to be kept. If two tableaux are identical only one has to be kept.
- Counting the labels from the first row from right to left, then the second row (from right to left) and so on, at any given box position there should be no more b than a, more c than b and so on (if it is not the case discard the tableau).

The tableau 

	a	b

 has to be discarded.

The adjoint representation is the irrep with dimension equal to the dimension of the the group (i.e.  $N^2-1$ ) and can be constructed by a direct product of the fundamental representation and its complex conjugate

$$\overline{N} \otimes N = (N^2 - 1) \oplus 1 \dots\dots\dots(2)$$

From the conjugation rule above it is clear that the adjoint representation is self conjugate

$$\overline{N^2 - 1} = N^2 - 1$$

### **4.4 UNITARY SYMMETRY**

Unitary symmetry refers to a type of symmetry that arises in the context of quantum mechanics. In quantum systems, the dynamics of the system are described by the Hamiltonian operator, which represents the total energy of the system. If a quantum system possesses a unitary symmetry, it means that the Hamiltonian operator commutes with a unitary symmetry operator. This symmetry operator leaves the Hamiltonian invariant, which leads to certain conservation laws and simplifies the analysis of the system.

Unitary symmetry plays a significant role in many areas of physics, such as quantum field theory, condensed matter physics, and nuclear physics. It allows for the classification and understanding of physical systems based on their symmetries, leading to powerful tools and techniques for solving quantum mechanical problems.

While Young tableaux are primarily used in representation theory and combinatorics, they can also be employed in the study of physical systems with unitary symmetry. For example, in the field of nuclear physics, Young tableaux can be used to classify the states of atomic nuclei based on their symmetry properties. The symmetries of the nuclear Hamiltonian can be analysed using Young tableaux techniques to predict various nuclear properties.

#### **4.4.1 Application of Young’s tableaux for unitary symmetry**

The group SU (N) has acquired an importance in particle physics because of the quark model. This necessitates calculating direct products of basis states to determine the characteristics of particle spectra. The methodology of the Young tableaux developed in the preceding section to SU (N), which turns out to be straight forward given the rules stated in the section. There is no change to the construction of the generic tableaux; the only changes

are in the labelling of the tableaux. Consider, for example the case of a twofold direct product of SU (3). There are six symmetric states

1	2
---	---

1	1
---	---

1	3
---	---

2	2
---	---

2	3
---	---

3	3
---	---

And three anti-symmetric states

1
2

1
3

2
3

As is evident from these constructions, the number of states associated with a tableau of a particular topology increase sharply with the number of basis states. The rules in the preceding section allow the number of such symmetric and anti-symmetric states to be calculated for SU (N). There  $\frac{1}{2}N(N + 1)$  symmetric and  $\frac{1}{2}N(N - 1)$  anti-symmetric states. The only other modification to our discussion of SU (2) is that for larger number of basis states, tableaux which make no contribution to SU (2), may make a contribution to SU (N). Consider for example the anti-symmetric three particle states. This states vanishes for SU (2) because there are only two basis states for SU (3), we have

1
2
3

In fact, this is a direct consequence of the rules for labelling Young tableaux, and we see that for SU (N), any column with more than N boxes makes no contribution.

➤ **3⊗3 Representations**

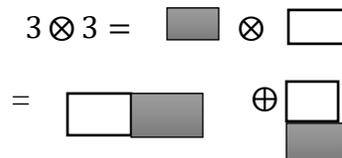
We know that the representation 3 is an inverted triangle in  $t_3$ -Y plane with  $t_3=0, Y=0$  as its centre. Graphically the  $3 \otimes 3$  representation can be obtained by placing the centre of the vertices of the other triangle. The figure describes process, in  $3 \otimes 3$  representation, we can identify a  $3^*$  representation and a 6 representation and we say  $3 \otimes 3$  is reducible into  $3^*$  and 6 i.e.

$$3 \otimes 3 = 3^* \oplus 6 \quad \dots\dots\dots(3)$$

Which in  $D^n(p, q)$  notation  $D^n = n$  dimension is

$$D^3(1,0) \otimes D^3(1,0) = D^3(0,1) \oplus D^6(2,0) \quad \dots\dots\dots(4)$$

Thus, when two triplets of SU (3) combine, we get an antitriplet and a hexad. The antitriplet is antisymmetric and 6 is symmetric. If the triplet is (d,u,s), then  $3^*$  is antisymmetric under the interchange of two quarks while the hexad is symmetric. The states of  $3^*$  and 6 are diquark states which do not belong to known elementary particles. Young tableau technique can provide the reduction of  $3 \otimes 3$  in a simple way. In SU (3) we denote



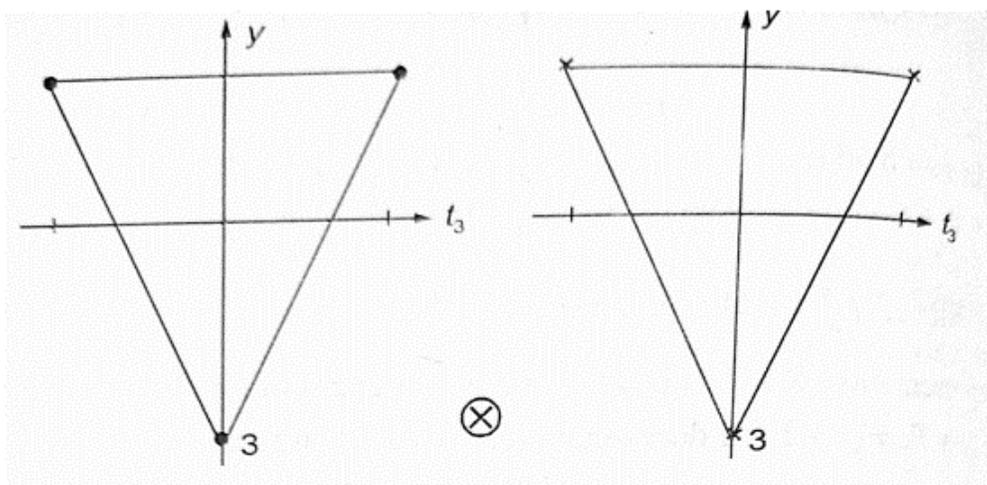


Fig01.:  $3 \otimes 3$  graphical representation showing two, 3 representation

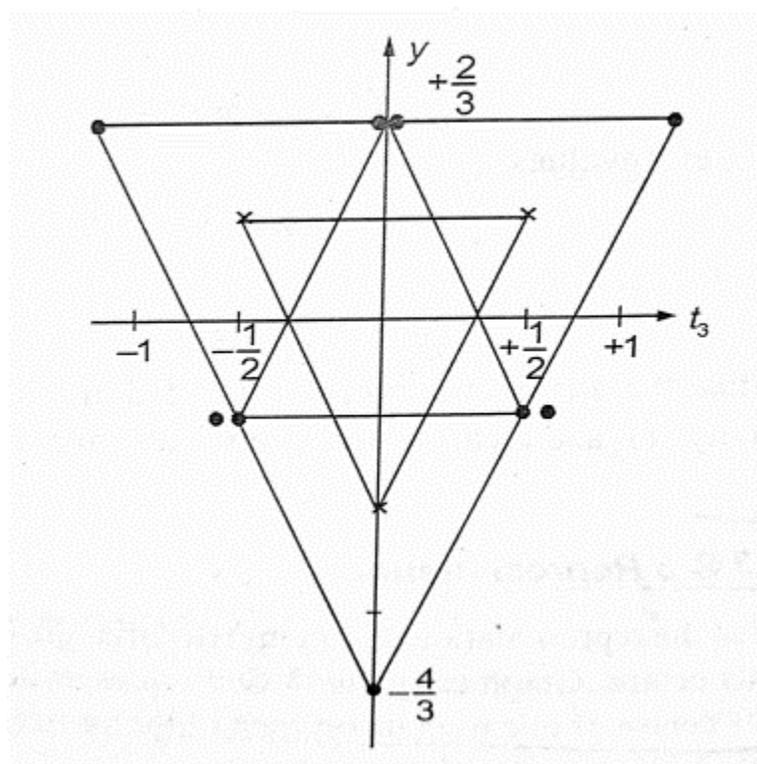


Fig 02.: On every vertex of other 3 (x sites) the first triangle is placed

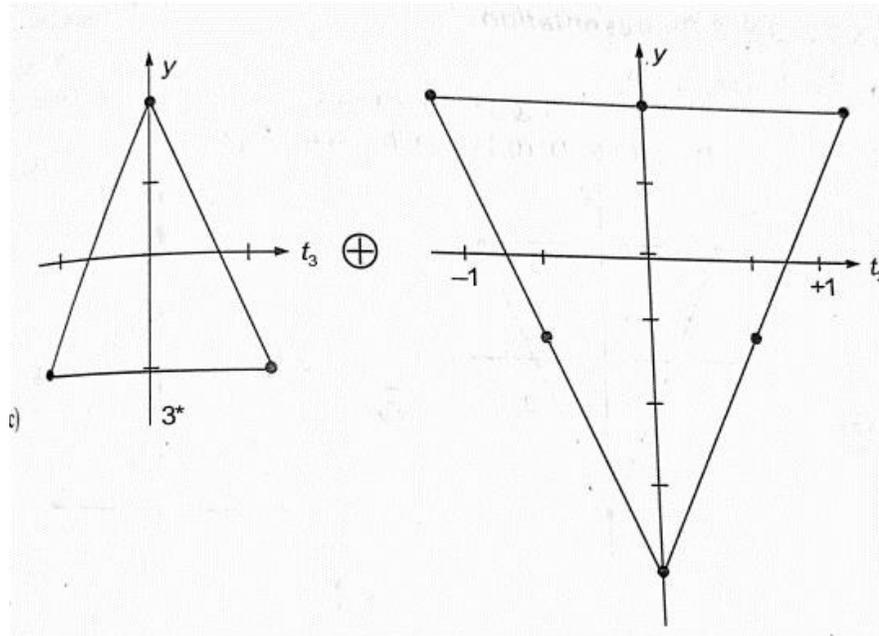


Fig03.: Above figure separated into 6, a six dimensional multiplet

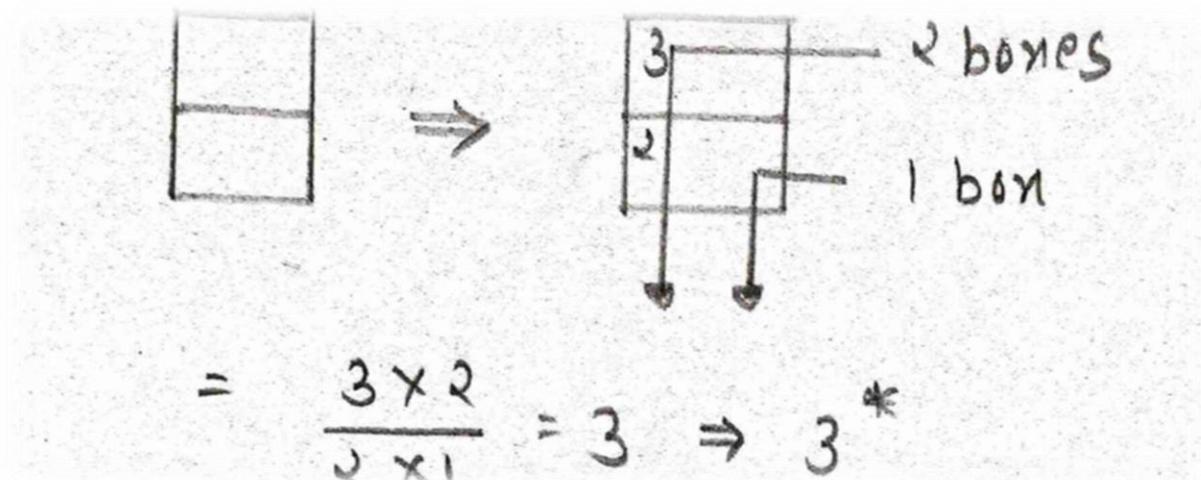
The graphical representation of  $3 \otimes 3 = 3^* + 6$  figure (01) shows two, 3 representations. In figure (02) on every vertex of other 3 ( $\times$ ) sites the first triangle is placed. Figure (02) can be separated into figure (03). 6 is a six dimensional multiplet.

In which the dimensions of  $\begin{bmatrix} \square & \square \end{bmatrix}$  are 6 and the dimension  $\begin{bmatrix} \square \\ \square \end{bmatrix}$  are though 3, it is defined as

$3^*$ . We calculated these dimensions in the following manner. The dimension of

$$= \frac{3 \times 4}{2 \times 1} = 6$$

And the dimension



Therefore equation (A) is defined as,

$$3 \otimes 3 = 6 \oplus 3^*$$

While assigning number to boxes the 3 of SU (3) was kept in mind. The first box was assigned the number 3. In SU (2), too, the number assigned to next box in the row increased from 2 by 1 while the number assigned to next box in the column decreased from 2 by 1.

Examples

❖ SU(2) (any irrep is self conjugate)

$$\square \otimes \square = \square \oplus \begin{matrix} \square \\ \square \end{matrix}, \quad (2 \otimes 2 = 3 \oplus 1);$$

$$\square \otimes \begin{matrix} a & a \\ a & a \end{matrix} = \begin{matrix} \square & a \\ & a \end{matrix} \oplus \begin{matrix} \square & \square \\ & a \end{matrix} \otimes \begin{matrix} a \\ a \end{matrix} = \square \oplus \square \oplus \begin{matrix} \square \\ \square \end{matrix},$$

$$(3 \otimes 3 = 5 \oplus 3 \oplus 1);$$

$$(2j_1 + 1) \otimes (2j_2 + 1) = \oplus_{j=|j_1-j_2|}^{j_1+j_2} (2j+1);$$

▪ SU(3)

$$\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} \otimes \begin{array}{|c|} \hline \square \\ \hline \end{array} = \begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} \oplus \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array}, \quad (\bar{3} \otimes 3 = 8 \oplus 1);$$

$$\begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} \otimes \begin{array}{|c|c|} \hline a & a \\ \hline b & \square \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline \square & \square & a \\ \hline \square & \square & \square \\ \hline \end{array} \oplus \begin{array}{|c|c|} \hline \square & \square \\ \hline \square & a \\ \hline \end{array} \oplus \begin{array}{|c|c|} \hline \square & \square \\ \hline \square & a \\ \hline \end{array} \otimes \begin{array}{|c|} \hline a \\ \hline b \\ \hline \end{array} =$$

$$\begin{array}{|c|c|c|c|} \hline \square & \square & a & a \\ \hline \square & \square & \square & \square \\ \hline \end{array} \oplus \begin{array}{|c|c|c|} \hline \square & \square & a \\ \hline \square & \square & a \\ \hline \end{array} \oplus \begin{array}{|c|c|c|} \hline \square & \square & a \\ \hline \square & \square & \square \\ \hline \end{array} \oplus \begin{array}{|c|c|} \hline \square & \square \\ \hline \square & a \\ \hline \end{array} \otimes \begin{array}{|c|} \hline b \\ \hline \end{array} =$$

$$\begin{array}{|c|c|c|c|} \hline \square & \square & a & a \\ \hline \square & \square & \square & \square \\ \hline \end{array} \oplus \begin{array}{|c|c|c|} \hline \square & \square & a \\ \hline \square & \square & a \\ \hline \end{array} \oplus \begin{array}{|c|c|c|} \hline \square & \square & a \\ \hline \square & \square & b \\ \hline \end{array} \oplus \begin{array}{|c|c|} \hline \square & a \\ \hline \square & a \\ \hline \end{array} \oplus \begin{array}{|c|c|} \hline \square & a \\ \hline b & \square \\ \hline \end{array} \oplus \begin{array}{|c|} \hline a \\ \hline b \\ \hline \end{array};$$

$$(8 \otimes 8 = 27 \oplus 10 \oplus \bar{10} \oplus 8 \oplus 8 \oplus 1);$$

▪ SU(6)

$$6 \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} \otimes 6 \begin{array}{|c|} \hline a \\ \hline b \\ \hline \end{array} \otimes \begin{array}{|c|} \hline \square \\ \hline \end{array}$$

$$= \begin{array}{|c|c|c|} \hline \square & a & b \\ \hline \square & \square & \square \\ \hline \end{array} \oplus \begin{array}{|c|c|} \hline \square & a \\ \hline \square & b \\ \hline \end{array} \oplus \begin{array}{|c|c|} \hline \square & b \\ \hline a & \square \\ \hline \end{array} \oplus \begin{array}{|c|} \hline \square \\ \hline a \\ \hline b \\ \hline \end{array}$$

$$= 56 \oplus 70 \oplus 70 \oplus 20$$

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### ***4.5 STANDARD ARRANGEMENTS OF YOUNG TABLEAUX***

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Standard Young tableaux are a special class of Young tableaux that have specific properties. A standard Young tableau is a Young tableau where the numbers in each row and column are strictly increasing from left to right and from top to bottom, respectively.

Here are some properties of standard Young tableaux:

1. Row and column properties:
  - In each row, the numbers increase from left to right.
  - In each column, the numbers increase from top to bottom.
  - The numbers in the entire tableau form a strictly increasing sequence.
2. Size and shape:
  - The size of a standard Young tableau is determined by the total number of boxes (or cells) in the tableau.
  - The shape of a standard Young tableau is determined by the arrangement and number of rows and columns.
3. Dominance order:
  - Standard Young tableaux can be compared using dominance order. Given two standard Young tableaux of the same shape, one tableau is said to dominate the other if its entries weakly increase along rows and columns compared to the other tableau.
4. Correspondence with irreducible representations:
  - Each standard Young tableau corresponds to a unique irreducible representation of the symmetric group, and vice versa.
  - The shape of the standard Young tableau corresponds to the Young diagram, which represents the shape of the irreducible representation.

Standard Young tableaux have important applications in representation theory, symmetric functions, and combinatorics. They provide a combinatorial tool to study the irreducible representations of symmetric groups and other related algebraic structures.

Note: It's worth mentioning that the concept of standard Young tableaux is specific to the theory of symmetric groups and may not be applicable to all applications of Young tableaux in other areas of mathematics or physics.

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## ***4.6 DIMENSIONALITY OF THE REPRESENTATIONS OF $SU(N)$***

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The dimensionality of the irreducible representations of the special unitary group  $SU(N)$  can be determined using the concept of Young tableaux and the associated Young diagrams. The dimension of an irreducible representation is given by the number of standard Young tableaux with a specific Young diagram.

In the case of  $SU(N)$ , the Young diagrams are characterized by  $N$  boxes arranged in rows, with each row having fewer or equal boxes than the row above it. The dimensionality of the irreducible representations is determined by the shape of the Young diagram.

To calculate the dimension of an irreducible representation, one can use the hook length formula. The hook length of a box in a Young diagram is the number of boxes to its right plus the number of boxes below it, plus 1 for the box itself. The dimension of the irreducible representation associated with the Young diagram is then given by the product of the hook lengths of all the boxes divided by the product of the hook lengths of the boxes in the first row.

The hook length formula provides an explicit formula for the dimension of irreducible representations of  $SU(N)$ . However, it can be computationally intensive to calculate the dimensions for large values of  $N$ . Alternatively, one can use computer software or reference tables to obtain the dimensionality of specific irreducible representations.

It is important to note that the dimensionality of irreducible representations of  $SU(N)$  depends on the choice of the fundamental representation and the specific labeling convention used for the representations. Different conventions may lead to different labeling schemes and thus different dimensionality results.

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## 4.7 MULTIPLETS OF $SU(N-1)$

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The multiplets of  $SU(N-1)$  can be described using the Young tableaux or Young diagrams, which represent the irreducible representations of the group. In the case of  $SU(N-1)$ , the Young diagrams are characterized by  $N-1$  boxes arranged in rows, with each row having fewer or equal boxes than the row above it.

The multiplets of  $SU(N-1)$  are labeled by their Young diagrams, which determine their transformation properties under the group. The dimensionality of each multiplet is given by the number of standard Young tableaux with the corresponding Young diagram.

To illustrate this, let's consider a few examples:

1. Fundamental representation  $(N-1, 1)$ : The Young diagram for the fundamental representation of  $SU(N-1)$  consists of a single row with  $N-1$  boxes. This representation corresponds to the  $(N-1, 1)$  Young diagram. The dimensionality of this multiplet is  $N-1$ .
2. Antifundamental representation  $(1, N-1)$ : The Young diagram for the antifundamental representation of  $SU(N-1)$  consists of a single column with  $N-1$  boxes. This representation corresponds to the  $(1, N-1)$  Young diagram. The dimensionality of this multiplet is  $N-1$ .
3. Symmetric representation  $(N-2, 2)$ : The Young diagram for the symmetric representation of  $SU(N-1)$  consists of two rows, with the first row having  $N-2$  boxes and the second row having 2 boxes. This representation corresponds to the  $(N-2, 2)$  Young diagram. The dimensionality of this multiplet can be calculated using the hook length formula.
4. Antisymmetric representation  $(2, N-2)$ : The Young diagram for the antisymmetric representation of  $SU(N-1)$  consists of two rows, with the first row having 2 boxes and the second row having  $N-2$  boxes. This representation corresponds to the  $(2, N-2)$  Young diagram. The dimensionality of this multiplet can also be calculated using the hook length formula.

These are just a few examples of the multiplets of  $SU(N-1)$  and their associated Young diagrams. The dimensionality and specific properties of the multiplets can be further determined using the hook length formula or other techniques in representation theory.

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## 4.8 SUBGROUP OF $SU(N)$

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A subgroup of  $SU(N)$  is a group that is contained within the special unitary group  $SU(N)$  and shares some of its properties. Here are a few examples of subgroups of  $SU(N)$ :

1.  **$U(N-1)$ :** The unitary group  $U(N-1)$  is a subgroup of  $SU(N)$  that consists of  $N-1$  dimensional unitary matrices. It can be seen as the subgroup of  $SU(N)$  where the last row and column of the matrix are fixed to be zero.  $U(N-1)$  is often relevant in quantum systems where a specific mode or degree of freedom is traced out.

2.  **$SU(N-1)$ :** The special unitary group  $SU(N-1)$  is a subgroup of  $SU(N)$  that consists of  $N-1$  dimensional special unitary matrices. It can be seen as the subgroup of  $SU(N)$  where the last row and column of the matrix are fixed to be zero and the determinant of the matrix is fixed to be 1.  $SU(N-1)$  is often encountered when dealing with quantum systems with a fixed total particle number.

3.  **$U(1) \times SU(N-1)$ :** The product group  $U(1) \times SU(N-1)$  is a subgroup of  $SU(N)$  that consists of diagonal unitary matrices ( $U(1)$ ) multiplied by  $N-1$  dimensional special unitary matrices ( $SU(N-1)$ ). This subgroup captures the global phase symmetry ( $U(1)$ ) and the remaining internal symmetries ( $SU(N-1)$ ) of the full  $SU(N)$  group. It is often relevant in quantum systems where both global and internal symmetries are present.

These are just a few examples of subgroups of  $SU(N)$ . There can be other subgroups depending on the specific context and requirements of the problem at hand. The study of subgroups and their properties is an important aspect of group theory and has applications in various areas of mathematics and physics, including quantum mechanics, particle physics, and condensed matter physics.

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## 4.9 BARYON MULTIPLETS IN DIFFERENT REPRESENTATIONS

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Baryon multiplets can be described using Young tableaux, which represent the irreducible representations of the flavor symmetry group (such as  $SU(3)$ ) that acts on the quark flavors. The specific Young tableaux associated with the baryon multiplets depend on the particular flavor representation being considered.

In the case of  $SU(3)$ , which is commonly used to describe the flavor symmetry of quarks, there are several irreducible representations that correspond to different baryon multiplets.

The most well-known representations include the octet and the decuplet. The Young tableaux associated with these multiplets are as follows:

1. Octet: The octet representation, also known as the 8-dimensional representation, corresponds to the baryon multiplet consisting of the nucleon (proton and neutron) and other baryons with similar quantum numbers. The Young diagram for the octet representation is:

This Young diagram has three rows, with the first row containing three boxes, the second row containing two boxes, and the third row containing one box. Each box represents a quark in the baryon.

2. Decuplet: The decuplet representation, also known as the 10-dimensional representation, corresponds to a baryon multiplet containing particles with higher spin and isospin values. The Young diagram for the decuplet representation is:

This Young diagram has three rows, with each row containing three boxes. The decuplet consists of baryons with spin  $3/2$  and isospin  $3/2$ .

These are two examples of baryon multiplets described by Young tableaux in the context of SU(3) flavor symmetry. The dimensions of the irreducible representations and the specific baryons within each multiplet can be determined using additional techniques, such as Clebsch-Gordan coefficients or other methods from representation theory.

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#### ***4.10 GENERAL RULE AND ITS APPLICATION FOR REDUCING KRONECKER PRODUCT OF TWO REPRESENTATIONS***

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When reducing the Kronecker product of two irreducible representations, a general rule known as the Clebsch-Gordan decomposition is often used. The Clebsch-Gordan decomposition allows us to express the tensor product of two representations as a direct sum of irreducible representations.

The Clebsch-Gordan decomposition is based on the concept of Young tableaux and Young diagrams. The Young diagrams associated with the irreducible representations are used to determine the decomposition of the tensor product.

Here's a general outline of the procedure for reducing the Kronecker product:

1. Determine the Young diagrams: Identify the Young diagrams associated with each irreducible representation being considered. These diagrams represent the shape of the irreducible representations.
2. Construct the Young diagram of the Kronecker product: To construct the Young diagram of the Kronecker product, combine the rows of the Young diagrams of the individual representations, ordering them from largest to smallest. This combined diagram represents the shape of the resulting representation.
3. Generate the Young tableaux: Construct all possible standard Young tableaux for the combined Young diagram. Each tableau corresponds to an irreducible representation that appears in the Clebsch-Gordan decomposition of the tensor product.
4. Calculate the dimensions: Calculate the dimensions of each irreducible representation by using the hook length formula or other techniques.
5. Decompose the tensor product: Express the tensor product as a direct sum of irreducible representations. The coefficients in the decomposition correspond to the number of standard Young tableaux for each irreducible representation.

The Clebsch-Gordan decomposition is widely used in various areas of physics, including quantum mechanics and particle physics, where it is applied to the composition of angular momenta, isospins, and other quantum numbers. It provides a systematic way to analyze the resulting representations and understand the symmetries present in the combined system.

It is worth noting that the Clebsch-Gordan decomposition can be computationally demanding, especially for large representations and high-dimensional systems. In practice, specialized algorithms and software packages are often employed to perform the decomposition efficiently.

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## 4.11 KRONECKER PRODUCT OF THREE PARTICLE STATE VECTORS

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To find the Young tableaux associated with the Kronecker product of three particle state vectors, we need to consider the Young tableaux corresponding to each individual state vector and combine them appropriately.

Let's denote the Young tableaux associated with the individual state vectors  $|A\rangle$ ,  $|B\rangle$ , and  $|C\rangle$  as  $Y(A)$ ,  $Y(B)$ , and  $Y(C)$ , respectively. To find the Young tableaux for the Kronecker product of the three particle state vectors, we overlay the Young diagrams and combine the boxes.

The Young tableaux for the Kronecker product can be constructed as follows:

1. Start with the Young diagram  $Y(A)$  associated with  $|A\rangle$ .
2. Overlay the Young diagram  $Y(B)$  associated with  $|B\rangle$  on top of  $Y(A)$ , aligning the boxes appropriately.
3. Overlay the Young diagram  $Y(C)$  associated with  $|C\rangle$  on top of the combined diagram from step 2, aligning the boxes appropriately.

The resulting combined Young diagram represents the Young tableaux associated with the Kronecker product of the three particle state vectors.

It's important to note that the specific procedure for combining the Young diagrams may depend on the ordering of the particle state vectors and the specific conventions used in the particular context. Also, the resulting Young tableaux may have different shapes and dimensions depending on the dimensions of the individual state vectors and the specific compositions considered.

By constructing the combined Young diagram and associated Young tableaux, we can describe the resulting irreducible representations and determine their properties, such as dimensions and transformation properties under the corresponding symmetry groups.

The calculation of the Young tableaux and associated properties can be complex, especially for large representations and high-dimensional systems. In practice, specialized algorithms, software packages, or symmetry reduction techniques may be employed to perform these calculations efficiently.

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### 4.12 SUMMARY

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After studying the unit learners will be able to explain

- Young Tableaux
- Unitary Symmetry
- Applications of Young Tableaux
- Standard arrangements of Young Tableaux
- Dimensionality of the representation of  $SU(N)$
- Multiplets of  $SU(N-1)$
- Baryon multiplets in different representation
- Kronecker product of the three particle state vectors

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### 4.13 REFERENCES

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### 4.14 SUGGESTED READINGS

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2. Quarks and Patrons by FE Close:
3. T.P.Cheng and G.LF Li by Gauge Field Theory.
4. Nuclear Physics by Irving Kaplan, Narosa Publishing House

5. Elementary Nuclear Theory, 2<sup>nd</sup> ed. by Bethe and Morrison, Wiley: New York.
6. Introduction to High Energy Physics by D. H. Perkins.

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#### ***4.15 TERMINAL QUESTIONS***

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1. Draw the Young tableaux diagrams for 8 and 6 under SU(3).
2. Find the product of 6 x 8 using the Young tableaux's.
3. Explain the standard arrangements of Young Tableaux.
4. Discuss the dimensionality of the representation of SU(n) and multiplets of SU(N-1).
5. Discuss the Kronecker product of three particle state vectors in detail.
6. What will be the Young tableaux diagram for SU(5) and its conjugate, i.e., 5 and 5. Find the dimension of SU(5) and the number of diagonal matrices.
7. Use Young,s tableau technique to prove
  - (a)  $3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$
  - (b)  $8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10^* \oplus 10 \oplus 27$

## UNIT 5

**NUCLEAR AND PARTICLE DETECTORS**

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**Structure of the Unit**

- 5.1 Introduction
- 5.2 Objectives
- 5.3 Classification of Detectors
- 5.4 Detection efficiency of Detectors
- 5.5 Ionization Chamber
  - 5.5.1 Construction
  - 5.5.1 Working Principle
- 5.6 Proportional Counter
  - 5.6.1 Principle
  - 5.6.2 Construction and Working
  - 5.6.3 Gas Amplification
  - 5.6.4 Uses
  - 5.6.5 Main Disadvantages
- 5.7 Geiger-Muller (GM) Counter
  - 5.7.1 Principle
  - 5.7.2 Construction and Working
  - 5.7.3 Quenching
  - 5.7.4 Characteristics of GM Tubes
- 5.8 Scintillation Detectors
  - 5.8.1 Scintillation detector/ Crystals
  - 5.8.2 Photomultiplier Tubes
  - 5.8.3 Electronic Circuitary
  - 5.8.4 Uses of Scintillator Detector
  - 5.8.5 Limitations of Scintillator Detector
- 5.9 Semiconductor Radiation Detector
- 5.10 Cloud Chamber
- 5.11 Bubble Chamber
- 5.12 Nuclear Emulsion Technique
- 5.13 Summary
- 5.14 References
- 5.15 Suggested Readings
- 5.16 Terminal Questions

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## ***5.1 INTRODUCTION***

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Nuclear radiation detectors are used to analyse the nature of the radiation, measure its intensity, analyse the energy spectrum of the particles, investigate the interactions between fast particles and atomic nuclei, and track the disintegration of unstable particles. Positively charged ions and electrons may be created when, and radiations interact with matter. The detectors are tools that measure this ionisation and generate an output that may be observed. Early detectors looked for "tracks" left by nuclear interactions on photographic plates. Electronic detectors were made possible by developments in electronics, particularly the transistor's discovery. New and improved detectors have been made possible by advancements in fabrication techniques and materials, particularly ultra-pure materials..

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## ***5.2 OBJECTIVES***

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After going through this unit learners should be able to understand the principle and working of :

- Ionization Chamber
- Proportional Counter
- Geiger-Muller Counter
- Scintillation Detector
- Semiconductor detector
- Nuclear Emulsion Technique
- Cloud Chamber
- Bubble Chamber

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## ***5.3 CLASSIFICATION OF DETECTORS***

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The detectors are mainly classified into two categories as shown below

### **1. Electrical detectors**

- Ionization Chamber
- Proportional Counter

- Geiger-Muller Counter
- Scintillation Counter
- Semi-conductor detector

2.Optical detectors

- Cloud Chamber
- Bubble Chamber
- Spark Chamber
- Photographic Emulsion

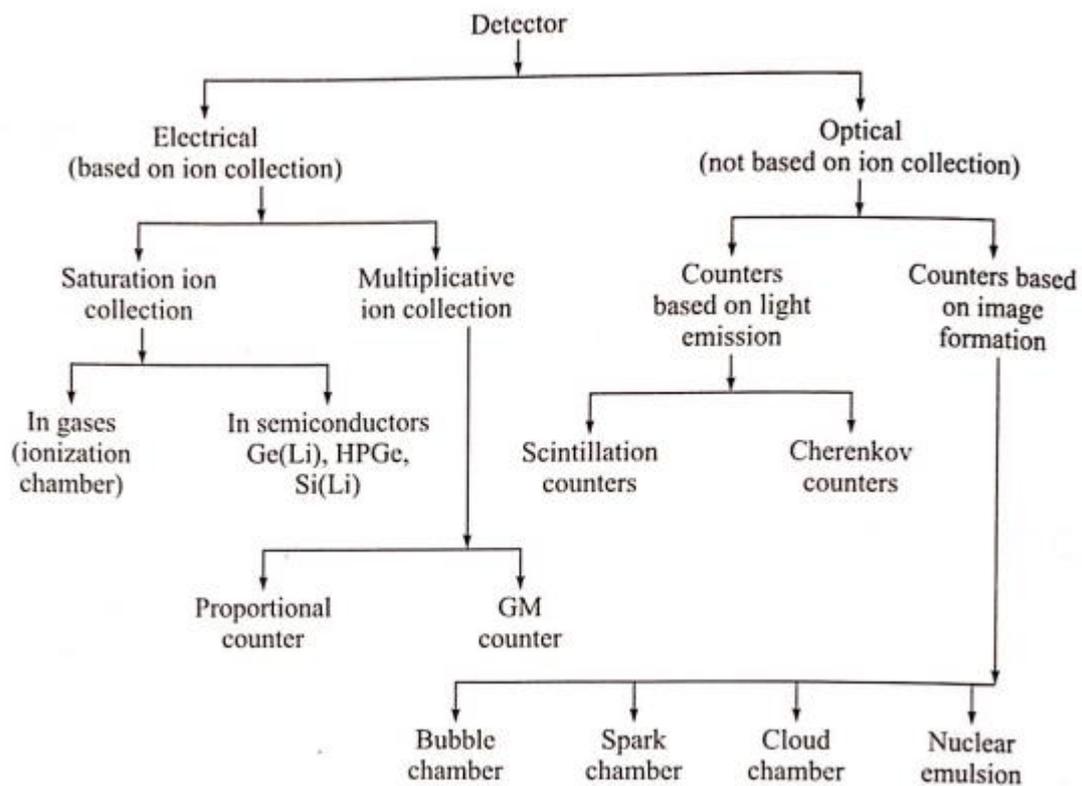


Fig.1: Classification of detectors

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## ***5.4 DETECTION EFFICIENCY OF DETECTORS***

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Efficiency, the probability that a particle will be registered upon entry into the effective volume of the detector, is a key feature of nuclear radiation detectors that register individual particles. Efficiency depends on the working medium's characteristics and the detector's design. However, a perfect detector should have the following features.

- 100% detection efficiency
- High-speed counting
- Good energy resolution
- Linearity of response
- Application to virtually to all types of particles and radiations
- Virtually no limit to the highest energy detectable
- Reasonably large solid angles of acceptance
- Discrimination between types of particles.
- Picturization of the event.

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## ***5.5 IONIZATION CHAMBER***

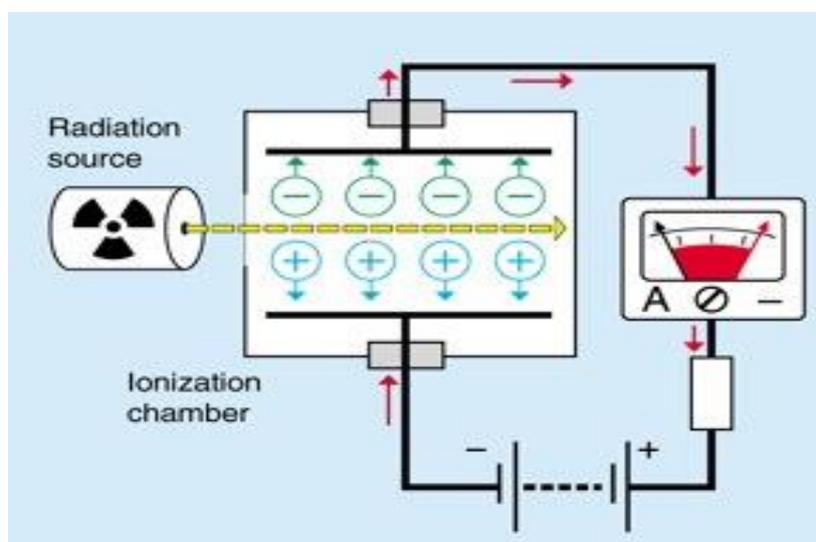
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An ionization chamber is a device used for detecting and measuring ionizing radiation, such as alpha particles, beta particles, and gamma rays. It operates on the principle of ionization, where incoming radiation interacts with the gas inside the chamber, producing ion pairs (positively charged ions and negatively charged electrons) in the process.

The term “ionisation chamber” is only used to refer to detectors that collect all the charges produced (i.e., current) by direct ionisation of the gas through the application of an electric field. In a perfect world, the quantity of electric current produced in an ionisation chamber would be directly proportional to the strength of the radiation field. Ionisation chambers are frequently employed in a variety of applications that require for radiation detection, including radiation monitoring in nuclear power plants, medical imaging and radiation therapy, environmental monitoring, and scholarly research. They have benefits including high sensitivity, a large dynamic range, and strong energy response for various kinds of radiation.

### 5.5.1 Construction:

The basic construction of an ionization chamber consists of a gas-filled chamber with two electrodes—an outer cylindrical electrode called the shell or chamber wall, and a central electrode called the collector. The chamber is typically filled with a gas, such as air or a specialized gas mixture, at a specific pressure.



**Fig.2:** Electric Circuit diagram

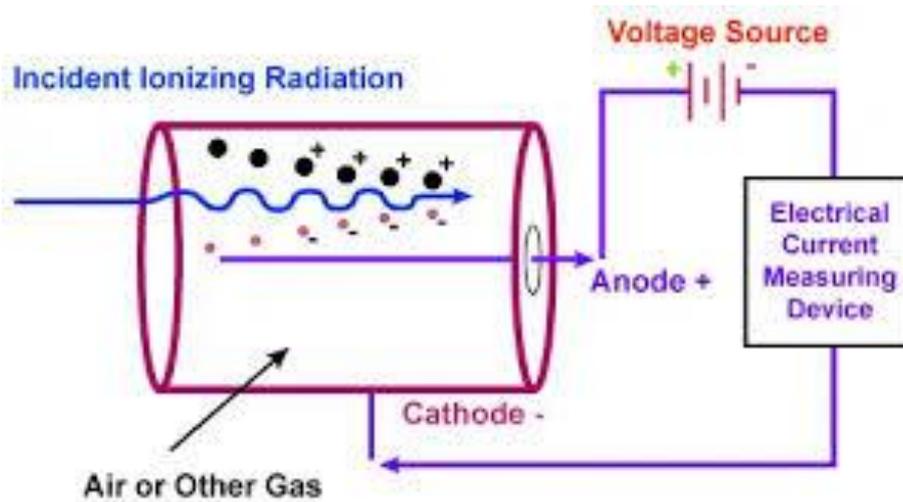
Following are the basic needs to construct an ionization chamber.

**Two collecting electrodes:** The anode and cathode (the anode is positively charged with respect to the cathode). In most cases, the outer chamber wall serves as the cathode. The electrodes may be in the form of parallel plates (Parallel Plate Ionization Chambers: PPIC), or a cylinder arrangement with a coaxially located internal anode wire. You can take the analogy from this if you have a bottle of cold drink which is behaving as cathode and if you insert the straw to suck the drinks it will behave as the anode.

A voltage source to create an electric field between the electrodes.

An electrometer circuit which is capable of measuring the very small output current which is in the region of femtoamperes to picoamperes.

### 5.5.2 Working Principle



**Fig.3:** Ionization Chamber Working

First the potential difference between the anode and cathode is often in the 100 to 500 volt range. The most appropriate voltage depends on a number of things such as the chamber size (the larger the chamber, the higher the required voltage). When an ionising radiation or a charged particle enters the chamber, it converts some of the gas molecules to positive ions and electrons; under the influence of the electric field, these particles migrate to the wall and the wire, respectively, and cause an observable current to flow through the circuit. This accumulated charge is proportional to the number of ion pairs created, and hence implies the strength the radiation dose which is a measure of the total ionizing dose entering the chamber. However, there is one problem with this set up. As the produced electrons move toward the anode, on its journey it may recombine with other ions to produce a neutral element. Thus, there is possibility that the ion current will diminish due to recombination. Thus, it can be seen that in the “ion chamber” operating region the collection of ion pairs is either effectively constant or less than the expected value over a range of applied voltage, as due to its relatively low electric field strength.

It's worth noting that while ionization chambers are effective for measuring radiation intensity, they do not provide information about the energy or type of radiation. For more detailed information about the radiation, other types of detectors like scintillation detectors or Geiger-Muller counters may be used in conjunction with ionization chambers.

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## ***5.6 PROPORTIONAL COUNTERS***

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A proportional counter counts and measures the energy of ionizes radiation particles. Although it employs a lower working voltage, it operates on the same principles as a Geiger-Müller counter. A proportional counter differs from an ionizes on chamber in that the operating voltage is high enough for the drifting electrons to gather energy over a mean free path to produce further ion-pairs when they collide with other neutral gas atoms. The kinetic energy of the particle can be calculated by measuring the total charge, which is the time integral of the electric current between the electrodes.

### **5.6.1 Principle**

The idea behind proportional counters is that nuclear radiation ionizes a medium as it passes through it.

### **5.6.2 Construction and Working**

The proportional counter is based on the principle of gas-filled detector that was introduced in the late 1940s. These detectors work in the proportional region III (Fig.4).

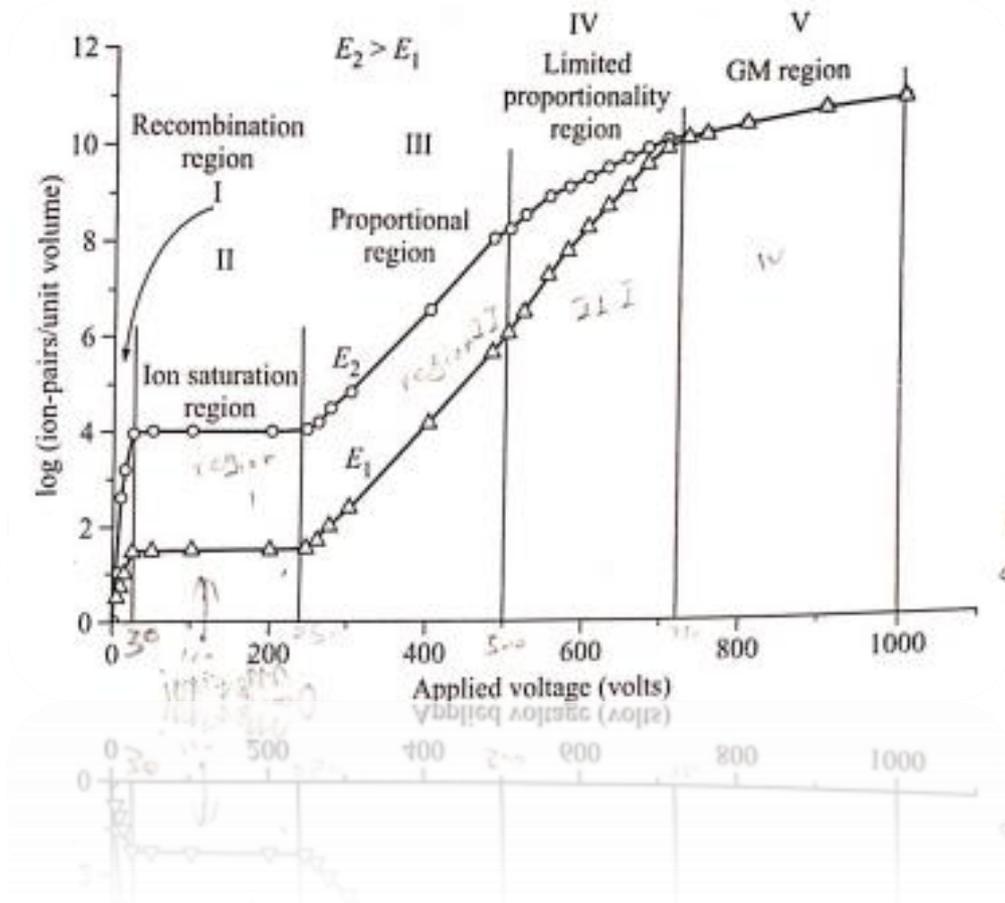


Fig.4: Various Operating regions in Gas Filled detectors

The proportional counters come in a variety of sizes and shapes. The cylindrical-shaped proportional counters are the most widely used ones. The cylinder which is made up of a metal act as the cathode. The inside walls of this cylinder are coated with a conducting metal, such as copper or silver, and built of glass. Anode is a straight conducting wire that is insulated from the outer cylinder and is located in the center of the cylindrical cathode. The schematic view of the proportional counter is shown in Fig.05

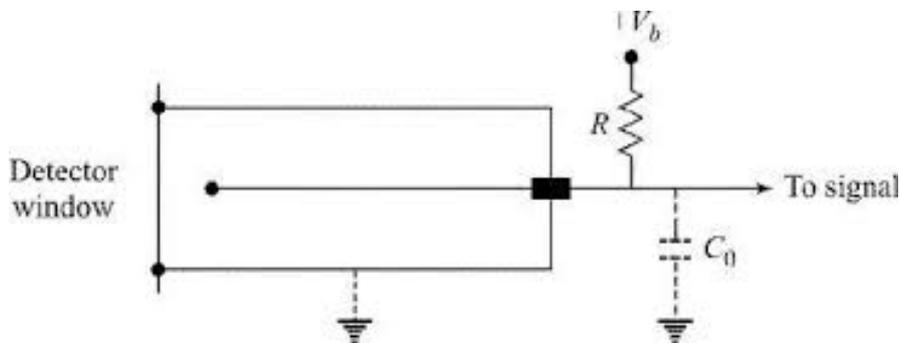


Fig.5: Proportional Counter Schematic view

Usually, a mixture of gases is used containing gases like neon, argon, krypton, etc. which favor high amplification and a complex gas like carbon dioxide, methane, pentane etc. which helps in stopping the phenomenon of secondary ionization. Common proportional counters used in the laboratories contain a mixture of about 90% argon and 10% methane. The total gas pressure in the tube is generally less than one atmosphere. A thin glass, mica or polymer window (10  $\mu\text{m}$ ) isolates the gases present in the proportional counter from the atmosphere.

As discussed earlier, the output current signal in the ionization chambers is very low. One common method of amplifying the signal is to increase the electric field till electrons gain sufficient energy between two successive collisions with gas atoms to cause further ionization.

A high voltage (V) applied to anode is so adjusted that the tube operates in the proportional region (Figure 4). For cylindrical geometry, the strength of electric field at a radial distance  $r$  from the central wire is given by

$$E(r) = \frac{V}{r \log\left(\frac{b}{a}\right)}$$

Where  $a$  is the radii of the central wire and  $b$  is the inner radius of the cylinder. The electric field close to the anode wire is sufficiently high. For the case, if  $a=0.005$  cm and  $b=1.0$  cm and  $V= 2000$ V, then  $E(r)$  takes the value of  $5.18 \times 10^6$  V/m near the wire. The preferred geometry for proportional counters is cylindrical. When the distance between the plates is 1cm for a parallel plate design, a voltage of roughly 51,800 V is needed to produce this field gradient. Practically this cannot be done. The Phenomenon of gas multiplication takes place for such a high electric field gradient.

In such a high electric field gradient, a phenomenon called gas multiplication sets in. Now we shall study about this phenomenon i.e. gas amplification.

### 5.6.3 Gas Amplification

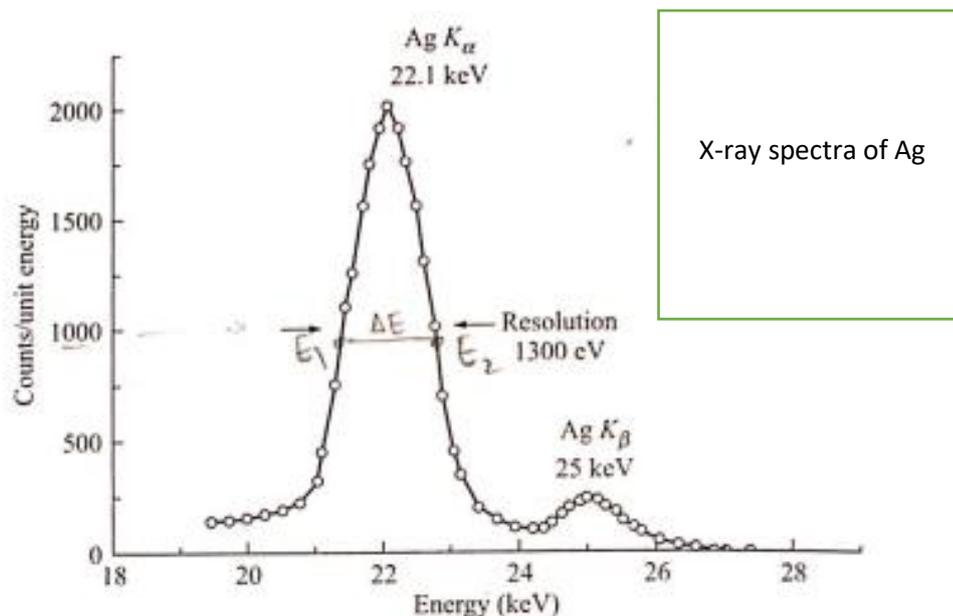
The electrons liberated by the original ionising event receive enough kinetic energy as a result of the increasing voltage. As they go towards the anode, these energising electrons collide with neutral gas atoms, ionising them in the process. The resulting electrons could lead to more ionisation as a result. Common names for this multiplying effect are Townsend avalanche and

Townsend cascade. The gas multiplication factor is the total number of ion pairs that one primary ion can produce. This factor ranges from  $10^2$  to  $10^4$  in proportional counters and is unity in an ionisation chamber. If the incident particle initially formed  $10^4$  ion-pairs, for instance, and those ion pairs were multiplied by a factor of 1000, the detector would now contain  $10^7$  ion-pairs instead of  $10^4$ .

At such a high field, the electrons acquire sufficiently high velocities. For example, if the field gradient is  $10^4$  V/m the velocity of the electrons is approximately  $4 \times 10^4$  m/s. With such high velocities electrons generated near the cathode take less than  $1 \mu\text{s}$  to reach the anode. Thus the proportional counter works at a much faster rate compared to the ionization chamber.

A proportional counter has two main features. First because of its built-in amplification, it can conveniently detect low energy radiations and second because of its proportionality, one can produce an energy spectrum of the incoming radiations.

Fig.6 shows a typical low energy X-ray spectrum obtained with a proportional counter. This shows two peaks, first at 22.1 KeV due to  $K_\alpha$  line of Ag and second at 25.0 KeV due to  $K_\beta$  line of Ag. One important parameter used in such detectors is Energy Resolution or Full Width at Half Maximum (FWHM). From the experimental spectrum, it is measured as under. Let us consider 22.1 KeV peak. The maximum counts at this peak are 2000. The width of the peak at exactly half of 2000, i.e.1000 counts, is about 1300 eV. This number 1300 eV is known as energy resolution or FWHM of this detector at 22.1 keV. The physical meaning of energy resolution is that if there are tweaks with energy difference greater than the resolution, the peaks are seen as two separate peaks; otherwise only a single peak is visible. For example, in the present detector where the energy resolution is 1300 eV. If we draw the spectrum with two X-rays at 22.1 and 22.2 keV, the detector shows only a single peak.



**Fig.6:** Low energy X-ray spectrum obtained by Proportional counter

#### 5.6.4 Uses

The main use of proportional counters is detecting charged particles and low energy X-rays and recording their energy spectrum.  $^{10}\text{BF}_3$ -filled proportional counters are used for detecting slow neutrons. Slow neutrons produce  $^{10}\text{B}(n,\alpha)\text{Li}^7$  reaction.  $\alpha$ -particles ejected in the reaction are detected in the proportional counter.

#### 5.6.5 Main Disadvantage

The main disadvantage of the proportional counter is the need for an expensive highly stabilized power supply to maintain the proportionality characteristics.

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### 5.7 GEIGER-MÜLLER (GM) COUNTER

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When we are studying about the Geiger-muller counter we have to look at fig.4 from here if we continue to increase the voltage across the electrodes, then we pass from the proportional region through the region of limited proportionality to the GM region. In this case, the electron cascade spreads from the immediate neighborhood of the initial track. until it encompasses the whole length of the anode. The resulting pulse is thus very much larger than

that obtained from a proportional counter, but is obviously independent of the initial particle energy. This means that all pulses from a GM tube are of the same amplitude regardless of the number of original ion-pairs that started the process or is independent of the energy brought into the tube by the incident particle. Therefore, a GM counter cannot be used for measuring energy of the radiations.

Hans Geiger and W. Müller, who invented this counter in the 1920s, are honoured by its name.

The portable radiation equipment that is most frequently used known by the names Geiger counter, GM counter, or GM tube. A GM tube counter can only detect radiation's presence and intensity, while being practical, affordable, and durable. Ionising radiation, primarily alpha and beta radiation as well as other types of radiation like low energy X-rays etc. are detected using Geiger counters.

### **5.7.1 Principle**

It is based on the idea or principle that nuclear radiations while passing through the gas contained in GM counter ionize it.

### **5.7.2 Construction and Working**

The schematic diagram of a GM counter is identical to that of a proportional counter and is shown in Fig.5. The GM tube is cylindrical in shape in most of the cases. This cylinder is either made up of a conducting metal like copper, which acts as a cathode or it is made up of glass, in which a metallic cylinder is supported, which acts as a cathode. However, in some cases when the tube is of glass, inner sides of the tube are coated with a conducting metal like copper or silver. This coating acts as a cathode. Tungsten wire acts as anode which stretches in the center of the tube and is isolated from the cylindrical tube. The thickness of this wire ranges from 0.02 to 0.05 mm.

The GM tubes are offered in a range of sizes depending on the need. The tube's length ranges from 1 cm to 100 cm, and their diameters range from 0.3 cm to around 15 cm. In GM tubes that are for commercial use is filled with 90% of the argon (as the filling gas) and 10% is ethyl alcohol (as the quenching gas) or 0.1% either chlorine or bromine (as the

quenching gas), and the remaining is neon or argon gas. The atmosphere is separated from the gases present in the GM counter by a thin (10 m) glass, mica, or polymer window.

The gas inside the GM tube is ionised by the charged particles entering the tube. Primary ionization is the term used to describe this phenomenon.

The main ionization's liberated electrons are accelerated in the direction of the centre anode wire. The electrons may acquire enough energy due to the high voltage on the wire to further ionise the neutral gas molecules (a process known as secondary ionisation). This causes a phenomenon called Townsend avalanche, which is a series of ionising events that have been discussed earlier in detail.

The possibility of excitation of molecules and atoms exists in addition to these activities. These excited molecules and atoms may release ultraviolet (UV) or visible photons when they de-excite. By ionizing gas molecules or atoms or by interacting photoelectrically with the counter's walls, these photons once more result in the generation of electrons. Again, a Townsend avalanche would be set on by each of these liberated electrons. The Geiger discharge would occur as a result of such a series of avalanches in the tube. A dense envelop of electron-ion pairs forms surrounding the anode in such a condition. Electrons from a single event that results in a Geiger discharge are all collected by the anode when a voltage is applied.

### **5.7.3 Quenching**

Generally, the process above discussed is not as simple. There is a dense envelop of ions and electrons around the anode during the Geiger discharge. Drifting of the electrons would be towards the anode and positive ions would be drifted towards the cathode. The positive ions drifting towards the cathode having ionization potential greater than the work function of the cathode material lead to exchange of electron from the cathode and become neutral. The excess energy may be dissipated with the emission of either a photon or an electron from the cathode the latter process take place only if excess energy is greater than the work function of the cathode material. This again initiates another Geiger discharge. The result of this is that the tube would always be in continuous Geiger discharge and hence is not able to measure any radiation.

The concept of quenching has been introduced to solve this problem. This is mainly of two types mentioned below

I. Organic quenching

II. Halogen quenching

### **I. Organic Quenching:**

A small amount of organic gas with a complicated molecular structure must be added in order to accomplish this. The charge transfer collision concept prevents the continuous Geiger discharge in this way. As they travel, the positive ions hit organic molecules, which neutralises them. This limits the ions of organic gas that can pass through the cathode and be neutralised.

Organic molecules would split apart if there was any extra energy released. Multiple Geiger discharges could thus be prevented. 90% argon and 10% ethyl alcohol are a typical filler for organic quenched GM tubes. Multiple discharges typically occur when the organic gas is sufficiently depleted, which causes the plateau length to decrease with an increase in slope. The organic quenched GM tubes are characterized by short life time and are not suitable for operation in very high fields which leads to large count rate. To overcome this, the technique of halogen quenching was introduced.

### **II. Halogen Quenching:**

This involves the addition of small quantity of halogen gas such as chlorine or bromine. A typical filling is about 0.1% of chlorine to argon or neon. The quenching action is the same as that of the organic quenching process. The diatomic halogen gas molecules also get dissociated in quenching but get recombined to replenish the gas molecules and thus the counter life gets extended.

### 5.7.4 Characteristics of GM Tubes

The important parameters that decide the quality of functioning of GM tubes are mentioned below:-

1. Dead time
2. Recovery time
3. Geiger plateau length and plateau slope

#### 1. Dead Time:

As was said earlier, compared to electrons, positive ions take a lot longer to reach the cathode. The explanation is that positive ions have a mobility that is around 1000 times greater than that of electrons. A cloud of positive ions forms as a result of the positive ions' slow drift velocity. Close to the anode, a sheath of positive ions (space charge) is formed by the electric field of these positive ions, considerably weakening the powerful electric field. As a result, incoming electrons don't develop enough energy to trigger additional avalanches.

The detector becomes inactive (dead) until the ion sheath has migrated sufficiently for the field gradient to rise over the avalanche threshold. The dead time of a Geiger tube is defined as the time between the initial pulse and the time at which a second Geiger discharge (regardless of its size) can be developed. In most Geiger tubes, dead time is of the order of 50-100  $\mu$ s.

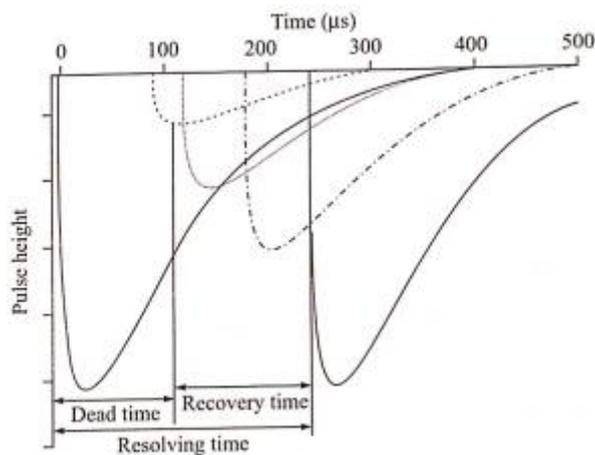
Dead time is the time immediately following the introduction of a particle during which the counter tube is insensitive for detection of another particle.

#### 2. Recovery Time:

Some of the electrons and +ve ions combine again once the dead time is over to produce neutral atoms.

Ionising radiation enters the tube at this moment, pulses with a smaller amplitude are created that the counter cannot count. The total time taken for the tube to recover to its fully sensitive state to give the next pulse is called resolving time. The sum of the dead time and recovery time is called Resolving time. The dead time, recovery time and resolving time are shown in Figure 7. During the dead time, no new pulses are formed. However, during the recovery time, pulses of lower amplitude are formed, which are not counted by the counting system. It

is only after the recovery time that the pulses of larger amplitude are formed. capable of getting counted.



**Fig.07:** Relationship between Dead time, Recovery time and Resolving time for GM counter.

### 3. Geiger Plateau:

In Fig.8, a plot of count rate versus the anode voltage has been shown. In this figure, the source of radiations is kept at a fixed distance from the counter. The potential difference at which the counting just starts is called threshold potential. The typical threshold potential depends upon the gas and its pressure in the counter. It generally varies between 400 and 900 volts. After a very sharp rise, the counting rate remains almost constant with increasing voltage. This flat region, the plateau, where the number of counts recorded per second remains independent of applied voltage is called Geiger region. The length of the plateau is about 100 to 300 volts. When the plateau region becomes shorter and steeper, it means that the GM tube is nearing the end of its useful life. If a Geiger-Müller counter is operated in this region, it eliminates the need of a costly and highly regulated power supply.

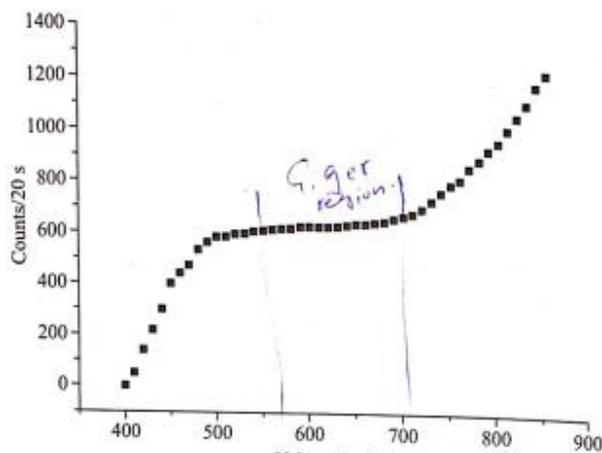


Fig.8: Plateau curve of a GM counter

**Main Uses of GM Counter:**

X-rays,  $\beta$ -particles,  $\alpha$ -particles etc. are all frequently detected by GM counters because of their high multiplication factor, which allows them for picking up even very small activities. Bigger size GM tubes are required for  $\gamma$ -ray detection. These nuclear radiation detectors are among the least expensive types available.

**Main Drawback:**

The major drawback of GM counters is that they cannot be used to measure the energy of radiations. Main differences between an ionization chamber and GM counter are shown in Table1, while the main differences between proportional counter and GM counter are shown in Table 2.

Table 1: Difference between Ionization chamber and GM counter

Ionization Chamber	GM Counter
It operates at relatively low voltages (~ 30 V to ~ 250 V). The output pulse height is low, so an amplifier is needed. Power supply used to feed voltage to ionization chamber must be highly regulated.	It operates at much higher voltages (~ 800 V to ~ 1000 V). The output pulse height is large, so no amplifier is needed. Power supply used to feed voltage to a GM counter need not be regulated.

**Table 2: Difference between Proportional counter and GM counter**

Proportional Counter	GM Counter
It generally operates at lower voltages (~200 V to ~500 V).	It generally operates at higher voltages (~ 800 V to ~1000 V).
Output pulse height depends upon the energy of the incident particle.	Output pulse height do not depends upon the energy of incident particle.
Energy of the incident radiation is measured by it.	Energy of the incident radiation cannot be measured by it. However, intensity/flux of incident radiation can be measured by it.
The output pulse height is low, so an amplifier is needed.	The output pulse height is large, so no amplifier is needed.
Power supply used to feed voltage to proportional counter must be highly regulated.	Power supply used to feed voltage to a GM counter <u>need not</u> be regulated.

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## 5.8 SCINTILLATION DETECTORS

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We are all familiar with the ZnS screen that Rutherford utilised in his renowned scattering experiment. When particles collide with ZnS, they cause light flashes or scintillations. With a microscope, one may observe these flashes. The first scintillation detector that was really used was ZnS. The television screen, which emits visible photons in a cathode ray tube under electron bombardment, is another very popular and well known application of the scintillation mechanism. Later, it was discovered that there are many more materials that display the scintillations feature in addition to ZnS. Anthracene, naphthalene, sodium iodide, and cesium iodide are a few of them. This feature is also present in several gases, particularly noble gases like xenon and others. With the invention of photo multiplier (PM) tubes, scintillation detectors became the most potent and popular radiation detectors. These PM tubes are incredibly sensitive light detectors. Therefore, a microscope is not required to see the scintillations.

Three parts make up a scintillation spectrometer:

1. Scintillation detector/crystal
2. Photomultiplier tube

## 3. Electronic circuitry

Let us discuss the above components in detail.

### 5.8.1 Scintillation Detector/Crystal

Scintillators are available in many forms. Some commonly used scintillators are

#### 1. Inorganic crystals

- ❖ **Nal (TI):** The most popular scintillator for  $\gamma$ -ray work is this one. Among all known scintillators, the efficiency for converting the energy deposited into light output is one of the highest. It comes in a range of sizes and shapes. This scintillator has a drawback in that it must be stored in airtight containers due to its hygroscopic nature. The low density ( $-3.67 \text{ g/cm}^3$ ) of this scintillator is another drawback.
- ❖ **CsI(TI) and CsI(Na):** These crystals are nonhygroscopic and have a higher light output than Nal(TI). Growing large single crystals is the only challenge with these scintillators.
- ❖ **CaF<sub>2</sub>(Eu):** Another nonhygroscopic crystal, this one is helpful for detecting low energy (100 ke) X- or  $\gamma$ -rays.
- ❖ **ZnS:** It served as the industry's first scintillator in nuclear physics. But growing large transparent single crystals is exceedingly challenging.
- ❖ **Bi<sub>4</sub>Ge<sub>3</sub>O<sub>12</sub>:** In comparison to Nal (TI), it is nonhygroscopic and more robust mechanically and chemically. Compared to Nal(TI) scintillator, its density is significantly higher ( $7.13 \text{ g/cm}^3$ ). In comparison to a Nal(TI) crystal with the same efficiency, this would allow us to employ a smaller crystal. However, it produces little light and has a lower energy resolution than Nal(TI). Compared to Nal(TI) detectors, they are more expensive.

**2. Organic scintillators:** Common organic scintillators are anthracene, stilbene, etc. These scintillators have very poor efficiency compared to Nal(TI) detector. However, in these scintillators, the decay time of light produced due to ionizing radiations is much shorter compared to that in Nal(TI) detector. These detectors can be operated at higher count rate. These detectors are practically more suitable to  $\beta$  counting.

**3. Liquid scintillators:** In these detectors, scintillator material is dissolved in certain suitable liquid. In some applications, radioactive source is thoroughly mixed with liquid scintillator. These detectors are mostly used to count very low activities specially that of B-particles.

Many scientists have studied the mechanism of light emission by scintillators in detail. Without going into the detailed mechanism of light emission, certain terms commonly used while working with scintillation detectors are discussed,

The process by which visible light is emitted by the scintillator within  $10^{-8}$ s of falling of radiations is known as prompt fluorescence's or simply foresee. There is another mechanism visible light emission is known as phosphorescence in which the light of longer wavelength is emitted and this process occurs in a time which is much longer (few seconds to few minutes) There is a third mechanism, which is known as delayed fluorescence in which the light emitted is of the same wavelength as in the fluorescence but its emission time is longer (hours to days).

Another term commonly used in scintillators is the scintillation efficiency. It is defined as the fraction of the energy deposited by the incident radiation that is converted into light.

A good scintillator should have high scintillation efficiency and should convert a larger fraction of the emitted light by the process of prompt fluorescence. Single crystal of sodium iodide with traces of thallium written as NaI (TI) is one such crystal, which is extensively used for detecting and measuring the energy of  $\gamma$ -rays. TI in scintillators is also known as wave length shifter. The emission spectrum of pure sodium iodide has maxima at a wavelength of 303 nm. Most of the photomultiplier tubes are not sensitive to this wavelength. Addition of a small quantity of TI (0.1 to 0.5% mole fraction of TI) shifts this wavelength to 410 nm. Most of the photomultiplier tubes are sensitive to this wavelength

Incident-charged particles falling on the scintillator lose all the energy in the scintillator and light photons are produced. However, if the incident radiations are X- or y-rays, they interact with the scintillator in mainly three different ways.

1. Photoelectric effect
2. Compton effect
3. Pair production

In all these three processes, electrons are produced and ultimately total energy of X- or  $\gamma$ -rays is converted into the kinetic energy of electrons. These electrons give up their kinetic energy in ionization and excitation in the scintillator material, resulting in production of scintillations in the scintillator.

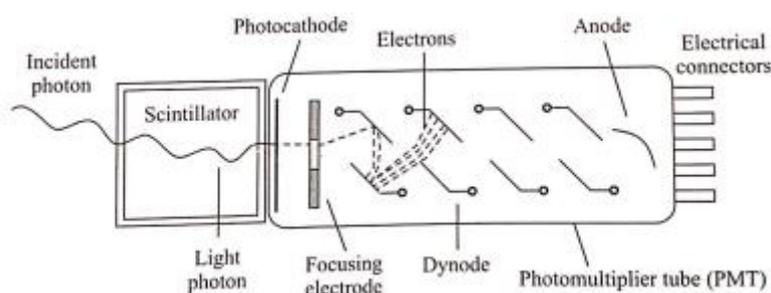
Scintillators are available in a variety of shapes and sizes, Scintillators commonly used in laboratories have dimensions as 1"x1" which means it is a cylinder of base diameter of 1" and height of 1". etc.

All the light produced in the scintillator should reach the photomultiplier tube. For this purpose, the scintillator is covered with light reflector MgO from all sides except one and this uncovered side is optically coupled to PM tube. Optical coupling means that the whole of the light photons produced in the scintillator reaches the photo cathode of the PM tube without any loss.

### 5.8.2 Photomultiplier Tubes

Photomultiplier tubes (PMTs) are extremely sensitive detectors of light in the ultraviolet, visible and near infrared. These detectors multiply the signal produced by incident light by as much as 10<sup>6</sup> to 10<sup>7</sup>. The combination of high gain, low noise and large area of collection means that these devices find applications in nuclear and particle physics.

Photomultipliers are constructed from a glass vacuum tube which houses a photocathode, several dynodes and an anode as shown in Figure 7.9. Incident photons strike the photocathode of the PM tube, and electrons are produced as a consequence of the photoelectric effect. The photocathode is generally coated with either antimony-rubidium-caesium or antimony-potassium-caesium. These electrons are directed by the focusing electrode towards the electron multiplier, where electrons are multiplied by the process of secondary emission.



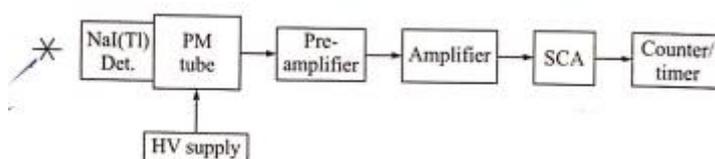
**Fig 9:** Photomultiplier Tube

The electron multiplier consists of a number of electrodes, called dynodes. Each dynode is held at a more positive-voltage than the previous one (50 to 100 V). The electrons are ejected by the photocathode. As they move towards the first dynode, they are accelerated by the electric field and arrive with much greater energy. On striking the first dynode, more low energy electrons are emitted and these, in turn, are accelerated towards the second dynode.

The geometry of the dynode chain is such that a cascade occurs with an ever-increasing number of electrons being produced at each stage. Finally, the anode is reached where the accumulation of charge results in a sharp current pulse indicating the arrival of a photon at the photocathode.

### 5.8.3 Electronic Circuitry

A block diagram of a complete NaI(Tl)  $\gamma$ -ray spectrometer is shown in Figure 10. In this figure, pre-amp is a pre-amplifier. The purpose of the pre-amplifier is to match impedance between photomultiplier and amplifier. Amplifier amplifies the signal arriving at its input. The amplified output of the amplifier is in direct proportion to the energy deposited by  $\gamma$ -rays in scintillation detector. For example, if energy absorbed in the NaI(Tl) detector is 100 keV, the amplifier gives output pulse of 1V (say) and if the energy deposited in the NaI(Tl) is 600 keV, the output pulse would be 6 V and so-a. The SCA stands for single channel analyzer. It gives the number of pulses in a certain voltage interval, for example, the number of pulses between voltage interval 500 mV and 600 mV, etc., depending upon the setting of SCA. These pulses are fed to counter/ timer, which counts the number of pulses for a fixed time. A typical  $\gamma$ -ray spectrum of  $^{137}\text{Cs}$  emitting one  $\gamma$ -ray of 662 keV is shown in Figure 11. The resolution of the scintillation spectrometer at 662 keV is 75 keV.



**Fig.10:** Block diagram of Scintillation spectrometer

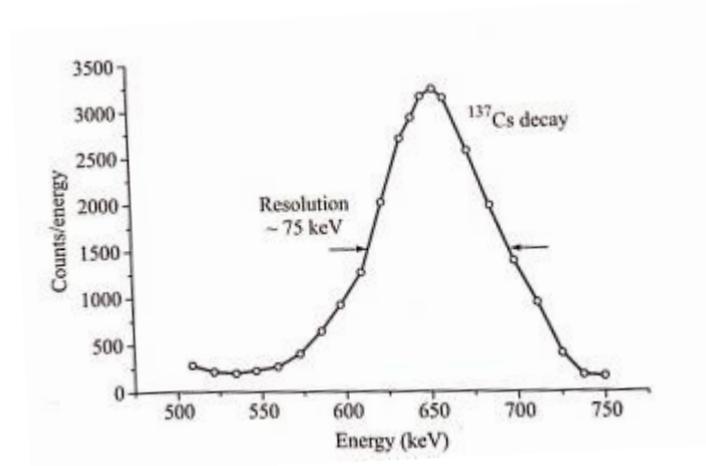


Fig.11: Photo peak (662 KeV) of  $\gamma$ -ray spectrum for  $^{137}\text{Cs}$

#### 5.8.4 Uses of Scintillation Detectors

- ❖ Inorganic scintillation detectors are used to measure the energy of X- and  $\gamma$ -rays.
- ❖ Organic scintillation detectors are used to detect  $\beta$  and  $\alpha$ -particles.
- ❖ They generally have high efficiency. Efficiency is nearly 100% in case of NaI(Tl)detectors.
- ❖ They are rugged and can with mechanical jerks.
- ❖ Scintillation detectors have very short dead time ( $\sim 10$  s) permitting very high counting rates.

#### 5.8.5 Limitations of Scintillation Detectors

- ❖ Energy resolution of these detectors is poor. For example, in case of NaI(Tl) detector, the energy resolution is 8-10% at 662 keV.
- ❖ The NaI(Tl) detector is hygroscopic. If it absorbs moisture, it is completely damaged.

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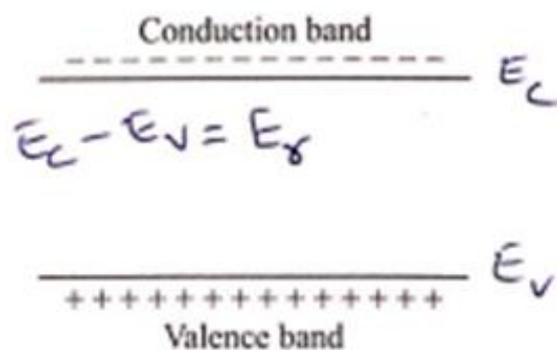
### 5.9 SEMICONDUCTOR RADIATION DETECTORS

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The field of nuclear physics has been completely revolutionized with the development of semiconductor radiation detectors. A semiconductor radiation detector is a solid-state version of a gaseous ionization chamber. It is simply a reverse biased p-n junction diode. In these diodes, ionizing radiations produce electron-hole pairs in the depletion region. This process is

just like creation of electron-ion pairs in ionization chamber. These pairs are collected by the electric field applied and thus the detector gives an electric pulse. The amplitude of this pulse is proportional to the energy of ionizing radiation. Since semiconductor detector is a reverse biased, therefore, the signal is much larger than the noise due to leakage current.

The electronic structure of semiconductors is such that, at ordinary temperatures, nearly all electrons are tied to specific sites in the crystalline lattice and are occupying the valence band. At any given time, a few electrons gain sufficient thermal energy to break loose from localized sites in the valence band and shift to conduction band and are called conduction electrons. Since some energy must be expended in freeing an electron from its normal place in the covalent lattice of a crystal as there is a band gap that separates bound valence electrons from free conduction electrons(Fig.12)



**Fig.12:** Energy gap between conduction band ( $E_C$ ) and valence band( $E_B$ ) for silicon.

In pure crystals no electrons can have energy within this gap. In germanium it is about 0.7 eV.

In perfect materials held at absolute zero temperature, all electrons are theoretically bound to specific lattice sites. so that the valence band is completely filled and the conduction band is empty. The thermal energy available at ordinary temperatures allows some electrons to be freed from specific sites and be elevated across the band gap to the conduction hand. Therefore, for each conduction electron that exists, an electron is missing from a normally occupied valence site. This electron vacancy is called a hole, and in many ways, it behaves as though it is a point positive charge. If an electron jumps from a nearby bond to fill the vacancy, the hole can be thought of as moving in the opposite direction. Both electrons in the conduction band and holes in the valence band can be made to drift in a preferred direction under the influence of an electric field. In semiconductor detectors, an electric field is present

throughout the active volume, i.e. the volume of the detector or total volume of the depletion region, where incident radiations are detected. The subsequent drift of the electrons and holes toward electrodes on the surface of the semiconductor material generates a current pulse, in much the same manner as the motion of ion- pairs in a gas-filled ion chamber.

The minimum energy required for the creation of an electron-hole pair is the band-gap energy of about 1 eV. Experimental measurements show that, as in the production of an ion- pair in a gas, about three times the minimum energy is required on the average to form an electron-hole pair. Thus, a 1-MeV charged particle losing all its energy in a semiconductor creates about 3.00.000 electron-hole pairs. This number is about 10 times larger than the number of ion-pairs that would be formed by the same particle in a gas. As a consequence, the charge packet for equivalent energy loss by the incident particle is 10 times larger, thus improving the signal-to-noise ratio as compared with a pulse-type ion chamber. More significant is the improvement in energy resolution. The statistical fluctuations in the number of charge carriers per pulse (that often limit energy resolution) become a smaller fraction as the total number of carriers increases. Thus, semiconductor detectors offer the best energy resolution provided by common detectors, and values of a few tenths of a per cent are not uncommon.

Another benefit is derived from the fact that the detection medium is a solid rather than a gas. In solids, the range of heavy charged particles such as alphas is only tens or hundreds of micrometers, as opposed to a few centimeters in atmospheric pressure gases. Therefore, the full energy of the particle can be absorbed in a relatively thin detector. More importantly, it is practical to fully absorb fast electrons such as beta particles. As opposed to ranges of meters in gases, fast electrons travel only a few millimeters in solids, and semiconductor detectors can be fabricated that are thicker than this range. Therefore, spectroscopic methods can be employed to measure the energies of fast electrons.

Semiconductor detectors offer many advantages over scintillation or gas-filled detectors.

Here are mentioned some of the advantages:

1. Excellent energy resolution.
2. Linear response over wide-energy range of incident radiation.
3. These detectors are generally compact and are small in size.

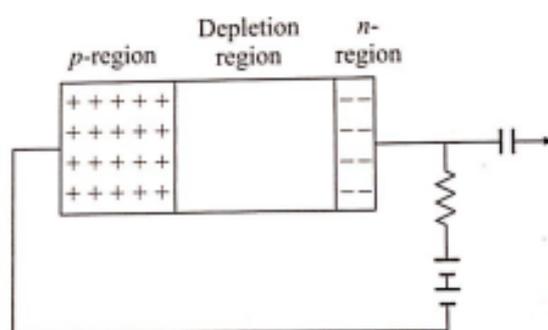
4. These detectors are fast detectors. The mobility of electrons is about  $1500 \text{ cm}^2/\text{Vs}$  and the mobility of holes is about  $500 \text{ cm}^2/\text{Vs}$ , which is much faster compared to mobility ions in gas counters.

5. These detectors can be fabricated in a wide range of sensitive depth and geometry.

Initially, these detectors were developed for detection of heavy charged particles. Later on, with the advancement in semiconductor technology, detectors were developed for detection of electrons, X- and  $\gamma$ -rays. Let us discuss some commonly used semiconductor detectors in detail.

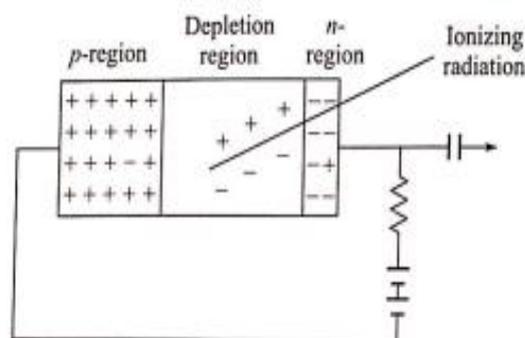
### ❖ Diffused Junction Detector

In a p-type silicon crystal (doped with boron) with a resistivity of about  $500\text{-}10,000 \text{ } \Omega \cdot \text{cm}$ . n-type impurity usually phosphorus is diffused. A p-n junction is formed at a depth of  $0.1\text{-}2.0 \text{ cm}$  from the surface. Generally, gold is evaporated on both sides of the crystal for good electrical connection.



**Fig.13:** Diffused p-n junction with reverse biasing

As shown in Fig.13, when this pn junction is reverse biased, a depletion region is formed which is free from majority charge carriers. This depletion region is majority charge carriers, This depletion region identical to a gas in the ionization chamber. It offers ideal conditions for the detection of ionizing radiation. Ionizing radiations entering the depletion region create electron-hole pairs as shown in Fig.14. Electrons and holes drift towards respective electrode due to applied field and a pulse is obtained in the output circuit. These detectors are rugged and are less prone to radiation damage. The main disadvantage of these detectors is that incident radiation has to pass through relatively thick n-type layer and the radiation may lose considerable amount of energy in the detector window.



**Fig.14:** Ionizing radiation in the depletion region

### ❖ Silicon Surface Barrier Detectors

Silicon surface barrier detectors have extremely thin p-type window and are fabricated by using a high purity n-type silicon crystal. One face of a high purity n-type silicon is etched with acid and is exposed to air. An extremely thin layer of oxide is formed on this face which acts as p-type layer (also called dead layer) and is generally  $<0.1$  micron thin. On this side a very thin layer ( $\sim 20\text{-}40 \mu\text{g}/\text{cm}^2$  or  $1\text{-}2 \times 10^{-6}$  cm) of gold is evaporated and on the back side  $200\text{-}400 \mu\text{g}/\text{cm}^2$  or  $70\text{-}150 \times 10^{-6}$  cm aluminium is evaporated for making electrical connections. They are small in size and have long-term stability. They are widely used in charged particle spectroscopy. These detectors are less rugged and need care while handling them. They are susceptible to damage due to vapors of most of organic chemicals. They are sensitive to light and, therefore, they are operated in the dark environment.

### ❖ Ion Implantation Detectors

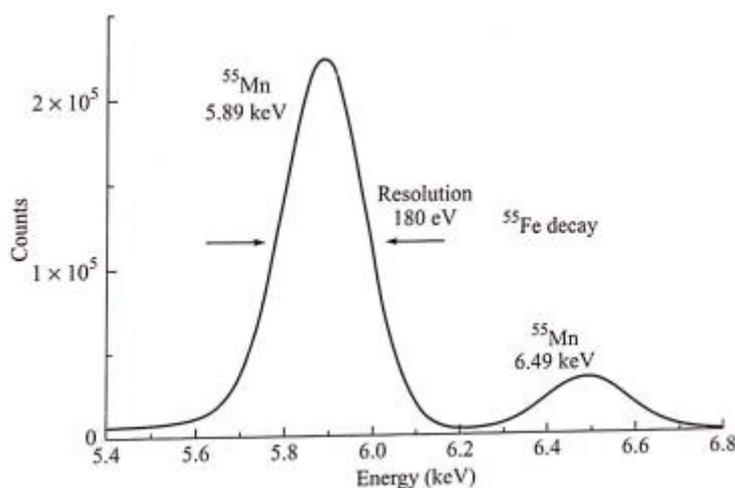
High purity p-type semiconductor is bombarded by say phosphorus ions accelerated to about  $10\text{-}20$  keV. In this method, the thickness of n-type dead layer can be precisely controlled by varying the energy of the phosphorus ions. These detectors are more stable and less sensitive to the ambient conditions.

### ❖ Silicon Lithium Si(Li) Detectors

Till 1968, the techniques available could not reduce the impurity concentration in silicon to less than about  $10^{12}$  atoms/cc. With these impurity concentrations, it was not possible to

fabricate a good semiconductor detector. This situation was overcome by using special techniques. One such commonly used technique is to begin with p-type silicon and diffuse into its surface Li atoms, which tend to act as donor atoms and this creates a thin n-type region. This p-n junction is now put under reverse bias of about 500 V and at a slightly higher temperature ( $-120^{\circ}\text{C}$ ). This causes the Li to drift into the p-type region, making a quite thick (3-5 mm) depletion region. Such a detector is known as Si(Li) (pronounced as sili) detector. These detectors must be kept at liquid nitrogen temperature ( $-196^{\circ}\text{C}$  or 77K), otherwise the Li will migrate out of its lattice sites, destroying the effectiveness of the detector. Keeping the detector at liquid nitrogen temperature also reduces the background electrical noise in the detector.

Si(Li) detectors are mostly used for measuring energy of X-rays. They offer excellent energy resolution in the X-ray region. Typical energy resolution of these detectors is of order of 160-180 eV for 5.9 keV X-rays from  $^{55}\text{Fe}$ . A typical X-ray spectrum of  $^{55}\text{Fe}$  recorded with Si(Li) detector is shown in Figure 7.16. The energy resolution of this detector is 180 eV at 5.9 keV.

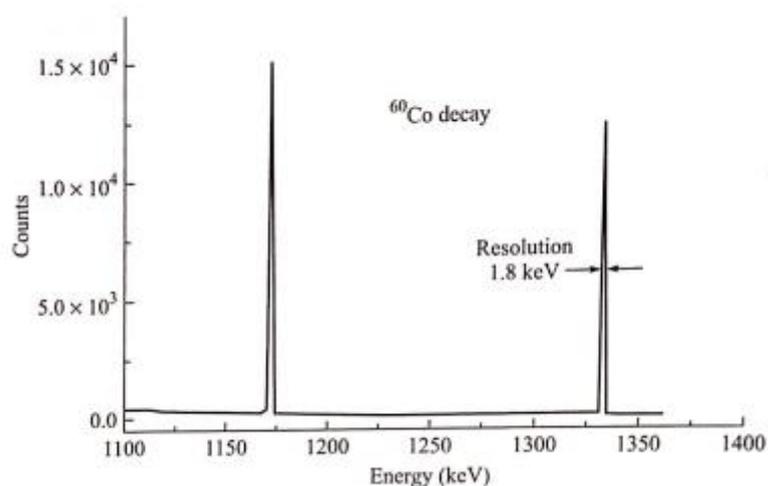


**Fig15.:** X-ray spectrum recorded by Si(Li) detector for  $^{55}\text{Fe}$

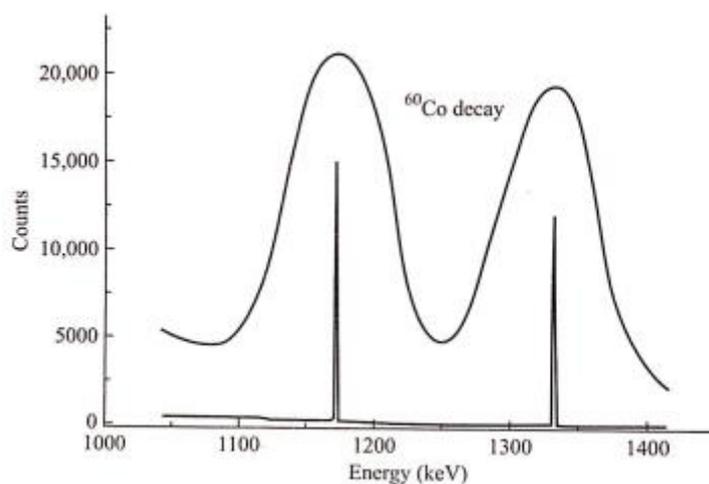
### ❖ Germanium Lithium Ge(Li) Detectors

The photoelectric cross-section varies as  $Z$ . therefore, germanium is a better detector for high energy gamma rays compared to silicon detector. The method of fabrication of Ge(Li) detector is similar to that of Si(Li) detector. In Ge(Li) detectors large active volume (i.e. the total volume of the depletion region) has been obtained like Si(Li) detectors. Ge(Li) detectors with active volume of 5 cc to 100 cc are commonly available.

Like Si(Li) detectors, Ge(Li) detectors are always maintained at the liquid nitrogen temperature, while in use or when they are stored. If by mistake, Ge(Li) detector comes to room temperature, then due to migration of Li, the detector is permanently damaged. The energy resolution of Ge(Li) detector is  $\sim 1.8$  keV for 1332 keV Co peak. The typical spectrum of  $^{60}\text{Co}$  is shown in Fig.16. In order to visualize the comparative energy resolution of Ge(Li) and NaI(Tl) detectors,  $\gamma$ -ray spectrum from both these detectors under similar conditions is shown in Fig.17. The energy resolution of Ge(Li) detector is about 2 keV, whereas for NaI(Tl) the energy resolution is 70 keV.



**Fig16.**  $\gamma$ - ray spectrum of  $^{60}\text{Co}$ .



**Fig. 17 :** Comparative  $\gamma$ -ray spectrum observed by Ge(Li) and Scintillation[NaI(Tl)] detector for  $^{60}\text{Co}$

### ❖ High Purity Germanium Detectors

With the advancement of semiconductor technology, it became possible to get germanium of very high purity, i.e. about 1 atom of impurity for about  $10^{13}$  to  $10^1$  atoms of germanium. Because of this high purity, it has become possible to fabricate thick detectors without lithium compensation. These germanium detectors are commonly referred as *High Purity Germanium (HPGe) detectors*. The biggest advantage of this is that the HPGe detectors can be maintained or stored at room temperature. There is no lithium in these detectors. However, HPGe detectors like Ge(Li) or Si(Li) detectors must be operated at liquid nitrogen temperature. Keeping the detector at liquid nitrogen temperature reduces the background electrical noise generated in the detector due to thermal energy. The energy resolution of this detector is also is 1.8 keV at 1332 keV. Like Ge(Li) detectors, these detectors are also commonly available with active volume of 5 to 100 CC.

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## 5.10 CLOUD CHAMBER

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In this section we shall study about is one of the detectors, which provides visual trajectory of a charged particle like electron, proton,  $\alpha$ -particles, etc. Cloud chamber also known as Wilson Chamber was built by CTR. Wilson in 1911

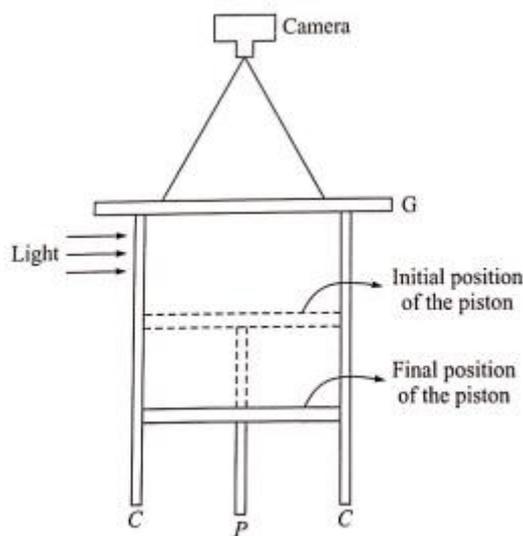
### ❖ Principle

It is based on the principle that when dust-free air saturated with vapours of a liquid (like water alcohol, ether, etc.) is allowed to expand adiabatically, supersaturation occurs. If at this stage an ionizing particle enters the chamber and creates ion-pairs, tiny droplets of liquid condense on these ions and form a visible track along the path of the ionizing radiation. These visible tracks can be photographed. In some cases, cloud chamber is subjected to a strong magnetic or electric field. Such a field causes the charged particles to travel in curved path. The curvature of the curved path gives information about the mass and charge of the ionizing particle.

### ❖ Construction and Working

A simplified form of a cloud chamber is shown in Figure 7.19. It consists of a transparent cylindrical chamber CC in which a piston P is fitted at the bottom. On the top of this

cylindrical chamber is an optically-flat glass plate G and a camera from the top views inside the chamber. This chamber is illuminated from a side with the help of a strong light source.



**Fig.18:** A Cloud Chamber

Air saturated with given liquid is filled in the space between the movable piston P and glass plate G. The pressure inside the chamber is kept high. The pressure in the chamber is lowered by moving the piston down suddenly due to which the temperature of the saturated liquid falls and vapours become super-saturated. If at this moment, a charged particle passes through the chamber, it will produce ion-pairs. The supersaturated vapours condense on the ions and a trail of droplets along the path of the charged particle is seen. These tracks are known as cloud tracks. These tracks have distinctive shapes (for example, an  $\alpha$ -particle track is broad and straight while an electron's trajectory is thinner and shows a zig-zag trajectory). If the chamber is illuminated with light, a camera can take a photograph of the track, which appears as a white line on a dark background. When a vertical magnetic field is applied, positively and negatively charged particles curve in opposite directions.

#### ❖ Advantages

1. When subjected to electric or magnetic field, cloud chamber is used to find charge on the ionizing particles and their momentum.
2. With cloud chamber, the range of high energy particles can easily be determined.

3. By seeing the broadness of a cloud track. we can immediately get an idea whether the track is due to heavy particle (like  $\alpha$ -particle) or light particle (like electron).

### ❖ Limitations

1. If the energy of the ionizing particle is high, it may not completely stop in the cloud chamber and may come out of the chamber. So, we will not get full information about the particle.
2. The recovery time of the cloud chamber is relatively very long. 10-60 s after the expansion, so it may miss many ionizing particles.

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## 5.11 BUBBLE CHAMBER

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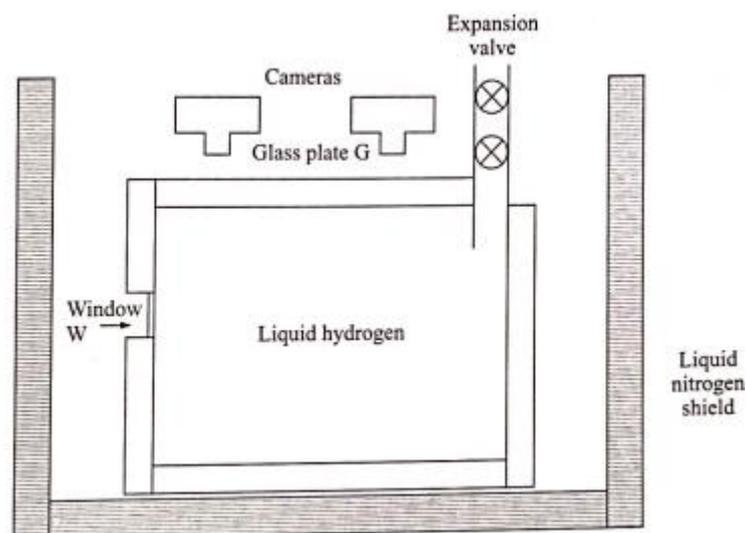
The basic drawback of cloud chamber is that because of the low density of the gas, it is not possible to observe high energy particles. In 1952. D.A. Glaser at the University of Michigan, conceived the idea of using superheated liquid to display the tracks of ionizing particles, just as a cloud chamber utilizes a supersaturated vapour. The instrument based on this concept is known as a Bubble Chamber, because the tracks in bubble chamber consist of a series of closely spaced bubbles, whereas in a cloud chamber there are tiny droplets of the liquid.

### ❖ Principle

It is based on the principle that under high pressure it is possible to heat a liquid without bubble formation well above its normal boiling point. If suddenly pressure is released, the liquid remains in a superheated state for some time. If such a superheated liquid is exposed to ionizing particles, the ionizing particles produce ion-pairs and these ions act as condensation centers for the formation of vapour bubbles along the path of the particle.

### ❖ Construction and Working

The schematic diagram of the bubble chamber is shown in Figure 7.20. The main body of the chamber is made up of stainless steel with thick glass ports at the top for a viewing camera. A box of thick-walled glass is filled with liquid hydrogen and is connected to the expansion pressure system. In order to maintain the chamber at constant temperature, it is surrounded by liquid nitrogen. High energy particles are allowed to enter the chamber from a side window W.



**Fig. 19:** Bubble Chamber

Initially, the liquid hydrogen is kept under high pressure, but when a charged particle is passing through it, the pressure is released so that the liquid is in superheated state. The liquid vapours get condensed in the form of bubbles on the ions formed by ionizing particle and photographs of the tracks formed are obtained by the cameras. Generally, bubble chamber is subjected to a strong magnetic field in order to distinguish the sign of the charge on the ionizing particles and to measure their momenta from the radius of curvature of the bubble tracks. Though most commonly used liquid in bubble chamber is liquid hydrogen, other liquids such as deuterium, helium, xenon, propane, pentane, etc, are also used in some cases.

### ❖ Advantages

1. Due to high density of the liquid, even high energy cosmic rays can also be recorded in bubble chamber.
2. The bubble chamber is sensitive to both high- and low-ionizing particles.
3. As bubbles grow rapidly, the tracks formed in bubble chamber are clean and undistorted.

### ❖ Limitations

1. The time during which bubble chamber is sensitive is only few milliseconds, the y of ionizing particles and photographing the tracks formed must take place during th short time.
- 2 Bubble chambers are the costliest detectors.

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## ***5.11 NUCLEAR EMULSIONS TECHNIQUE***

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Henry Becquerel first used photographic plates for detection of radioactivity in 1886. When an ionizing radiation passes through an emulsion, it interacts with the silver halide grains in the gelatin present on photographic plate. When this plate is developed, the affected silver halide grains change into black grains of metallic silver. It has been found experimentally that optical photographic emulsions are not suitable for qualitative work with nuclear radiations. The sensitivity of ordinary photographic plates is low and the tracks formed do not show clear range because the developed crystal grains are large and widely spaced. In nuclear emulsions, the grain size of silver halide is kept much smaller (0.1-0.6 micron) compared to optical emulsions (1-3.5 micron). Similarly, the thickness of nuclear emulsions is kept large (50-2000 micron) compared to that of optical emulsions (2-4 micron). After exposing the nuclear emulsions to charged particles the emulsion plate is developed and fixed (like ordinary photographic plate). The tracks can be viewed under a microscope.

### **❖ Advantages**

1. These emulsion plates are very light in weight and for exposing them to a beam of charged particles, no electronic circuitry is required.
2. The emulsions are extensively used in cosmic-ray studies. They can be exposed to cosmic rays in upper atmosphere using balloon flights.
3. From the observed range of ionizing particles in nuclear emulsion, their energy can easily be calculated.
4. Different ionizing particles, form tracks which are markedly different, hence the nature of the interacting particle can easily be deduced.

### **❖ Limitations**

1. The sensitivity of nuclear emulsions is affected by atmospheric conditions like temperature, humidity, etc.
2. The sensitivity of nuclear emulsions depends upon the age of nuclear emulsion.

3. The length of the track is relatively small compared to the one recorded in cloud chamber or bubble chamber, hence measurements are difficult.

4. Measurements of the tracks are to be done manually; no automation is possible.

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### ***5.12 SUMMARY***

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After going through this unit learners should be able to understand the principle and working of :

- Ionization Chamber
- Proportional Counter
- Geiger-Muller Counter
- Scintillation Detector
- Semiconductor detector
- Nuclear Emulsion Technique
- Cloud Chamber
- Bubble Chamber

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### 5.15 TERMINAL QUESTIONS

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1. What are the limitations of a GM counter?
2. What are the dead time and recovery time of a GM counter?
3. What is the principle and significance of a photomultiplier tube in a scintillation counter?
4. Give the basic principle of conversion of light into an electrical pulse in a scintillation counter.
5. Explain the principle of ionization chamber.
6. What are the advantages of a semiconductor detector? 7. Give the properties of a good quality photomultiplier tube.
7. How are Ge(Li) detectors more suitable over Si(Li) detectors for electromagnetic radiations?
8. Explain the phenomenon of quenching in GM counter.
9. On what factors, does the efficiency of a detector depend?
10. Why are solid-state detectors preferred over scintillation detectors?
11. What are gas-filled, ionization-based nuclear detectors? Discuss the curve between pulse height and applied voltage for a gas-filled counter serving as (i) an ionization chamber. (ii) a proportional counter, (iii) a Geiger counter.

12. Describe the construction, principle and working of an ionization chamber.
13. Explain the difference between the ionization chamber and the GM counter.
14. Explain the principle and operation of a scintillation counter.
15. Explain the principle and operation of a nuclear emulsion techniques
16. . Discuss the principle, construction and working of a proportional counter. Give its applications and explain how it can be used for neutron detection.
17. Discuss completely the construction, working and theory of a GM counter. Give its drawbacks.
18. What is the principle of a bubble chamber? Discuss its construction.
19. Describe the principle and working of an ionization chamber and compare it with a semiconductor detector? Explain its working.
20. Briefly explain the principle, construction and working of a GM counter, What are its dead time and recovery time?
21. Discuss the principle, construction and working of a semiconductor detector. Give its main advantages over other detectors
22. Write the principle, working and construction of a proportional counter. How is it different from a GM counter?
23. Write short notes on:
  - (i) Cloud chamber.
  - (ii) Bubble chamber



